A high-statistics study of the nucleon axial charge and quark momentum fraction

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Motivation

Nucleon axial charge g_A

- Experimental value is well determined: [PDG, 2013]

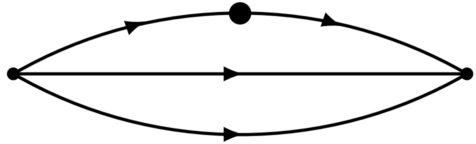
$$g_A = 1.2701(25)$$

- Ideal benchmark quantity for Lattice QCD
  - Simple matrix element $\rightarrow$ local operator with quark bilinears
  - No momentum involved at initial and final state
  - Isovecor quantity $\rightarrow$ No disconnected diagrams
- So far Lattice results are typically $\sim 10\%$ below experimental value

Quark momentum faction $\langle x \rangle_{u-d}$

- Benchmark quantity for Lattice QCD calculations
- Lattice computations tend to overestimate $\langle x \rangle_{u-d}$
- Important quantity to understand hadron structure

Nucleon axial charge in Lattice QCD



Nucleon axial charge g_A

- Improved local axial current:

$$O_3(x) = \overline{\psi}(x)\gamma_3\gamma_5\psi(x) + \underbrace{a c_a \partial_3 P}_{0} + \mathcal{O}(a^2)$$

- Build ratio of 3-pt and 2-pt: $R\,(t,\,t_s) := \frac{C_3^A(t,t_s)}{C_2(t_s)}$
- Extract g_A^{bare} from ratio $R\,(t,\,t_s)$

$$R\,(t,t_s) \xrightarrow{t,(t_s-t)\gg 0} g_A^{\text{bare}} + \mathcal{O}(e^{-\Delta t}) + \mathcal{O}(e^{-\Delta(t_s-t)})$$

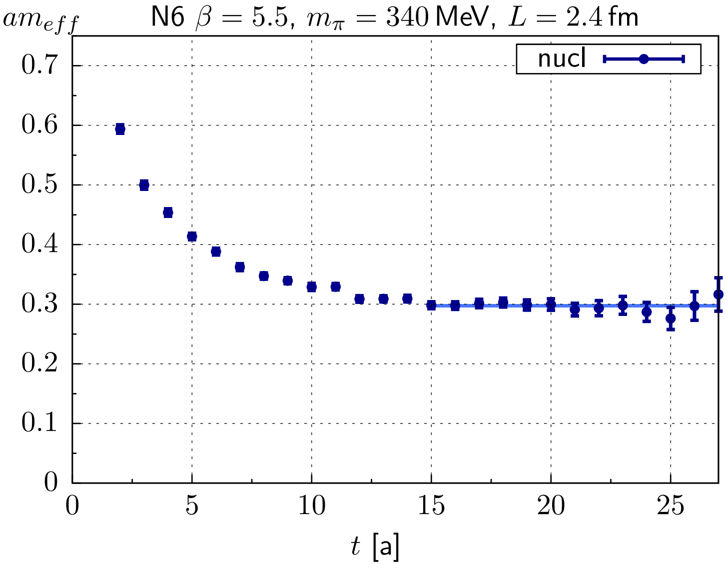
- Ratio should be independent of t and t_s
- Renormalize $g_A = Z_A(1+b_a m_q)\,g_A^{\text{bare}}$ [Della Morte et al., 2008]

Simulation details

- $\mathcal{O}(a)$ improved Wilson fermions (Wilson clover) with $N_f = 2$
- CLS ensembles:

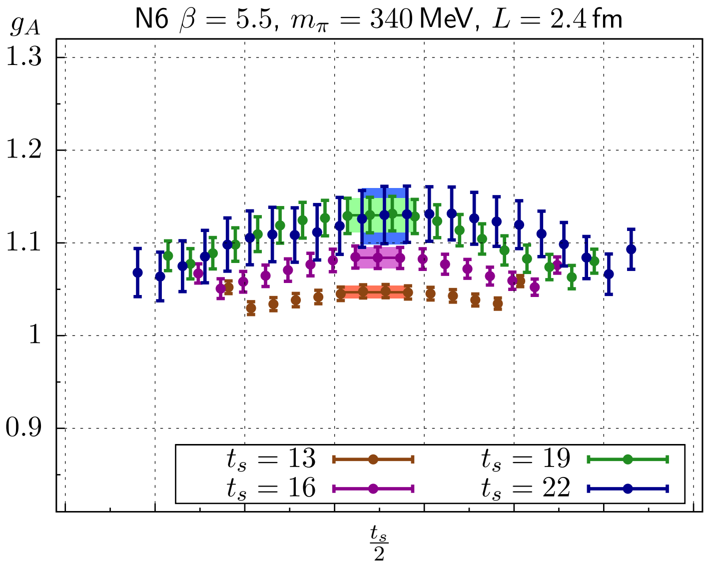
β	a [fm]	lattice	L [fm]	m_π [MeV]	$m_\pi L$	Label	# meas.
5.20	0.079	64×32^3	2.5	473	6.0	A3	2128
5.20	0.079	64×32^3	2.5	363	4.7	A4	3200
5.20	0.079	64×32^3	2.5	312	4.0	A5	4000
5.20	0.079	96×48^3	3.8	262	5.0	B6	2544
5.30	0.063	64×32^3	2.0	451	4.7	E5	4000
5.30	0.063	96×48^3	3.0	324	5.0	F6	3600
5.30	0.063	96×48^3	3.0	277	4.2	F7	3000
5.30	0.063	128×64^3	4.0	195	4.0	G8	4176
5.50	0.050	96×48^3	2.4	536	6.5	N4	600
5.50	0.050	96×48^3	2.4	430	5.2	N5	1908
5.50	0.050	96×48^3	2.4	340	4.0	N6	3784
5.50	0.050	128×64^3	3.2	270	4.4	O7	1960

Nucleon 2-pt function



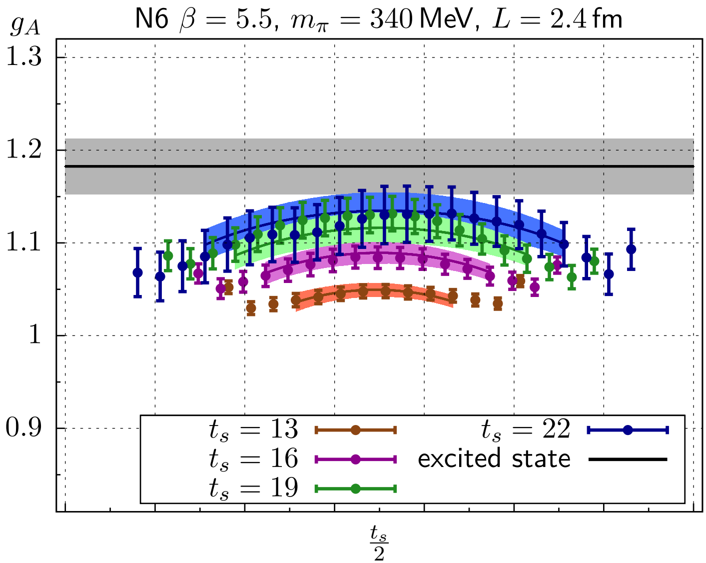
- 2-pt function: excited states have died out $t \sim 12$

Nucleon axial charge g_A



- Excited states still present from source and sink
- Simple plateau fits depend on source-sink separation t_s

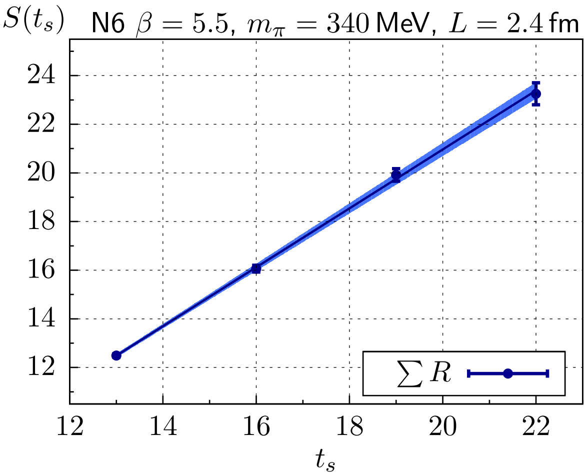
Nucleon axial charge g_A



- Included excites states to fit ansatz

$$f(t,\, t_s) = g_A + c_1\, e^{-\Delta t} + c_2\, e^{-\Delta(t_s - t)} + c_3\, e^{-\Delta t_s}$$

Nucleon axial charge g_A

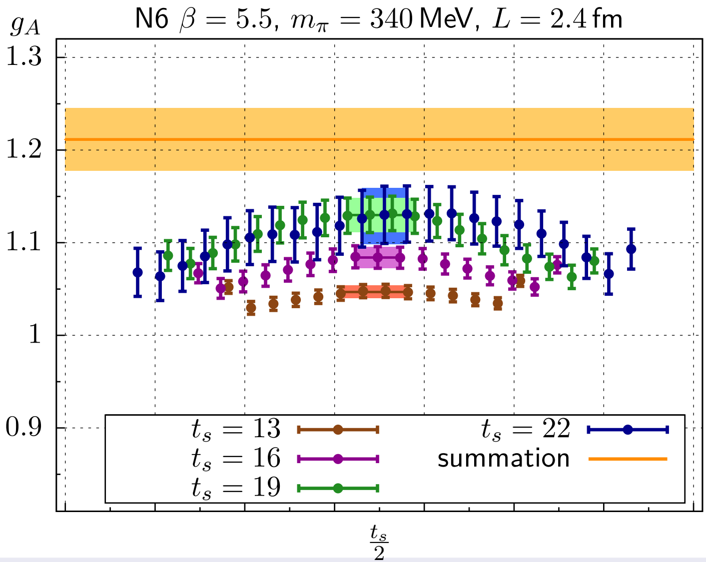


- Summed operator insertion method [L.Maiani et al., 1987]

$$S(t_s) := \sum_{t=1}^{t_s-1} R(t, t_s) \xrightarrow{t_s \gg 0} c + t_s \left(g_A^{\mathrm{bare}} + \mathcal{O}(e^{-\Delta t_s}) \right)$$

- Extract g_A from the slope of a linear fit

Nucleon axial charge g_A



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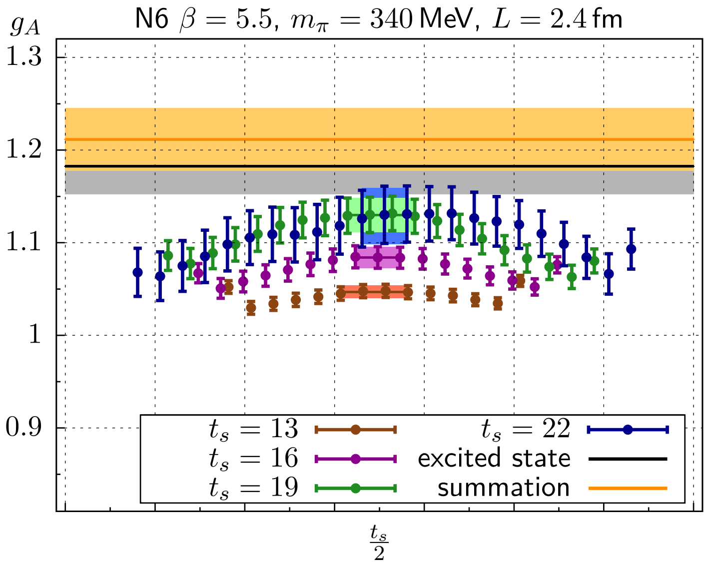
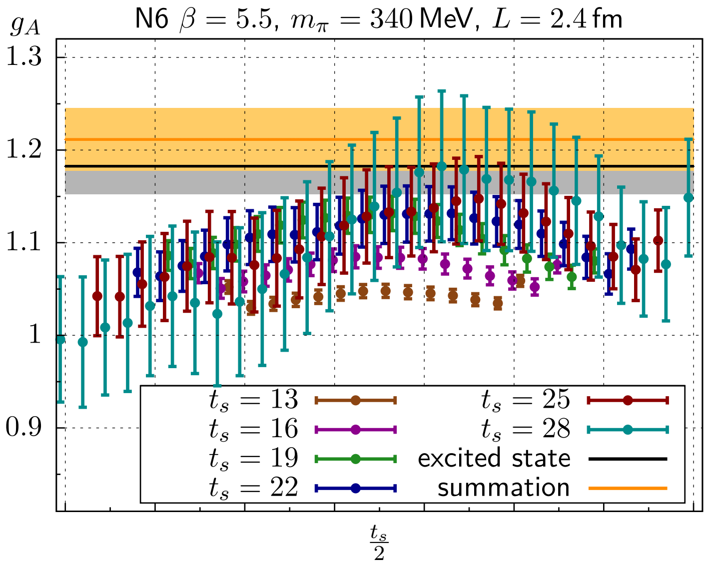
Nucleon axial charge g_A



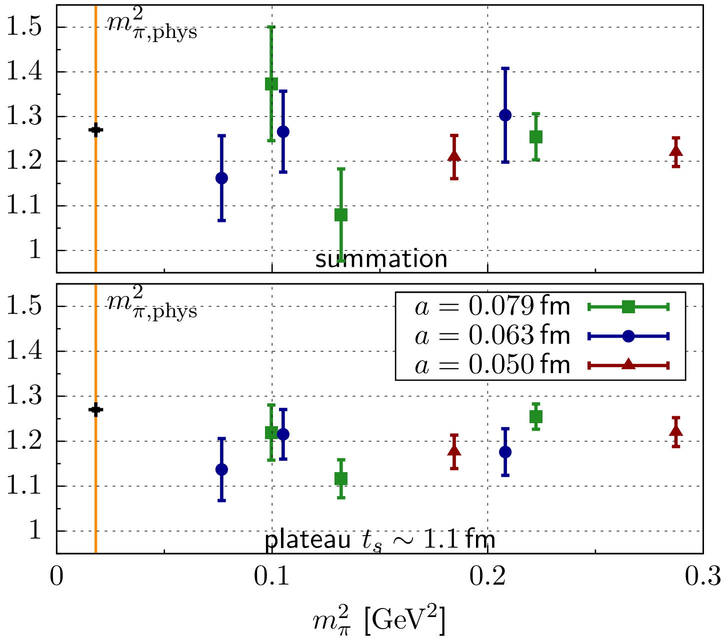
- Including excited states lead to higher value for g_A
- Summation and excited state fit agree

Nucleon axial charge g_A



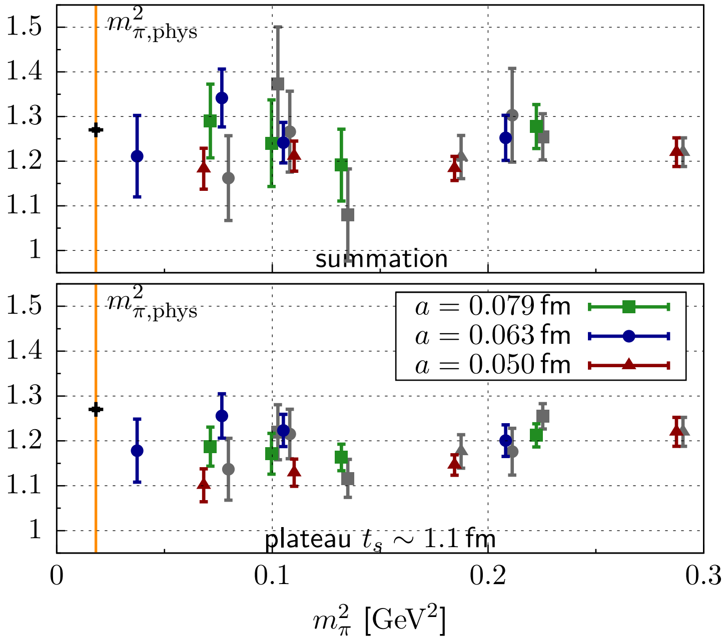
- Check summation and excited state fit by larger t_s (up to $t_s \sim 1.4$ fm)
- Signal-to-Noise ratio deteriorates quickly for large t_s

Chiral behaviour of nucleon axial charge g_A



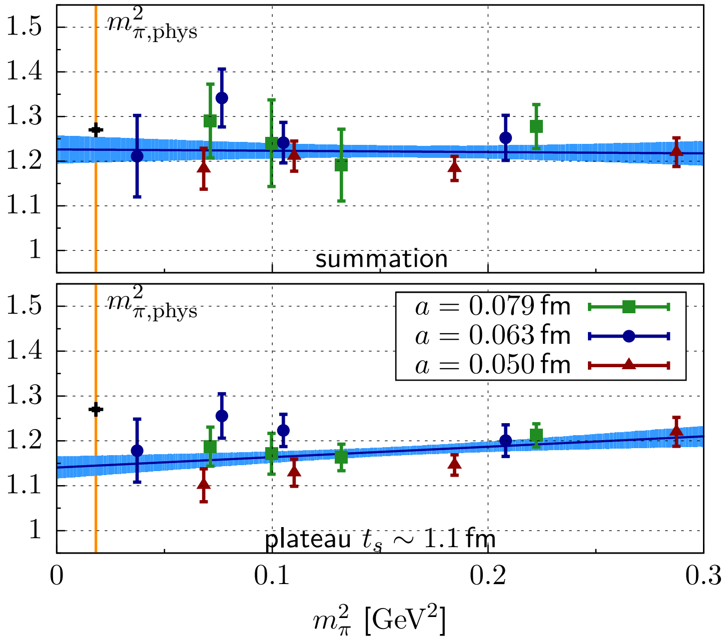
- Results for g_A a year ago arXiv:1205.0180

Chiral behaviour of nucleon axial charge g_A



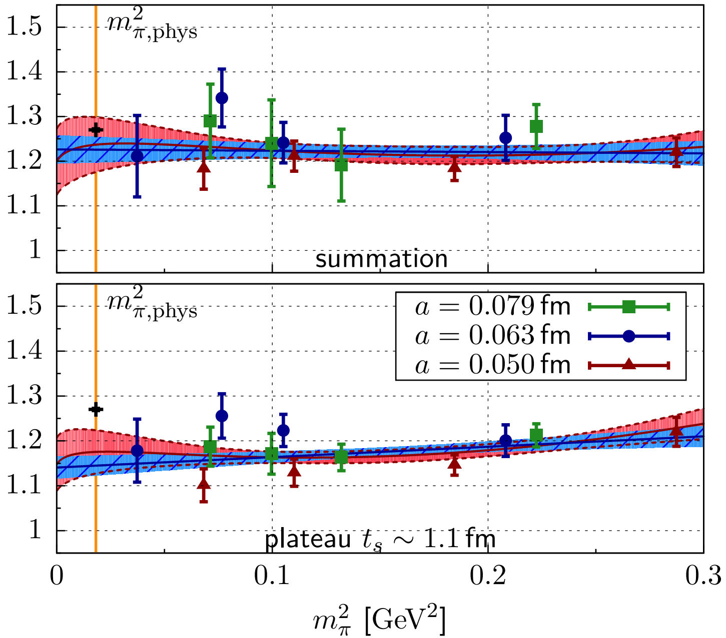
- Update: increased statistics and more chiral ensemble

Chiral behaviour of nucleon axial charge g_A



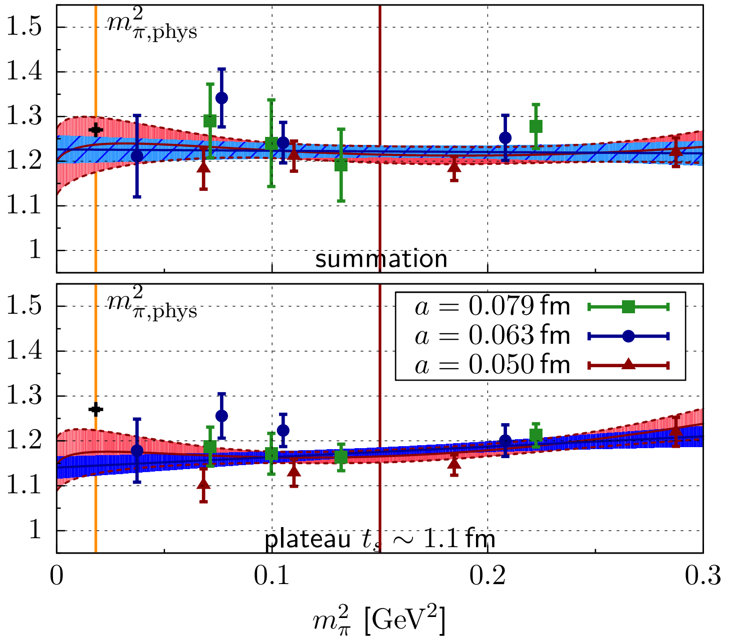
- No strong dependence on m^2_π → linear fit

Chiral behaviour of nucleon axial charge g_A



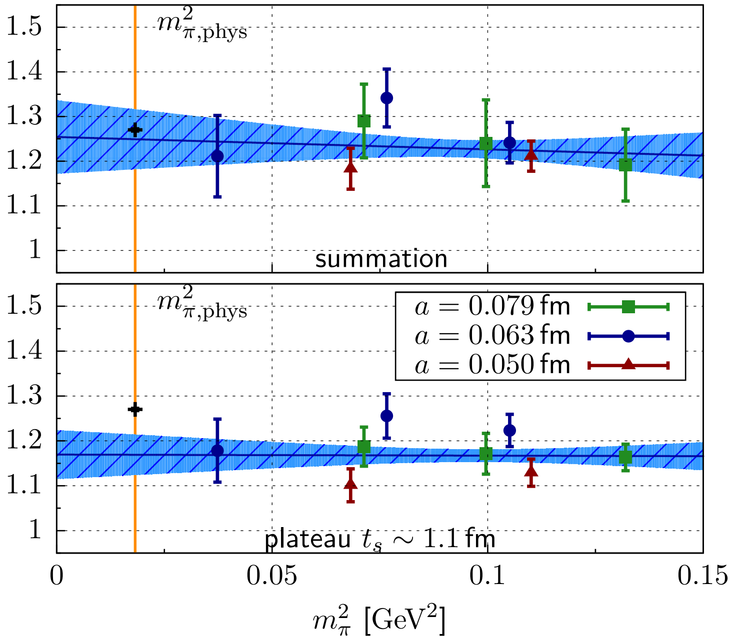
- Heavy Baryon ChPT inspired fit [T.R. Hemmert et al., 2003]

Chiral behaviour of nucleon axial charge g_A



- Restrict fit to chiral ensembles ($m_\pi \leq 365$ MeV)

Chiral behaviour of nucleon axial charge g_A



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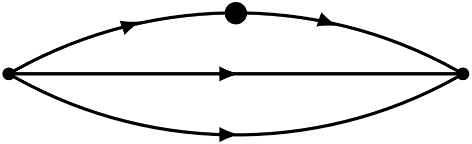
Outline

1. Nucleon axial charge
2. Quark momentum fraction of the nucleon $\langle x \rangle_{u-d}$

Quark momentum fraction of the nucleon $\langle x \rangle_{u-d}$



Quark momentum fraction $\langle x \rangle_{u-d}$

- Insert operator with derivatives (with zero momentum transfer):

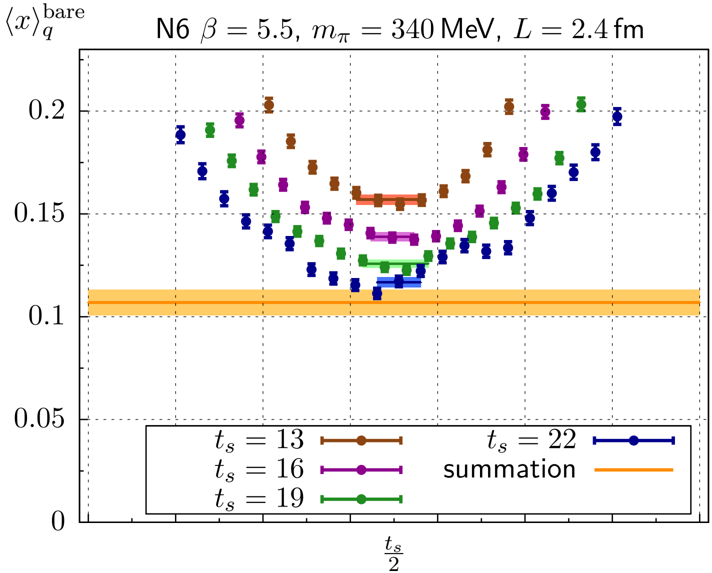
$$O(x) = \overline{\psi}(x)\left(\gamma_0 \overleftrightarrow{D}_0 - \frac{1}{3}\gamma_k \overleftrightarrow{D}_k\right)\psi(x)$$

- Build ratio of 3-pt and 2-pt: $R\,(t,\, t_s) := \frac{C_3^O(t,t_s)}{C_2(t_s)}$
- Extract $\langle x \rangle_{u-d}^{\text{bare}}$ from ratio $R\,(t,\, t_s)$:

$$R\,(t,t_s) \xrightarrow{t,\,(t_s-t)\gg 0} m_N \langle x \rangle_{u-d}^{\text{bare}} + \mathcal{O}(e^{-\Delta t}) + \mathcal{O}(e^{-\Delta(t_s-t)})$$

- Ratio should be independent of t and t_s
- Renormalize $\langle x \rangle_{u-d}^{\text{bare}} \rightarrow \langle x \rangle_{u-d}$ using RI-MOM (not yet included)

Quark momentum fraction of the nucleon $\langle x \rangle_{u-d}$



- Plateaus depend on source-sink separation t_s
  $\rightarrow$ Very clear sign for excited states
- Summed operator insertion method works as for g_A

Outlook and Conclusion

Conclusion

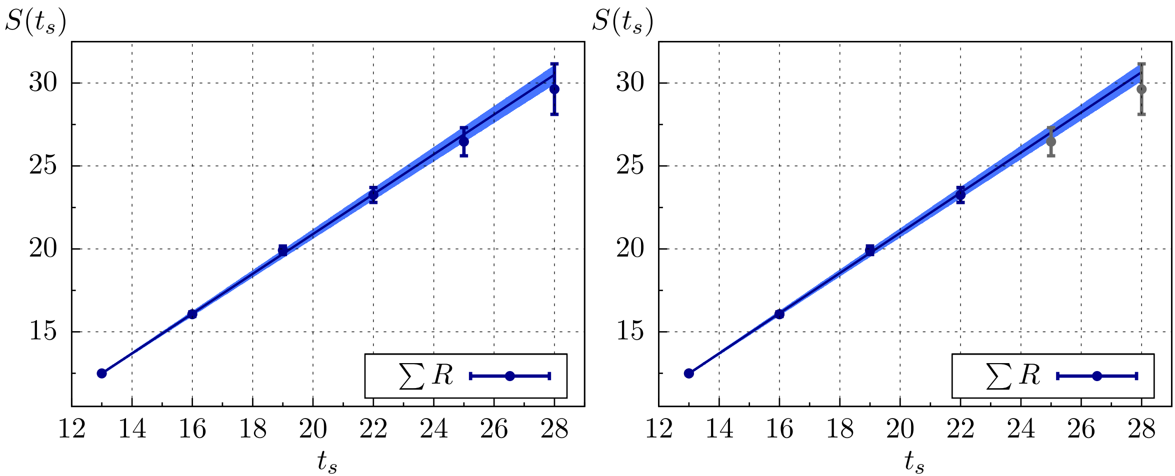
- Summed operator insertion method allows a systematic control of excited states
- Including excited states leads to agreement for the nucleon axial charge
- Chiral extrapolation improved by additional ensembles ($m_\pi^2 < 200$ MeV)
- The quark momentum fraction suffers even more from excited states

Outlook

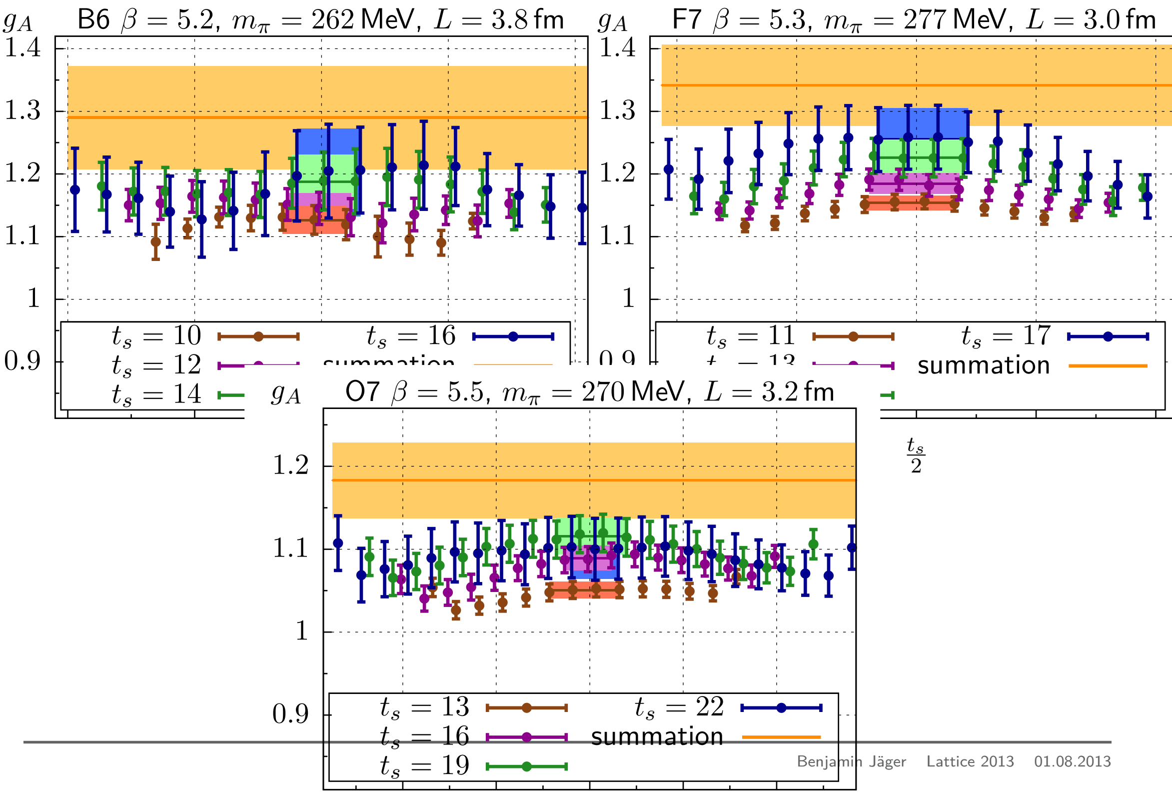
- Further improvements
  - Include renormalization for $\langle x \rangle_{u-d}$ using RI-MOM
  - Study finite size and volume effects (so far mild effect)
  - Simulations at the physical pion mass
  - Include a dynamical strange (and charm) quark
- Electromagnetic form factors $\rightarrow$ T. Rae's talk

Thank you for your attention!

Backup

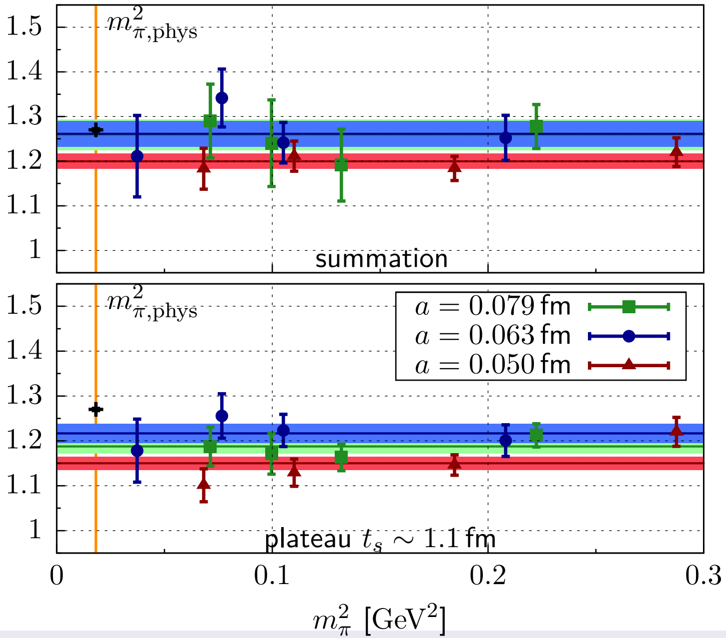


- Checking summation method on N6 for larger t_s
- $g_A\,[\text{all } t_s] = 1.201(0.037) \leftrightarrow g_A\,[t_s \leq 22] = 1.211(0.034)$



g_A
B6 $\beta = 5.2$, $m_\pi = 262$ MeV, $L = 3.8$ fm
1.4
1.3
1.2
1.1
1
0.9
$t_s = 10$
$t_s = 12$
$t_s = 14$
$t_s = 16$
g_A
F7 $\beta = 5.3$, $m_\pi = 277$ MeV, $L = 3.0$ fm
$t_s = 11$
$t_s = 17$
summation
$\frac{t_s}{2}$
g_A
O7 $\beta = 5.5$, $m_\pi = 270$ MeV, $L = 3.2$ fm
$t_s = 13$
$t_s = 16$
$t_s = 19$
$t_s = 22$
summation

Backup



- Fit individual β to a constant
- No clear sign of lattice artefacts ($\beta = 5.5$ tends to smaller g_A)

Backup

Heavy Baryon ChPT formula

[T.R. Hemmert et al., 2003]

- 6 free parameters: 3 are fixed to (physical) values

$$c_A = 1.5\,\text{GeV}\ ,\ \Delta_0 = 0.2711\,\text{GeV and } \lambda = 1\,\text{GeV}$$

$$\begin{aligned} g_A(m_\pi^2) = g_A^0 &- \frac{(g_A^0)^3 m_\pi^2}{16\pi^2 f_\pi^2} + 4\Big(C_{SSE}(\lambda) + \frac{c_A^2}{4\pi^2 f_\pi^2}\left[\frac{155}{972}g_1 - \frac{17}{36}g_A^0\right] \\ &+ \gamma \ln\frac{m_\pi}{\lambda}\Big)m_\pi^2 + \frac{4c_A^2 g_A^0}{27\pi^2 f_\pi^2 \Delta_0}m_\pi^2 + \frac{8}{27\pi^2 f_\pi^2}c_A^2 g_A^0 m_\pi^2 R(m_\pi) \\ &+ \frac{c_A^2\Delta_0^2}{81\pi^2 f_\pi^2}(25g_1 - 57g_A^0)\left(\ln\frac{2\Delta_0}{m_\pi} - R(m_\pi)\right) \end{aligned}$$

with

$$\gamma = \frac{1}{16\pi^2 f_\pi^2}\left(\frac{50}{81}c_A^2 g_1 - \frac{1}{2}g_A^0 - \frac{2}{9}c_A^2 g_A^0 - (g_A^0)^3\right)$$

$$R(m_\pi) = \sqrt{1 - \frac{m_\pi}{\Delta_0}}\left(\frac{\Delta_0}{m_\pi} + \sqrt{\frac{\Delta_0}{m_\pi} - 1}\right)$$