Calculating the K_L-K_S mass difference and ε_K to sub-percent accuracy

Lattice 2013

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RBC and UKQCD Collaboration

Outline

- Review $\boldsymbol{m_{K_L} - m_{K_S}}$ aspect and $\boldsymbol{\varepsilon_K}$ in the standard model.
- Box and disconnected contributions
  - Six types
  - Computational strategy
- Size of contributions to
  - $m_{K_L} - m_{K_S}$
  - ε_K
- Conclusion

Based on

Long distance contribution to the K_L–K_S mass difference

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Standard Model Review

- $K^0 - \overline{K^0}$ mixing requires two ΔS=1 interactions:

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left\{ \begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix} \right\} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

where

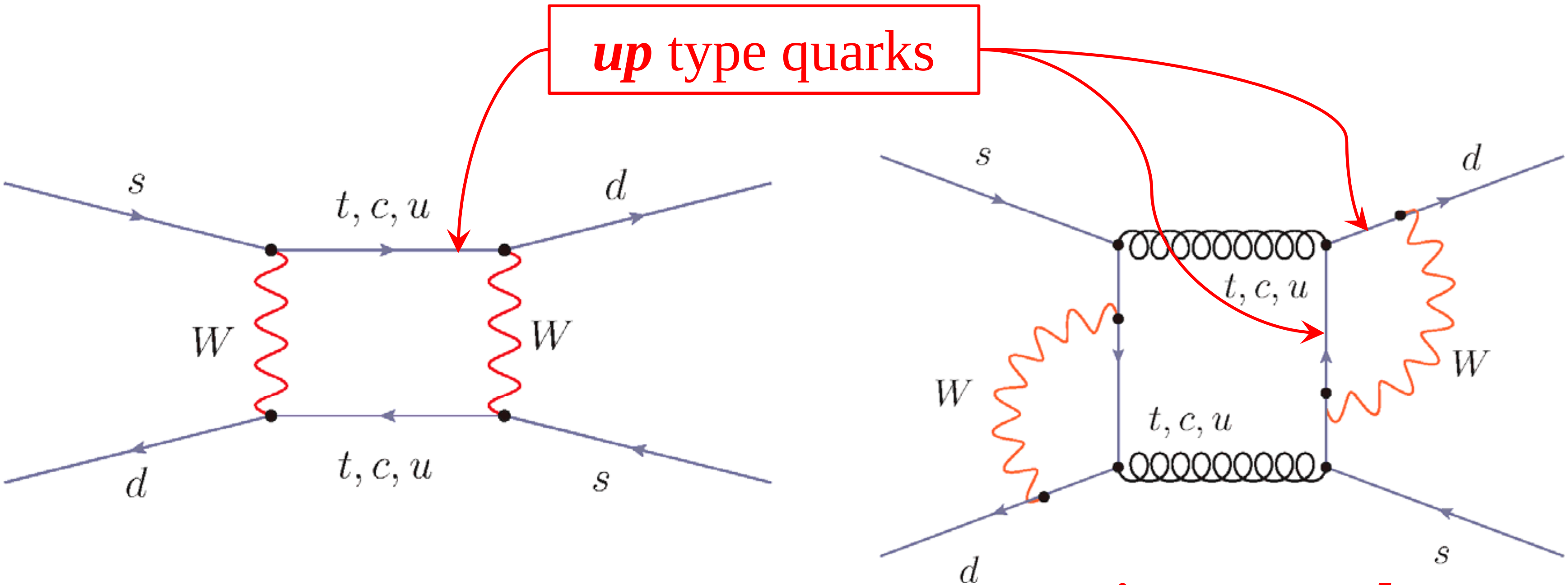
$$\Gamma_{ij} = 2\pi \sum_\alpha \int_{2m_\pi}^{\infty} dE \langle i|H_W|\alpha(E)\rangle\langle\alpha(E)|H_W|j\rangle\delta(E-m_K)$$

$$M_{ij} = \sum_\alpha \mathcal{P}\int_{2m_\pi}^{\infty} dE \frac{\langle i|H_W|\alpha(E)\rangle\langle\alpha(E)|H_W|j\rangle}{m_K - E}$$

$$\epsilon_K = \frac{i}{2}\left\{\frac{\mathrm{Im}M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\Gamma_{0\bar{0}}}{\mathrm{Re}M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\Gamma_{0\bar{0}}}\right\} + i\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \qquad m_{K_S} - m_{K_L} = 2\mathrm{Re}\{M_{0\bar{0}}\}$$

Standard Model Review

- Two types of diagram (most gluons not shown):



Connected

(two quark lines are connected by W's)

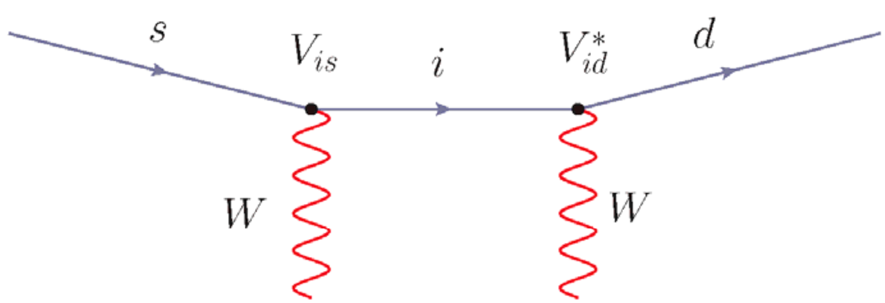
Disconnected

(each quark line is connected to itself by W's)

Standard Model Review

- Three up-type propagators:

$$\sum_{i=u,c,t} \left\{ \quad \right\}$$



- GIM subtraction:

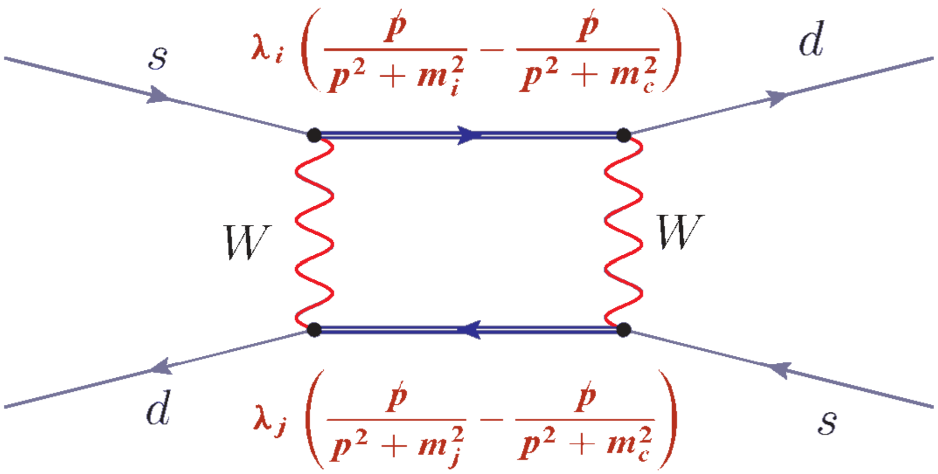
$$\sum_{i=u,c,t} \left\{ V^*_{i,d} \frac{\not{p}}{p^2+m_i^2} V_{i,s} - V^*_{i,d} \frac{\not{p}}{p^2+m_c^2} V_{i,s} \right\}$$

$$= \lambda_t \left\{ \frac{\not{p}}{p^2+m_t^2} - \frac{\not{p}}{p^2+m_c^2} \right\} + \lambda_u \left\{ \frac{\not{p}}{p^2+m_u^2} - \frac{\not{p}}{p^2+m_c^2} \right\}$$

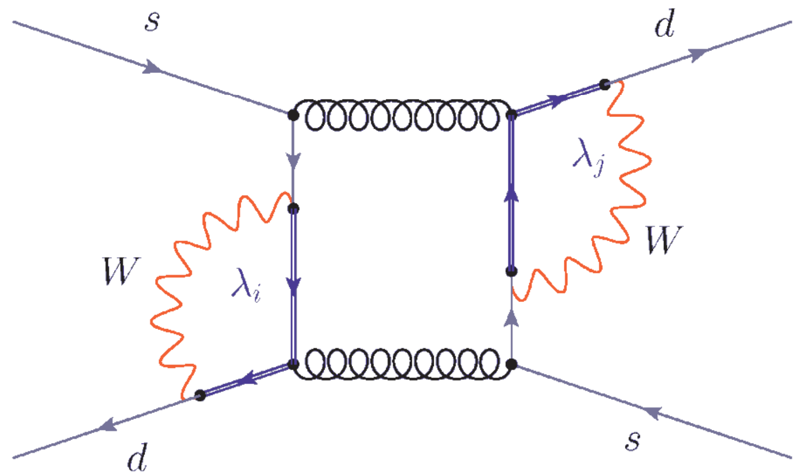
$$\lambda_i = V^*_{i,d} \, V_{i,s}$$

Six contributions to ΔM_K and ε_K

- Six types of box diagram:



i j = *t t*, *u u* and *t u*



i j = *t t*, *u u* and *t u*

- Masses & CKM coefficients: $(m_t/m_c)^2 = 2.1 \times 10^4$

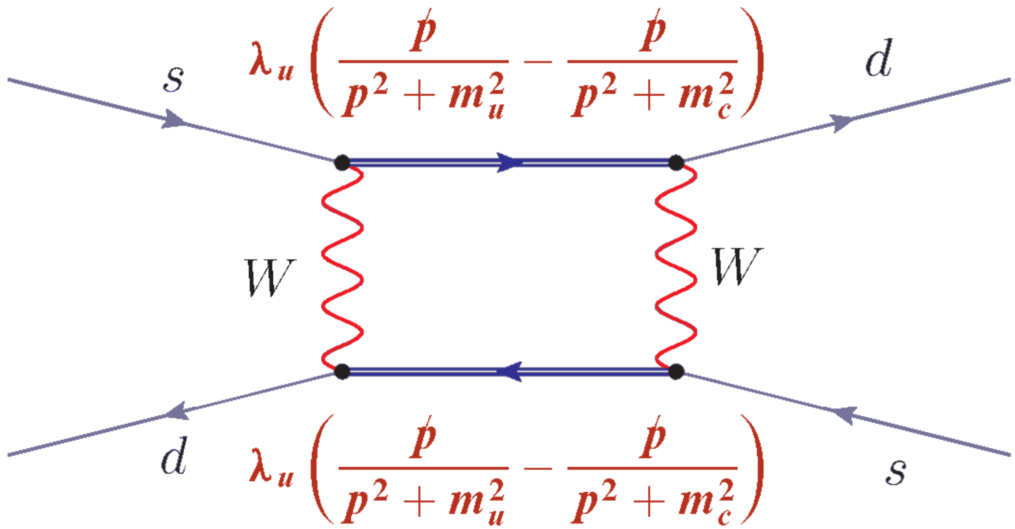
$\lambda_u = 0.22 \qquad \lambda_c = -0.22 + 1.34 \times 10^{-4}\, i$

$\lambda_t = 3.2 \times 10^{-4} - 1.34 \times 10^{-4}\, i$

uu box diagram

$\sim \lambda_u \lambda_u$

$\sim (0.22 + i\,0)^2$



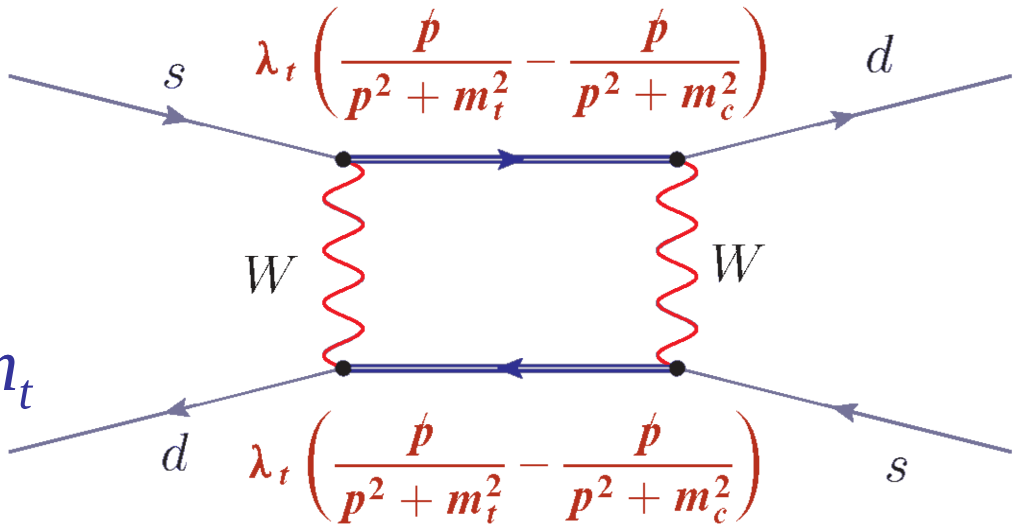
$$\lambda_u^2 G_F^2 \int d^4k \left[\frac{\not k\,(m_c^2 - m_u^2)}{(k^2 + m_u^2)(k^2 + m_c^2)} \right]^2 \sim G_F^2\, m_c^2\, [0.22 + i\,0]^2$$

- Large contribution to ΔM_K (see Jianglei Yu's talk)
- No imaginary part!

tt box diagram

$\sim\lambda_t\ \lambda_t$

$\sim(3.2 - i\ 1.34)^2 \mathrm{x} 10^{-8}$

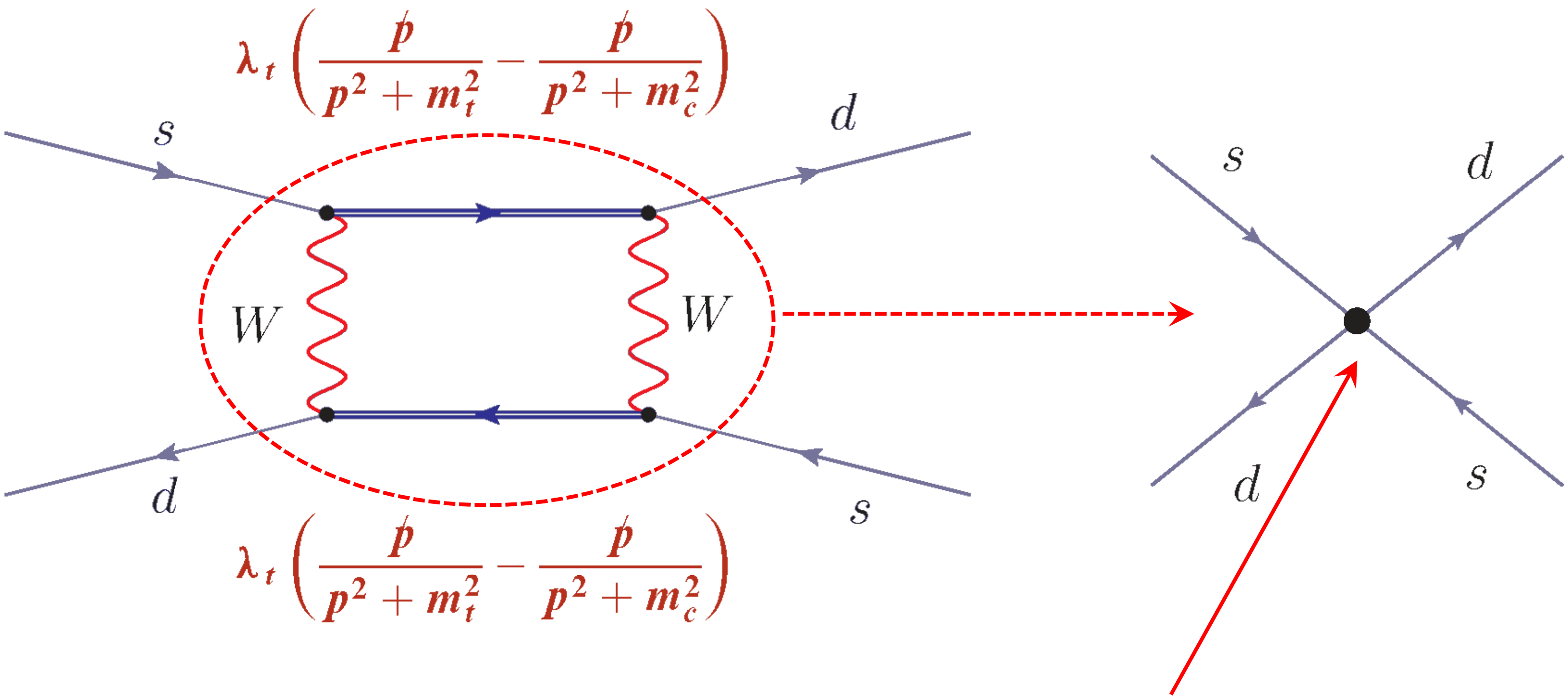


- Dominated by p ~ m_t

$$\lambda_t^2 G_F^2 \int d^4k \left[\frac{\not k\,(m_c^2 - m_t^2)}{(k^2 + m_t^2)(k^2 + m_c^2)} \right]^2 \sim G_F^2 m_t^2 \left[(3.2 - 1.34i) \times 10^{-4} \right]^2$$

- Largest contribution to ε_K
- Contributes ~4% to ΔM_K, familiar Pert. Th. x B_K

tt box diagram



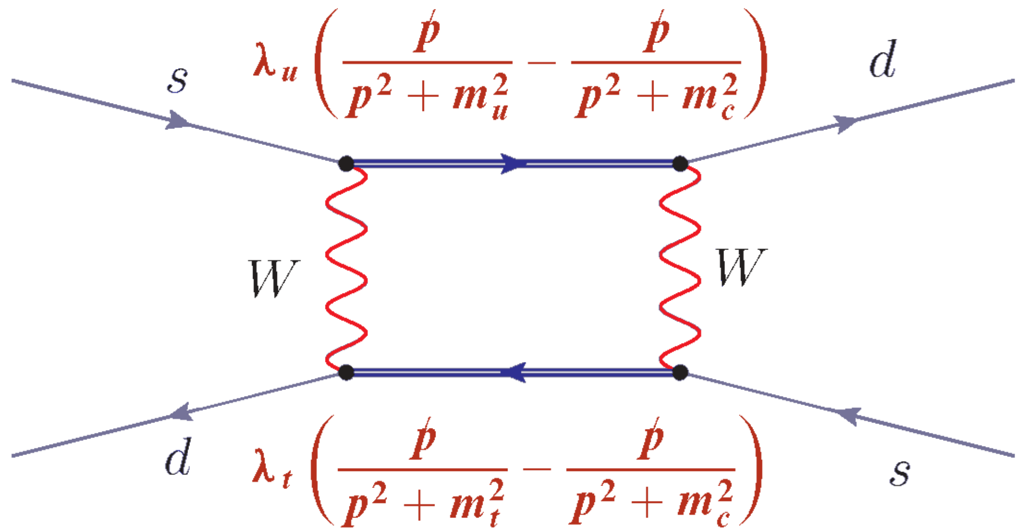
$$O_{LL} = \overline{s}_a \gamma^\mu (1 - \gamma^5) d_a \, \overline{s}_b \gamma^\mu (1 - \gamma^5) d_b$$

- Requires only the usual lattice calculation of B_K

ut box diagram

$\sim\lambda_u\ \lambda_t$

$\sim 0.22\text{x}(3.2 - i\ 1.34)\text{x}10^{-4}$



- Treat top and charm pieces separately

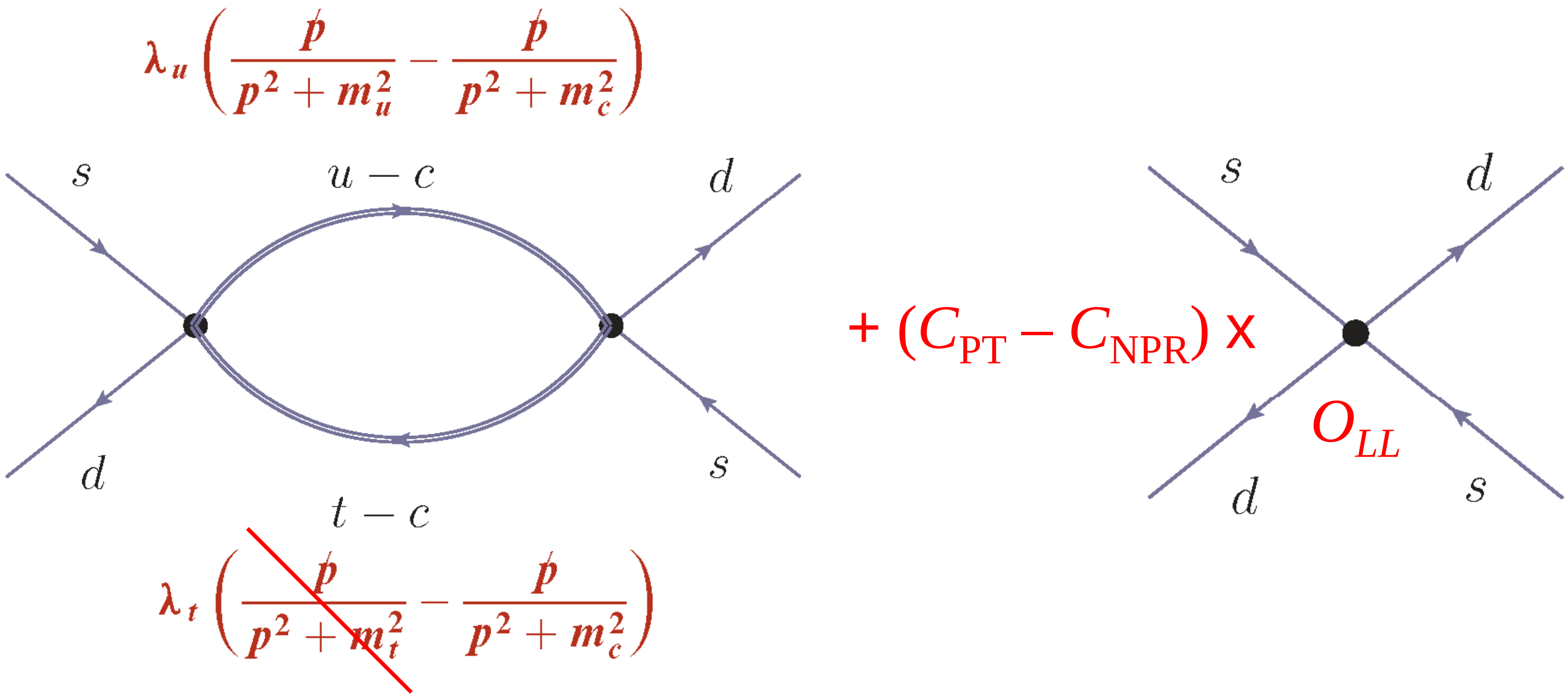
- Charm $\lambda_u \lambda_t G_F^2 \int d^4k \frac{\not k\,(m_c^2 - m_u^2)}{(k^2 + m_u^2)(k^2 + m_c^2)} \frac{\not k}{(k^2 + m_c^2)}$

  $\sim G_F^2 m_c^2 \log(m_c a) \left[0.22 \times (3.2 - 1.34i) \times 10^{-4}\right]$

- Top $\lambda_u \lambda_t G_F^2 \int d^4k \frac{\not k\,(m_c^2 - m_u^2)}{(k^2 + m_u^2)(k^2 + m_c^2)} \frac{\not k}{m_t^2} \sim G_F^2 m_c^2 \left[0.22 \times (3.2 - 1.34i) \times 10^{-4}\right]$

- Contributes ~ 2% to ε_K, ~ 0.2% to ΔM_K

ut box diagram

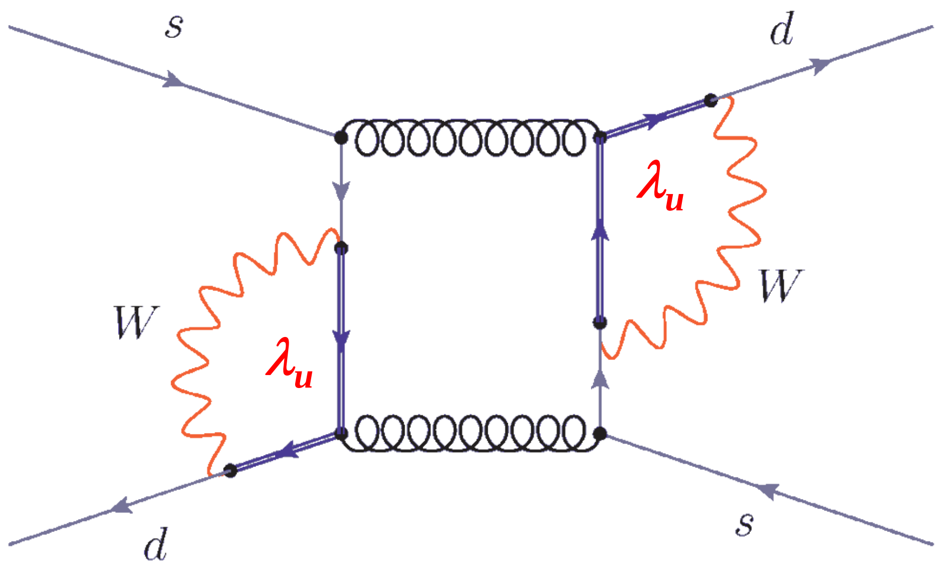


- O_{LL} subtraction replaces lattice short distance piece with perturbative short distance piece.

uu disconnected diagram

$\sim\lambda_u\ \lambda_u$

$\sim(0.22 + i\ 0)^2$

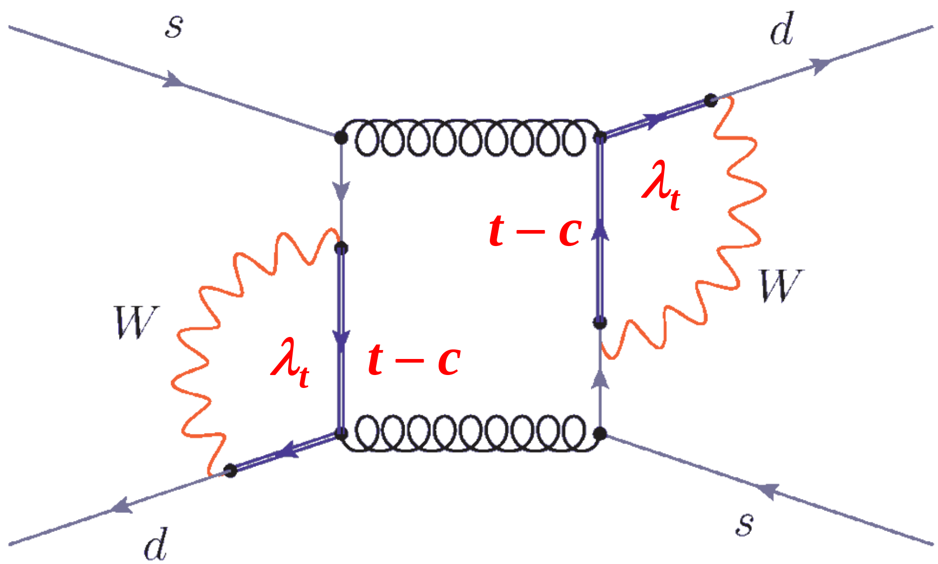


- Convergent $\sim \alpha_s^2\ G_F^2\ m_c^2\ (0.22 + i\ 0)^2$
- Large contribution to ΔM_K (see Jianglei Yu's talk)
- No imaginary part!

tt disconnected diagram

$\sim\lambda_t\,\lambda_t$

$\sim(3.2 - i\ 1.34)^2 \times 10^{-8}$

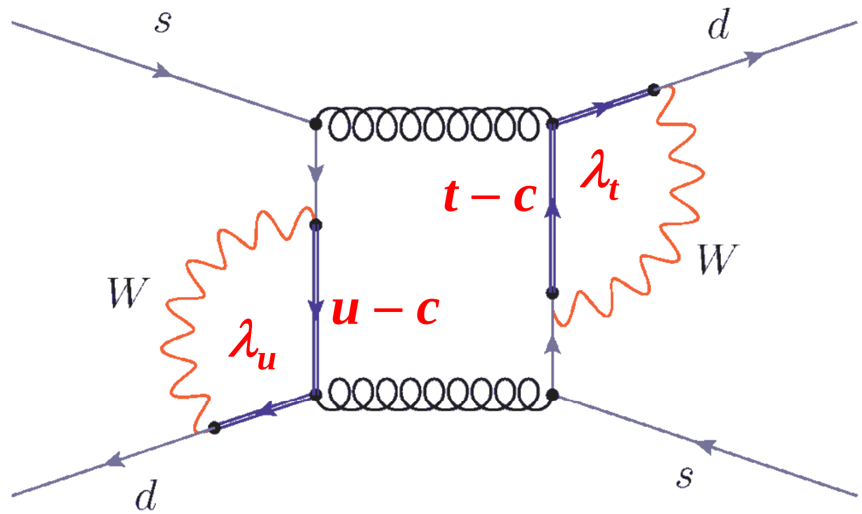


- Given by O_{LL} matrix element, accurate to 10^{-4}
- ~4% correction to ΔM_K
- Standard NNLO contribution to ε_K

ut disconnected diagram

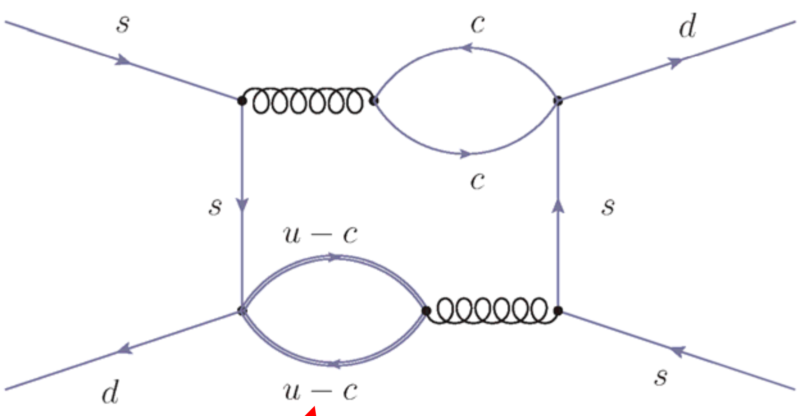
$\sim\lambda_u\,\lambda_t$

$\sim 0.22 \times (3.2 - i\,1.34) \times 10^{-4}$



- Up factor: convergent ($\sim m_c^2/k^2$)
- Top factor:
  - charm vertex: requires gluonic penguin subtraction
  - top vertex: represented by gluonic penguin operator
- Final overall log(a) will require O_{LL} subtraction.

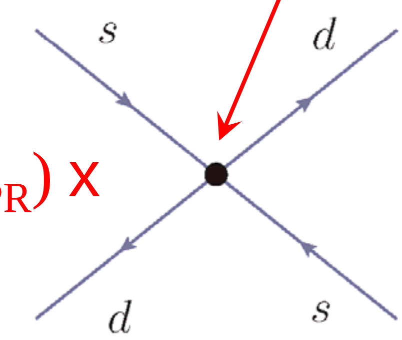
ut disconnected diagram



convergent

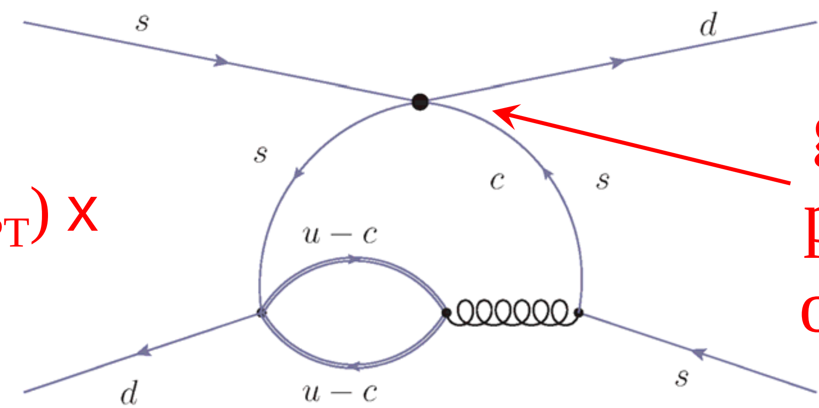
$+ (C_{\mathrm{PT}} - C_{\mathrm{NPR}})$ x

O_{LL} subtraction



O_{LL}

$+ (C^{\mathbf{c}}_{\mathrm{PT}} - C^{\mathbf{c}}_{\mathrm{NPR}} + C^{\mathbf{t}}_{\mathrm{PT}})$ x



gluonic penguin operator

Conclusion

quarks	$M_{\bar{0}0}$	$\mathrm{Re}(M_{\bar{0}0})$	$\mathrm{Im}(M_{\bar{0}0})$
$(u\text{-}c)(u\text{-}c)$	$\lambda_u^2\,(m_c/m_w)^2$	1.1×10^{-5}	0
$(t\text{-}c)(t\text{-}c)$	$\lambda_t^2\,(m_t/m_w)^2$	4.0×10^{-7}	4.1×10^{-7}
$(u\text{-}c)(t\text{-}c)$	$\lambda_u\lambda_t\,(m_c/m_w)^2$	1.6×10^{-8}	6.6×10^{-9}

- While elaborate, all six types of diagrams can be computed using lattice methods.
- Computing $(\lambda_u/2\mathrm{Re}(\lambda_t))\,(m_c/m_t)^2$ ~2% corrections to ε_K is an important next step!
- Including α_{EM} effects will then be the next barrier.