LATTICE 13

JULY 29 – AUGUST 03 2013
MAINZ, GERMANY

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

A high-statistics study of nucleon electromagnetic form factors

Thomas Rae

University of Mainz

arXiv hep-lat: 1205.0180, 1211.1282, 1208.0189

Collaborators: Stefano Capitani, Andreas Jüttner, Michele Della Morte, Georg von Hippel, Benjamin Jäger, Bastian Knippschild, Harvey Meyer, Hartmut Wittig

1 August, 2013

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Motivation

- Baryonic form factors
    - provide information on hadron structure
        - distribution of electric charge and magnetisation
        - charge radii
    - accurate experimental data available
    - relatively simple to compute on the lattice
        - large systematic uncertainties remain and need to be controlled

Form factors

- Rosenbluth formula describes electron-nucleon scattering

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) \propto \left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\left(\frac{\theta}{2}\right)\right], \quad \tau = \frac{Q^2}{4M^2}$$

- Form factors measured experimentally
  - e.g at MAMI here in Mainz

Form factors

- The matrix element of a nucleon interacting with an electromagnetic current is decomposed by the Dirac and Pauli form factors - F_1 and F_2 respectively

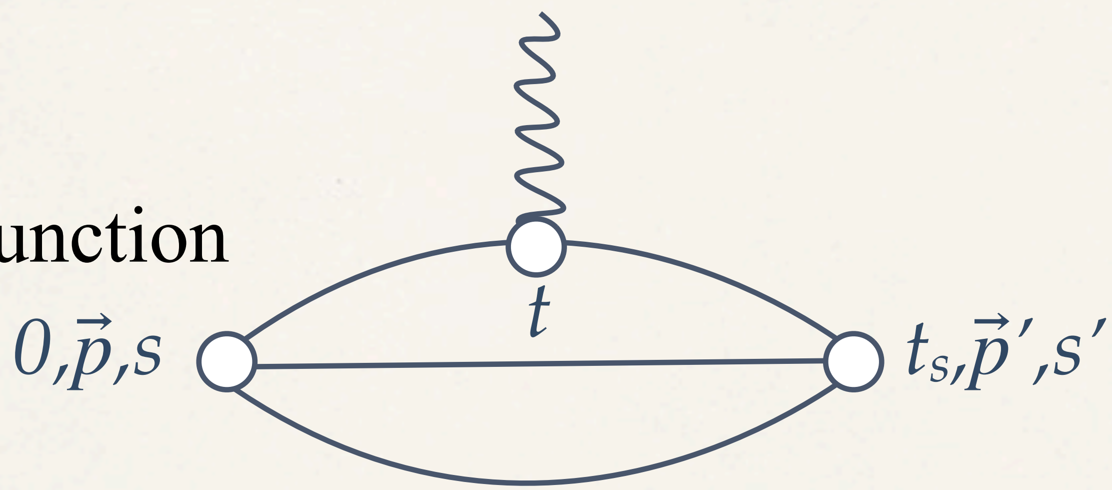
$$\langle N(p', s')|V_\mu|N(p, s)\rangle = \overline{u}(p', s')\left[\gamma_\mu F_1(Q^2) + i\frac{\sigma_{\mu\nu}q_\nu}{2m_N}F_2(Q^2)\right]u(p, s)$$

- These are related to the Sachs form factors G_E and G_M that are measured in scattering experiments

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2}F_2(Q^2), \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$

Lattice formulation

- 3pt function



$$\sim \langle N(p', s')|V_\mu|N(p, s)\rangle$$

- Build a ratio

$$R_{\gamma_\mu}(\vec{q}, t, t_s) = \frac{C_{3,\gamma_\mu}(\vec{q}, t, t_s)}{C_2(\vec{q}, t, t_s)} \sqrt{\frac{C_2(\vec{q}, t_s - t)C_2(\vec{0}, t)C_2(\vec{0}, t_s)}{C_2(0, t_s - t)C_2(\vec{q}, t)C_2(\vec{q}, t_s)}}$$

- from which the form factors may be extracted

$$\mathrm{Re}[R_{\gamma_0}] = \sqrt{\frac{M + E}{2E}} G_E(Q^2) \qquad \mathrm{Re}[R_{\gamma_i}]_{i=1,2} = \epsilon_{ij} p_j \sqrt{\frac{1}{2E(M + E)}} G_M(Q^2)$$

Lattice setup

- Gauge field configurations generated as part of the CLS project
  - $N_f = 2$ non-perturbatively $O(a)$ improved Wilson fermions
- Perform a systematic study of baryonic form factors with controlled systematics
  - lattice artefacts
  - finite volume effects
  - excited state contamination

Run	β	a [fm]	$L^3 \times T$	m_π [MeV]	L [fm]	$m_\pi L$ [MeV]	N_{meas}
A3	5.2	0.079	$32^3\times64$	473	2.5	6.0	2128
A4				363	2.5	4.7	3200
A5				312	2.5	4.0	4000
B6			$48^3\times96$	262	3.8	5.0	2544
E5	5.3	0.063	$32^3\times64$	451	2.0	4.7	4000
F6			$48^3\times96$	324	3.0	5.0	3600
F7				277	3.0	4.2	3000
G8			$64^3\times128$	195	4.0	4.0	4176
N5	5.5	0.050	$48^3\times96$	430	2.4	5.2	1908
N6				340	2.4	4.0	3784
O7			$64^3\times128$	270	3.2	4.4	1960

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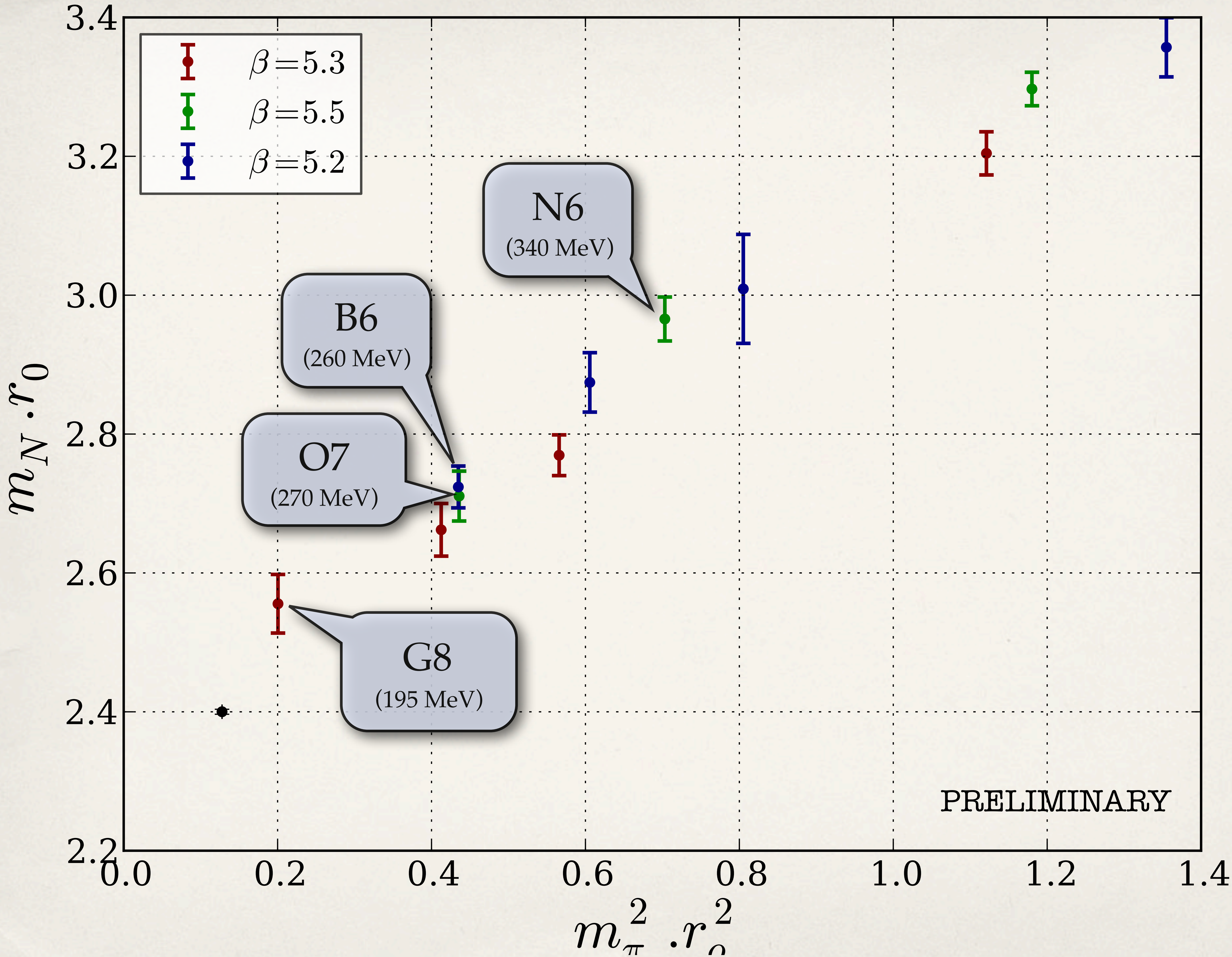
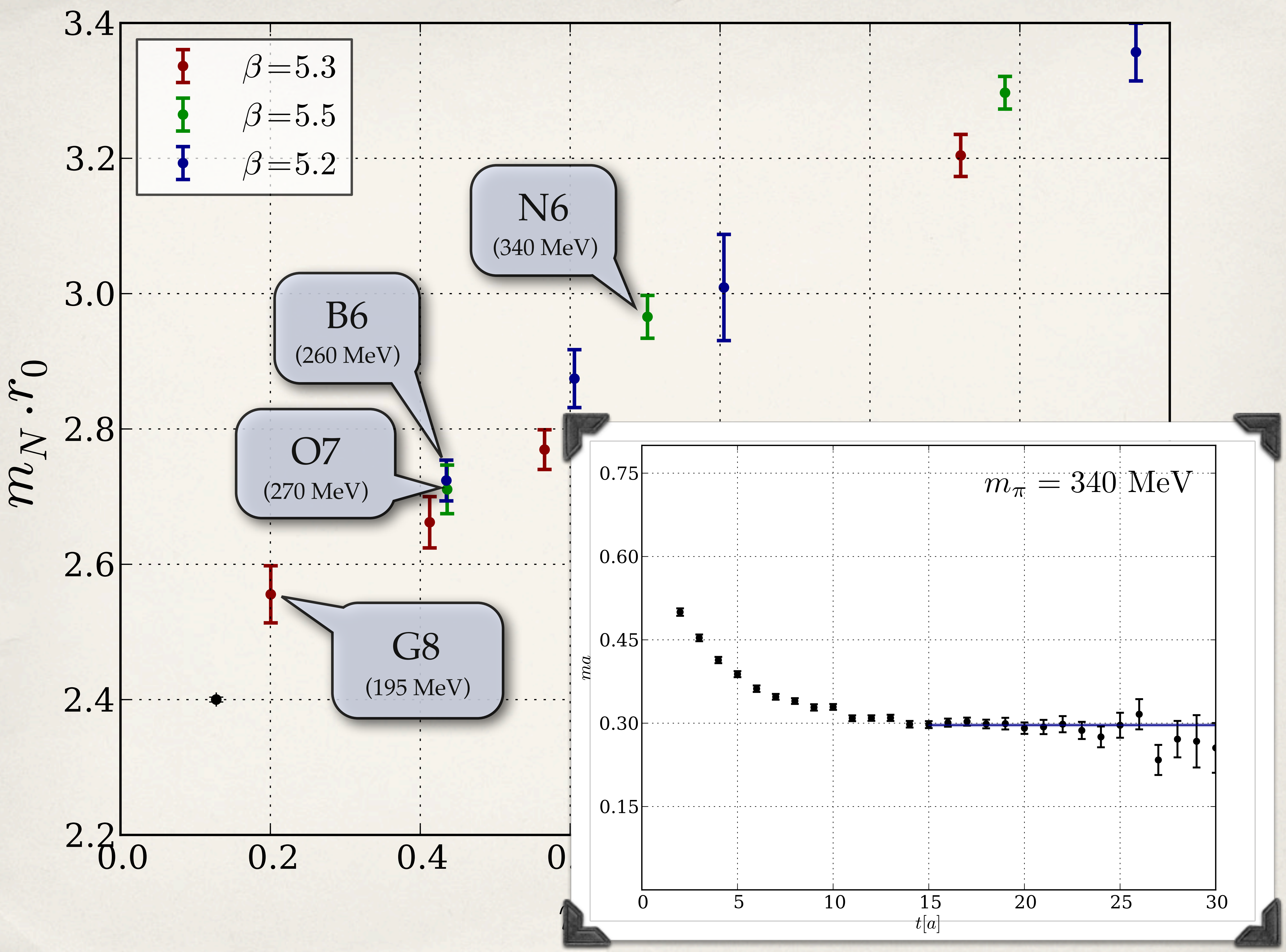
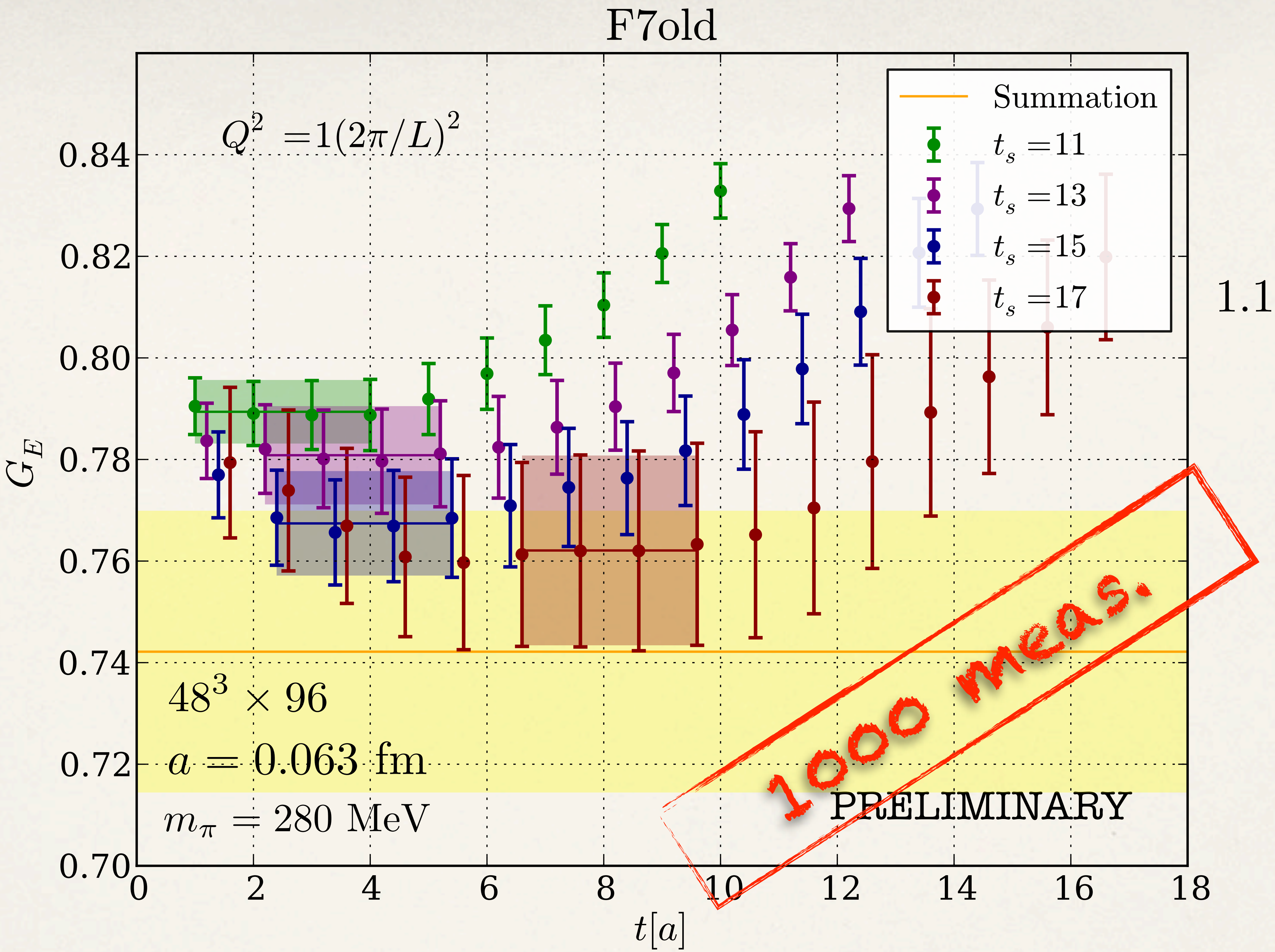


$\beta = 5.3$
$\beta = 5.5$
$\beta = 5.2$
N6
(340 MeV)
B6
(260 MeV)
O7
(270 MeV)
G8
(195 MeV)
PRELIMINARY
$m_N \cdot r_0$
$m_\pi^2 \cdot r_0^2$
3.4
3.2
3.0
2.8
2.6
2.4
2.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

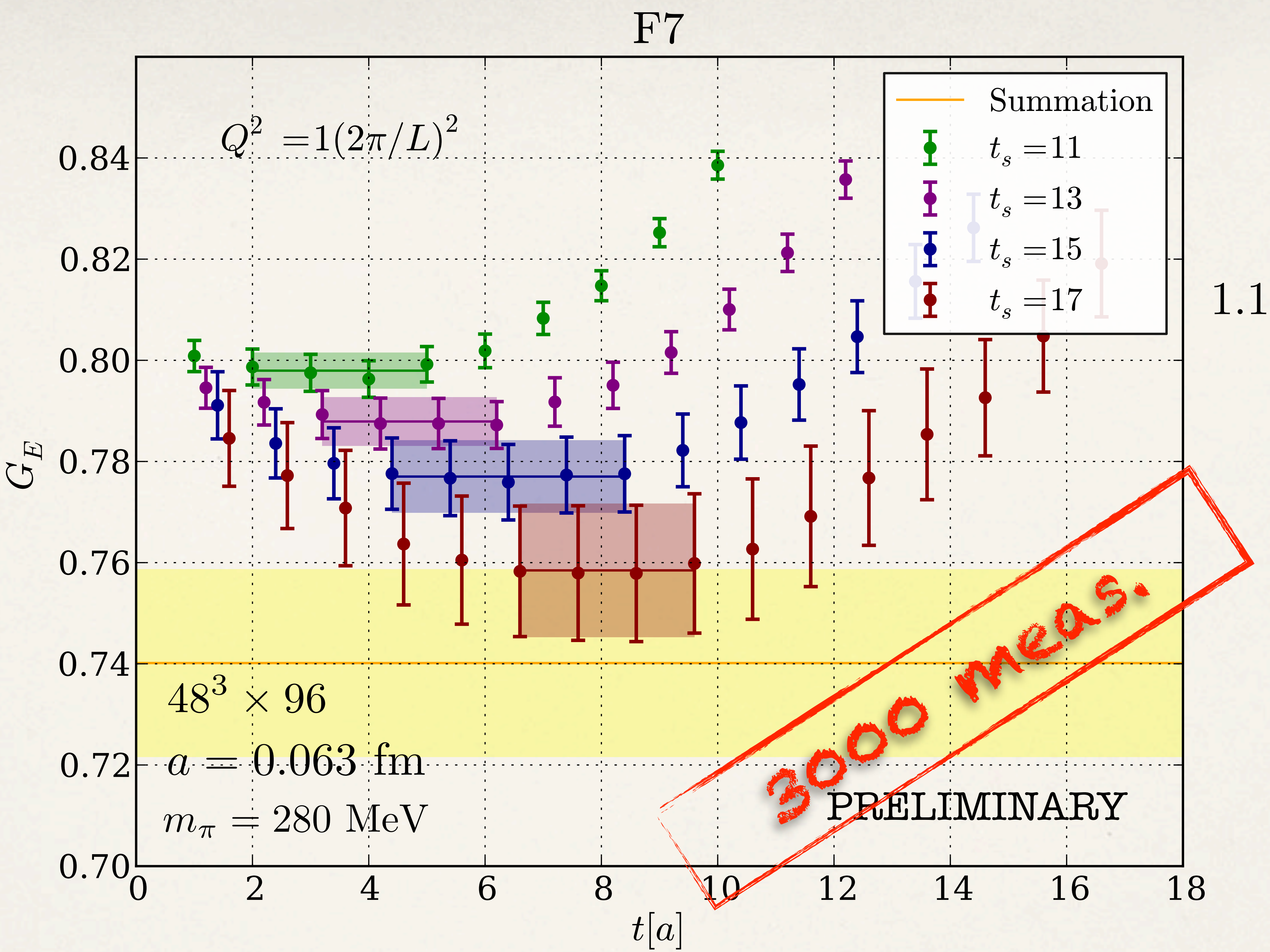


$m_N \cdot r_0$
$\beta = 5.3$
$\beta = 5.5$
$\beta = 5.2$
N6
(340 MeV)
B6
(260 MeV)
O7
(270 MeV)
G8
(195 MeV)
$m_\pi = 340$ MeV
ma
$t[a]$



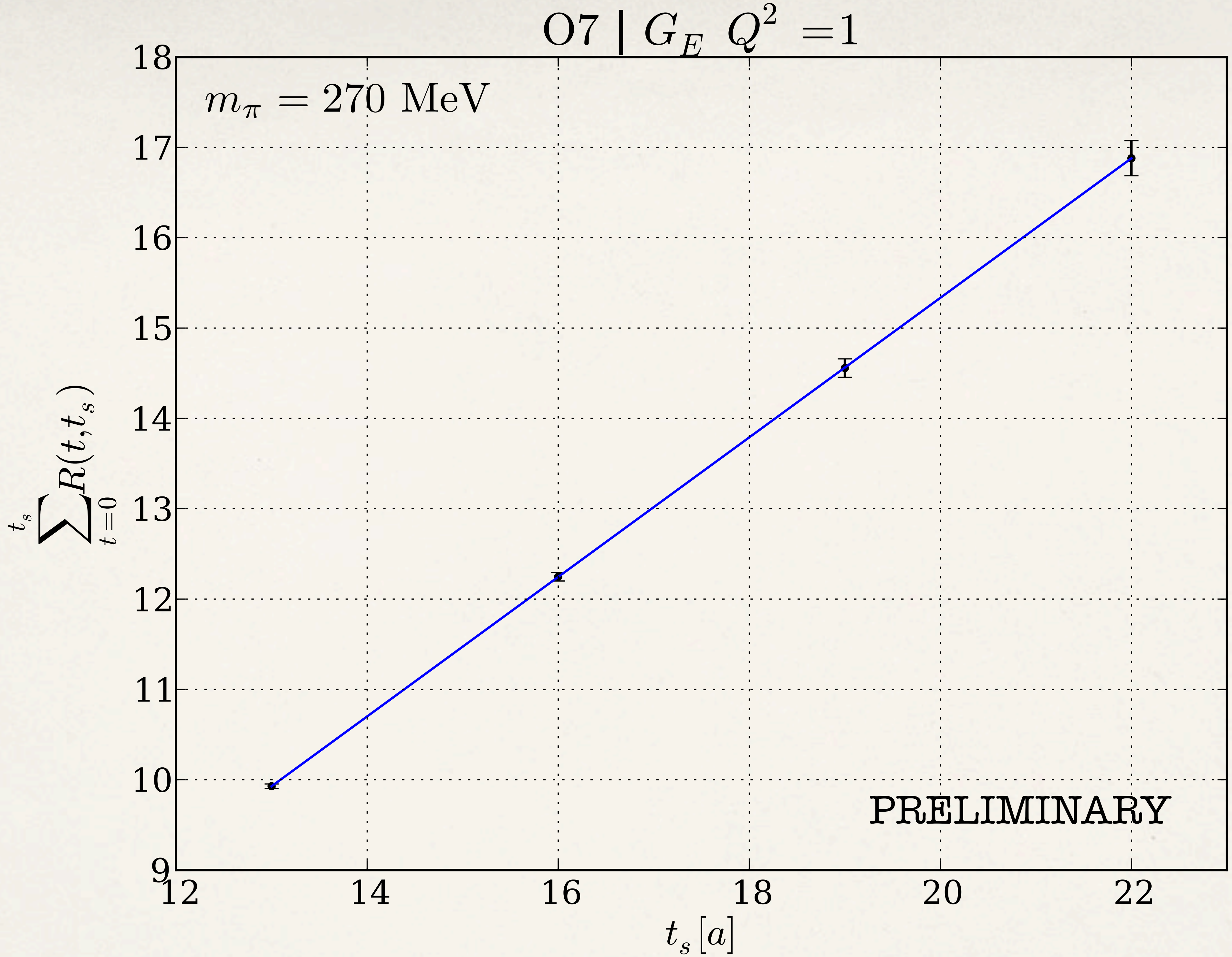
$1.1 fm$

- Form factors should be independent of time and source position
  - Simple plateau fits show a trend to step as source-sink separation is changed
- Exponentially decaying excited states from source and sink
  - Look at several src/snk combinations



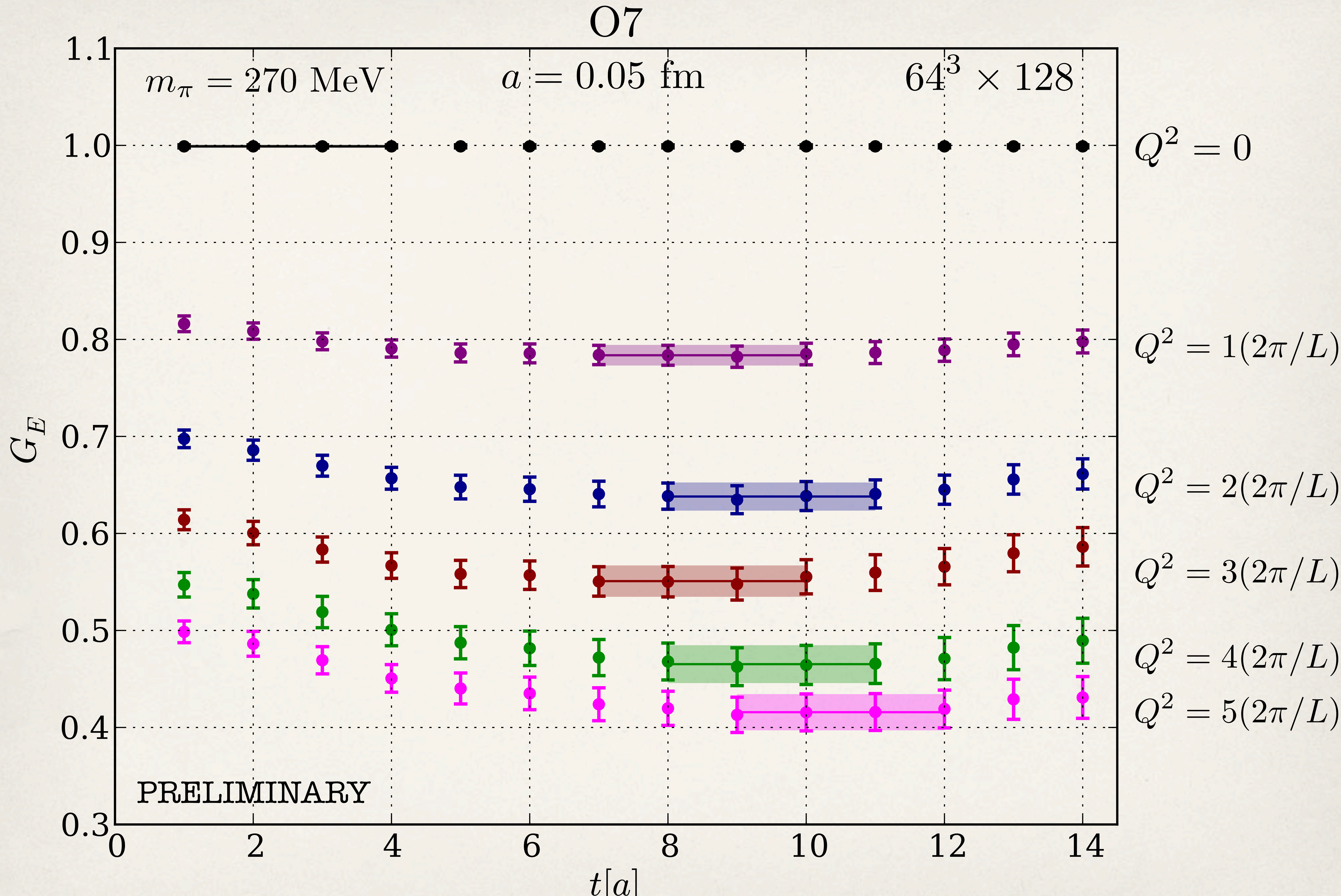
$1.1 fm$

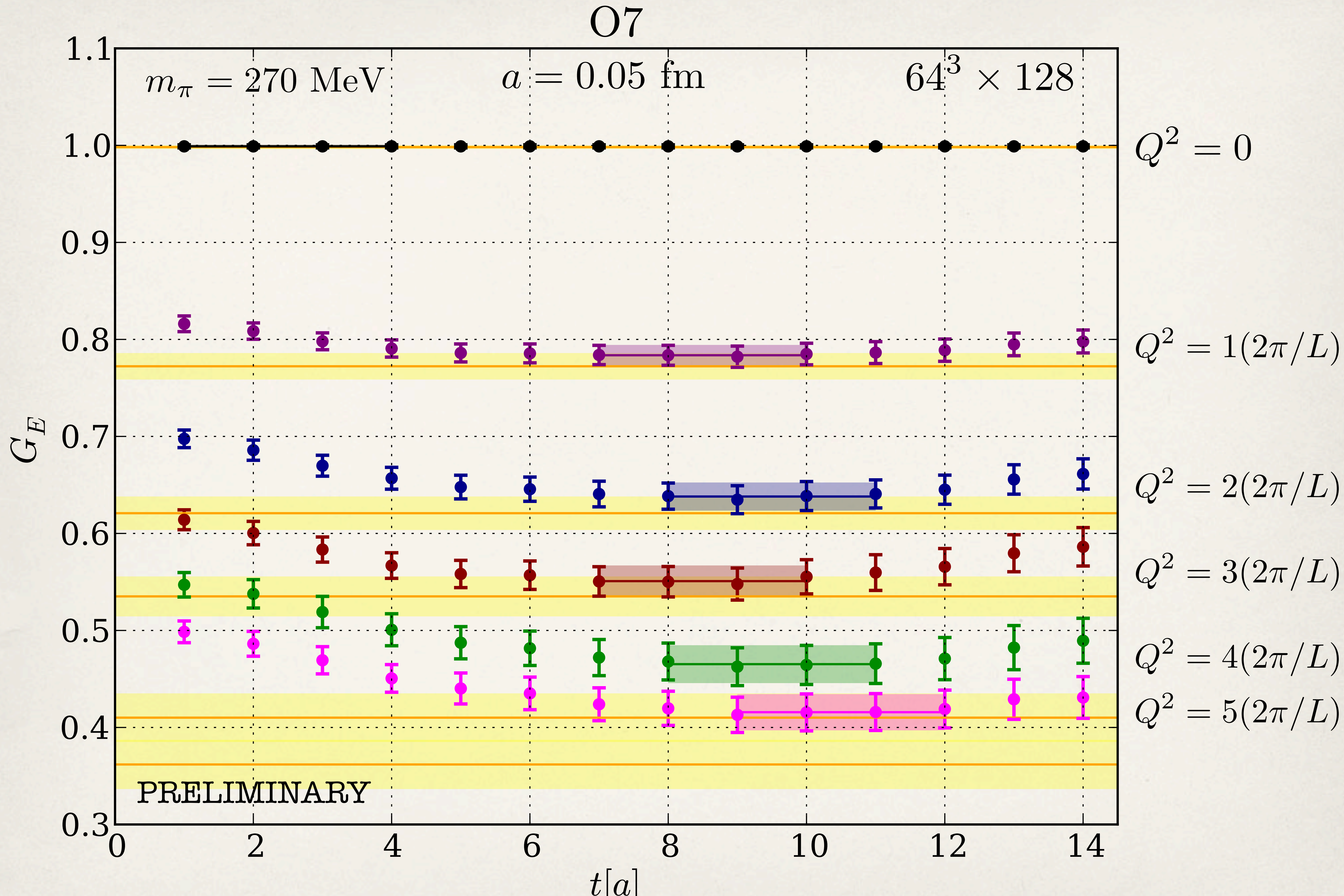
- Unclear as to whether t_s=1.1 fm is sufficient to rule out bias



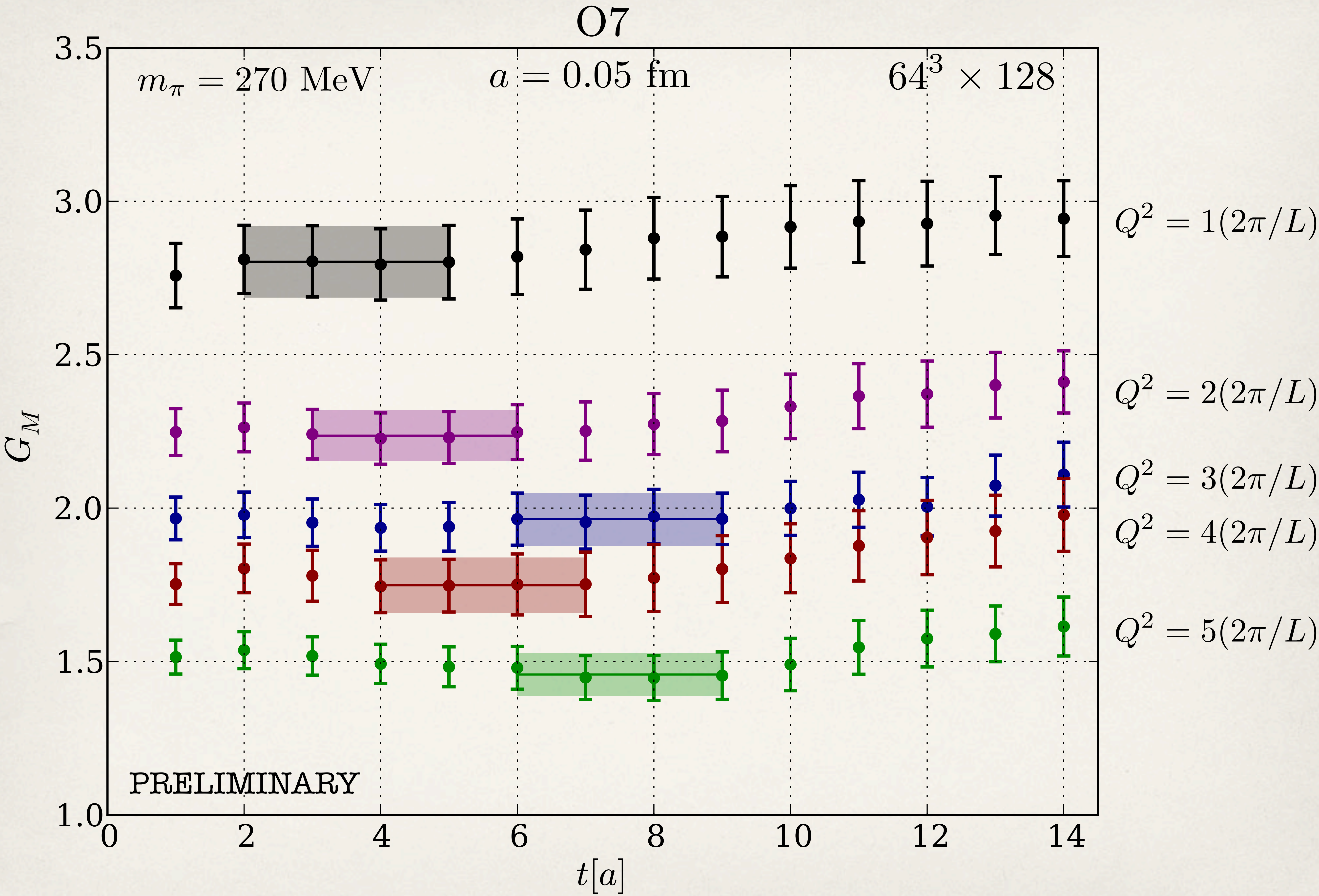
- Compute the sum $S(t_s)$ for several t_s

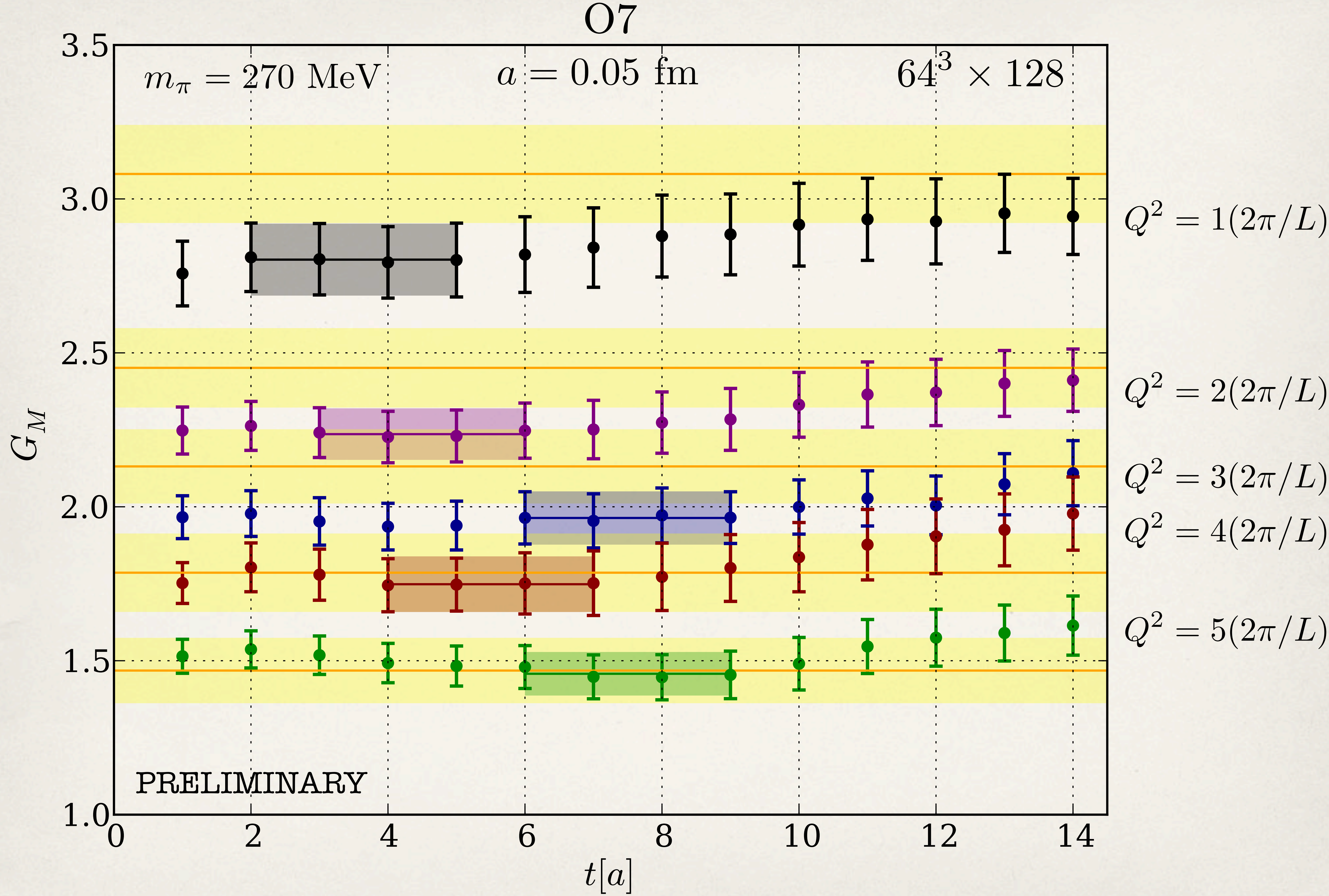
$$S(t_s) = \sum_{t=0}^{t_s} R(\vec{q},t,t_s) \to c(\Delta,\Delta') + t_s\left(G + \mathcal{O}\left(e^{-\Delta t_s}\right) + \mathcal{O}\left(e^{-\Delta' t_s}\right)\right)$$


O7
$m_\pi = 270$ MeV
$a = 0.05$ fm
$64^3 \times 128$
$Q^2 = 0$
$Q^2 = 1(2\pi/L)$
$Q^2 = 2(2\pi/L)$
$Q^2 = 3(2\pi/L)$
$Q^2 = 4(2\pi/L)$
$Q^2 = 5(2\pi/L)$
G_E
$t[a]$
PRELIMINARY
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
2
4
6
8
10
12
14



O7
$m_\pi = 270$ MeV
$a = 0.05$ fm
$64^3 \times 128$
$Q^2 = 0$
$Q^2 = 1(2\pi/L)$
$Q^2 = 2(2\pi/L)$
$Q^2 = 3(2\pi/L)$
$Q^2 = 4(2\pi/L)$
$Q^2 = 5(2\pi/L)$
G_E
$t[a]$
PRELIMINARY
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
2
4
6
8
10
12
14


O7
$m_\pi = 270$ MeV
$a = 0.05$ fm
$64^3 \times 128$
$Q^2 = 1(2\pi/L)$
$Q^2 = 2(2\pi/L)$
$Q^2 = 3(2\pi/L)$
$Q^2 = 4(2\pi/L)$
$Q^2 = 5(2\pi/L)$
PRELIMINARY
G_M
$t[a]$
3.5
3.0
2.5
2.0
1.5
1.0
0
2
4
6
8
10
12
14


O7
$m_\pi = 270$ MeV
$a = 0.05$ fm
$64^3 \times 128$
$Q^2 = 1(2\pi/L)$
$Q^2 = 2(2\pi/L)$
$Q^2 = 3(2\pi/L)$
$Q^2 = 4(2\pi/L)$
$Q^2 = 5(2\pi/L)$
G_M
$t[a]$
PRELIMINARY
3.5
3.0
2.5
2.0
1.5
1.0
0
2
4
6
8
10
12
14

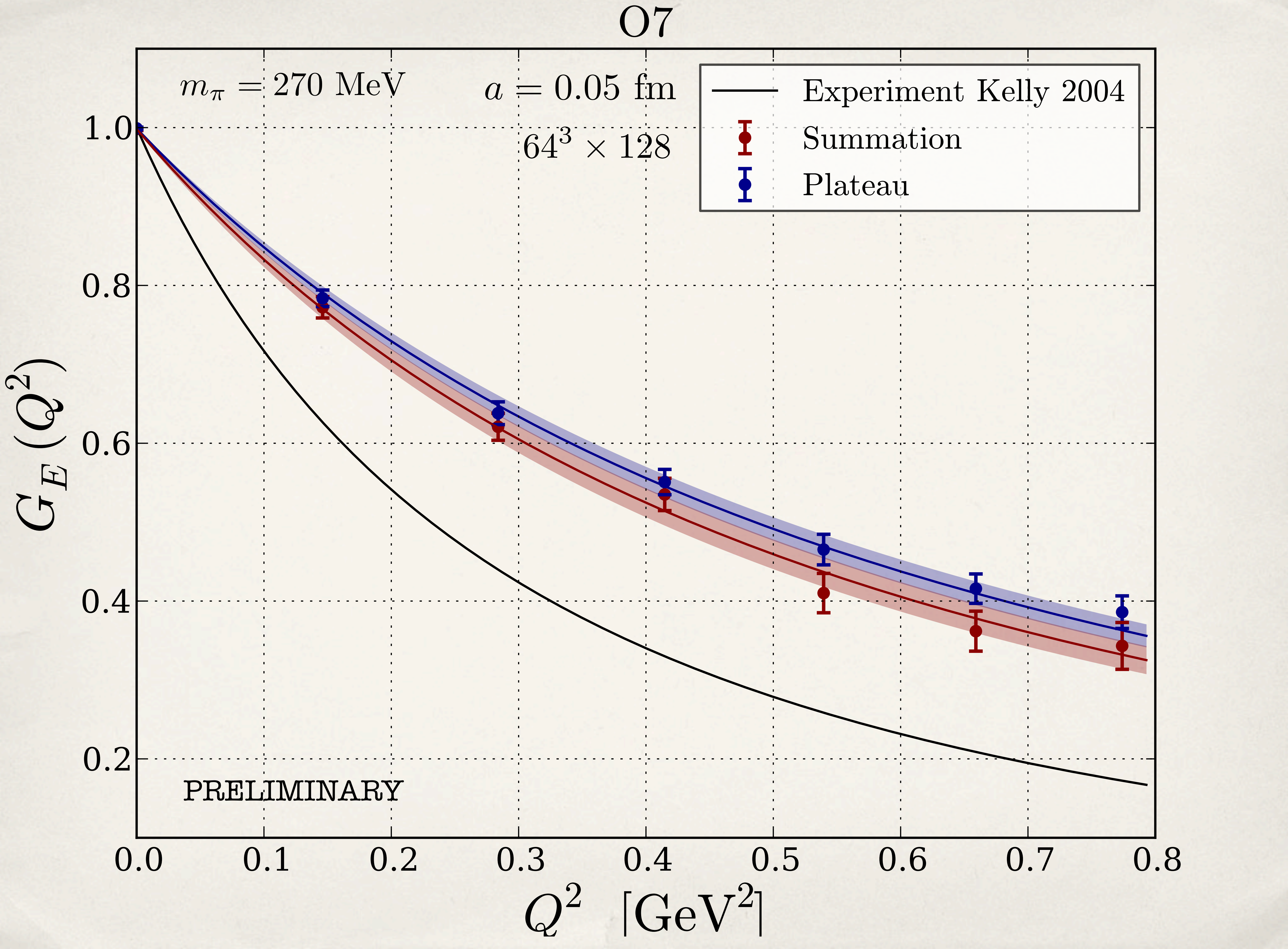
Vector form factors

- Model the Q^2 dependence
  - dipole ansatz:

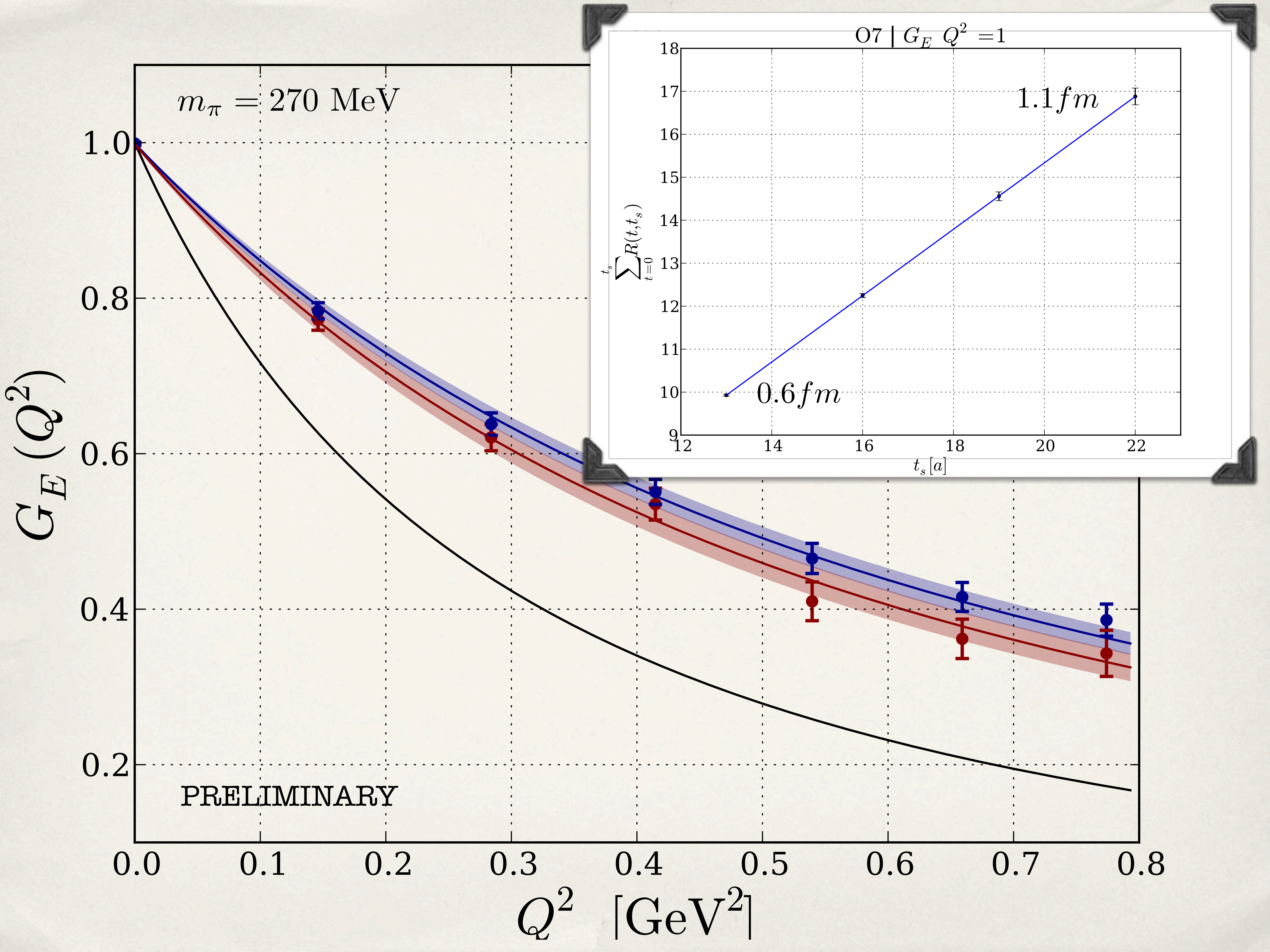
$$G_{E,M}(Q^2) = \frac{G_{E,M}(0)}{\left(1 + \frac{Q^2}{M_{E,M}^2}\right)}$$

  - used to determine the radius
  - and to determine the magnetic moment, $\mu = G_M(0)$

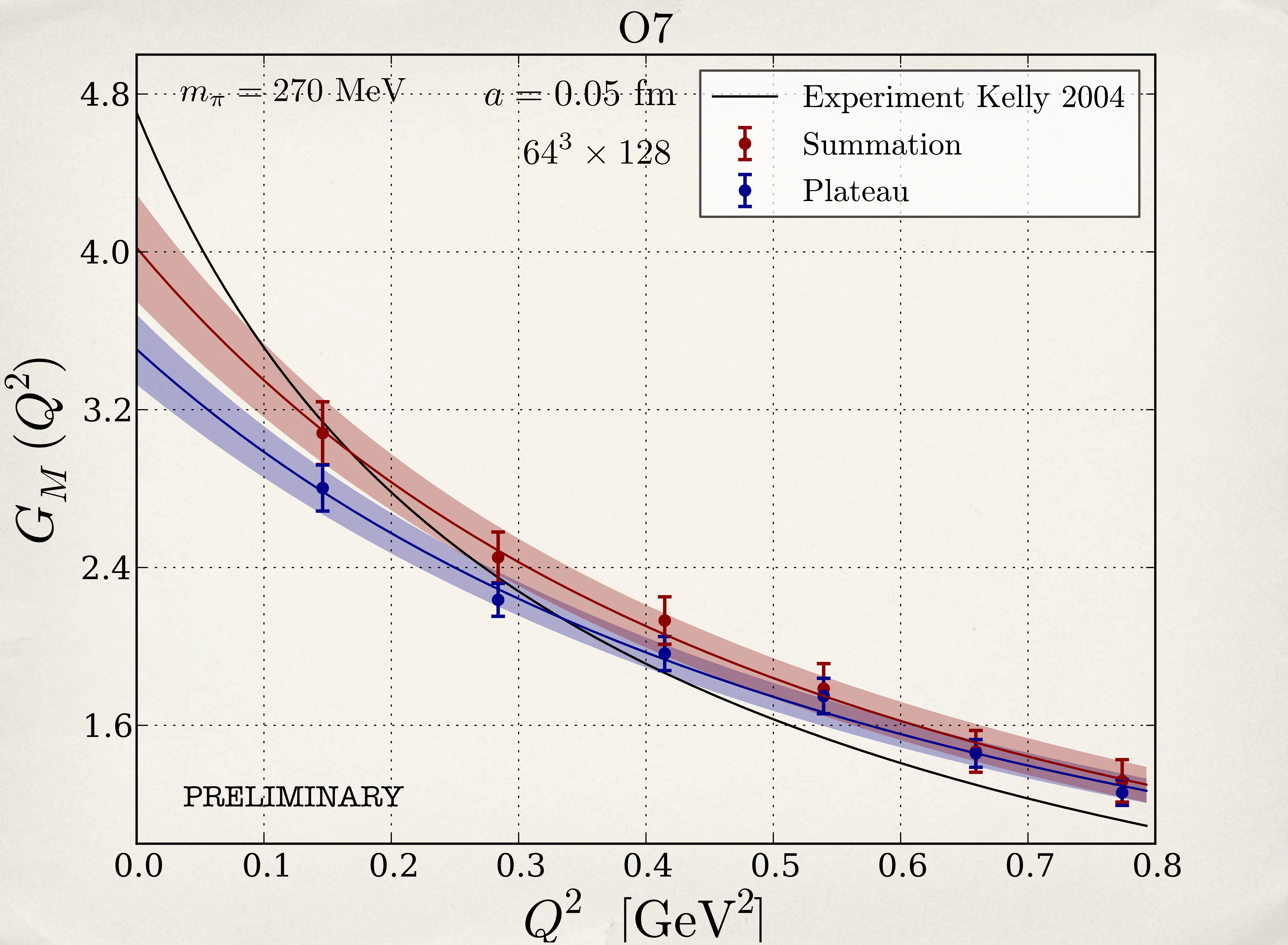
$$\mu = \lim_{Q^2 \to 0} \frac{G_M(Q^2)}{G_E(Q^2)}$$

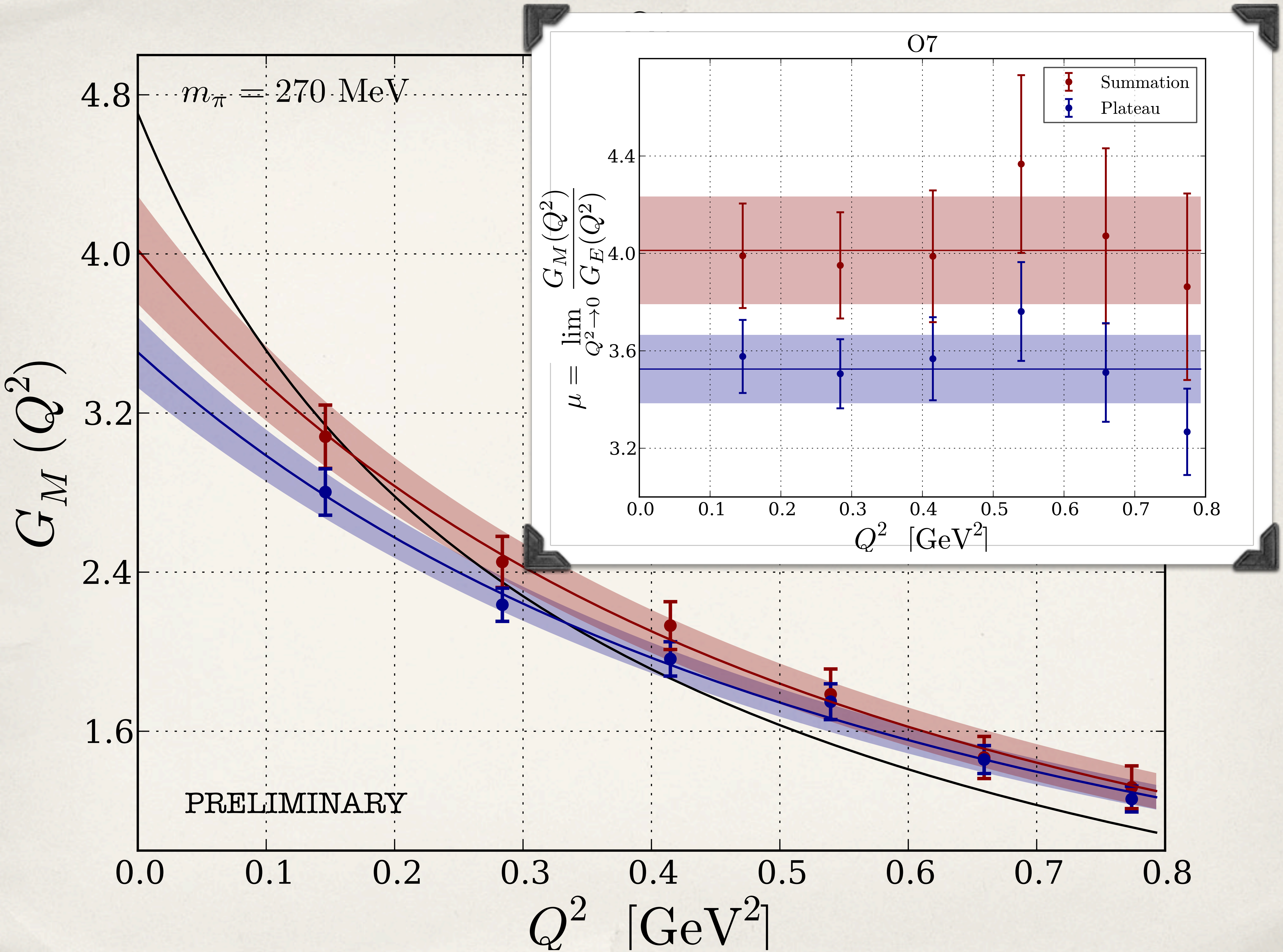


O7
$m_\pi = 270$ MeV
$a = 0.05$ fm
$64^3 \times 128$
Experiment Kelly 2004
Summation
Plateau
PRELIMINARY
$G_E(Q^2)$
Q^2 [GeV^2]
1.0
0.8
0.6
0.4
0.2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

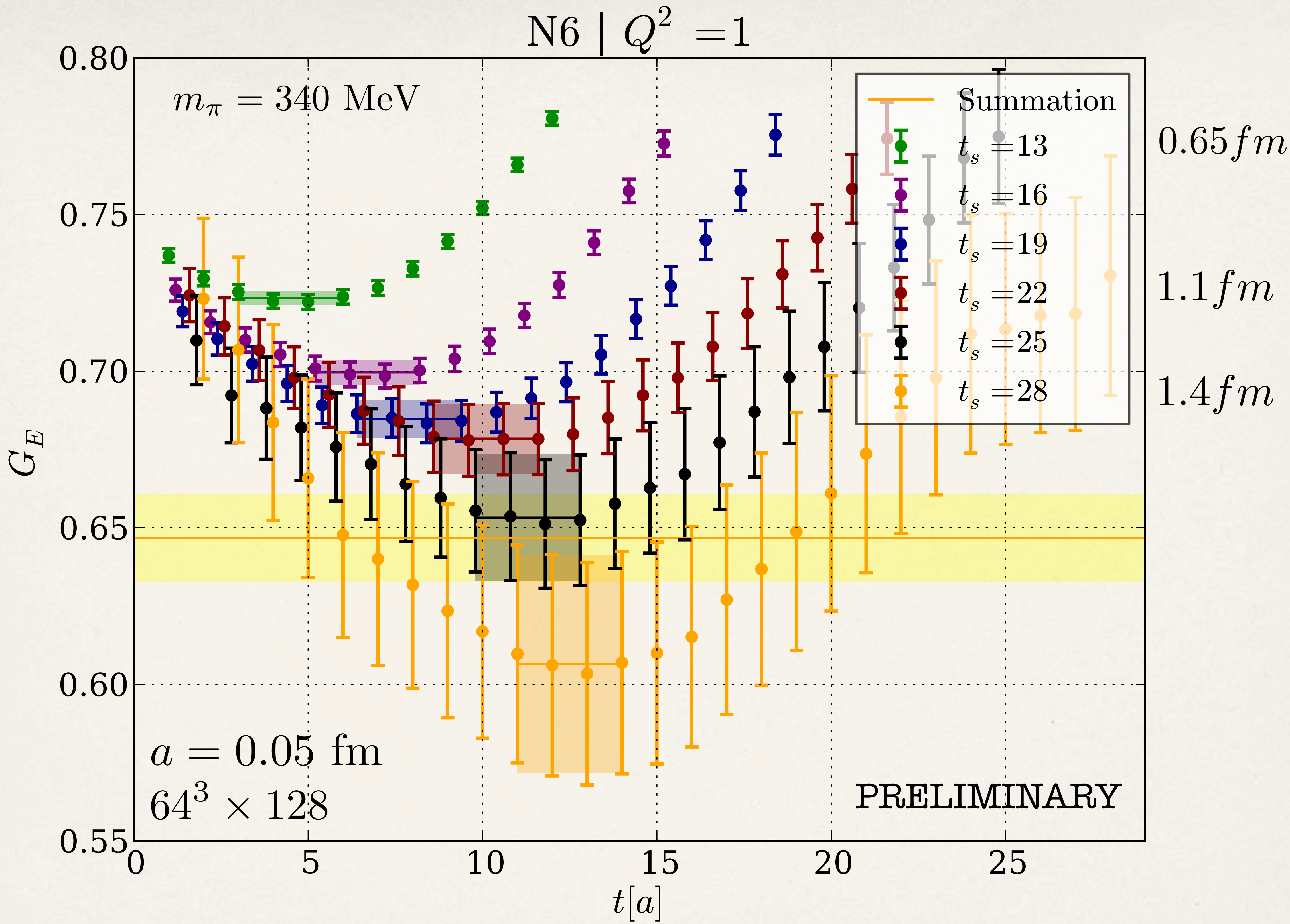


$m_\pi = 270$ MeV
1.0
0.8
0.6
0.4
0.2
$G_E\,(Q^2)$
PRELIMINARY
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Q^2 [GeV^2]
O7 | G_E Q^2 =1
18
17
16
15
14
13
12
11
10
9
$\sum_{t=0}^{t_s} R(t,t_s)$
$1.1fm$
$0.6fm$
12
14
16
18
20
22
$t_s\,[a]$

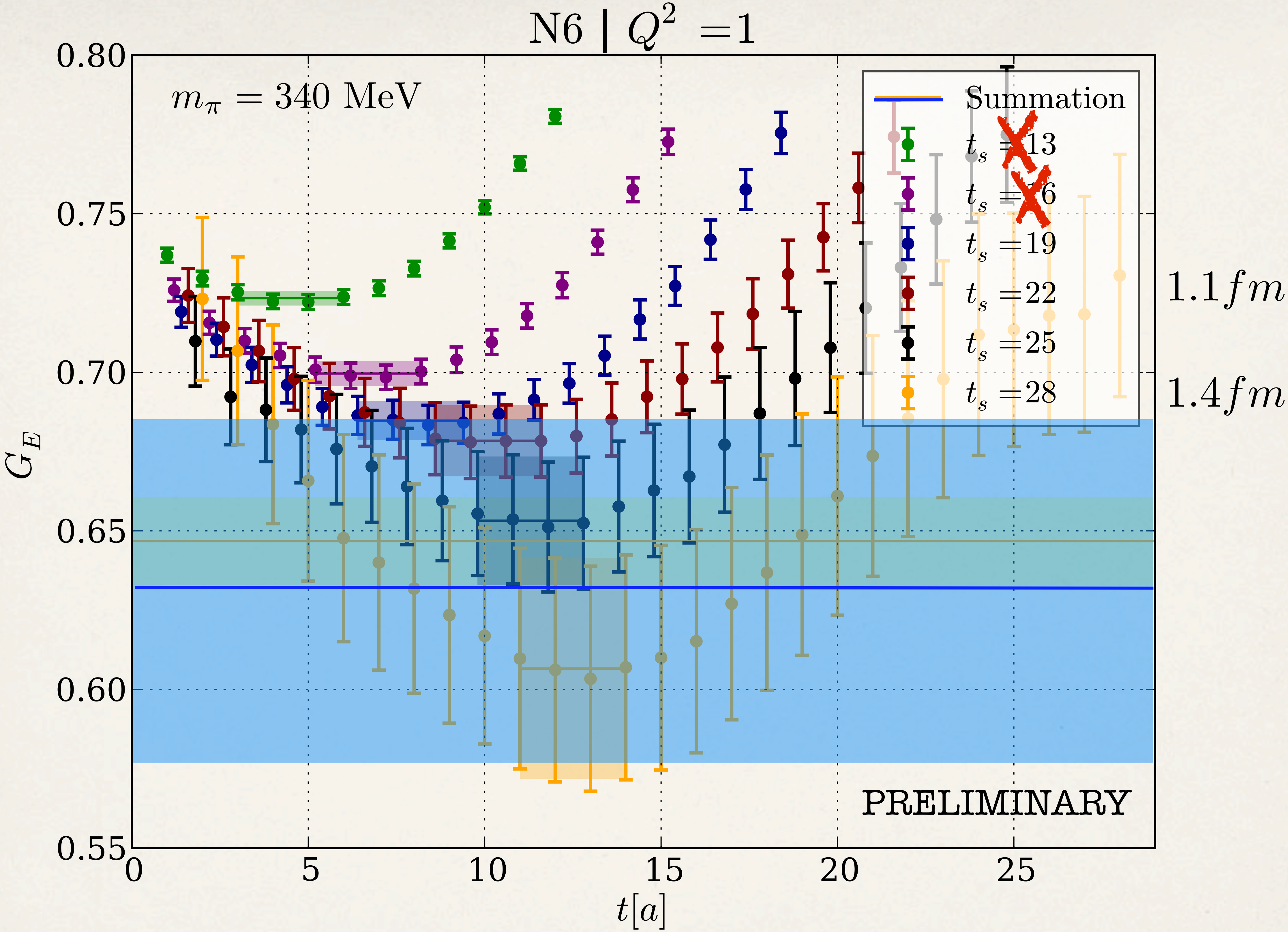

O7
$m_\pi = 270$ MeV
$a = 0.05$ fm
$64^3 \times 128$
Experiment Kelly 2004
Summation
Plateau
PRELIMINARY
$G_M\,(Q^2)$
$Q^2\;[\mathrm{GeV}^2]$
4.8
4.0
3.2
2.4
1.6
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8



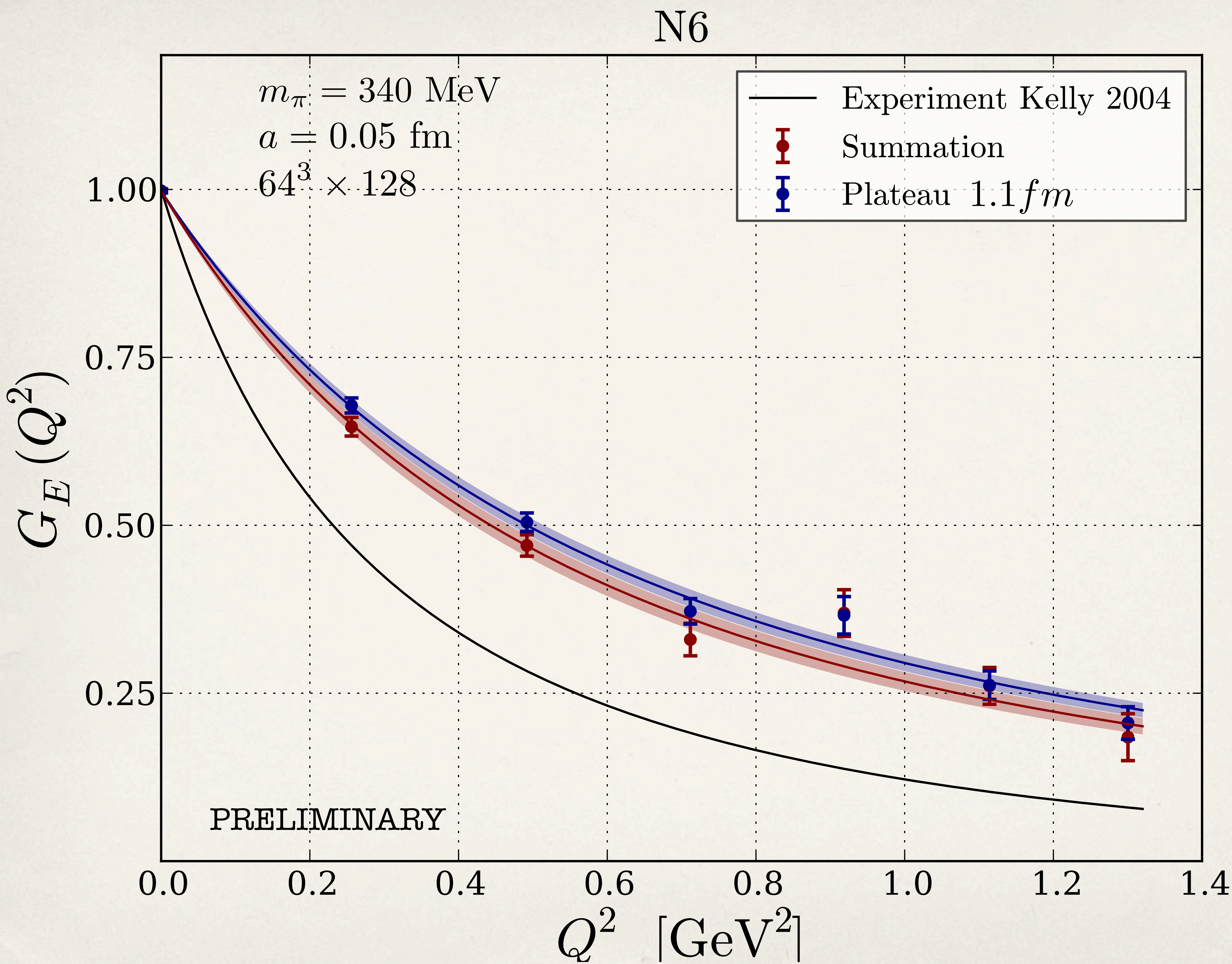
$m_\pi = 270$ MeV
PRELIMINARY
$G_M\,(Q^2)$
Q^2 [GeV^2]
O7
Summation
Plateau
$\mu = \lim_{Q^2\to 0} \frac{G_M(Q^2)}{G_E(Q^2)}$
Q^2 [GeV^2]

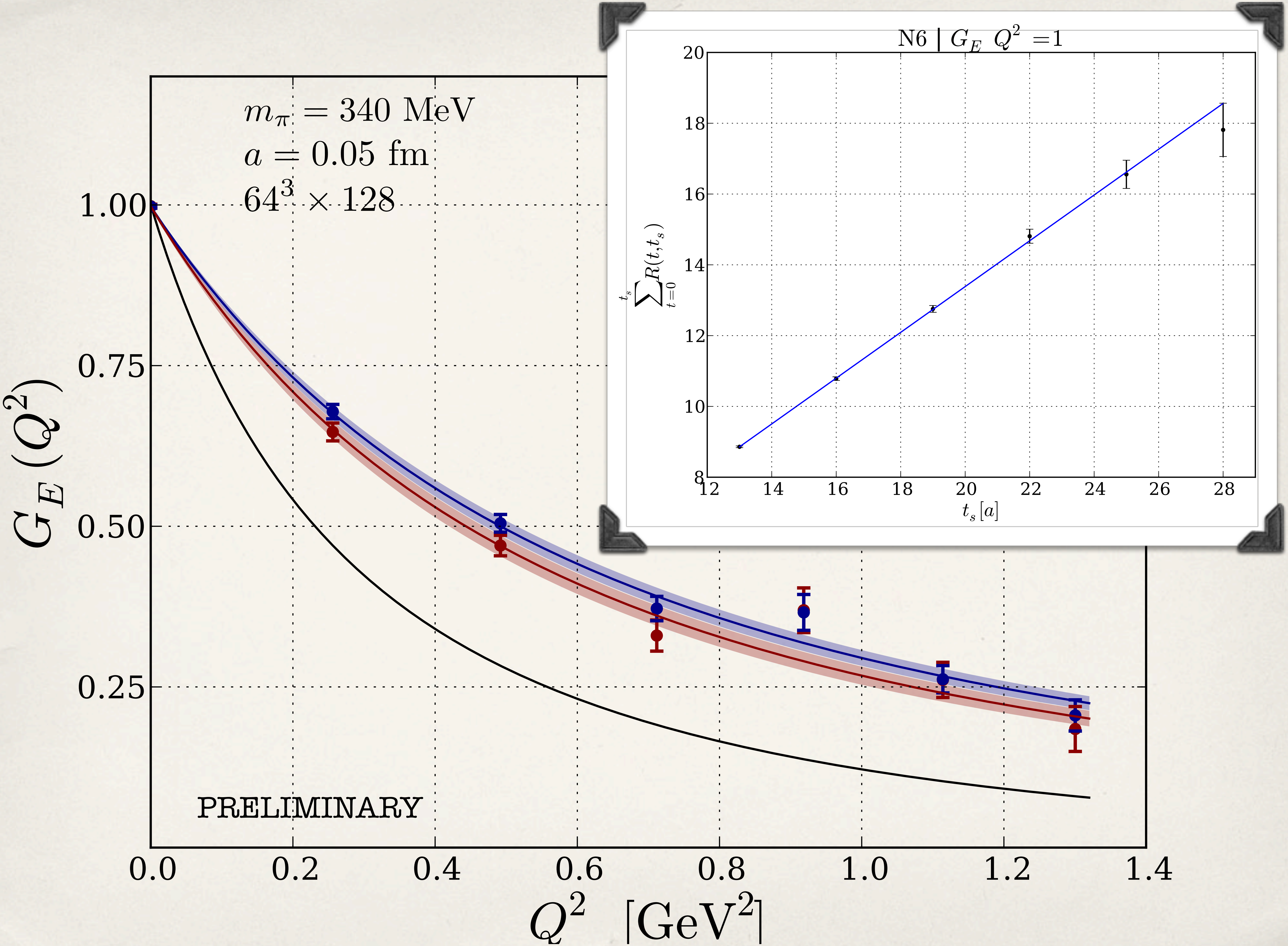


- N6: measured 6 different source-sink separations

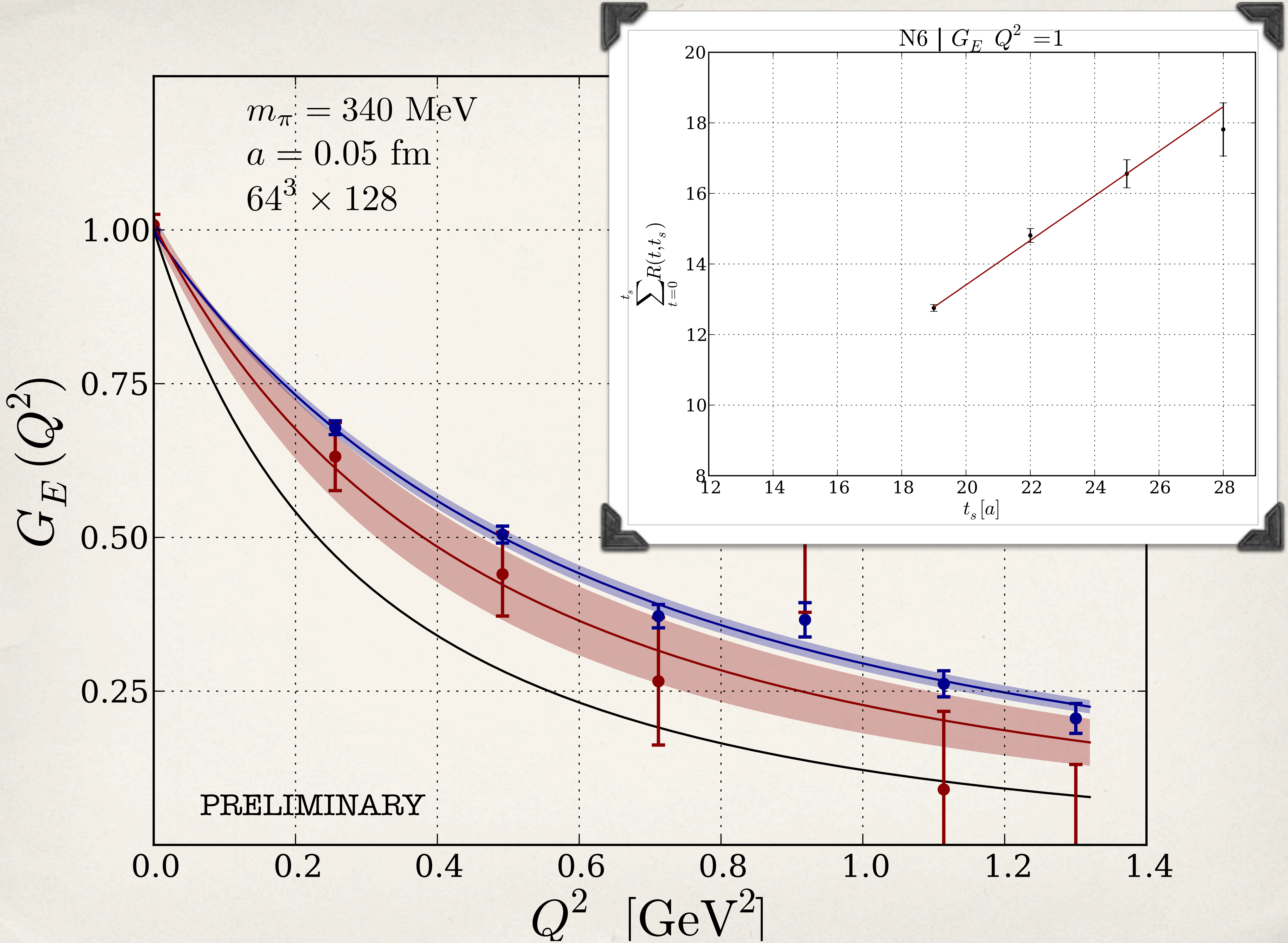


- shift of summation method using only 4 largest t_s


N6
$m_\pi = 340$ MeV
$a = 0.05$ fm
$64^3 \times 128$
Experiment Kelly 2004
Summation
Plateau $1.1 fm$
PRELIMINARY
1.00
0.75
0.50
0.25
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
$G_E\,(Q^2)$
Q^2 [GeV^2]



$m_\pi = 340$ MeV
$a = 0.05$ fm
$64^3 \times 128$
1.00
0.75
0.50
0.25
PRELIMINARY
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
$G_E(Q^2)$
Q^2 [GeV^2]
N6 | G_E Q^2 =1
20
18
16
14
12
10
8
12
14
16
18
20
22
24
26
28
$\sum_{t=0}^{t_s} R(t,t_s)$
$t_s\,[a]$



$m_\pi = 340$ MeV
$a = 0.05$ fm
$64^3 \times 128$
PRELIMINARY
$G_E(Q^2)$
Q^2 [GeV^2]
1.00
0.75
0.50
0.25
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
N6 | G_E Q^2 =1
$\sum_{t=0}^{t_s} R(t,t_s)$
$t_s\,[a]$
8
10
12
14
16
18
20
12
14
16
18
20
22
24
26
28

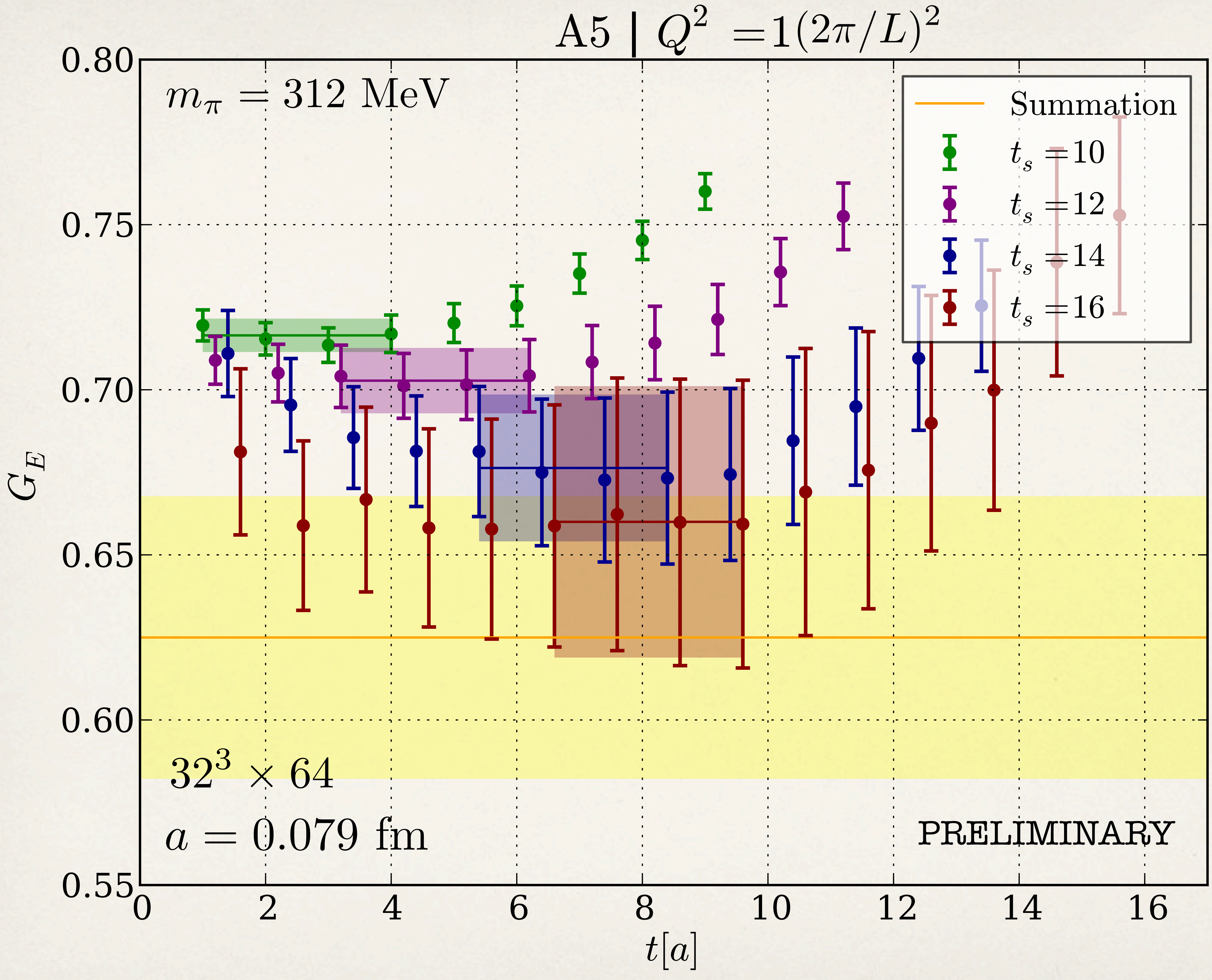
Summary

- We observe a systematic variation in the form factors for the plateau method for different source-sink separations t_s
  - clearer for large statistics
- Summed insertions help control excited state contamination
  - remove the need to fit plateaus
- Important to consider range of t_s
  - small t_s have smallest statistical errors but most effected by excited state contamination

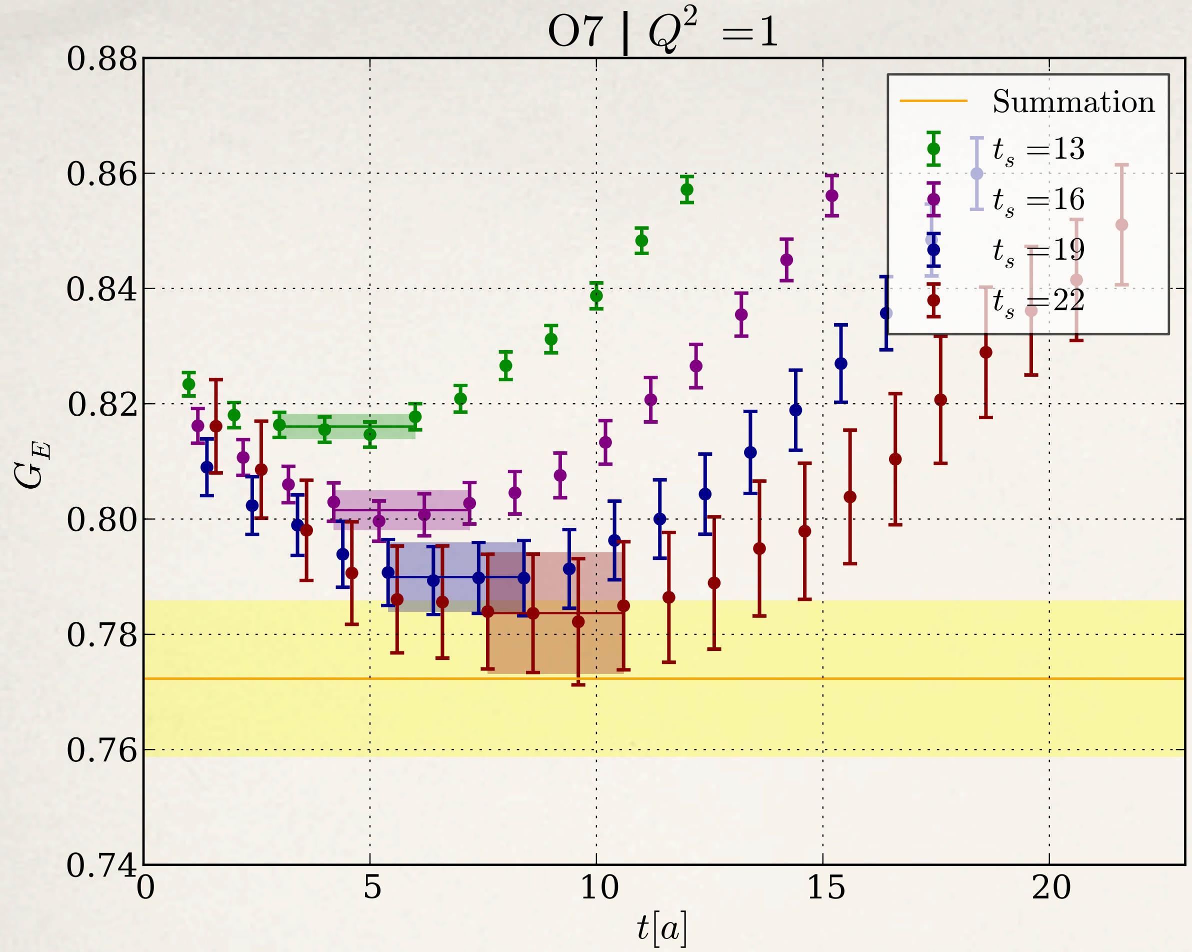
Outlook

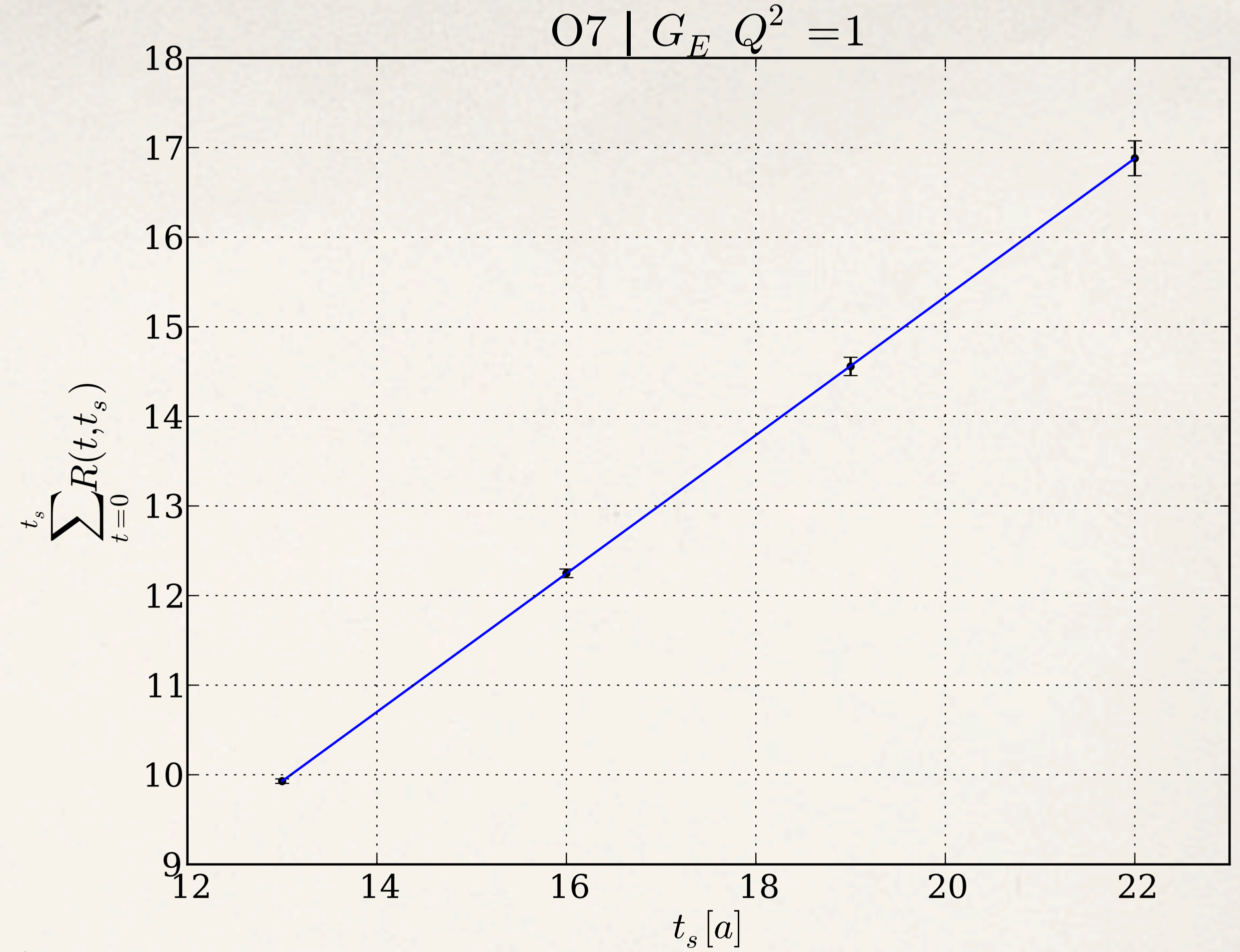
- N6 ensemble hints at bias for plateau method even for t_s=1.1fm
  - check for the most chiral ensembles
- Chiral behaviour of the form factors and derived quantities
- look at axial form factors $G_P(Q^2)$ and $G_A(Q^2)$
- Introduce dynamical strange quark
- Simulations at the physical pion mass

Thank you

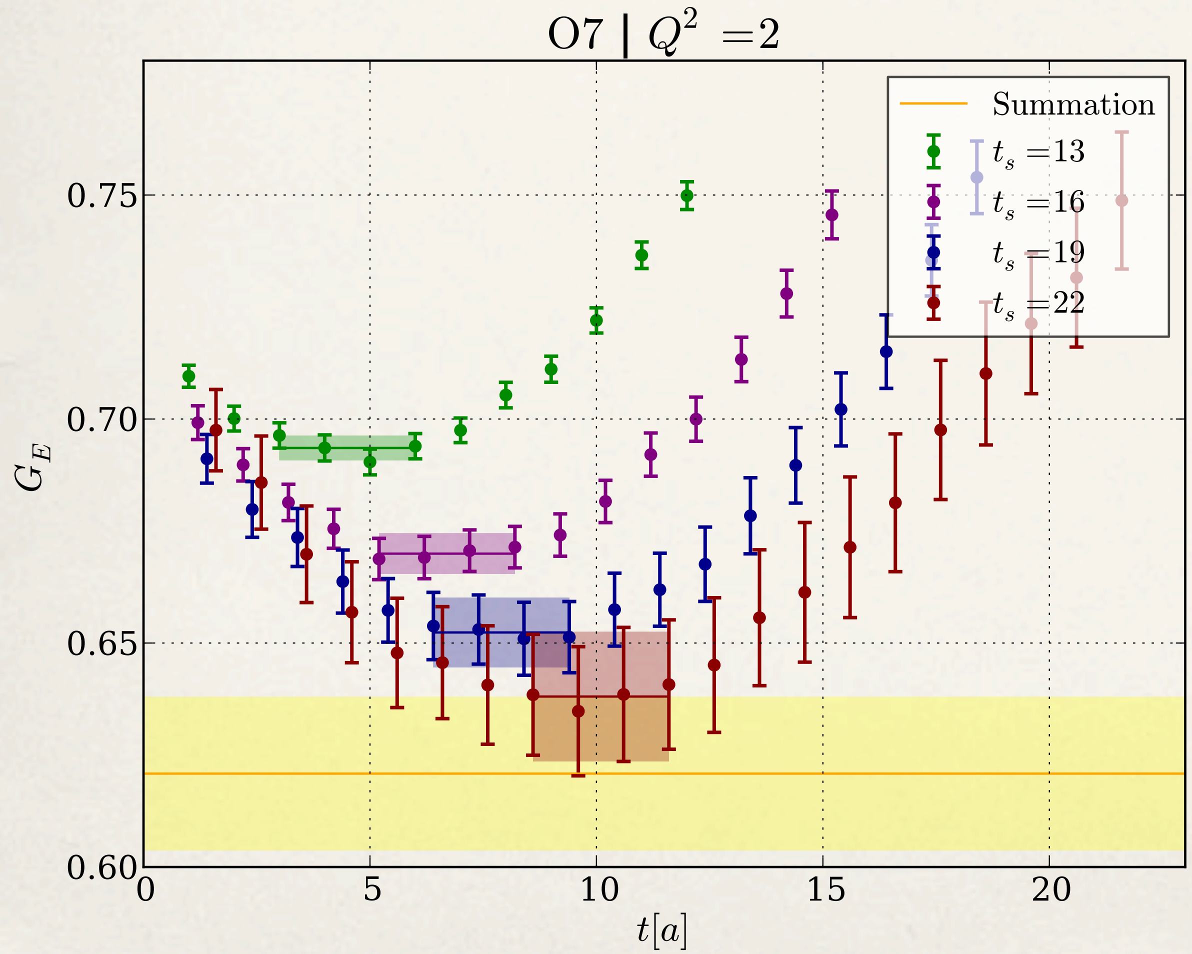


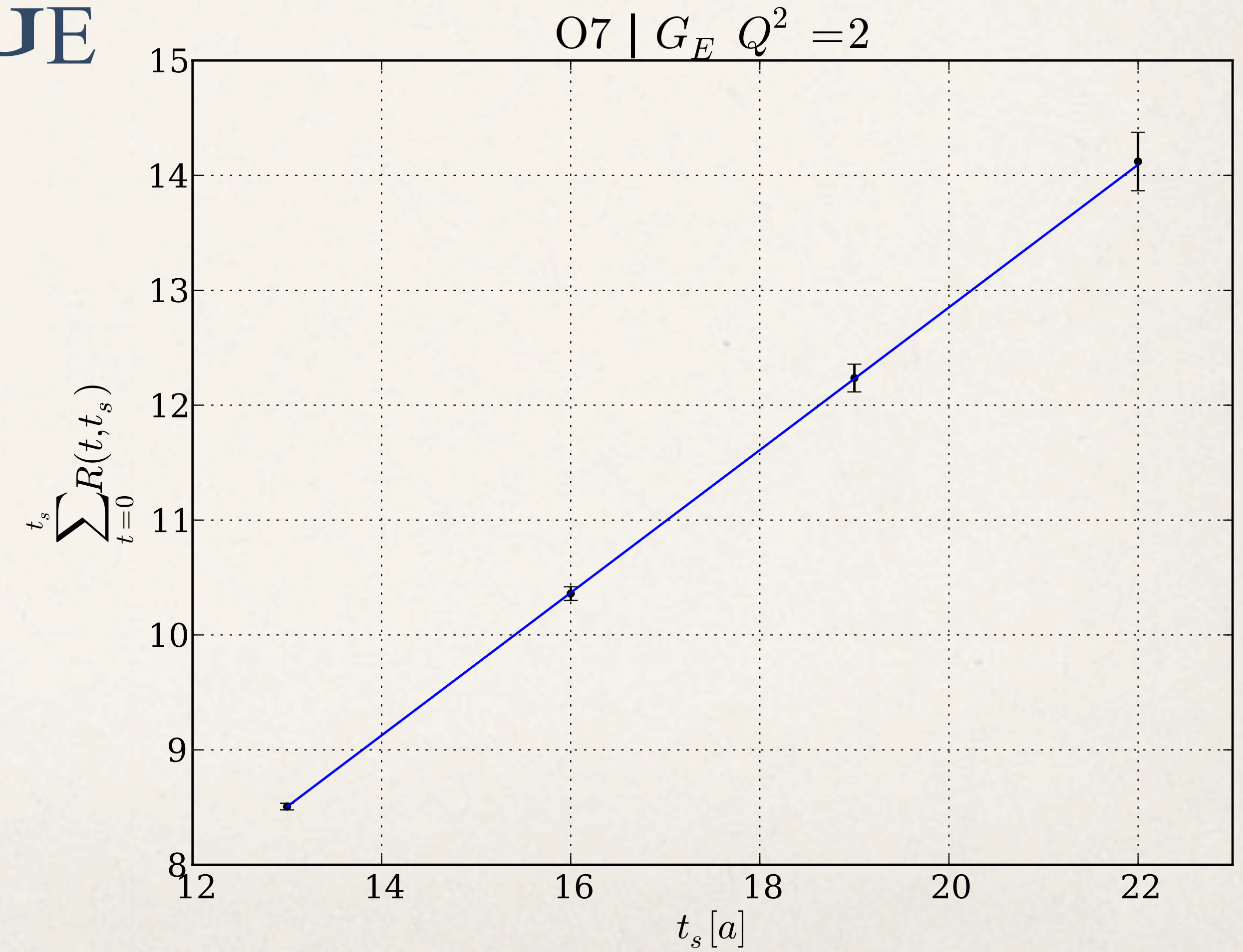
$A5 \mid Q^2 = 1(2\pi/L)^2$
$m_\pi = 312$ MeV
Summation
$t_s = 10$
$t_s = 12$
$t_s = 14$
$t_s = 16$
$32^3 \times 64$
$a = 0.079$ fm
PRELIMINARY
G_E
$t[a]$
0.80
0.75
0.70
0.65
0.60
0.55
0
2
4
6
8
10
12
14
16

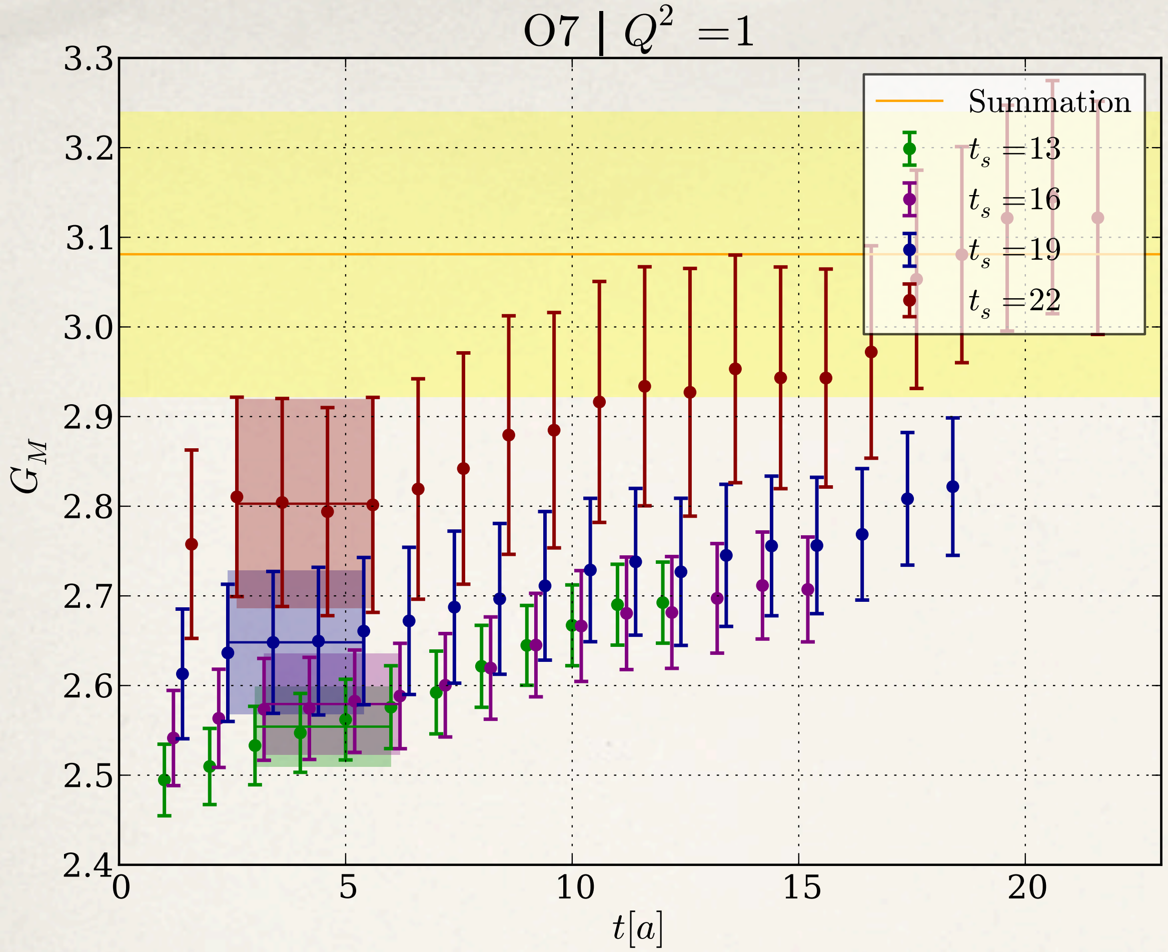

O7 | Q^2 =1
Summation
t_s =13
t_s =16
t_s =19
t_s =22
G_E
$t[a]$

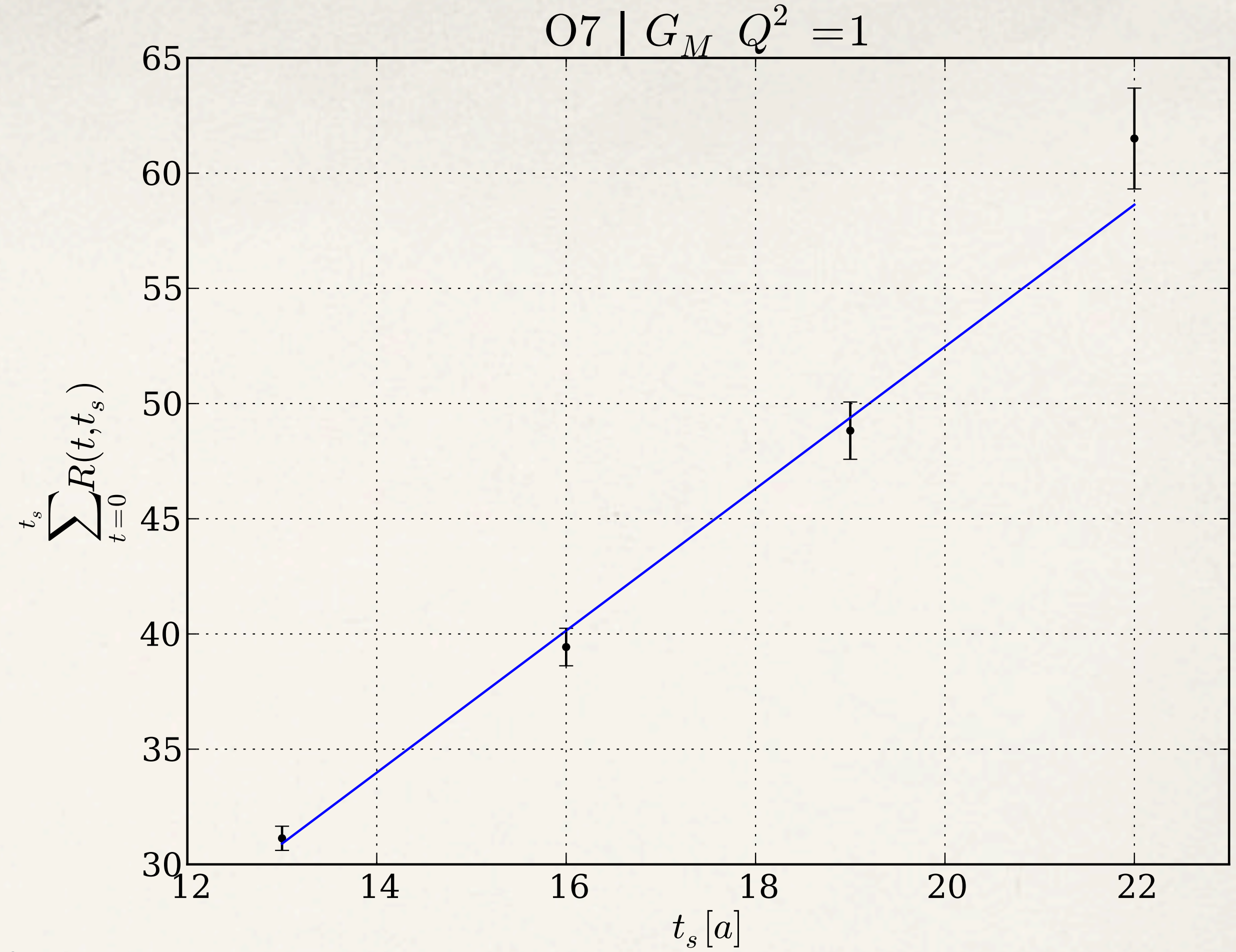

O7 | G_E Q^2 =1
$\sum_{t=0}^{t_s} R(t,t_s)$
$t_s\,[a]$

G_E

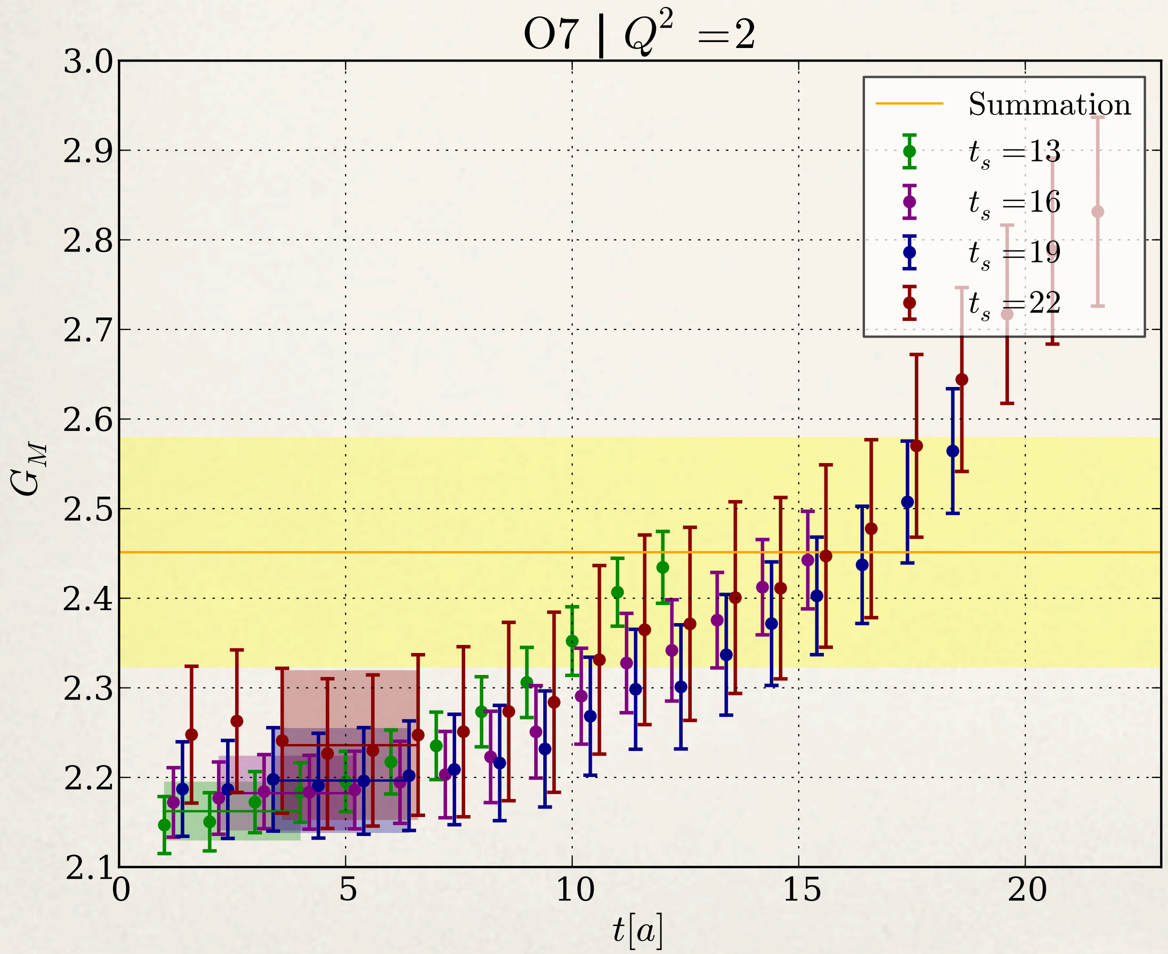

O7 | Q^2 =2
Summation
t_s =13
t_s =16
t_s =19
t_s =22
G_E
$t[a]$


O7 | G_E Q^2 =2
$\sum_{t=0}^{t_s} R(t,t_s)$
$t_s\,[a]$


O7 | Q^2 =1
Summation
t_s =13
t_s =16
t_s =19
t_s =22
G_M
$t[a]$


O7 | G_M Q^2 =1
$\sum_{t=0}^{t_s} R(t,t_s)$
$t_s[a]$

GM


O7 | Q^2 =2
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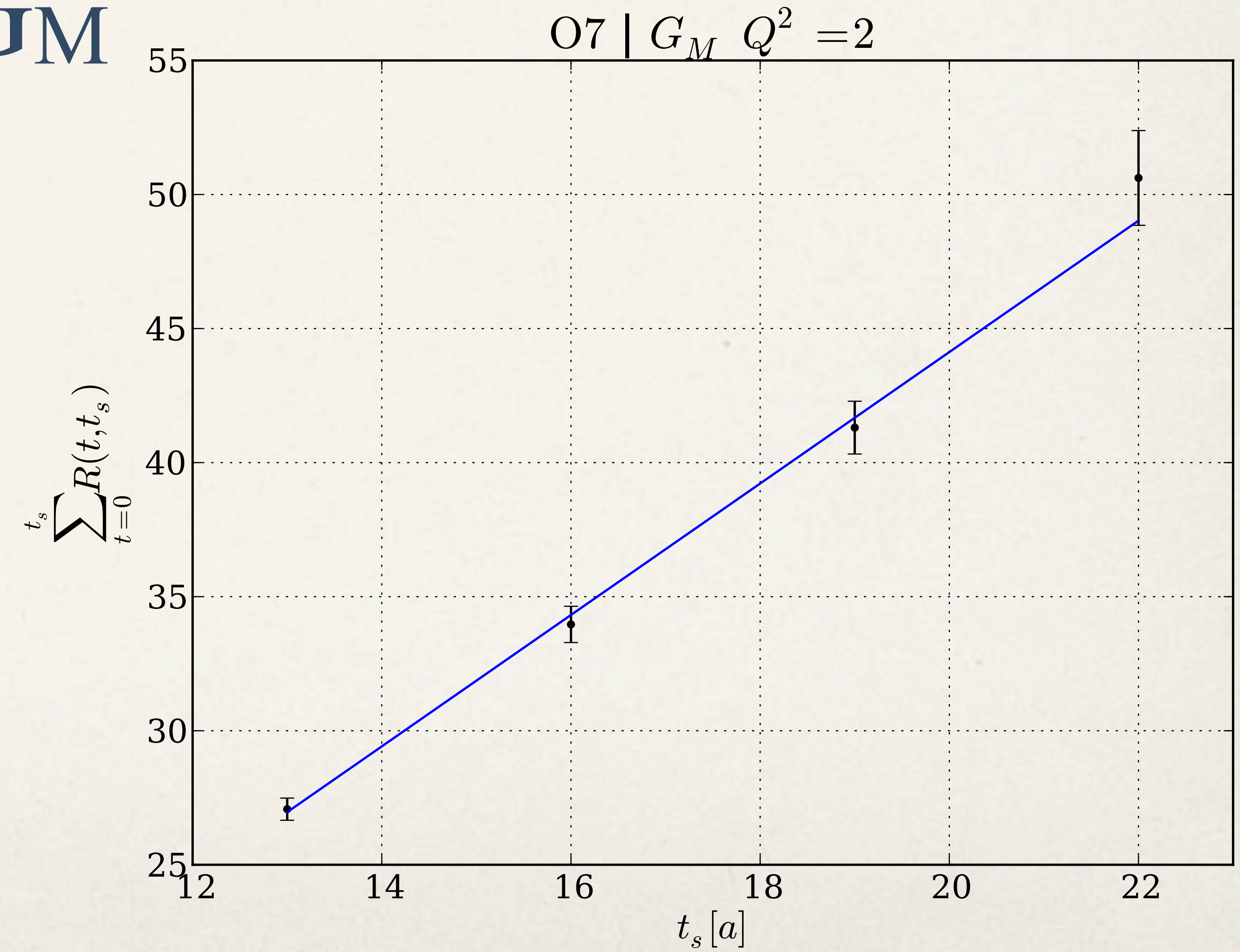
t_s =13
t_s =16
t_s =19
t_s =22
G_M
$t[a]$


O7 | G_M Q^2 =2
$\sum_{t=0}^{t_s} R(t,t_s)$
$t_s[a]$

Sachs to Dirac form factors

- Relate Sachs form factors to the Dirac and Pauli form factors

$$\frac{1}{M_E^2} = \frac{r_1^2}{12} + \frac{\kappa}{8m_N^2} \qquad \frac{1}{M_M^2} = \frac{r_1^2 + \kappa r_2^2}{12(1+\kappa)}$$

- where

$$\kappa = G_M(0) - 1 = \mu - 1 \qquad G_{E,M}(Q^2) = \frac{G_{E,M}(0)}{\left(1 + \frac{Q^2}{M_{E,M}^2}\right)}$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2} F_2(Q^2), \qquad G_M(Q^2) = F_1(Q^2 + F_2(Q^2).$$