Nucleon axial charge in 2+1-flavor dynamical DWF lattice QCD

Shigemi Ohta [*†‡] for RBC and UKQCD Collaborations

Lattice 2013 in Mainz, August 1, 2013

RBC and UKQCD collaborations have been generating dynamical Domain-Wall Fermions (DWF) ensembles:

- good chiral and flavor symmetries,

that allowed us do a lot of good pion and kaon physics as well as nucleon.

We are now much closer to physical pion mass with large volume, than the previous sets of ensembles:

- light, $m_\pi \sim 171$ and 248 MeV, quarks ($m_{ud}a = 0.001$ and 0.0042, and $m_{\rm res}a \sim 0.002$),
- a large, $(4.6\text{fm})^3$, volume ($a^{-1} \sim 1.371(10)$ GeV),

made possible by Iwasaki + dislocation suppressing determinant ratio (DSDR) gauge action.

Here we report the current status of our nucleon calculations, by

- Meifeng Lin, Yasumichi Aoki, Tom Blum, Taku Izubuchi, Chulwoo Jung, SO, Shoichi Sasaki, Eigo Shintani, Takeshi Yamazaki, ...

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RBC/UKQCD $N_f = 2+1$ dynamical DWF ensembles, with good flavor and chiral symmetries:

- extrapolations to chiral and continuum limits are disentangled,
- with fully non-perturbative renormalizations.
- Also, reweighing allows calculations with exact strange mass.

With Iwasaki gauge action at $a^{-1} = 1.75(4)$ and $2.31(4)$ GeV with volumes larger than 2.7 fm across, and m_π as light as 330 MeV[1],

- $f_\pi = 124(5)$ MeV, $f_K/f_\pi = 1.204(26)$;
- $m_{\rm s}^{\overline{\rm MS}(2{\rm GeV})} = 97(3)$ MeV, $m_{\rm ud}^{\overline{\rm MS}(2{\rm GeV})} = 3.6(2)$ MeV, $B_K^{\overline{\rm MS}(3{\rm GeV})} = 0.529(20)$.

Chiral perturbation is useless from this heavy mass range, $m_\pi \sim 300$ MeV: e.g. NLO $\sim 0.5\times$LO.

Systematics arising from heavy pion dominated. So we added lighter pions with Iwasaki+DSDR action[2]:

- $a^{-1} \sim 1.371(10)$ GeV, $m_\pi \sim 250$ and 170 MeV, $L \sim 4.6$ fm,
- $f_\pi = 127.1(3.8)$ MeV, $f_K/f_\pi = 1.199(18)$;
- $m_{\rm s}^{\overline{\rm MS}(3{\rm GeV})} = 83.5(2.0)$ MeV, $m_{\rm ud}^{\overline{\rm MS}(3{\rm GeV})} = 3.05(10)$ MeV, $B_K^{\overline{\rm MS}(3{\rm GeV})} = 0.535(16)$.

[1] URL: http://link.aps.org/doi/10.1103/PhysRevD.83.074508, DOI: 10.1103/PhysRevD.83.074508.
[2] URL: http://link.aps.org/doi/10.1103/PhysRevD.87.094514, DOI: 10.1103/PhysRevD.87.094514.

Nucleon form factors, measured in elastic scatterings or β decay or muon capture:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p \left[\gamma_\mu F_V(q^2) + \frac{i\sigma_{\mu\lambda}q_\lambda}{2m_N} F_T(q^2)\right] u_n e^{iq\cdot x},$$

$$\langle p|A_\mu^+(x)|n\rangle = \bar{u}_p \left[\gamma_\mu\gamma_5 F_A(q^2) + q_\mu\gamma_5 F_P(q^2)\right] u_n e^{iq\cdot x}.$$

$$F_V = F_1, F_T = F_2; G_E(q^2) = F_1 - \frac{q^2}{4m_N^2} F_2, G_M = F_1 + F_2.$$

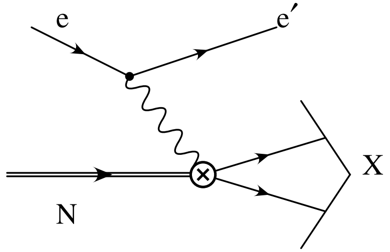
Related to mean-squared charge radius, magnetic moment, $g_V = F_V(0) = G_{\rm Fermi}\cos\theta_{\rm Cabibbo}$, $g_A = F_A(0) = 1.2701(25) g_V$, Goldberger-Treiman relation, $m_N g_A \propto f_\pi g_{\pi NN}$, ... determine much of nuclear physics.

On the lattice, with appropriate nucleon operator, for example, $N = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$, ratio of two- and three-point correlators such as $\dfrac{C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t)}{C_{\rm 2pt}(t_{\rm sink})}$ with

$$C_{\rm 2pt}(t_{\rm sink}) = \sum_{\alpha,\beta}\left(\frac{1+\gamma_t}{2}\right)_{\alpha\beta}\langle N_\beta(t_{\rm sink})\bar{N}_\alpha(0)\rangle,$$

$$C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t) = \sum_{\alpha,\beta}\Gamma_{\alpha\beta}\langle N_\beta(t_{\rm sink})O(t)\bar{N}_\alpha(0)\rangle,$$

give a plateau in t for a lattice bare value $\langle O\rangle$ for the relevant observable, with appropriate spin ($\Gamma = (1+\gamma_t)/2$ or $(1+\gamma_t)i\gamma_5\gamma_k/2$) or momentum-transfer (if any) projections.

Deep inelastic scatterings 

: $\left|\frac{\mathcal{A}}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu}$, $W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$

- unpolarized: $W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{\nu}{q^2} q^\mu\right)\left(P^\nu - \frac{\nu}{q^2} q^\nu\right) \frac{F_2(x,Q^2)}{\nu}$,
- polarized: $W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q\cdot S P_\sigma}{\nu^2} g_2(x,Q^2)\right)$,

with $\nu = q\cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$.

Moments of the structure functions are accessible on the lattice:

$$\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
\int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_1(x,Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{1}{2}\frac{n}{n+1} \sum_{q=u,d} [e_{2,n}^{q}(\mu^2/Q^2, g(\mu))\, d_n^q(\mu) - 2e_{1,n}^{q}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2)
\end{aligned}$$

- c_1, c_2, e_1, and e_2 are the Wilson coefficients (perturbative),
- $\langle x^n\rangle_q(\mu)$, $\langle x^n\rangle_{\Delta q}(\mu)$ and $d_n(\mu)$ are forward nucleon matrix elements of certain local operators,
- so is $\langle 1\rangle_{\delta q}(\mu) = \langle P,S|\bar\psi i\gamma_5\sigma_{\mu\nu}\psi|P,S\rangle$ which may be measured by polarized Drell-Yan and RHIC Spin.

Unpolarized (F_1/F_2): on the lattice we can measure: $\langle x\rangle_q$, $\langle x^2\rangle_q$ and $\langle x^3\rangle_q$.

$$\frac{1}{2}\sum_s \langle P,S|\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2\langle x^{n-1}\rangle_q(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{trace})]$$

$$\mathcal{O}^q_{\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{trace})\right]q$$

Polarized (g_1/g_2): on the lattice we can measure: $\langle 1\rangle_{\Delta q}$ (g_A), $\langle x\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$, d_1, d_2, $\langle 1\rangle_{\delta q}$ and $\langle x\rangle_{\delta q}$.

$$-\langle P,S|\mathcal{O}^{5q}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{n+1}\langle x^n\rangle_{\Delta q}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_\sigma\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1} - S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_{[\sigma}\overleftrightarrow{D}_{\mu_1]}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

and transversity (h_1):

$$\langle P,S|\mathcal{O}^{\sigma q}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{m_N}\langle x^n\rangle_{\delta q}[(S_\rho P_\nu - S_\nu P_\rho)P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \bar{q}[\left(\frac{i}{2}\right)^{n}\gamma_5\sigma_{\rho\nu}\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})]q$$

Higher moment operators mix with lower dimensional ones: Only $\langle x\rangle_q$, $\langle 1\rangle_{\Delta q}$, $\langle x\rangle_{\Delta q}$, d_1, and $\langle 1\rangle_{\delta q}$ can be measured with $\vec{P}=0$.

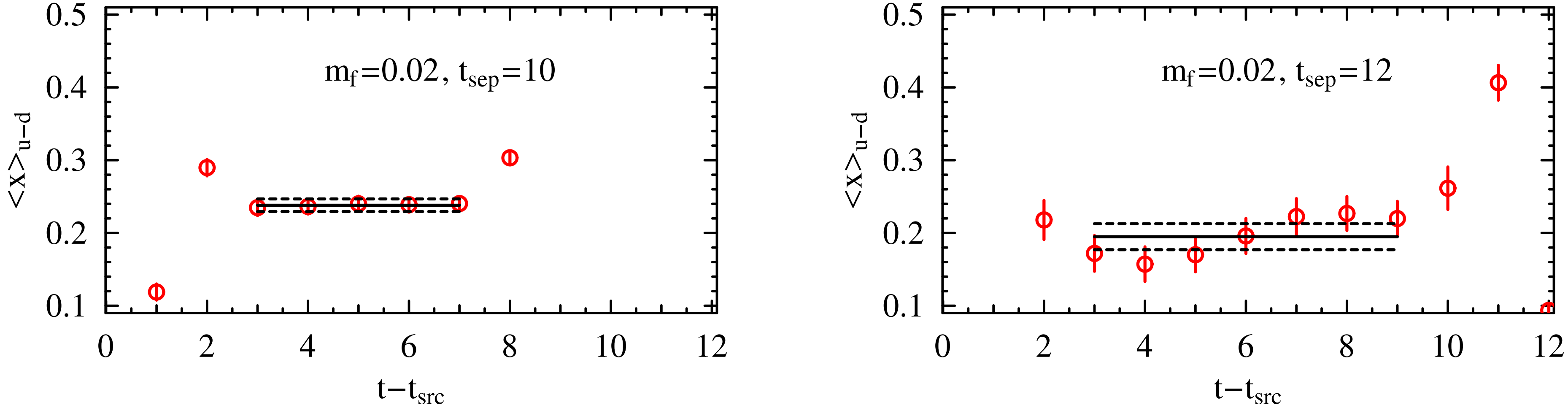
Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

And a better understanding of quark mass dependence is necessary.

Source/sink time separation:

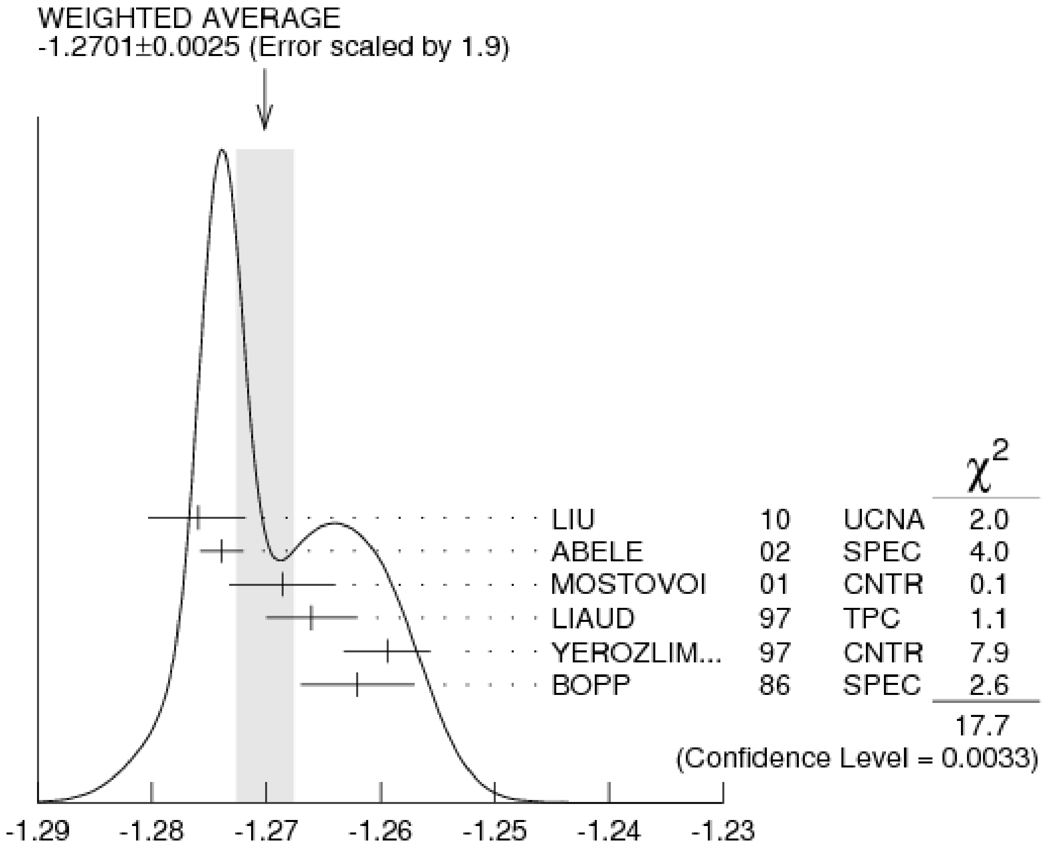
- If too short, too much contamination from excited states, but if too long, the signal is lost. In an earlier RBC 2-flavor DWF study at $a^{-1} \sim 1.7$ GeV, separation of 10 or 1.1 fm appeared too short.



In the previous (2+1)-flavor study we choose separation 12 or 13 , ∼1.4 fm:

Present study: two separations, 7 and 9 lattice units or 1.0 and 1.3 fm.

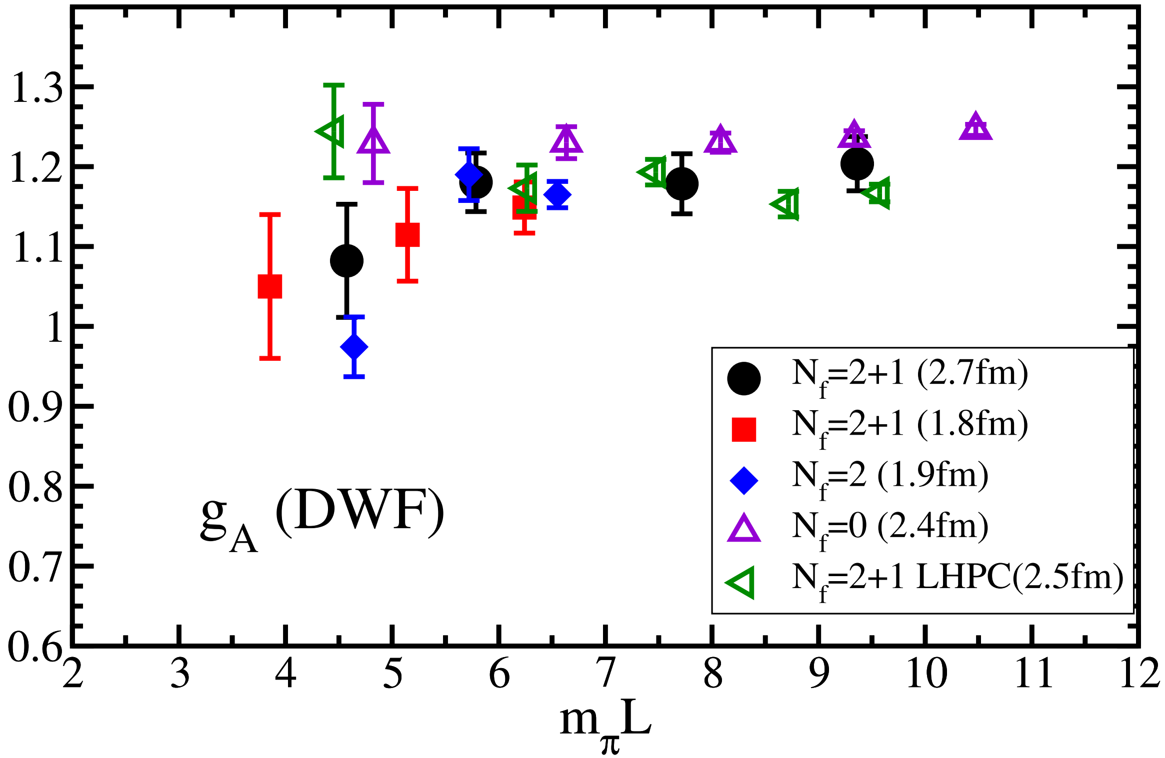
Spatial volume: let's look at nucleon isovector axial charge, g_A/g_V=1.2701(25),



Experimental value has been almost monotonically increasing since Maurice Goldhaber's first measurement.

Lattice calculations appeared to follow the same path, but, Spatial volume. In Lattice 2007 Takeshi Yamazaki reported unexpectedly large finite-size effect:

- in axial charge, $g_A/g_V = 1.2701(25)$, measured in neutron β decay, decides neutron life.



Our DWF on quenched and LHPC DWF on MILC calculations are presented for comparison.

- Heavier quarks: almost consistent with experiment, no discernible quark-mass dependence.
- Lighter quarks: finite-size sets in as early as $m_\pi L \sim 5$, appear to scale in $m_\pi L$:
- If confirmed, first concrete evidence of pion cloud surrounding nucleons.

Many in the past pointed out this is a fragile quantity as pion mass is set light: Adkins+Nappi+Witten, Jaffe, Kojo+McLerran+Pisarski, ...

RBC and UKQCD collaborations jointly generated (2+1)-flavor DWF ensembles:

- with Iwasaki and dislocation-suppressing-determinant-ratio (DSDR) gauge action, $\beta = 1.75$,
- and DWF fermion action, $L_s = 32$ and $M_5 = 1.8$, with $m_{\rm strange} = 0.045$, $m_{\rm ud} = 0.0042$ and 0.001,

using FNAL ALCF, a BG/P facility.

We have reasonable topology distribution while maintaining small residual mass, $m_{\rm res}a \sim 0.002$:

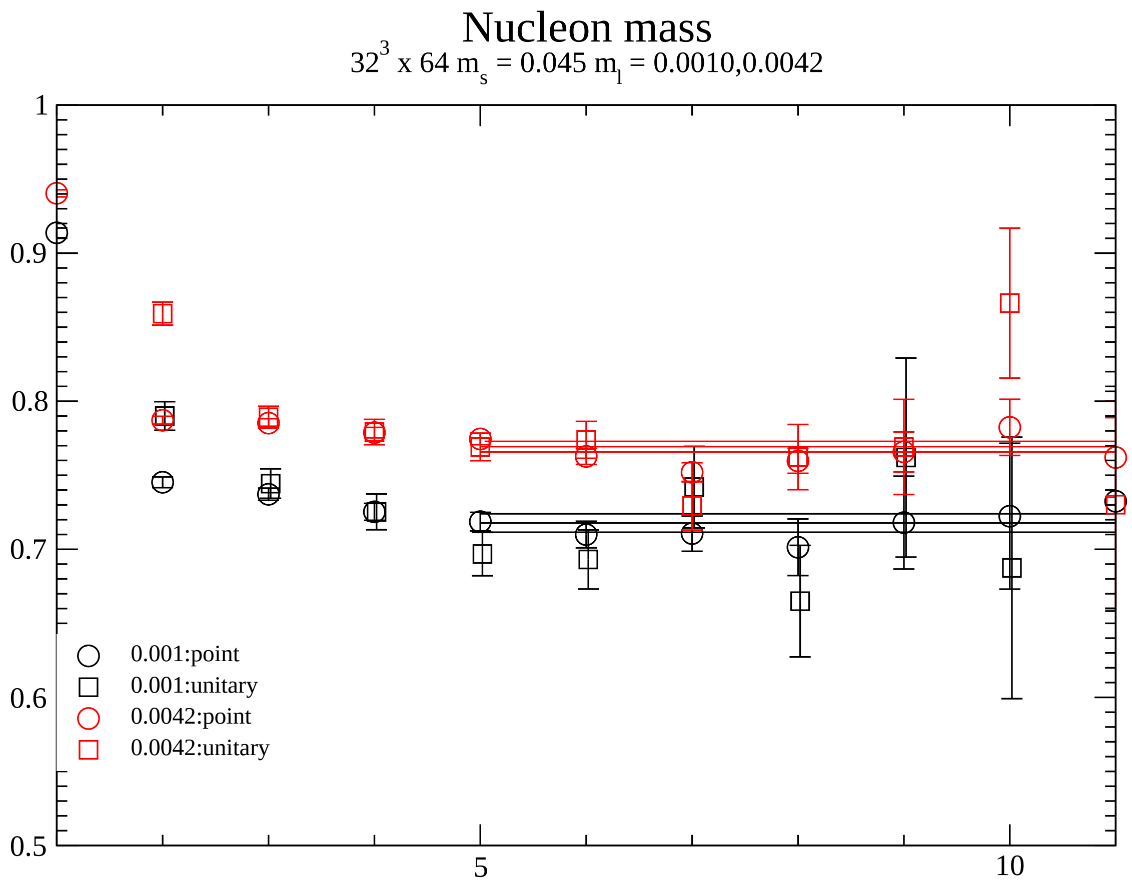
- lattice scale from Ω^-: $a^{-1} = 1.371(10)$ GeV,
- $m_\pi = 0.1816(8)$ and $0.1267(8)$, or $\sim$ 250 and 170 MeV,
- $32^3 \times 64$ volume is about 4.6 fm across in space, 9.2 fm in time.

Using RICC/RIKEN and Teragrid/XSEDE clusters, we started nucleon structure calculations:

- Gaussian smearing, width 6 favored over 4,
- sink separated by 7 or 9 lattice units from the source,
- 608–1920/8 for 250-MeV, 508–1412/8 for 170-MeV so far analyzed for 3pt.

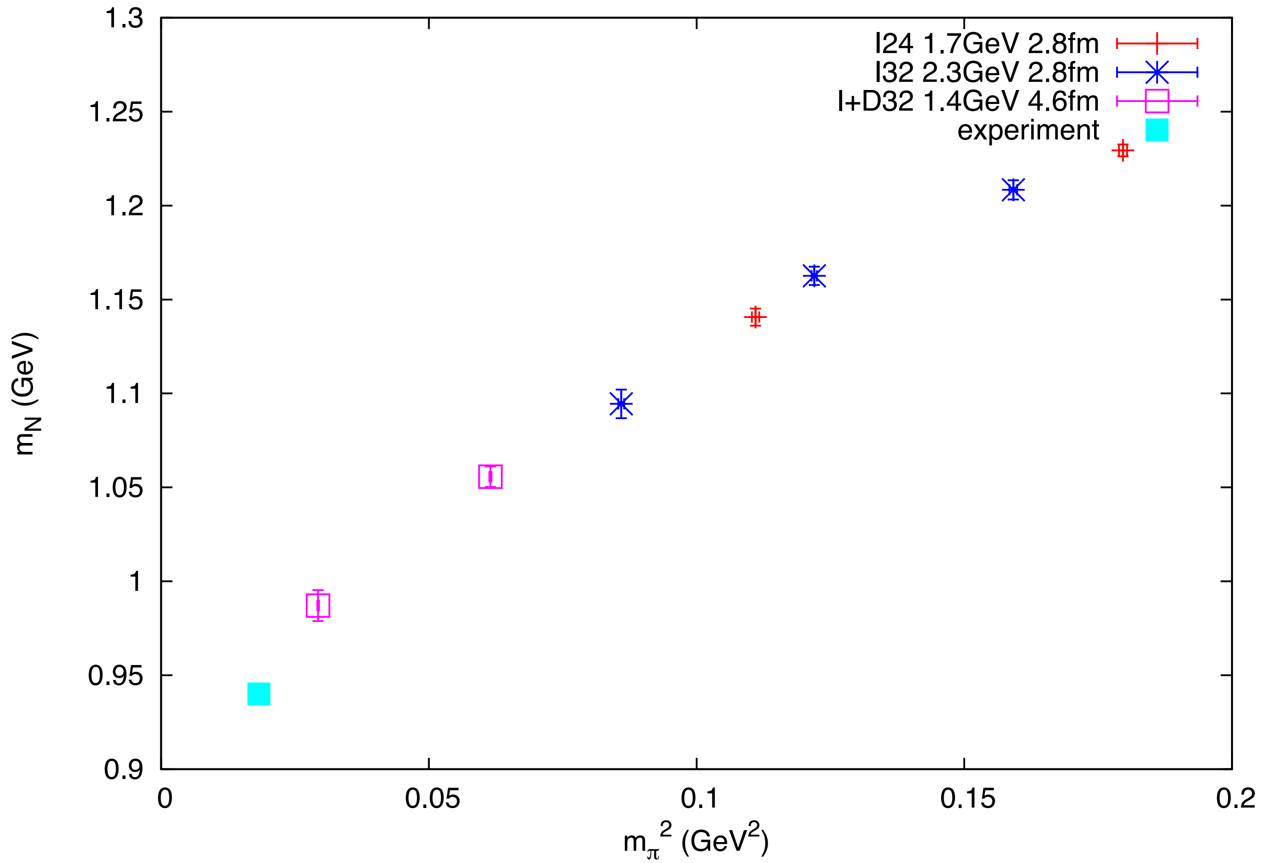
We also increased statistics of m_π=330 and 420 MeV ensembles with $a^{-1} \sim 1.75$ GeV and $L \sim 2.7$ fm.

RBC/UKQCD (2+1)-flavor, ID+DWF dynamical, $a^{-1} = 1.371(10)$ GeV,



$m_N = 0.718(6)$ or ~ 0.98 GeV for $m_\pi \sim 170$MeV, and $m_N = 0.769(5)$ or ~ 1.05 GeV for $m_\pi \sim 250$MeV.

Nucleon mass: RBC/UKQCD (2+1)-flavor, ID+DWF ensembles are being analyzed for nucleon physics.



with $a^{-1} = 1.371(10)$ GeV, $(\sim 4.6\text{fm})^3$ spatial volume.

Closer to physical mass, $m_\pi = 170$ and 250 MeV, $m_N < 1.0$ GeV,

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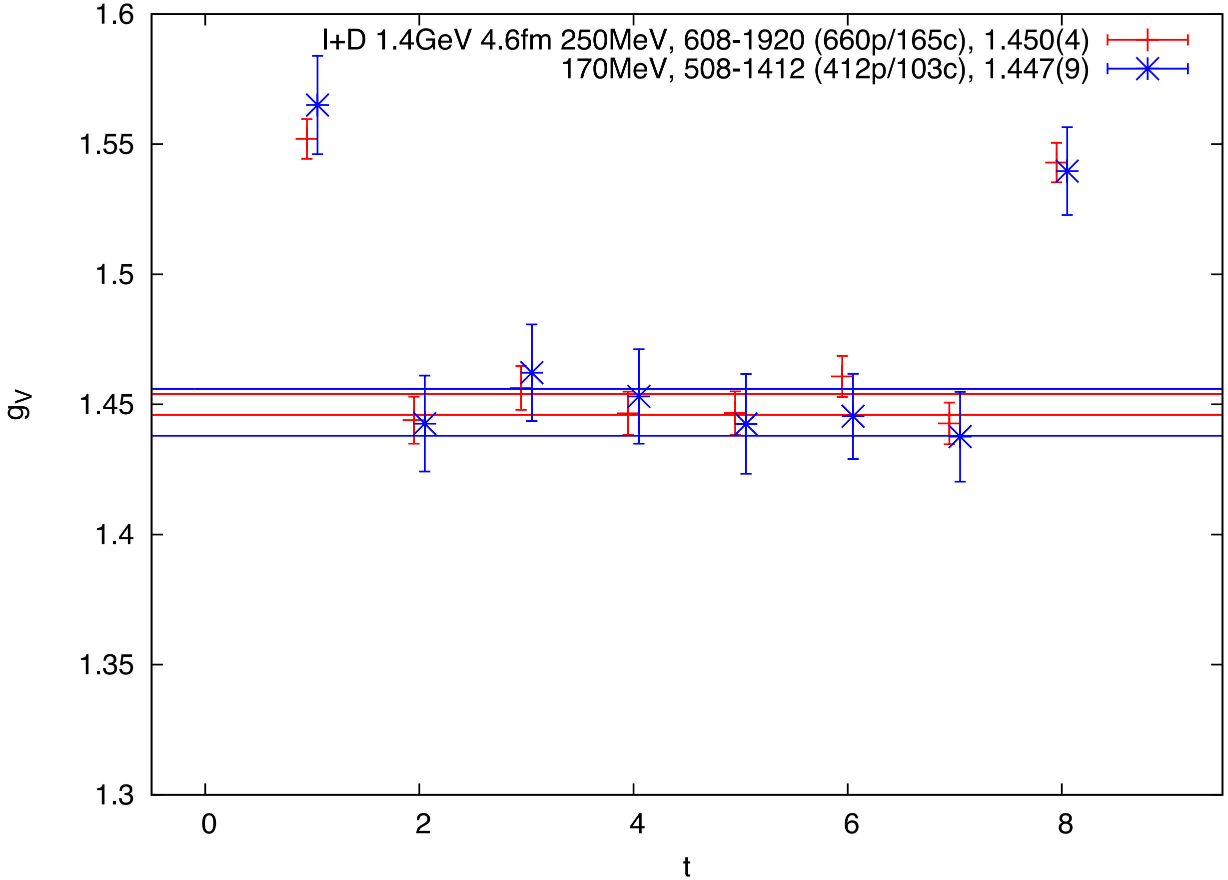
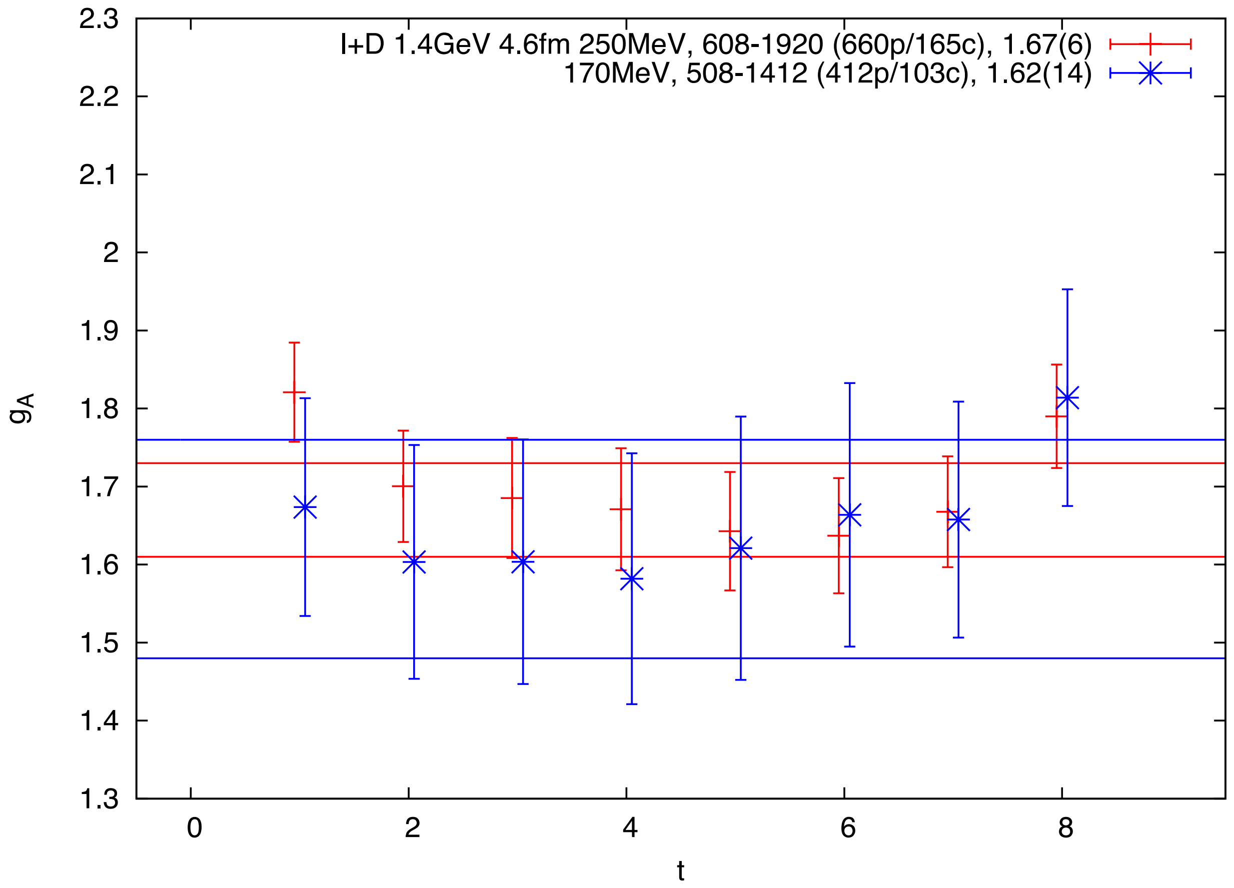
Nucleon isovector 3-pt functions are being obtained: 608-1920 for 250-MeV, 508-1412 for 170-MeV.



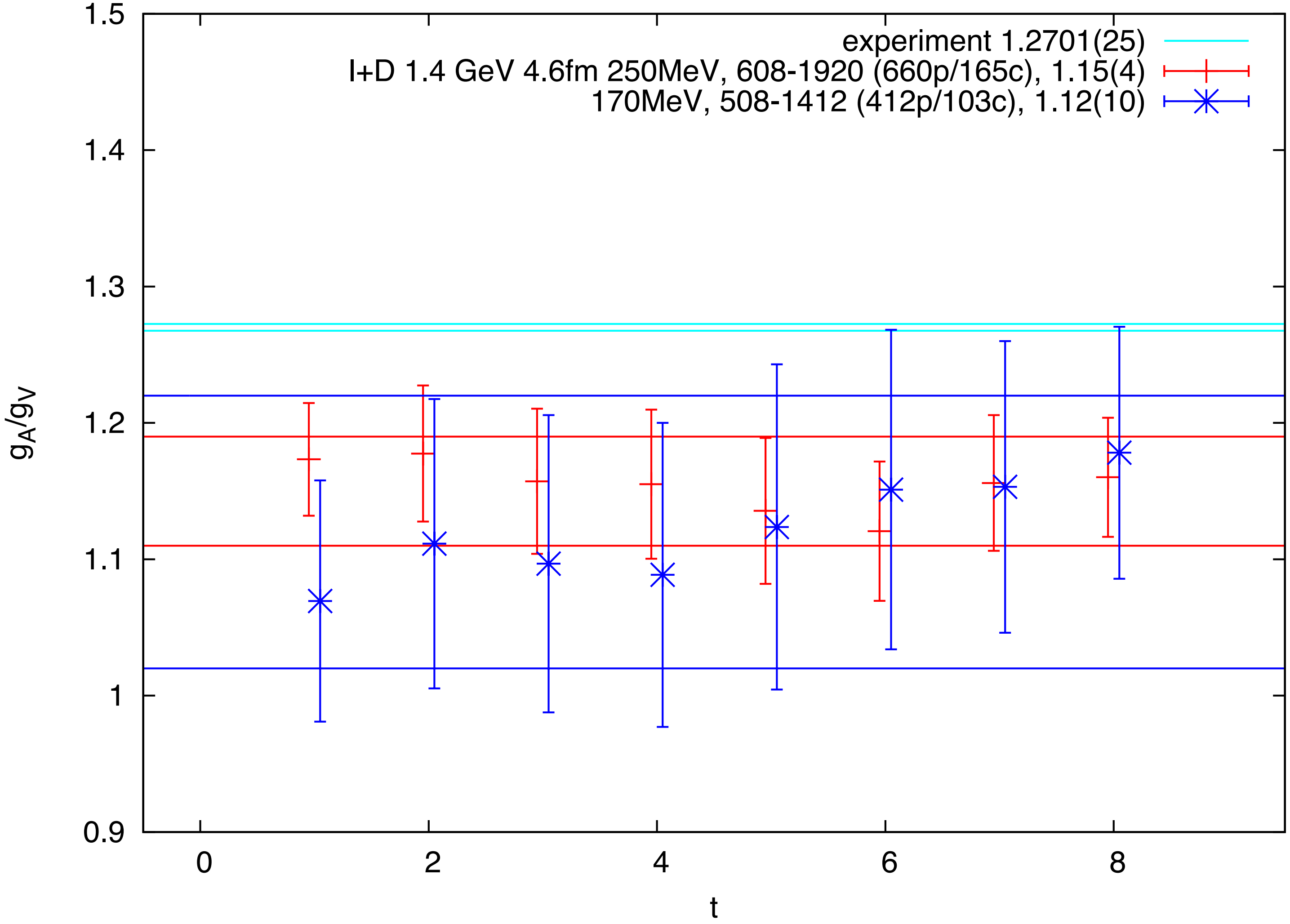
Local-current isovector vector charge, $g_V = 1.450(4)$ or $1.447(9)$, is obtained, corresponding to $Z_V = 0.692(7)$,

- in good agreement with $Z_V = 0.673(8)$ and $Z_A = 0.6878(3)$ obtained in the meson sector,
- yet again proving good chiral and flavor symmetries up to $O(a^2)$.

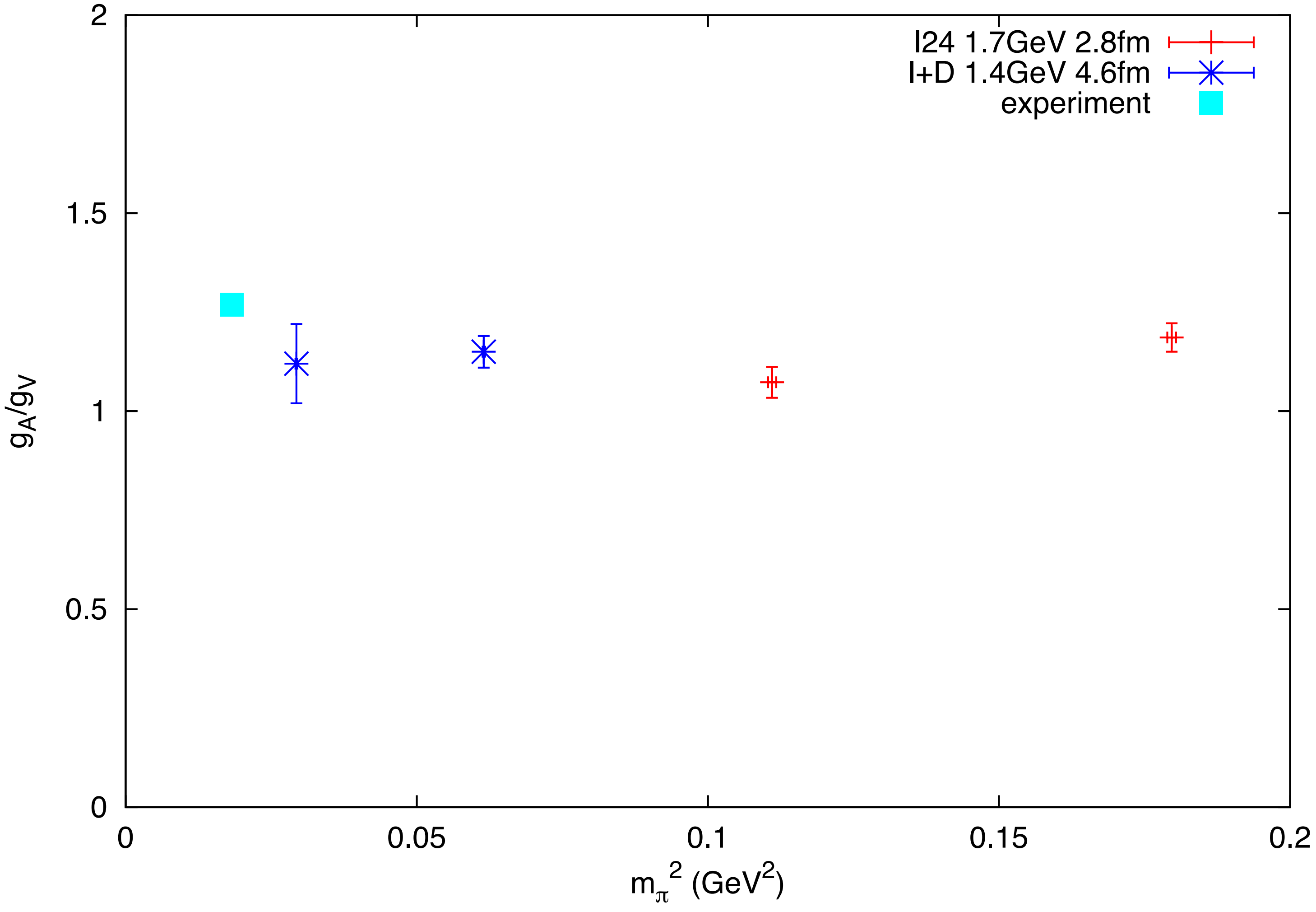
Axialvector current: Noisier than vector current, as expected,



g_A/g_V, ratio of isovector axial and vector charges, is less noisy, again as expected,

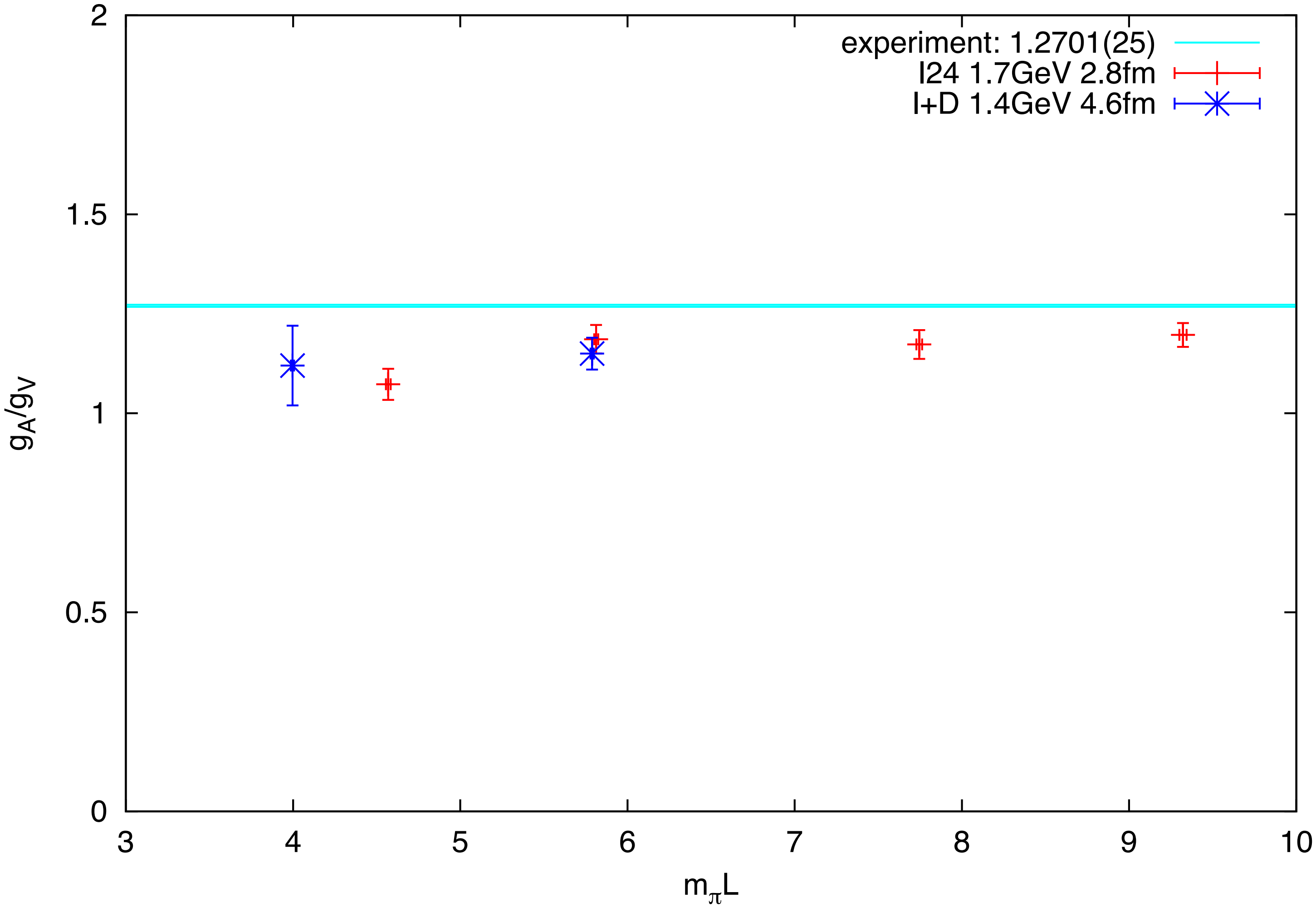


g_A/g_V: seems to stay away from the experiment as we set the pion mass lighter.



Not monotonic: appears to be a finite-size effect.

g_A/g_V: appears to show finite-size effect that is consistent with scaling in $m_\pi L$.



Results from two ensembles, 1.19(4) from I24 and 1.15(5) from ID, agree with each other, despite very much different m_π that significantly alter mass spectrum. There does not seem excited-state contamination above our statistics.

Results from two ensembles, I24 and ID32, which differ in

- quark/pion mass, m_π of 420 MeV and 250 MeV,
- spatial volume, L of 2.8 fm and 4.6 fm,
- lattice cut off, a^{-1} of 1.7 GeV and 1.4 GeV,
- gauge actions,

that should give different source, $A_0 e^{-E_0 t}|0\rangle + A_1 e^{-E_1 t}|1\rangle + ...$, agree well in g_A/g_V when $m_\pi L$ agree:

Do the differences magically conspire, or do we see scaling in $m_\pi L$?

Yet we liked to improve the statistical significance:

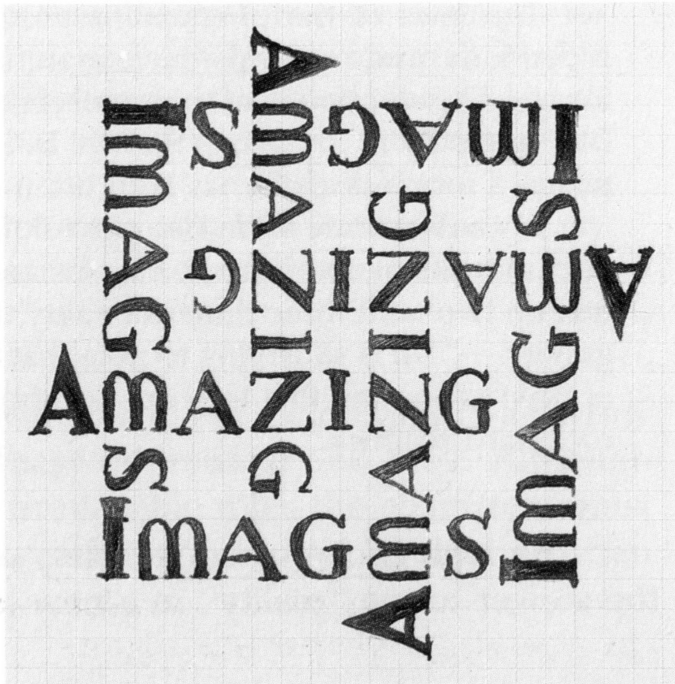
Not so trivial a task,

as the results took a few years using US and Japanese national clusters, XSEDE and RICC.

A new statistical technique, "AMA[3]," offer $\times$ 10–20 acceleration: by allowing

- cruder,
- but cheaper,

independent statistical sampling at much higher frequency, by taking advantage of point-group symmetries of the lattice to organize many such cruder but independent and equivalent measurements:



$$\langle O \rangle_{\text{AMA}} = \frac{1}{N_{\text{sloppy}}} \sum_{s}^{N_{\text{sloppy}}} \langle O \rangle^{s}_{\text{sloppy}} + \frac{1}{N_{\text{accurate}}} \sum_{a}^{N_{\text{accurate}}} \left(\langle O \rangle^{a}_{\text{accurate}} - \langle O \rangle^{a}_{\text{sloppy}} \right)$$

[3]T. Blum, T. Izubuchi and E. Shintani, arXiv:1208.4349; PoS Lattice 2012, 262.

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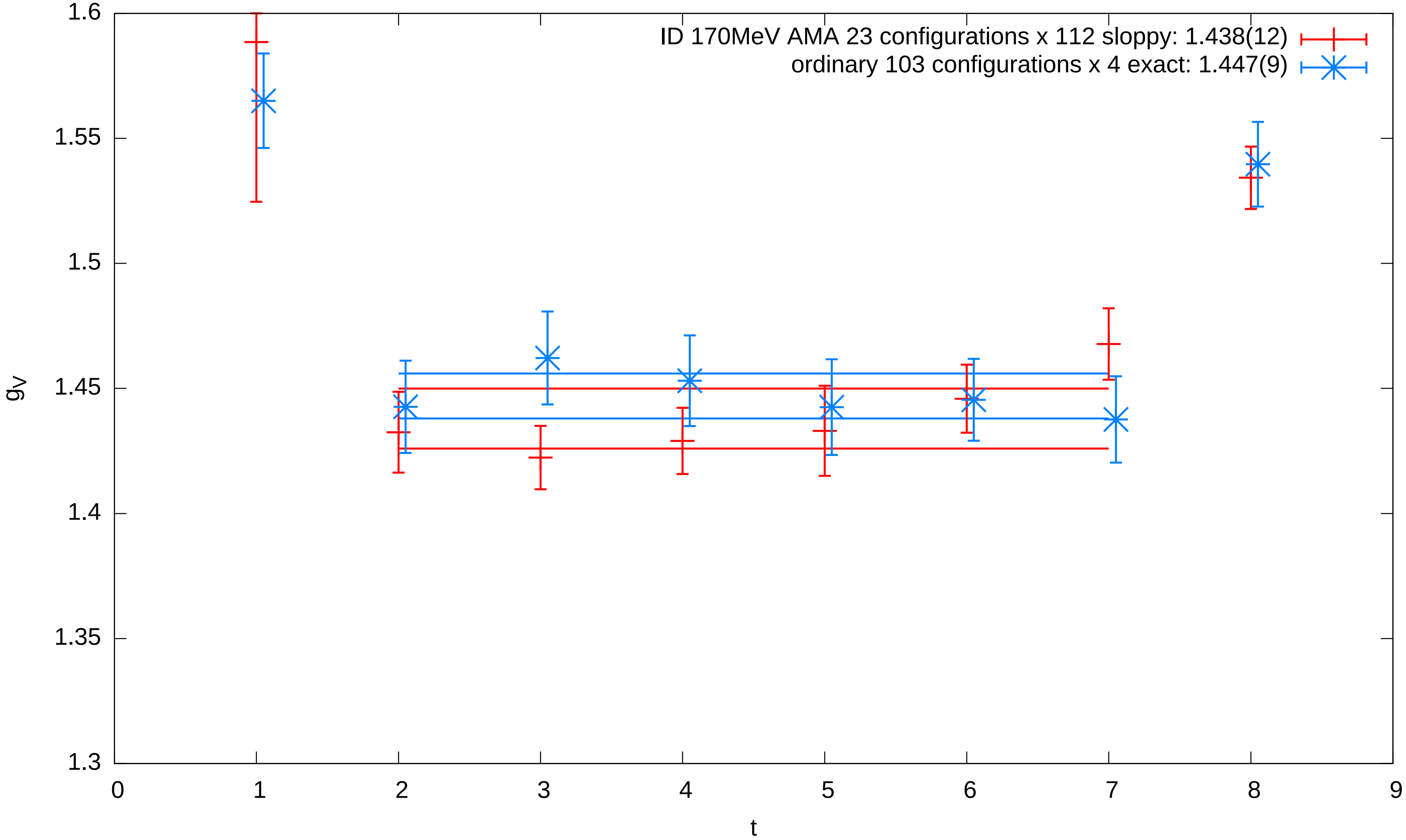

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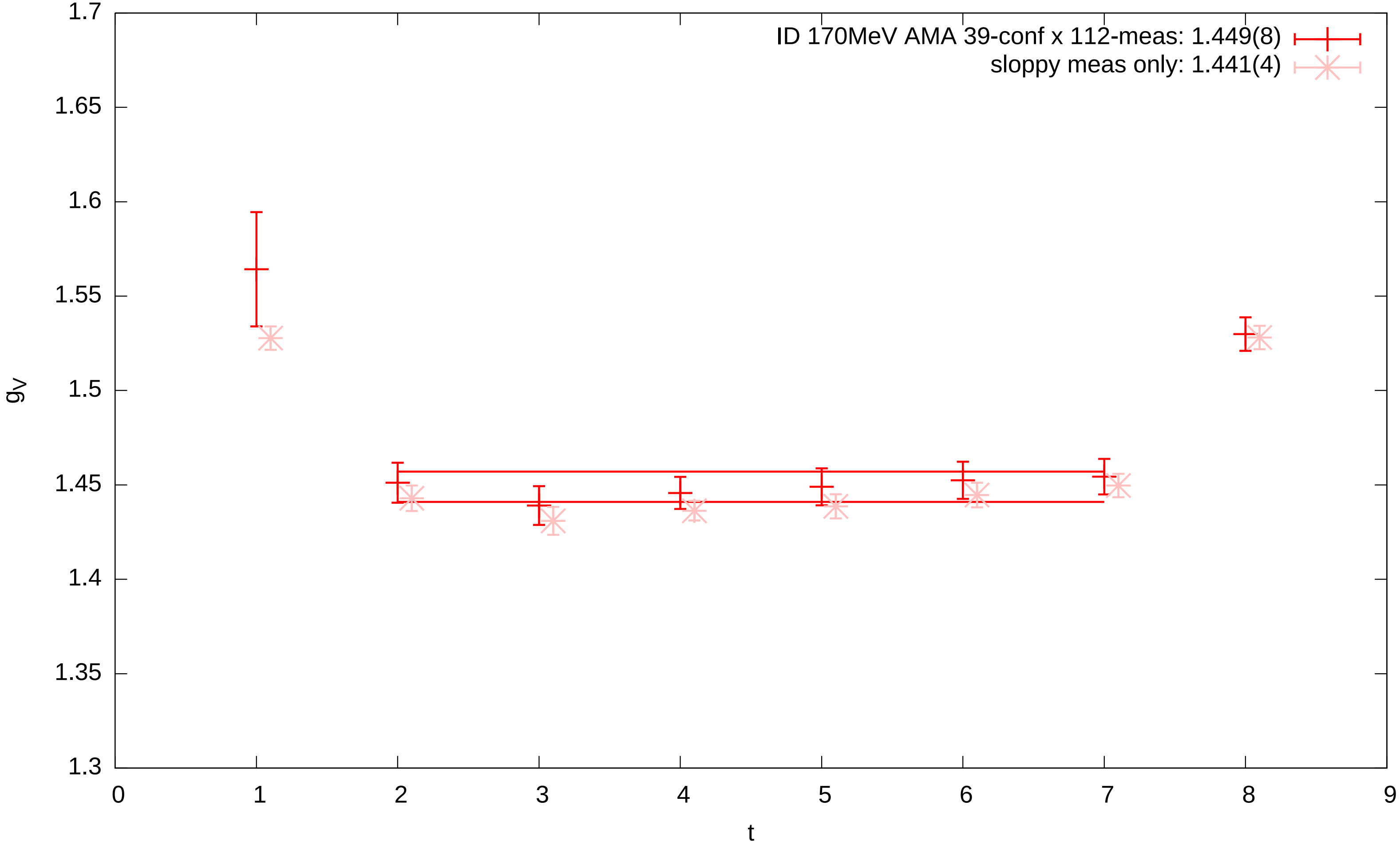

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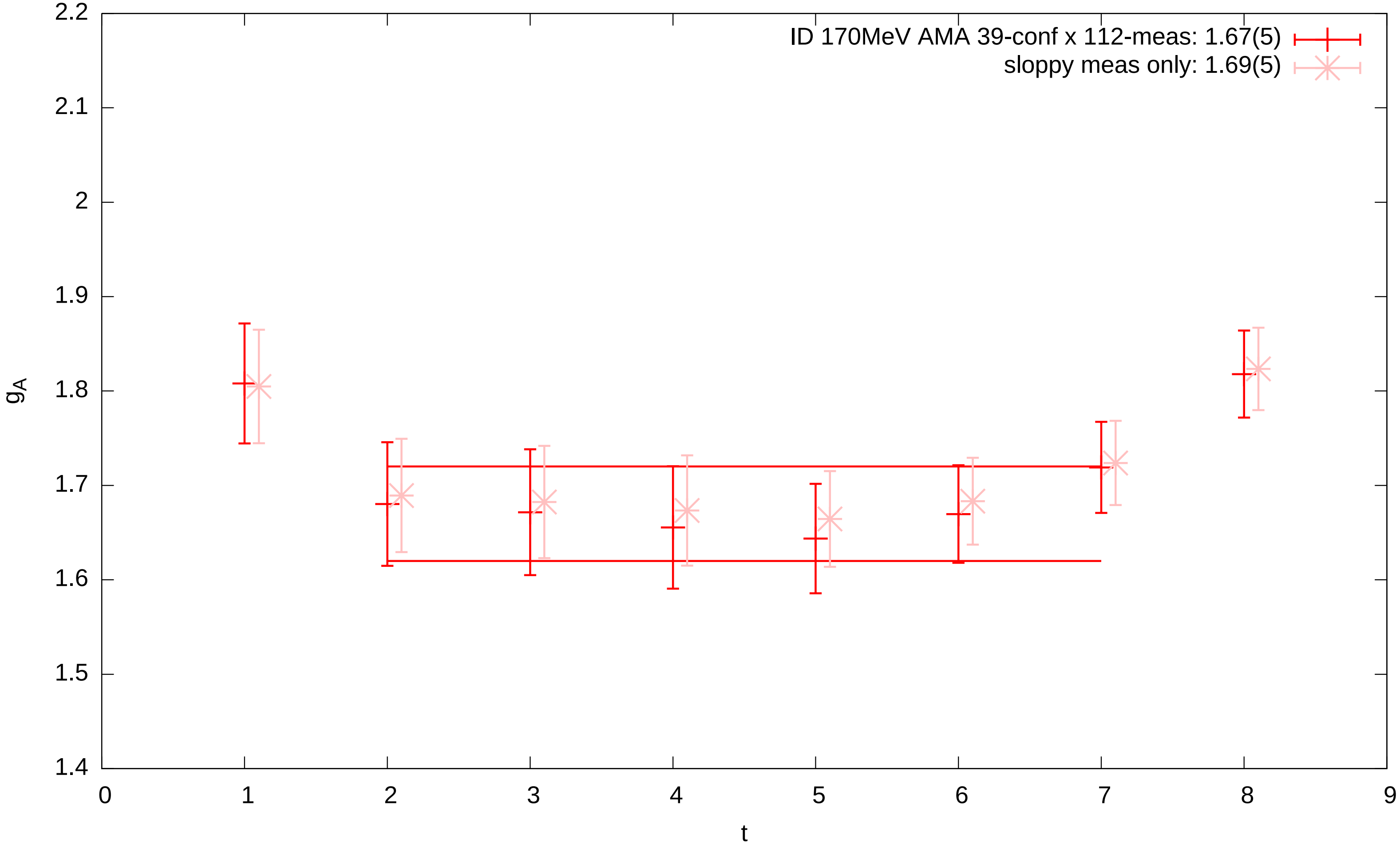

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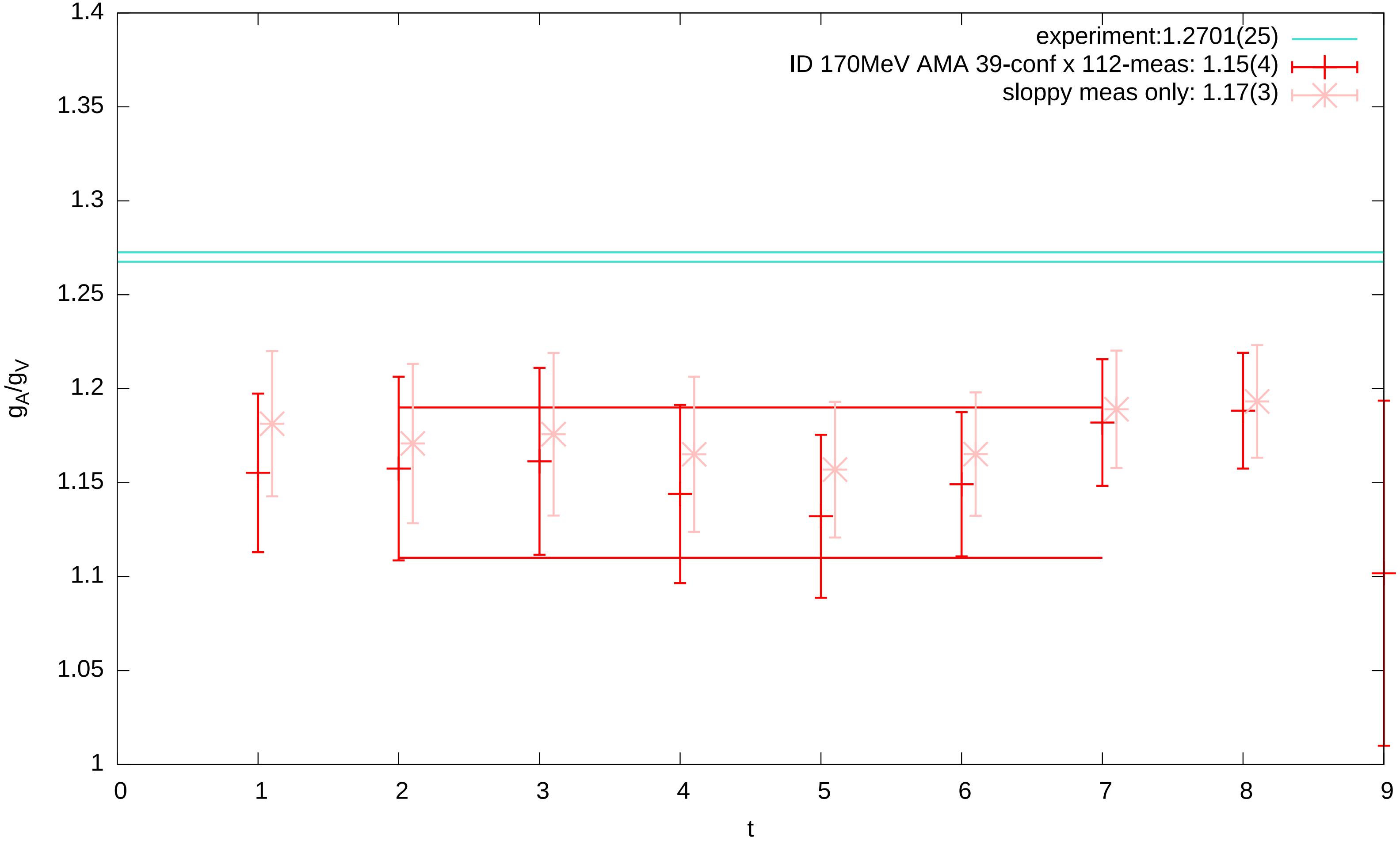

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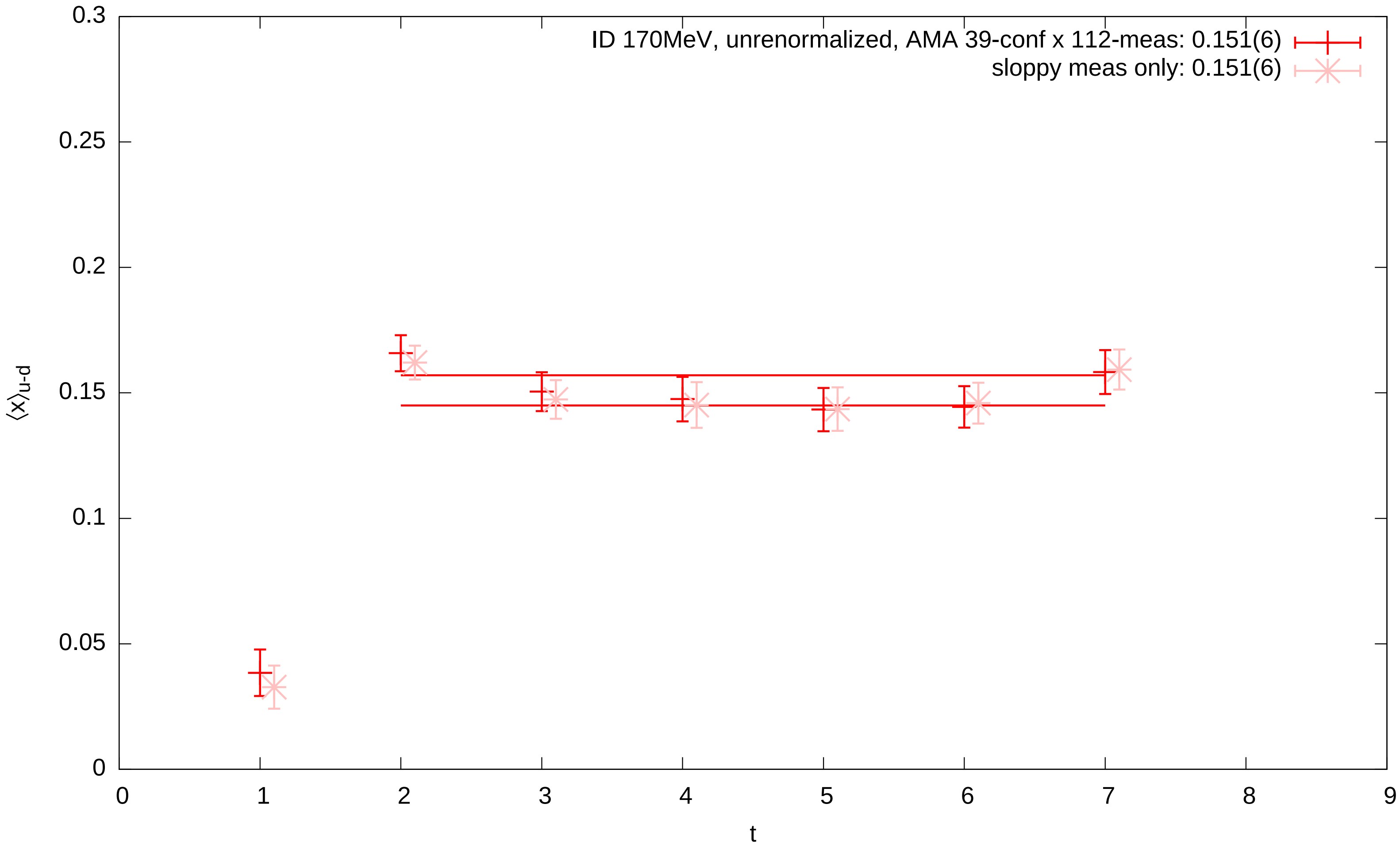

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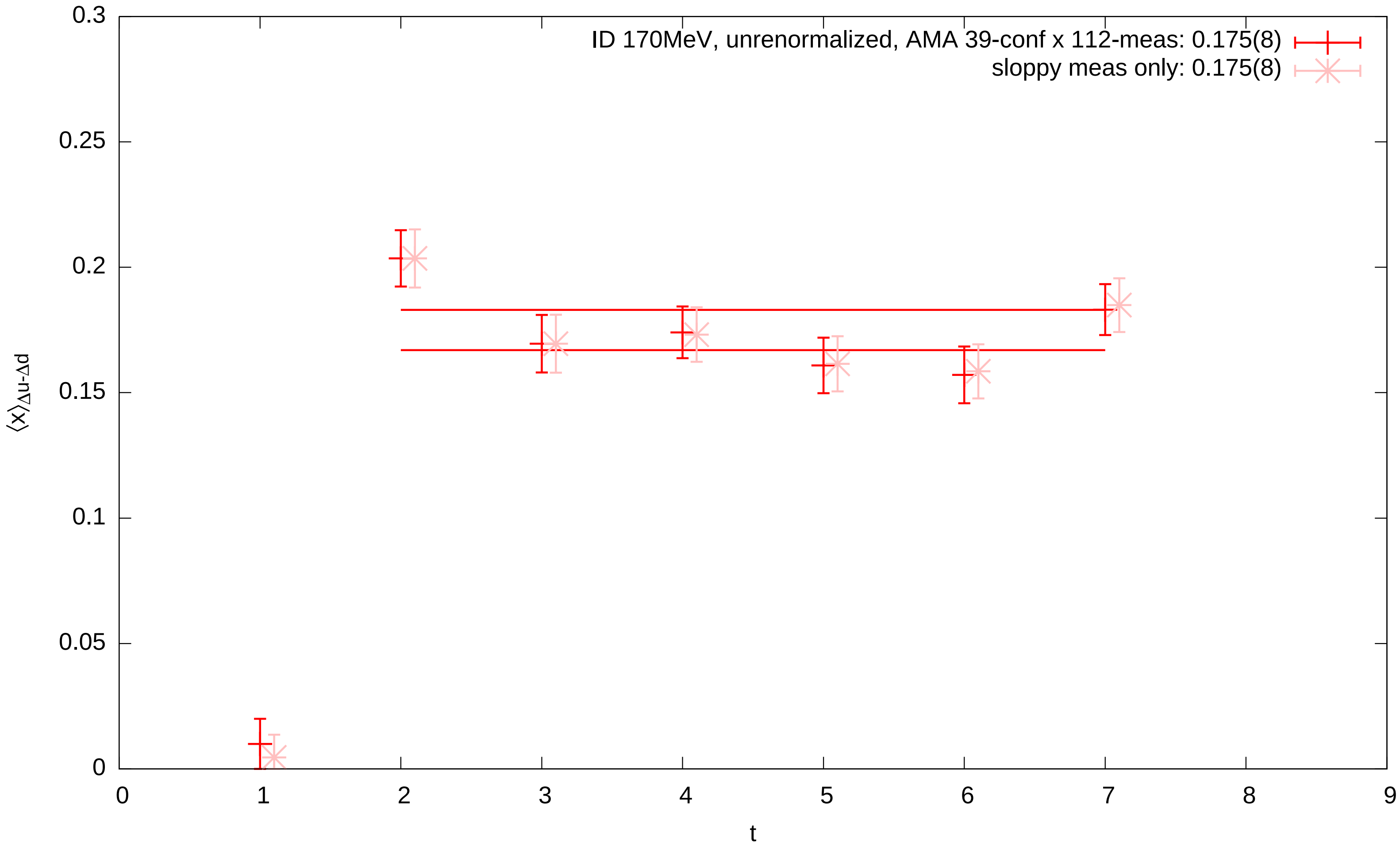
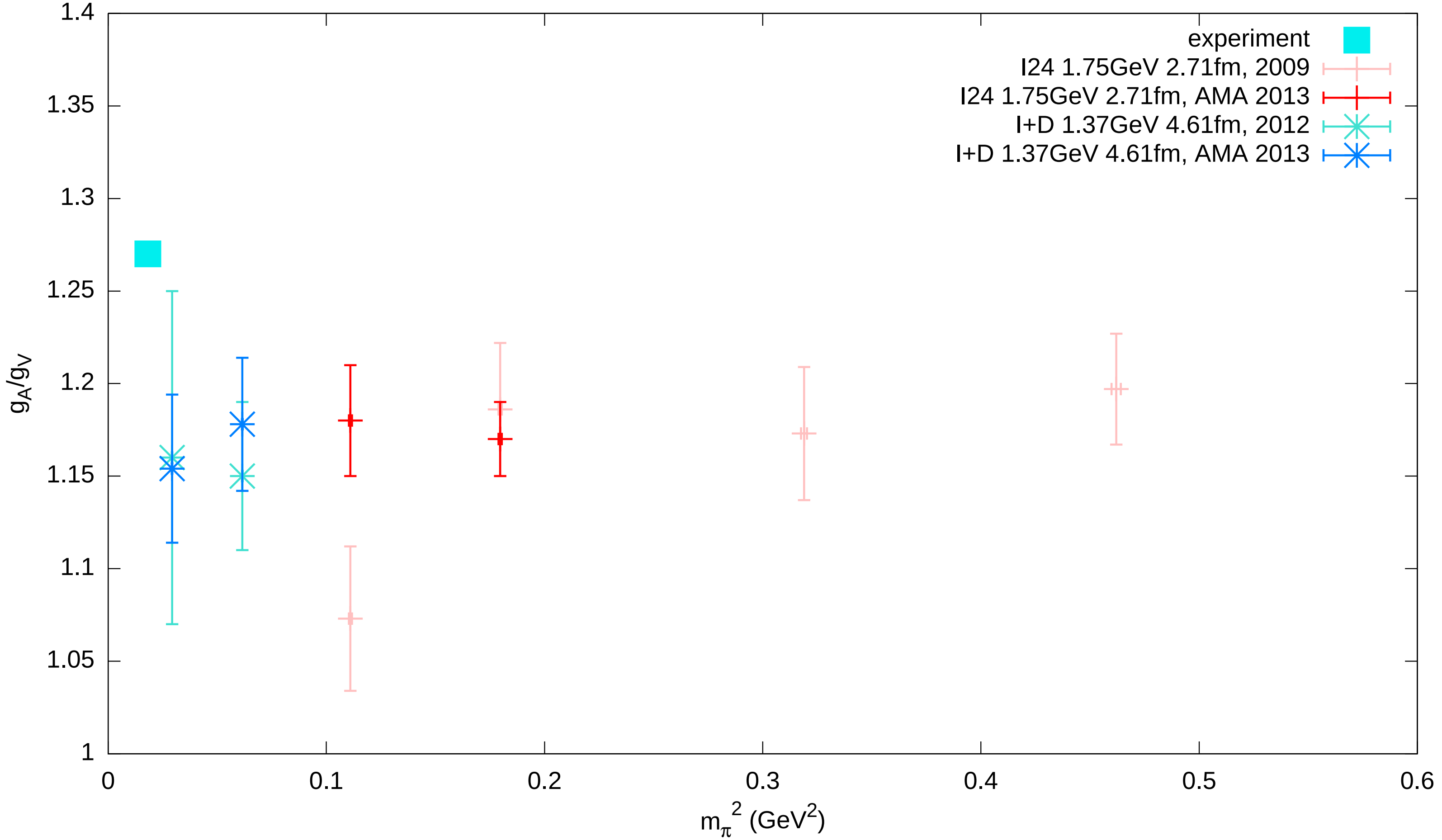
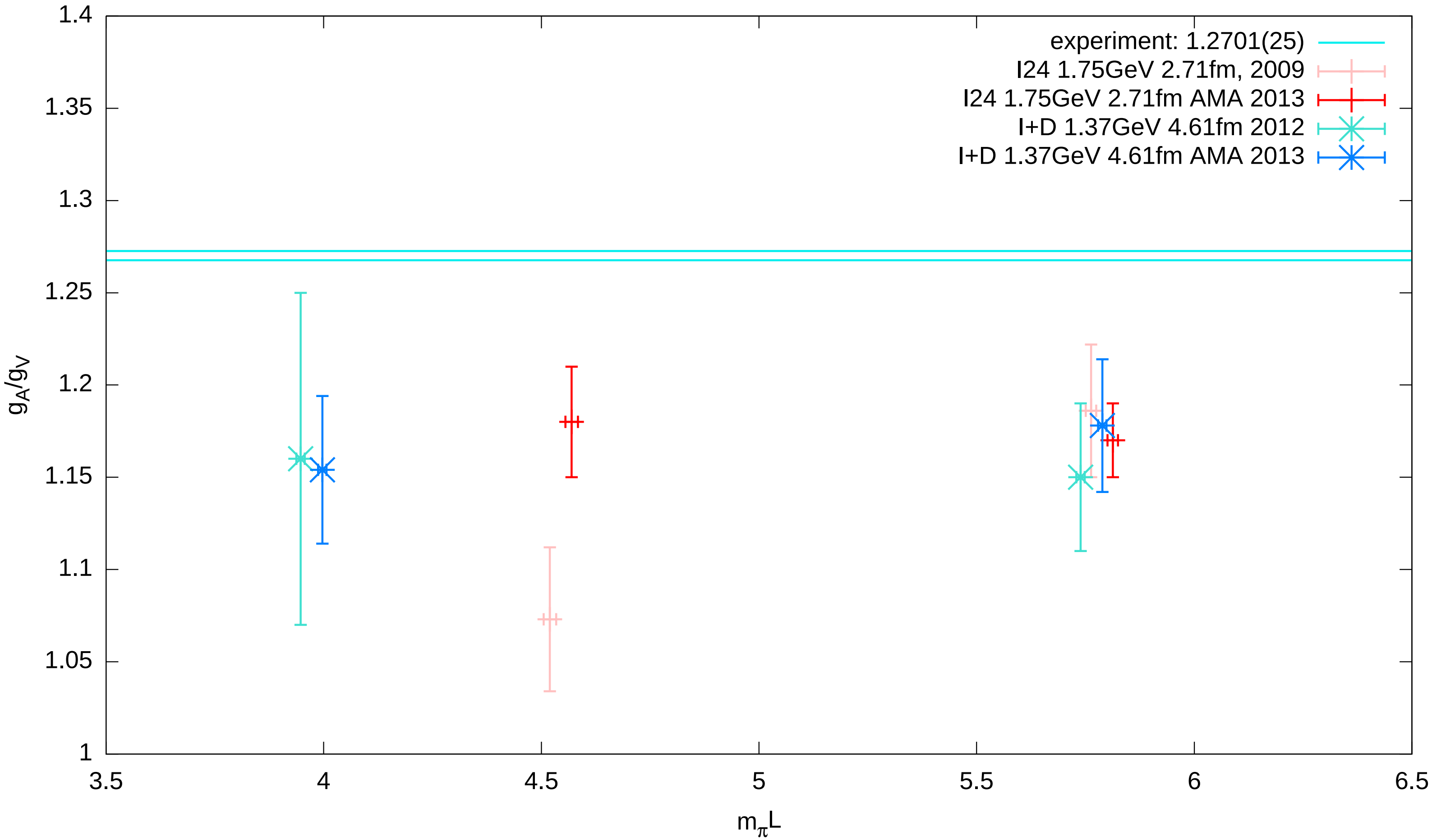


With AMA and other statistical improvements, g_A/g_V vs m_π^2 now looks like the following:



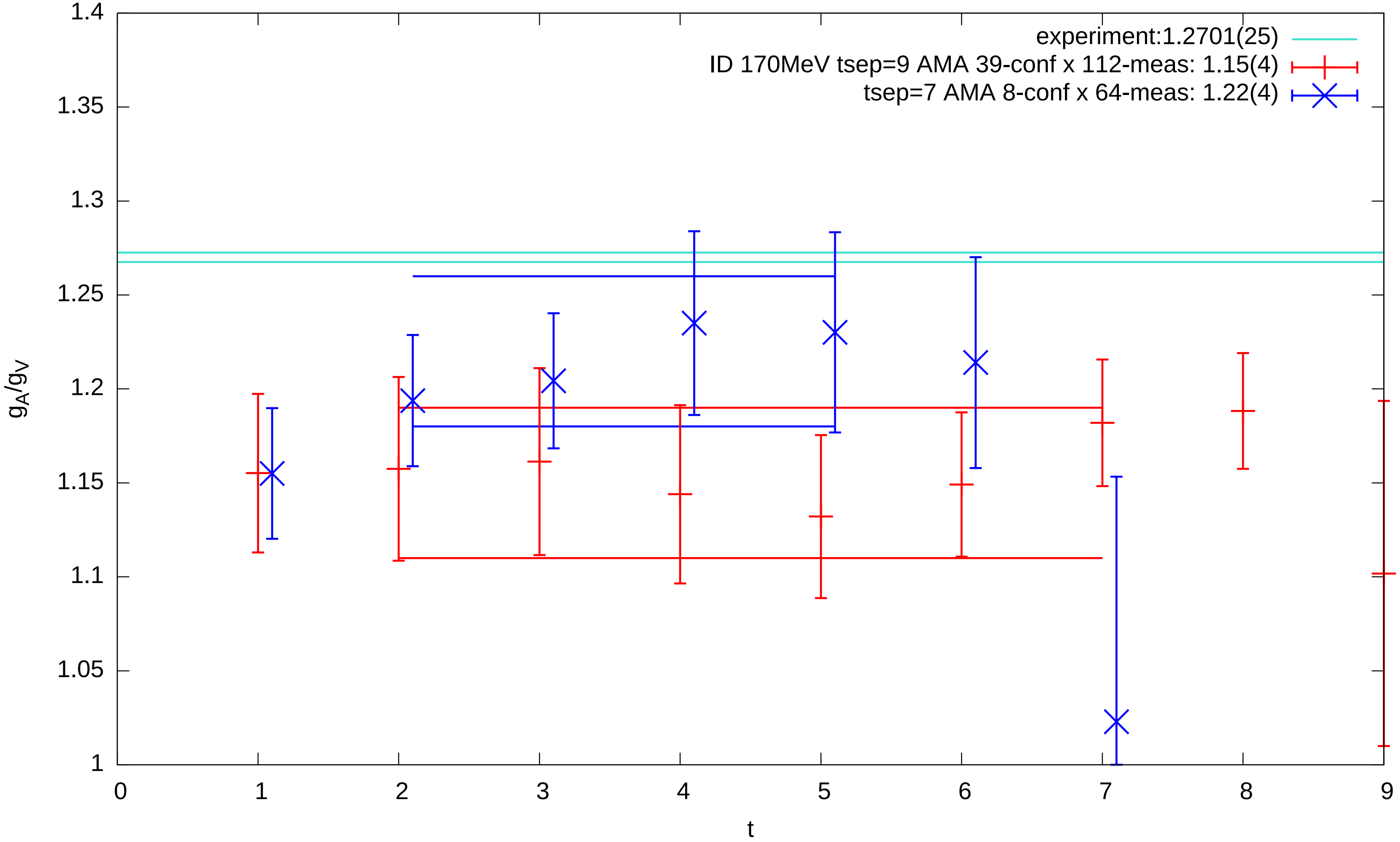
Moves away from the experiment as m_π approaches the experimental value.

With AMA and other statistical improvements, g_A/g_V agreement at $m_\pi L = 5.8$ is more significant: 1.17(2) and 1.18(4)



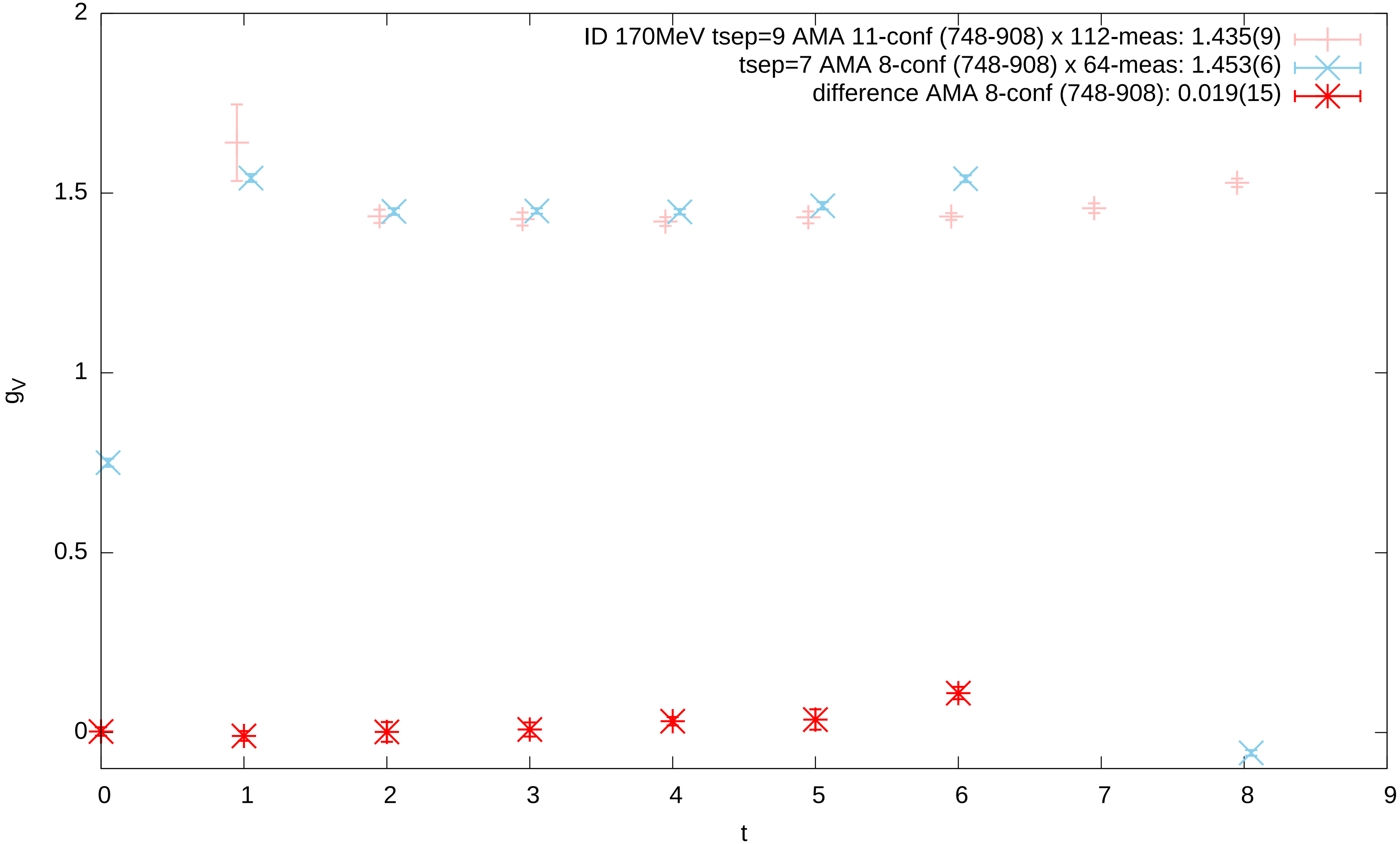
About 10-% deficit?

Also with new AMA calculations, this deficit in g_A/g_V seem less likely from excited states:



Results at shorter $t_{\text{sink}} - t_{\text{source}} = 7$ should suffer more excited-state:
though statistically not significant, they seem to give (systematically) higher g_A/g_V.
$t_{\text{sep}} = 9$ result is lower than experiment even when we consider excited-state contamination.

In fact, no excited-state contamination is seen in any of our 170-MeV calculations:



When compared with the same configurations, the difference is always consistent with 0. $A_1\langle 1|O|0\rangle \sim 0$ for any observable we look at: A_1 is negligible for these small $\langle 1|O|0\rangle$.

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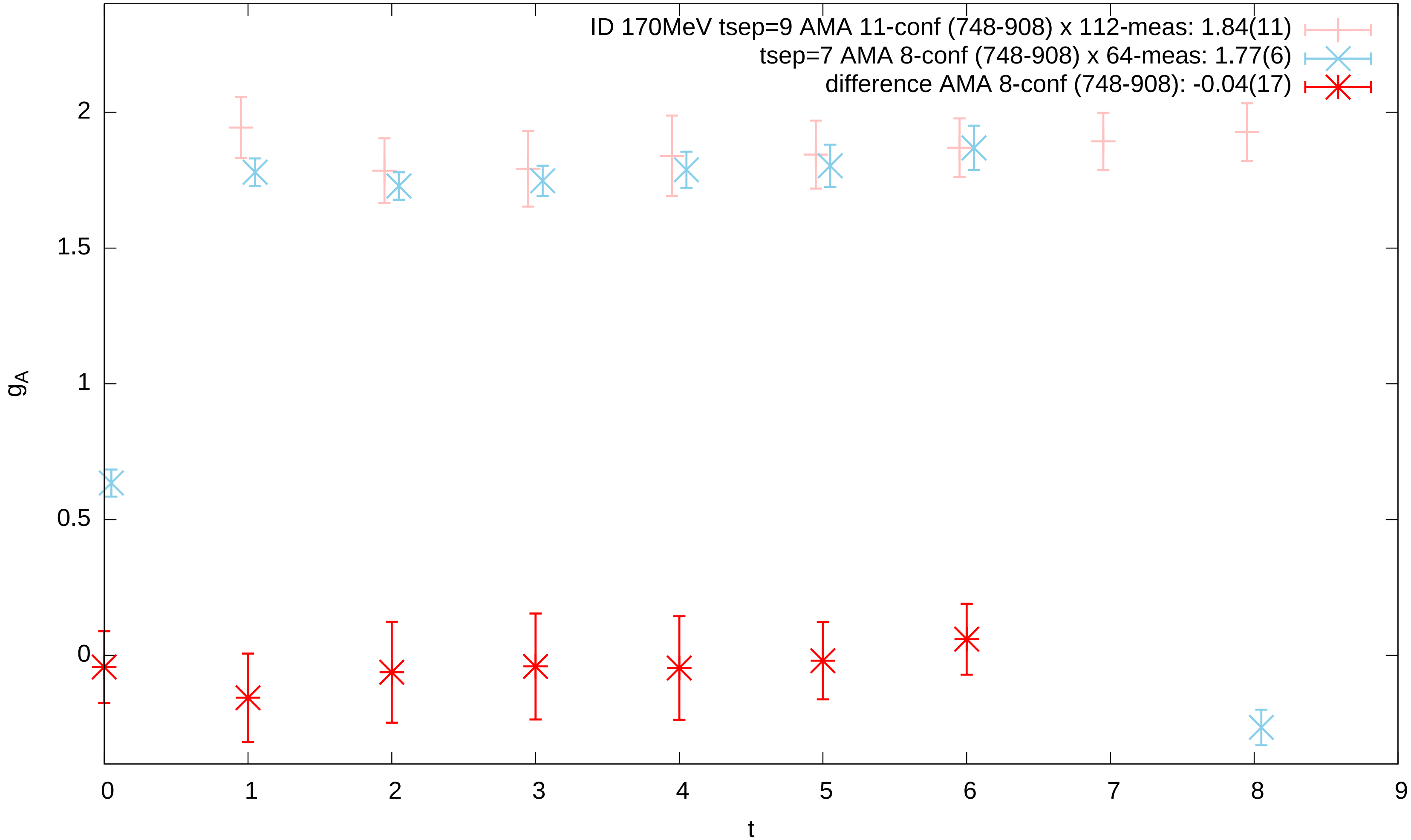

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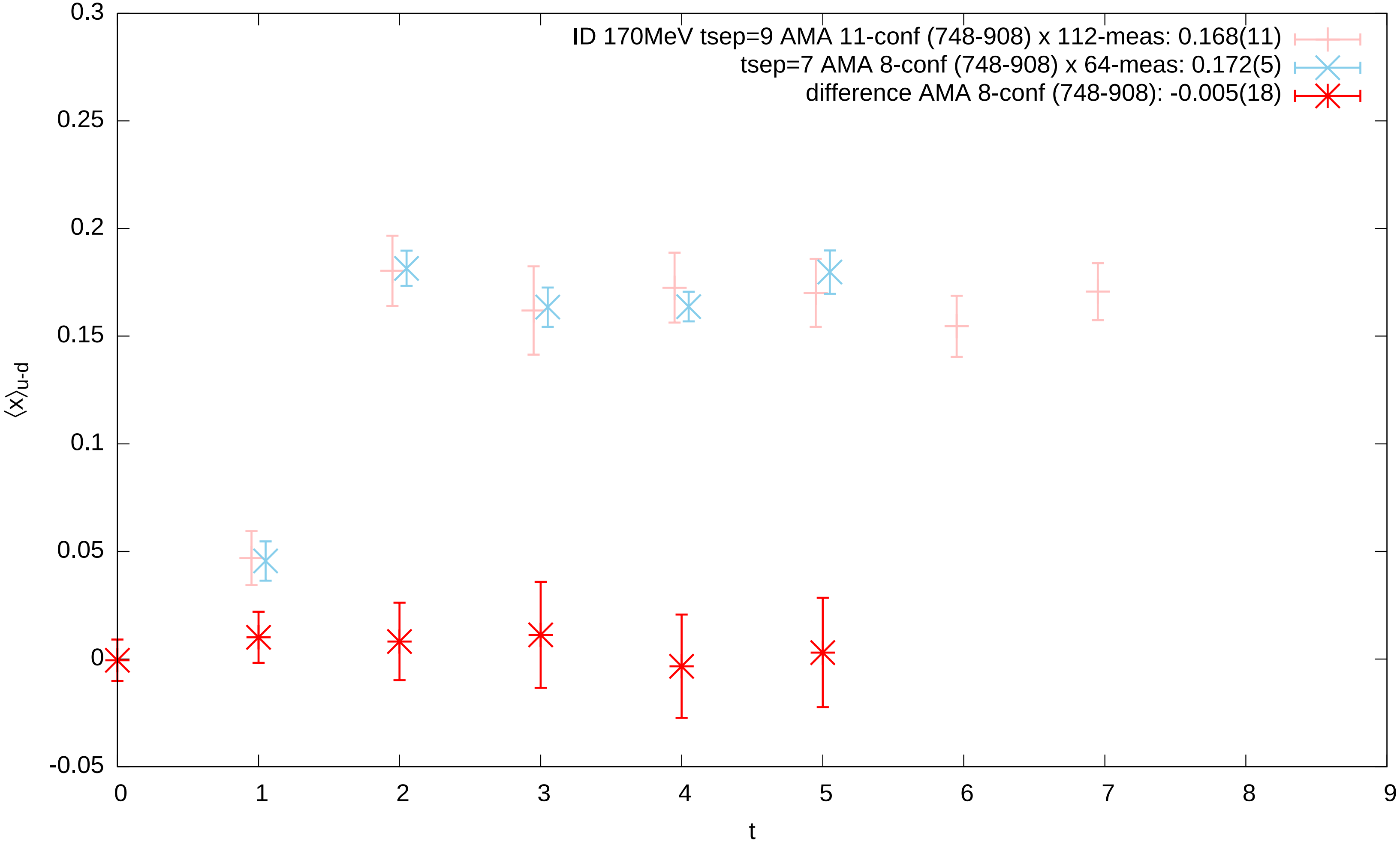

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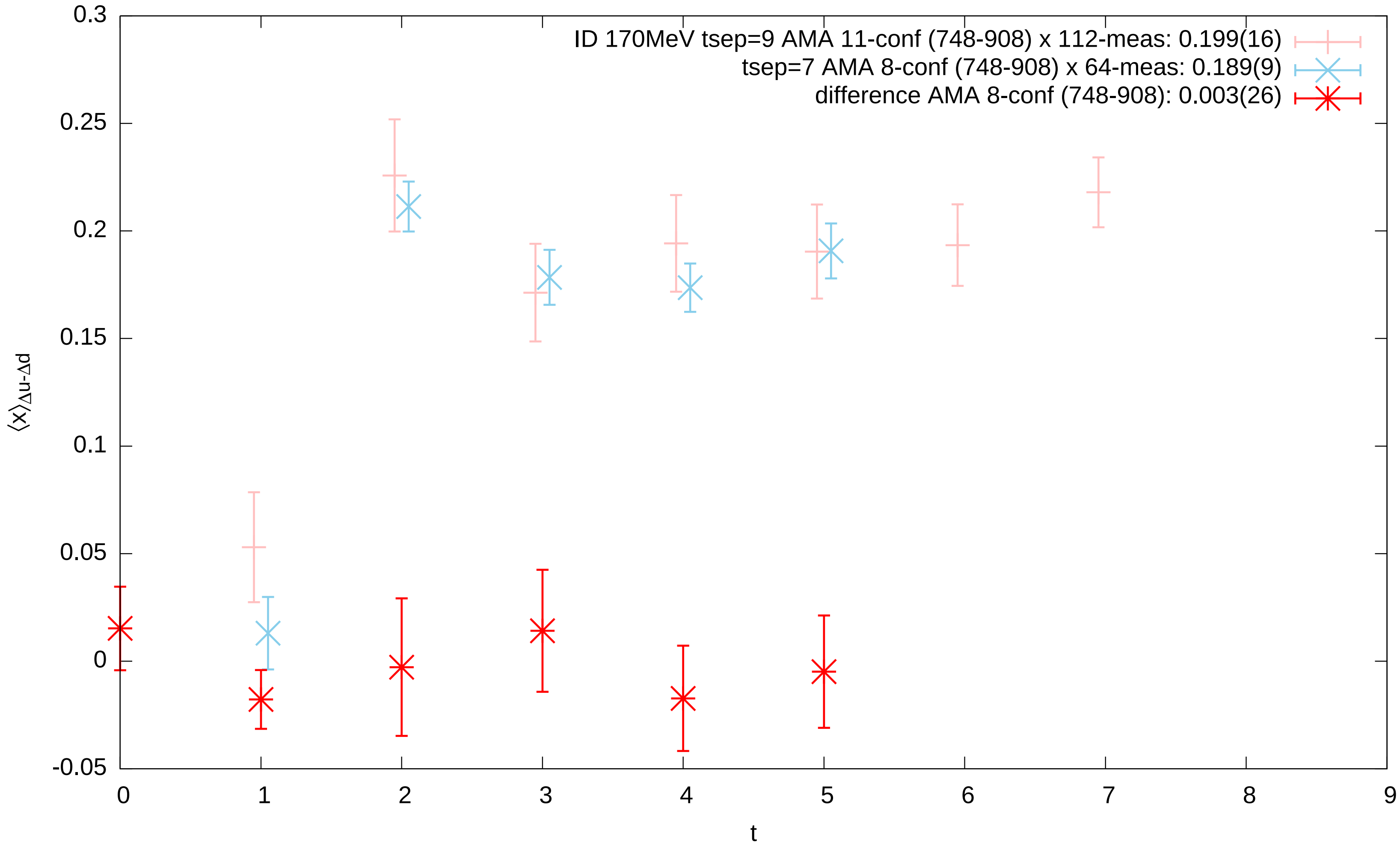
In fact, no excited-state contamination is seen in any of our 170-MeV calculations:



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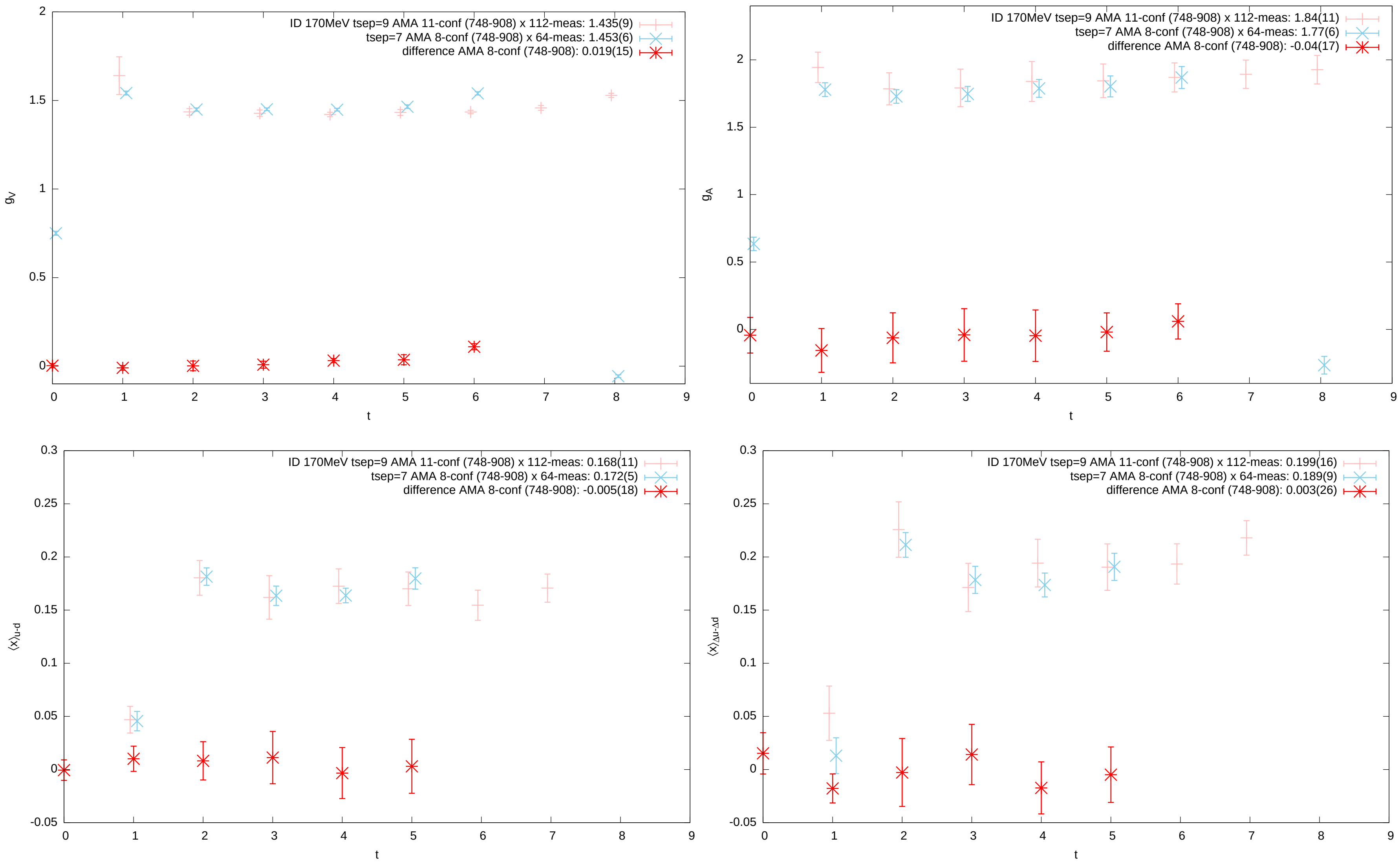

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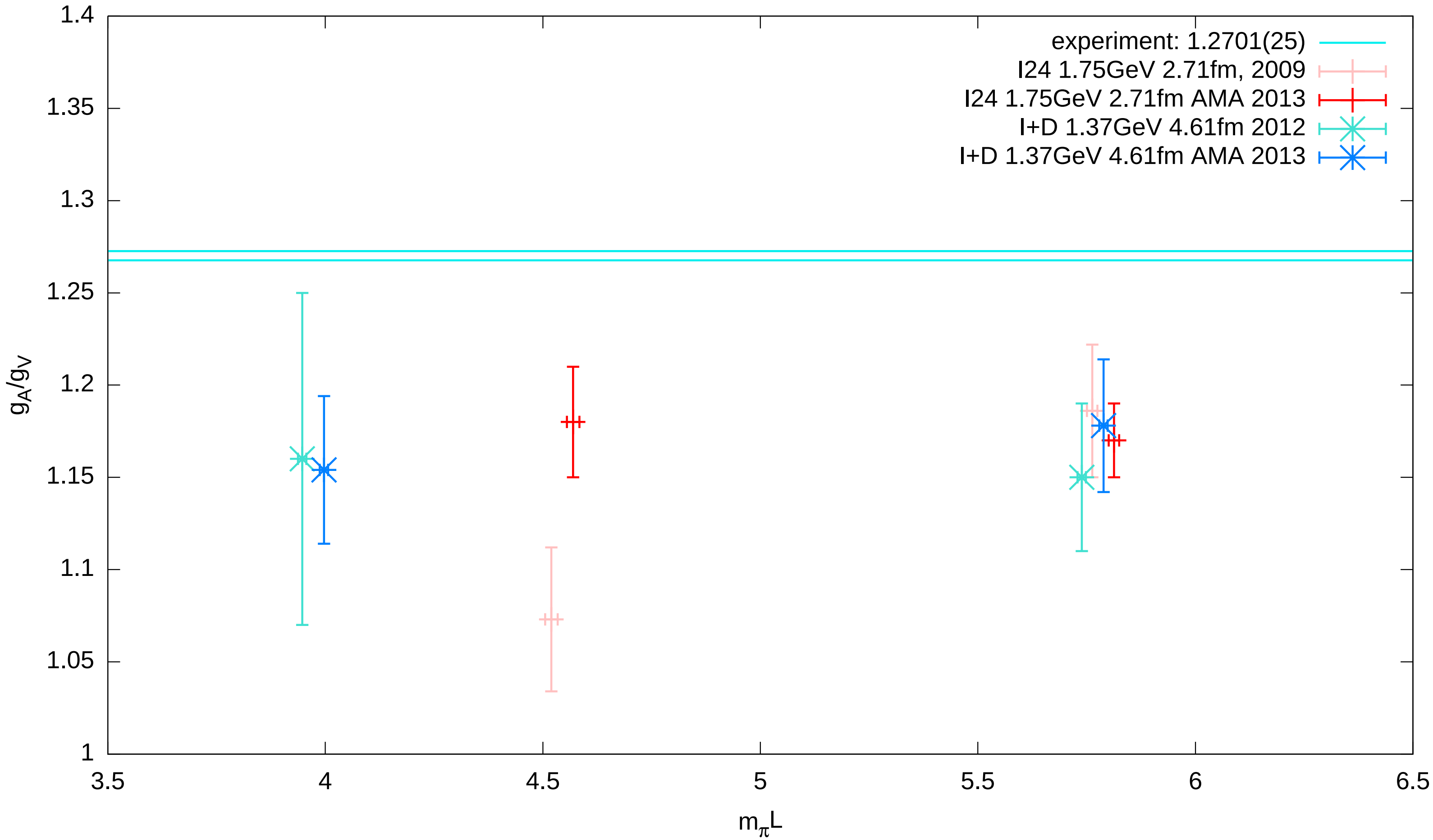
About 10-% deficit in g_A/g_V seen:

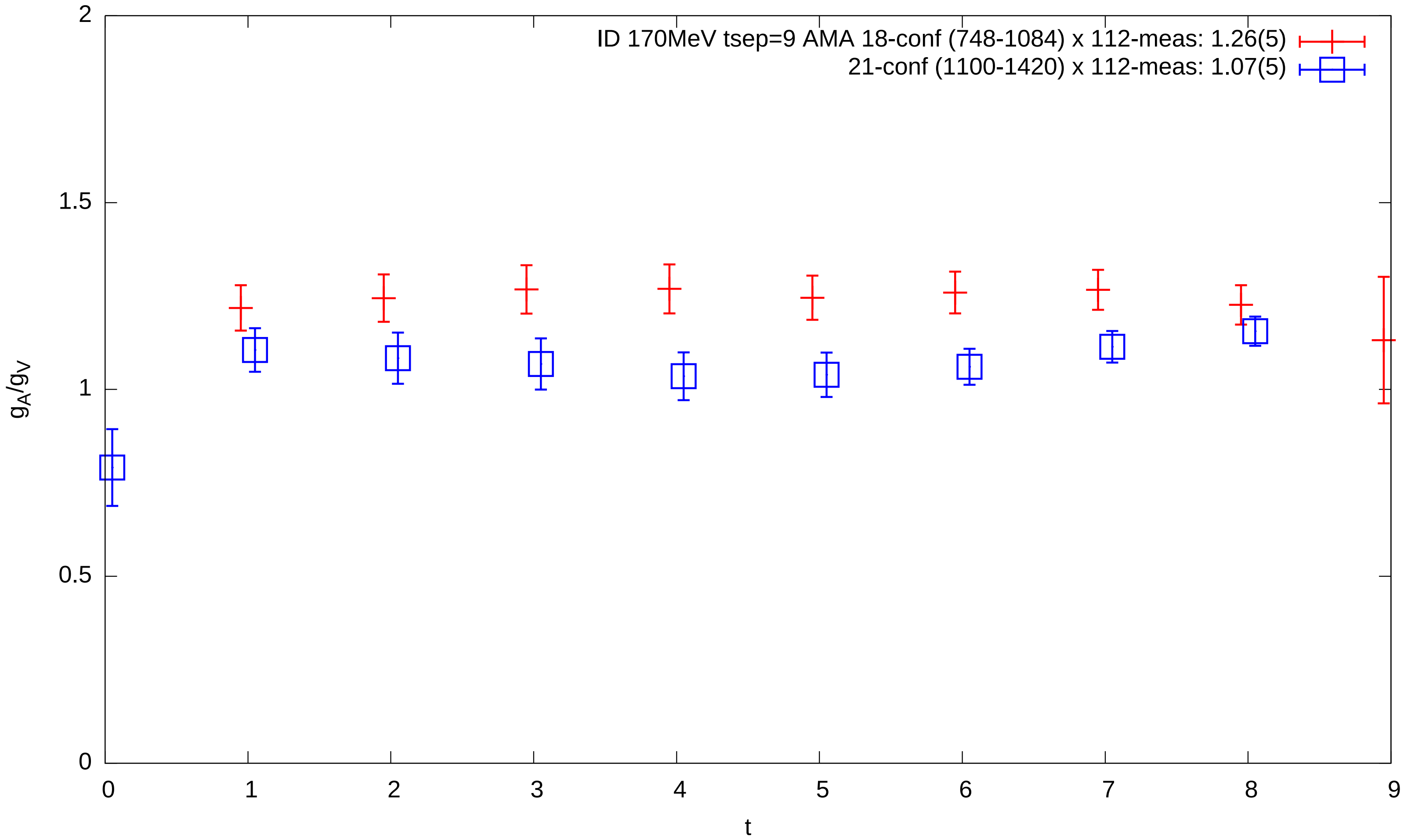


Excited-state contamination now is unlikely the cause.

Almost 5-standard-deviation significance at $m_\pi L \sim 5.8$.

Appears like monotonically decreasing with $m_\pi L$.

Long-range auto-correlation seen in g_A/g_V at $m_\pi = 170$ MeV:



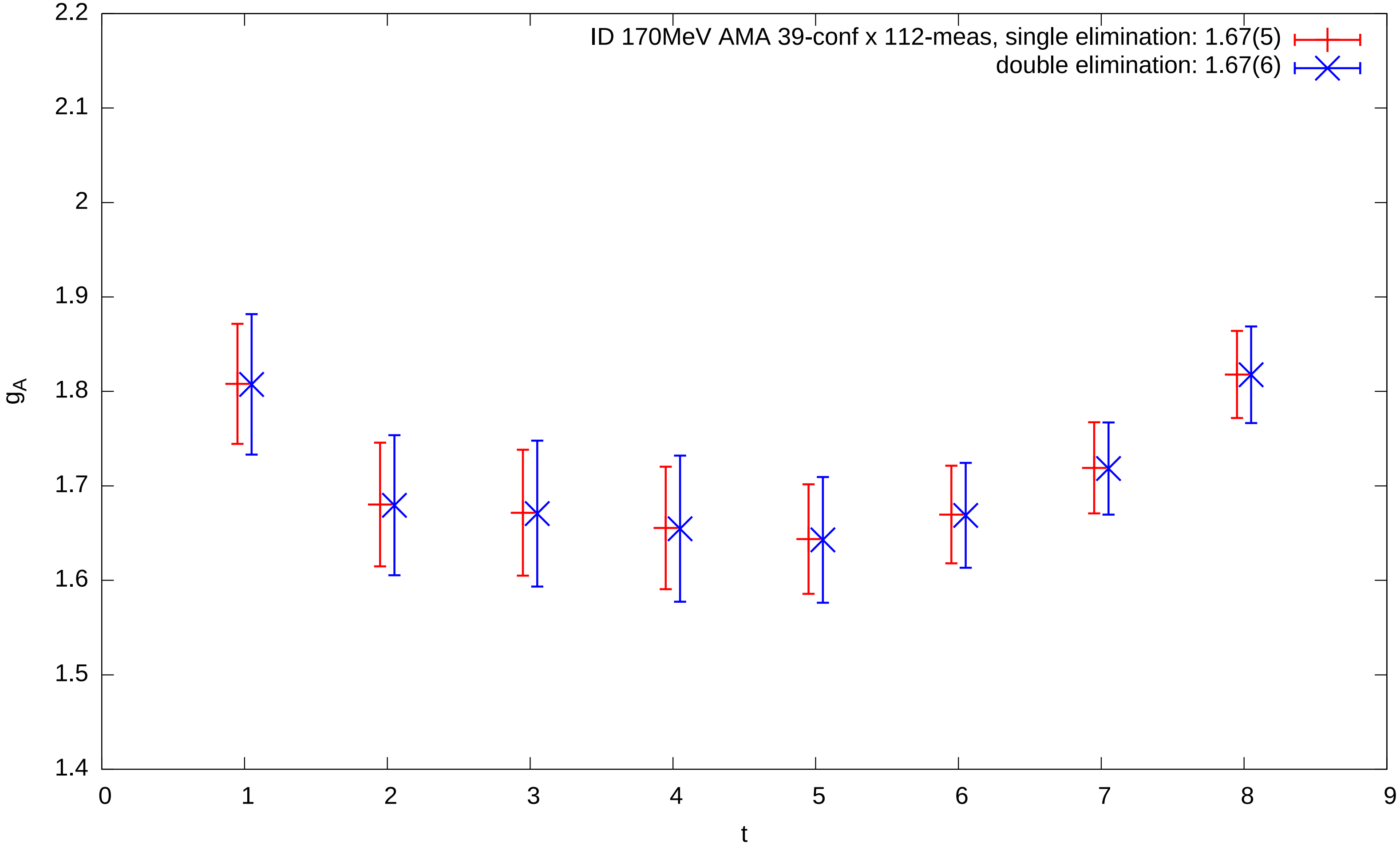
Indicative of insufficient spatial volume.

Systematics other than the spatial volume have been more or less dismissed, in particular the excited states.

$L > 8$ fm is required at the physical point, $m_\pi \sim 140$ MeV, to fully contain g_A and so $g_{\pi NN}$.

Can this be reconciled with conventional nuclear theory with point-like and non-relativistic nucleons?

Indeed the estimated errors grow from single- to double-elimination jack knife for g_A and g_A/g_V:



Two successive configurations, separated by 16-trajectory interval, are almost completely correlated.

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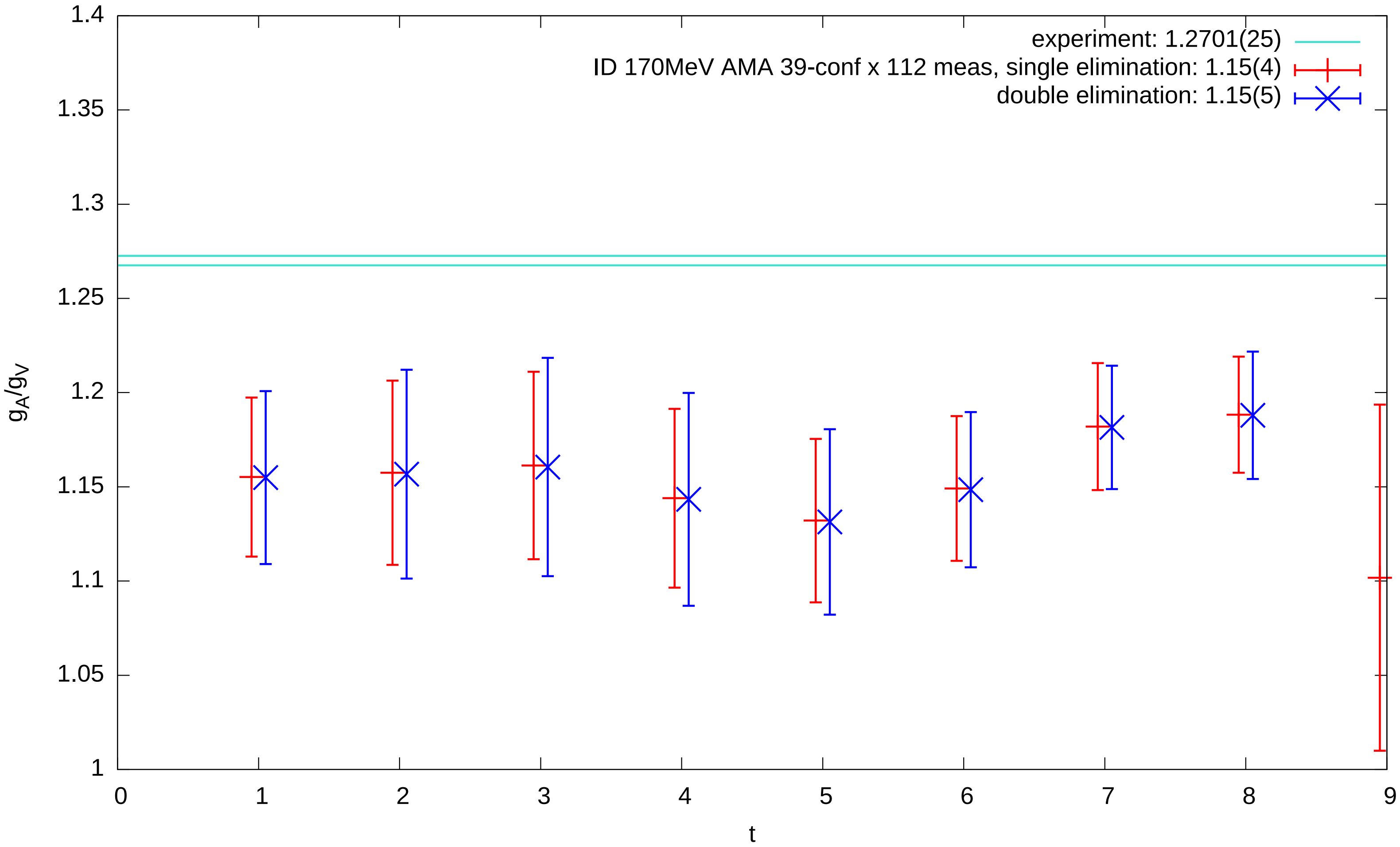

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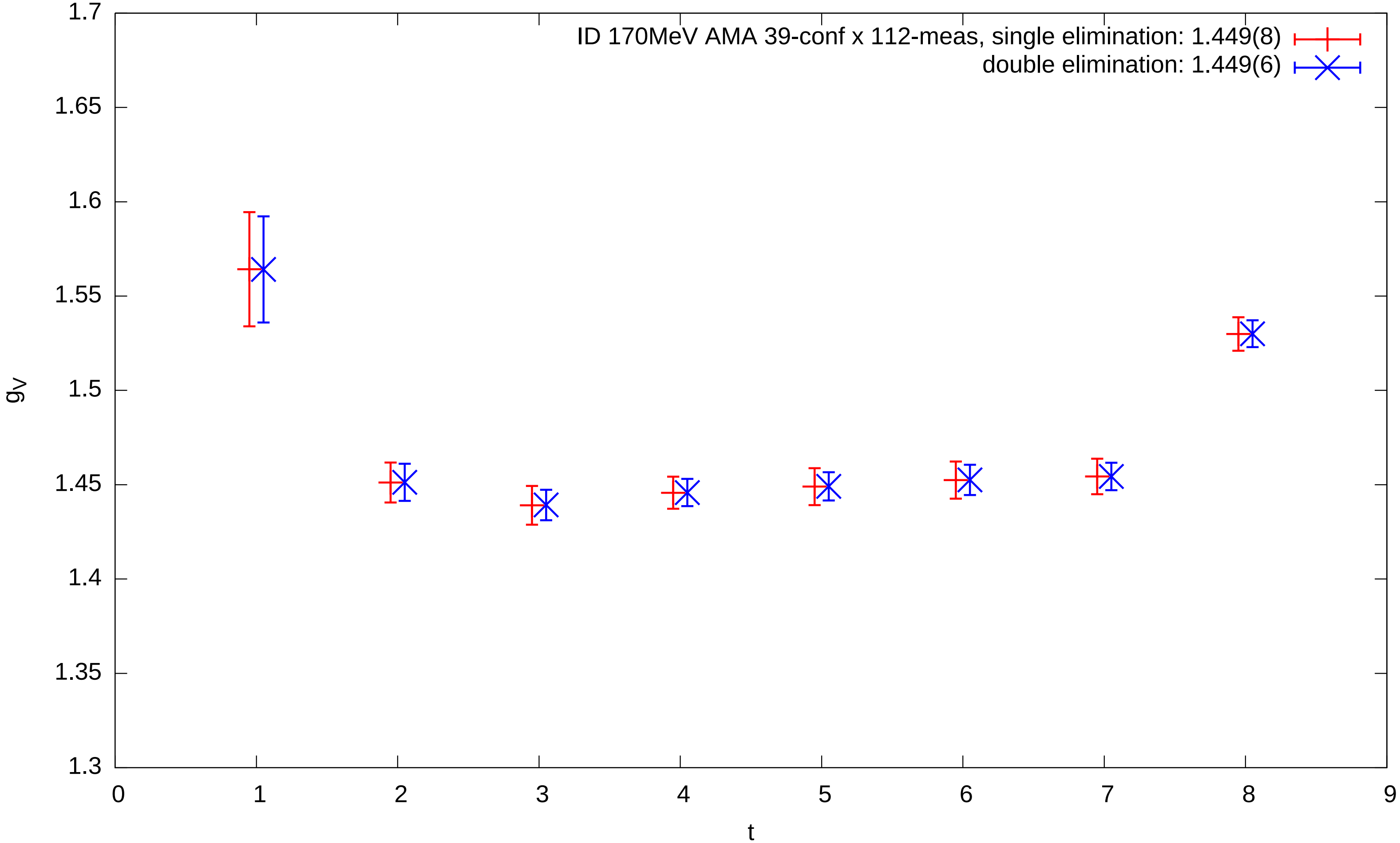
But no such auto-correlation is seen in other observables, g_V, $\langle x \rangle_{u-d}$ or $\langle x \rangle_{\Delta u - \Delta d}$:



Double-elimination JK sampling does not differ from single-elimination except for g_A.
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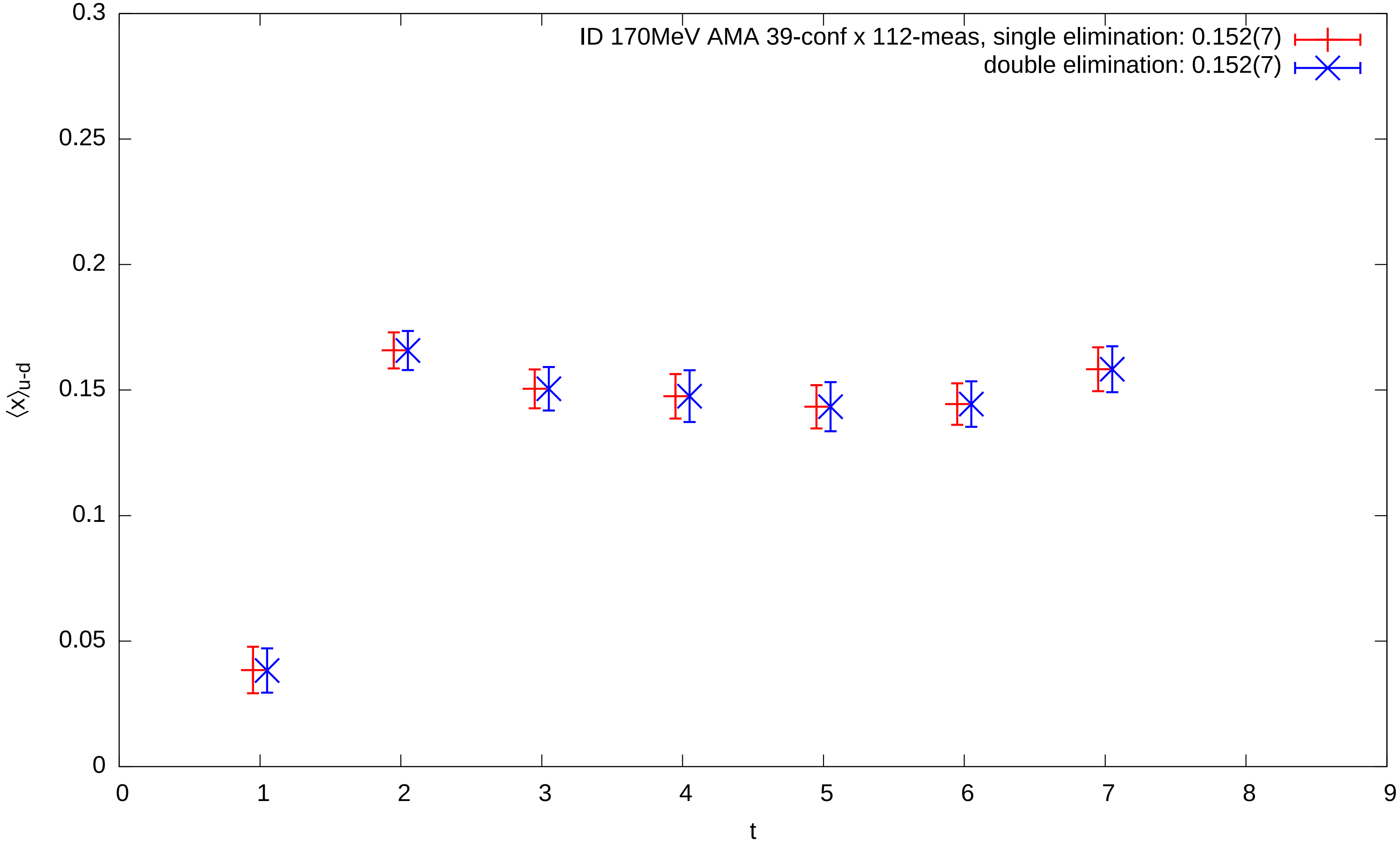

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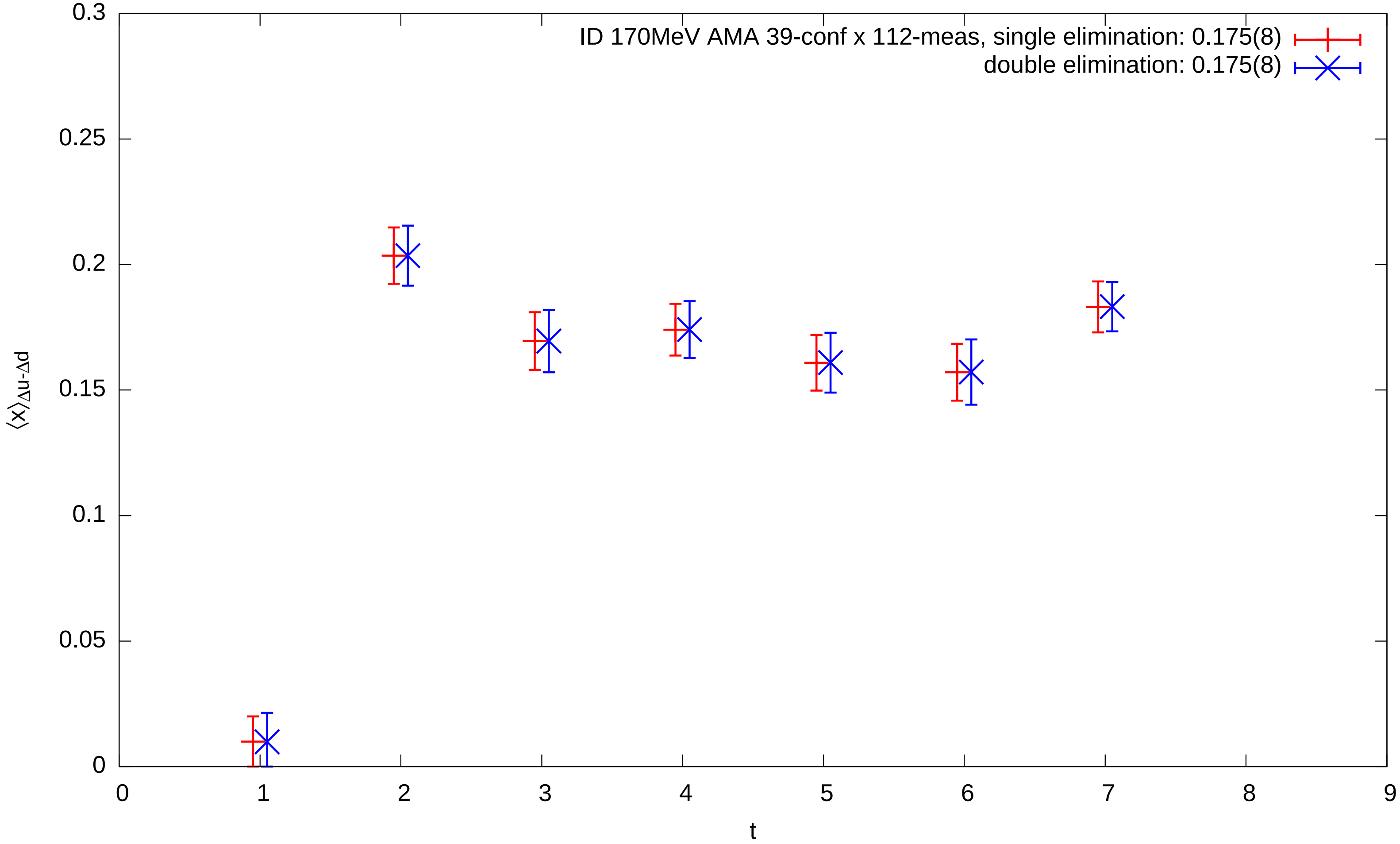

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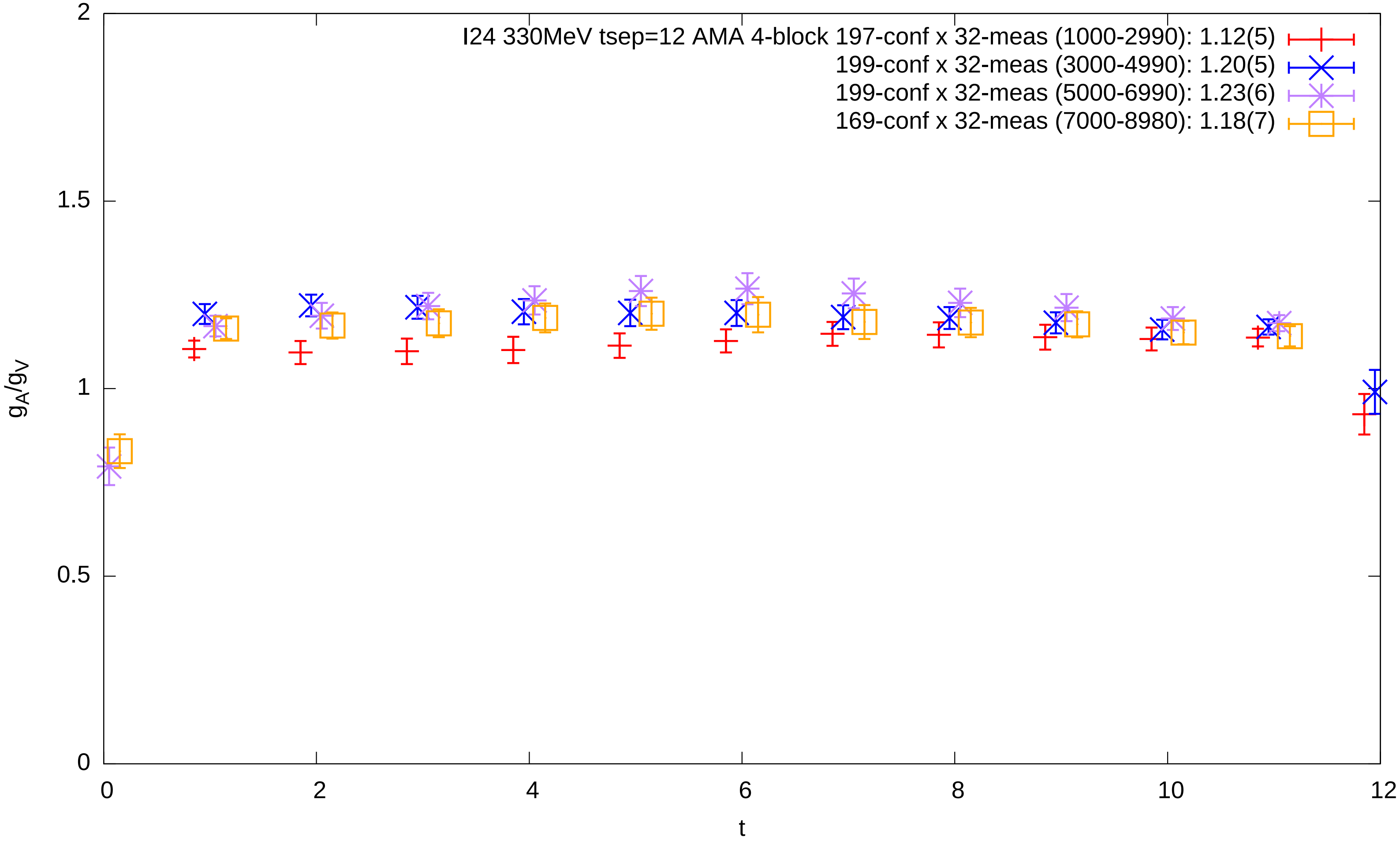
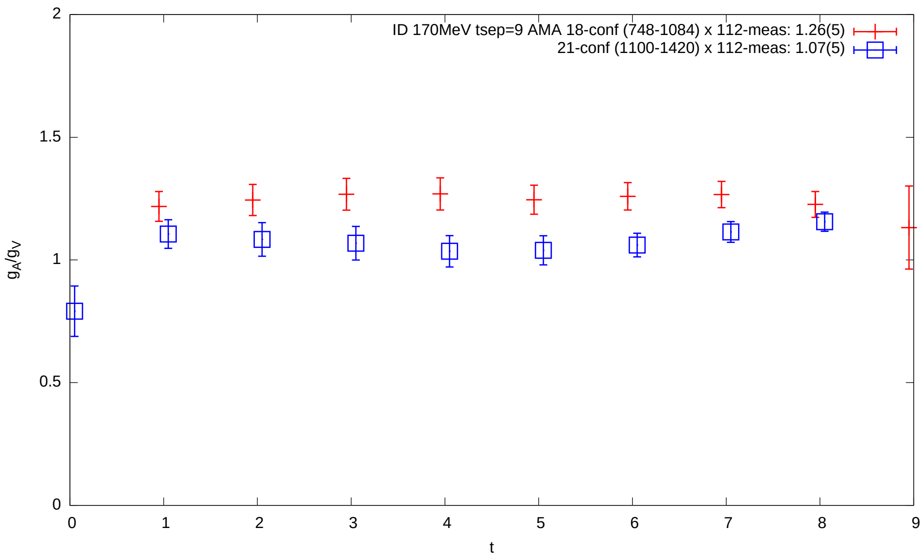
16-trajectory sampling interval is adequate for observables other than g_A.

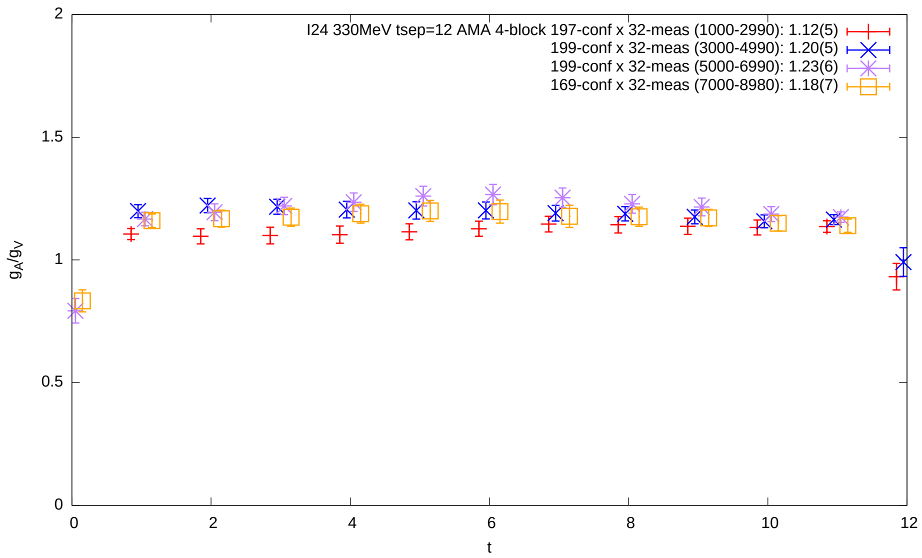
Long-range auto-correlation also seen in g_A/g_V also at $m_\pi = 330$ MeV:



Indicative of insufficient spatial volume.

Long-range auto-correlation seen in g_A/g_V by AMA:





Non-AMA analyses are much noisier, but not inconsistent with these either:

Indicative of insufficient spatial volume.

Systematics other than the spatial volume have been more or less dismissed, in particular the excited states.

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Can this be reconciled with conventional nuclear theory with point-like and non-relativistic nucleons?

Conclusions: RBC+UKQCD work on nucleon structure using the 2+1f dynamical DWF ensembles,

- lattice cutoff $\sim$ 1.4 GeV, $(4.6\text{fm})^3$ spatial volume,
- good chiral and flavor symmetries up to $O(a^2)$, $m_{\text{res}}a \sim 0.002$,
- $m_\pi \sim 170$ and 250 MeV, $m_N \sim 0.98$ and 1.05 GeV.

Successful deployment of the AMA technique resulted in 10-20 times more efficient collection of statistics.

Signals for g_V, g_A, $\langle x \rangle_{u-d}$, and $\langle x \rangle_{\Delta u - \Delta d}$ are solid:

- no excited-state contamination is seen,
- deficit in g_A/g_V with 3–5 standard deviation significance,
- long-range autocorrelation is seen in g_A, but not anything else,
- suggesting insufficient volume for the quantity.
- Systematics from finite lattice volume (L) and cut off $(O(a^2))$ least investigated.
- g_V, $\langle x \rangle_{u-d}$, and $\langle x \rangle_{\Delta u - \Delta d}$ do not suffer.

Nucleon is hardly point-like: How does this reconcile with the conventional nuclear models?

Now we are starting to calculate at physical mass!