Kaon semileptonic form factors with $N_f = 2+1+1$ HISQ fermions and physical light quark masses

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1. Introduction

The photon-inclusive decay rate for all $K \to \pi l \nu$ decay modes can be related to $|V_{us}|$ via

$$\Gamma_{K_{l3(\gamma)}} = \frac{G_F^2 M_K^5 C_K^2}{128\pi^3} S_{\rm EW} |V_{us}|^2 f_+^{K^0\pi^-}(0)^2 I_{Kl}^{(0)} \left(1 + \delta_{\rm EM}^{Kl} + \delta_{\rm SU(2)}^{K\pi}\right)$$

with $C_K = 1(1/\sqrt{2})$ for neutral (charged) K, $S_{EW} = 1.0223(5)$, $I_{Kl}^{(0)}$ a phase integral depending on shape of $f_\pm^{K\pi}$, and $\delta_{\rm EM}^{Kl}$, $\delta_{\rm SU(2)}^{K\pi}$ are long-distance em and strong isospin corrections respectively

$\delta_{\rm EM}^{Kl} = ([1.40 - 0.02] \pm [0.22 - 0.25])\,\%$ **Cirigliano et al**, JHEP11(2008)006

$\delta_{\rm SU(2)}^{K^0\pi^+} = 0$ and $\delta_{\rm SU(2)}^{K^\pm\pi^0} = 0.058 \pm 0.008$ (with experimental data 0.054 ± 0.008)

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Experimental average, Moulson, 1209.3426 (includes uncertainties above)

$$|V_{us}| f_+^{K\pi}(0) = 0.2163(\pm 0.23\%) \qquad f_+^{K\pi}(0) : \sim 0.4\% \text{ error}$$

FNAL/MILC, 1212.4993,

RBC/UKQCD, 1305.7217

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Check unitarity in the first row of CKM matrix (using FNAL/MILC result)

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Bounds on the scale of new physics: (from fits to K_{l3}, K_{l2} experimental data and lattice results for $f_+^{K\pi}(0)$ and f_K/f_π)

$\rightarrow$ scale of new physics larger than $\mathcal{O}(11 \text{ TeV})$ **Cirigliano et al**, 0908.1754

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Look for new physics effects in the comparison of $|V_{us}|$ from helicity suppressed $K_{\mu 2}$ versus helicity allowed K_{l3}

$$R_{\mu 23} = \left(\frac{f_K/f_\pi}{f_+^{K\pi}(0)}\right) \times \mathrm{experim.\,data\,on}\, K_{\mu 2}\pi_{\mu 2}\, \mathrm{and}\, K_{l3}$$

* In the SM $R_{\mu 23} = 1$. Not true for some BSM theories (for example, charged Higgs)

* With **FNAL/MILC** inputs: $R_{\mu 23} = 1.005(7)$. Limited by lattice inputs

2. Methodology

Follow HPQCD method developed for D semileptonic decays

$$f_+^{K\pi}(0) = f_0^{K\pi}(0) = \frac{m_s - m_l}{m_K^2 - m_\pi^2} \langle \pi | S | K \rangle_{q^2=0}$$

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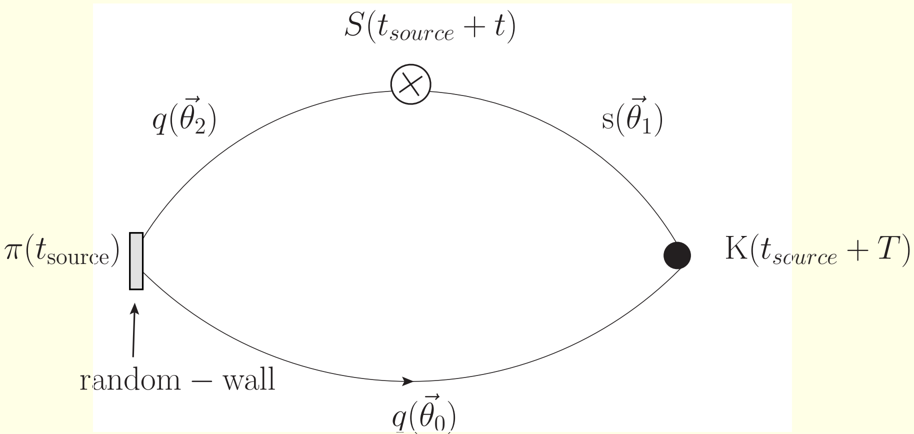

* **Twisted boundary conditions** $\rightarrow$ allow generating correlation functions with non-zero external mom. such that $q^2 \simeq 0$

Avoids extrapolation $q^2 \rightarrow 0$

Twisted boundary conditions: $\psi(x_k + L) = e^{i\theta_k}\psi(x_k)$
(with k a spatial direction and L the spatial length of the lattice).

$\rightarrow$ the propagator carries a momentum $p_k = \pi\frac{\theta_k}{L}$

* We inject momentum in either K (moving K) or π (moving pion).

3. Analysis on the asqtad $N_f = 2+1$ MILC ens.

Phys. Rev. D. 87 (2013) 073012

HISQ valence quarks on $N_f = 2+1$ Asqtad MILC configurations

$\approx a$ (fm)	am_l/am_s	Volume	N_{conf}	$N_{sources}$	N_T	$aM_{\pi,P}^{val}$
0.12	0.4	$20^3 \times 64$	2052	4	5	0.31315
	0.2	$20^3 \times 64$	2243	4	8	0.22587
	0.14	$20^3 \times 64$	2109	4	5	0.18907
	0.1	$24^3 \times 64$	2098	8	5	0.15657
0.09	0.4	$28^3 \times 96$	1996	4	5	0.20341
	0.2	$28^3 \times 96$	1946	4	5	0.14572

with N_T the number of source-sink separations. (need even and odd values of T to eliminate contamination with wrong-spin states (lattice artifacts)).

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* Strange valence quark masses are tuned to their physical values

C.T.H. Davies et al, PRD81(2010)

* Light valence quark masses: $\frac{m_l^{val}(HISQ)}{m_s^{phys}(HISQ)} = \frac{m_l^{sea}(Asqtad)}{m_s^{phys}(Asqtad)}$

3.1. Chiral and continuum extrapolation

The form factor $f_+(0)$ can be written in ChPT as

$$f_+(0) = 1 + f_2 + f_4 + f_6 + ... = 1 + f_2 + \Delta f$$

$f_+(0)$ goes to 1 in the $SU(3)$ limit due to vector current conservation

Ademollo-Gatto theorem $\rightarrow$ SU(3) breaking effects are second order in $(m_K^2 - m_\pi^2)$ and f_2 is completely fixed in terms of experimental quantities.

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Ademollo-Gatto theorem $\rightarrow$ SU(3) breaking effects are second order in $(m_K^2 - m_\pi^2)$ and f_2 is completely fixed in terms of experimental quantities.

* At finite lattice spacing systematic errors can enter due to violations of the dispersion relation needed to derive

$$f_+(0) = f_0(0) = \frac{m_s - m_q}{m_K^2 - m_\pi^2} \langle S \rangle_{q^2=0}$$

Dispersion relation violations in our asqtad data are $\leq 0.15\%$.

($\leq 0.1\%$ in our HISQ data)

3.1. Chiral and continuum extrapolation

* One-loop (NLO) partially quenched Staggered ChPT +

** Staggered ChPT: logs are known non-analytical functions of $m_{K,\pi}$ containing dominant taste-breaking a^2 effects

$\rightarrow$ remove the dominant light discretization errors (remain $a^2\alpha_s^2, a^4$)

$$f_+^{K\pi}(0) = 1 + f_2^{PQ\,,stag.}(a) + K_1^{(a)}\left(\frac{a}{r_1}\right)^2 +$$

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* Two-loop (NNLO) continuum ChPT by **Bijnens & Talavera**, arXiv:0303103.

$$f_+^{K\pi}(0) = 1 + f_2^{PQ,stag.}(a) + K_1^{(a)}\left(\frac{a}{r_1}\right)^2 + f_4^{cont.}(\text{logs}) + f_4^{cont.}(L_i's) + r_1^4(m_\pi^2 - m_K^2)^2 C_6^{\prime(1)}$$

where $C_6^{\prime(1)} \propto C_{12} + C_{34} - L_5^2$. L_5 is an $\mathcal{O}(p^4)$ LEC and $C_{12,34}$ are $\mathcal{O}(p^6)$ LECs

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* Free parameters of the fit: $C_6^{\prime(1)}$, $K_1^{(a)}$, $L_i's$ (priors equal to values in **Amorós et al**, 0101127, with enlarged errors), $\delta_A^{mix}, \delta_V^{mix}$ ($\mathcal{O}(a^2)$ SChPT param.)

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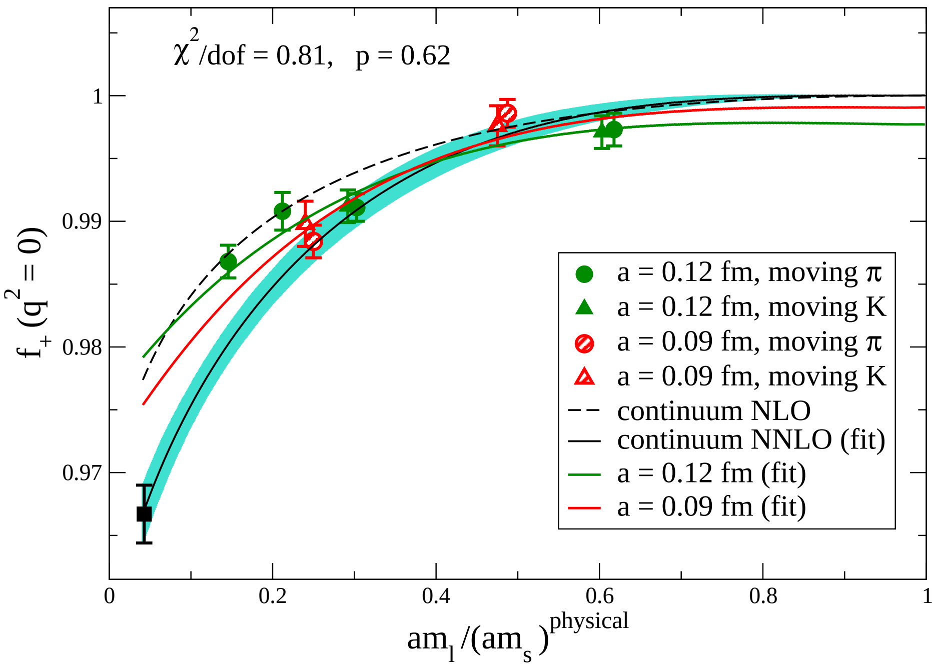
* Check: Use analytical parametrization for NNLO contribution

  → central value changes by less than 0.2%

3.2. Results



Source of uncertainty	Error $f_+(0)$ (%)
Statistics	0.24
Chiral ext. & fitting*	0.3
Discretization	0.1
Scale	0.06
Finite volume	0.1
Total Error	0.42

* Difference between m_s^{sea} and m_s^{val} at two loops

$$f_+(0) = 0.9667 \pm 0.0023 \pm 0.0033$$

$$(C_{12}^r + C_{34}^r)(M_\rho) = (4.57 \pm 0.44 \pm 0.90) \cdot 10^{-6}$$

3.2. Results: Comparison with previous work and unitarity

this work	$0.9667(23)(33)$	$N_f = 2+1$
RBC/UKQCD 13	$0.9670(20)^{+(18)}_{-(46)}$	$N_f = 2+1$
RBC/UKQCD 10	$0.9599(34)\left(^{+31}_{-43}\right)$	$N_f = 2+1$
ETMC	$0.9560(57)(62)$	$N_f = 2$
Kastner & Neufeld	$0.986(8)$	ChPT
Cirigliano	$0.984(12)$	χPT
Jamin, Oller, & Pich	$0.974(11)$	ChPT
Bijnens & Talavera	$0.976(10)$	ChPT
Leutwyler & Roos	$0.961(8)$	Quark model

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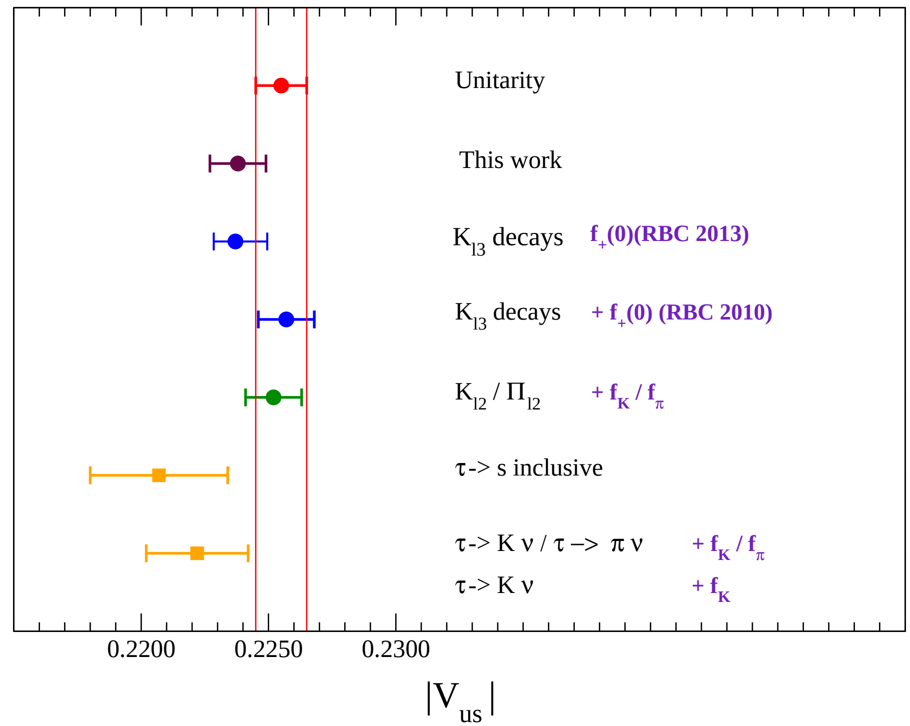

With this value of $f_+^{K\pi}(0)$ and latest experimental data ($|V_{us}|f_+(0) = 0.2163(5)$ **Moulson**, 1209.3426):

$$|V_{us}| = 0.2238 \pm 0.0009 \pm 0.0005$$

$$\rightarrow \Delta_{\rm CKM} \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0008(6)$$

4. Analysis on the HISQ $N_f = 2+1+1$ MILC ens.

a(fm)	m_l/m_s	Volume	$N_{conf.} \times N_{t_s}$	am_s^{sea}	am_s^{val}	
0.15	0.035	$32^3 \times 48$	1000×4	0.0647	0.0691	
0.12	0.2	$24^3 \times 64$	1053×8	0.0509	0.0535	
	0.1	$32^3 \times 64$	993×4	0.0507	0.053	
	0.1	$40^3 \times 64$	391×4	0.0507	0.053	FV check
	0.035	$48^3 \times 64$	945×8	0.0507	0.0531	
0.09	0.2	$32^3 \times 96$	775×4	0.037	0.038	
	0.1	$48^3 \times 96$	853×4	0.0363	0.038	
	0.035	$64^3 \times 96$	625×4	0.0363	0.0363	
0.06	0.2	$48^3 \times 144$	362×4	0.024	0.024	

* Physical quark mass ensembles

* HISQ action on the sea: smaller discretization effects.

* Charm quarks on the sea.

* Better tuned strange quark mass on the sea.

4.1. Simulation details

We avoid autocorrelations blocking by 4.

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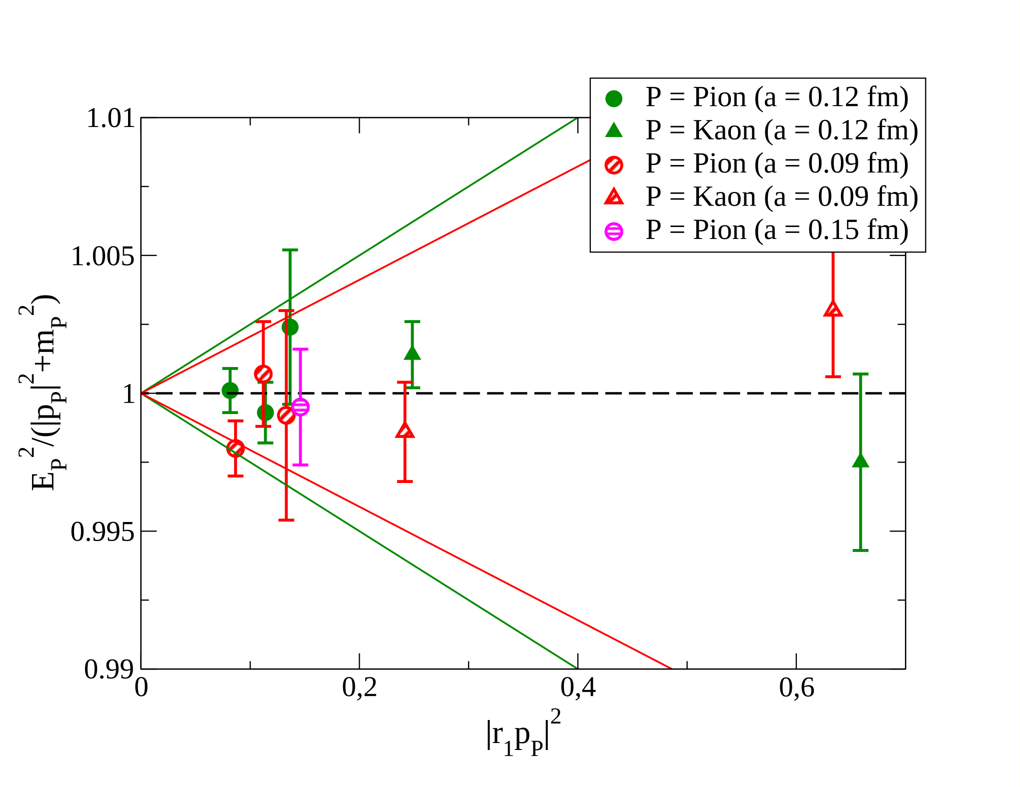
Dispersion relation: Good behaviour

Ratio of the measured (lattice) and the continuum dispersion relation



Lines are the power counting estimates of discretization errors for each lattice spacing, $\alpha_s |r_1 \vec{p}|^2$

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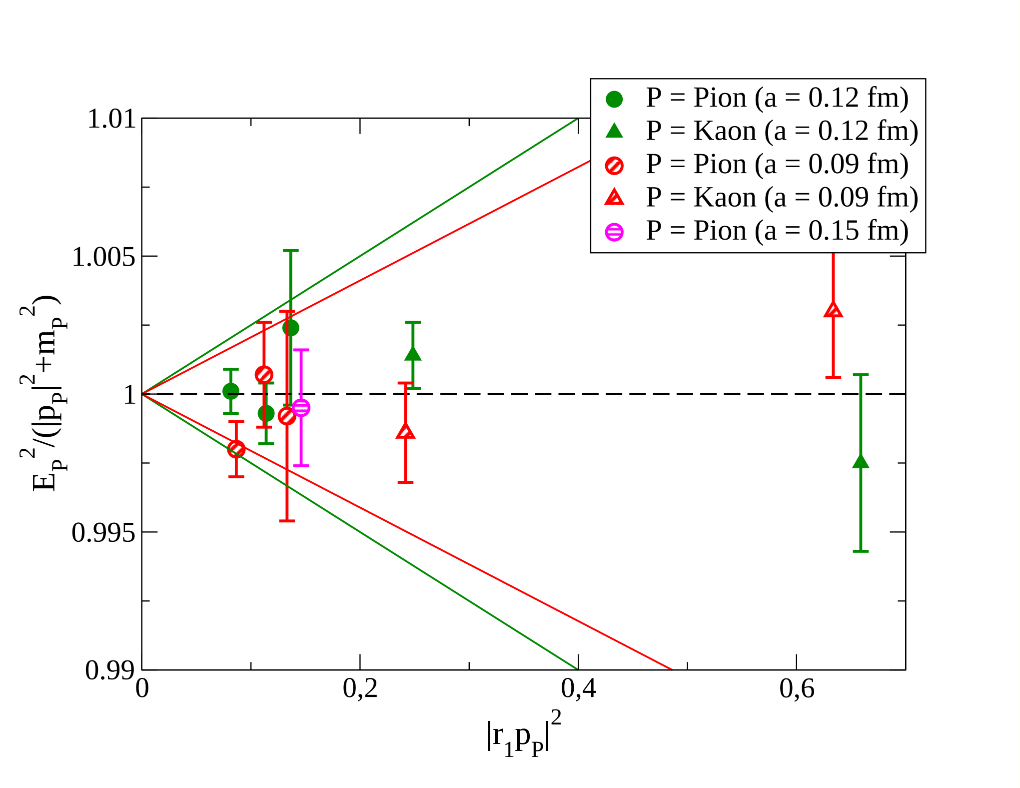

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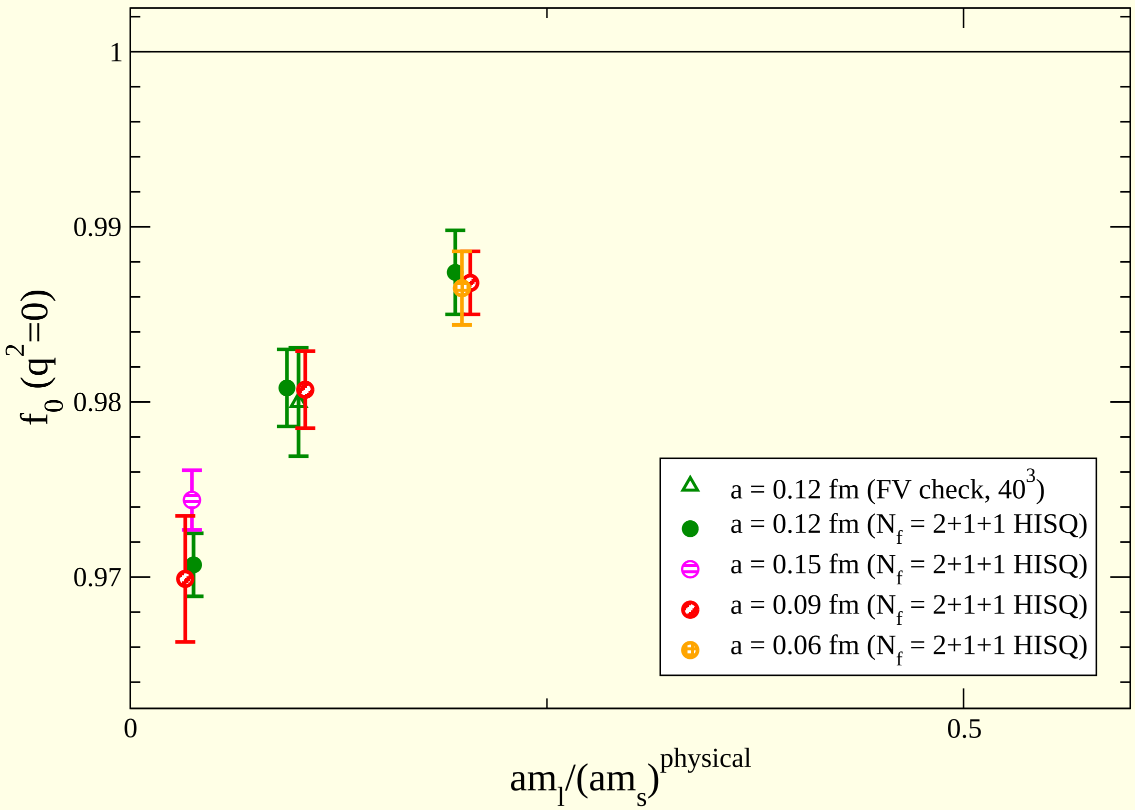
Only moving π data: for the smaller masses the momentum needed for the moving K is much larger and thus the statistical errors are much larger than for moving π.

4.2. Statistical errors and discretization effects



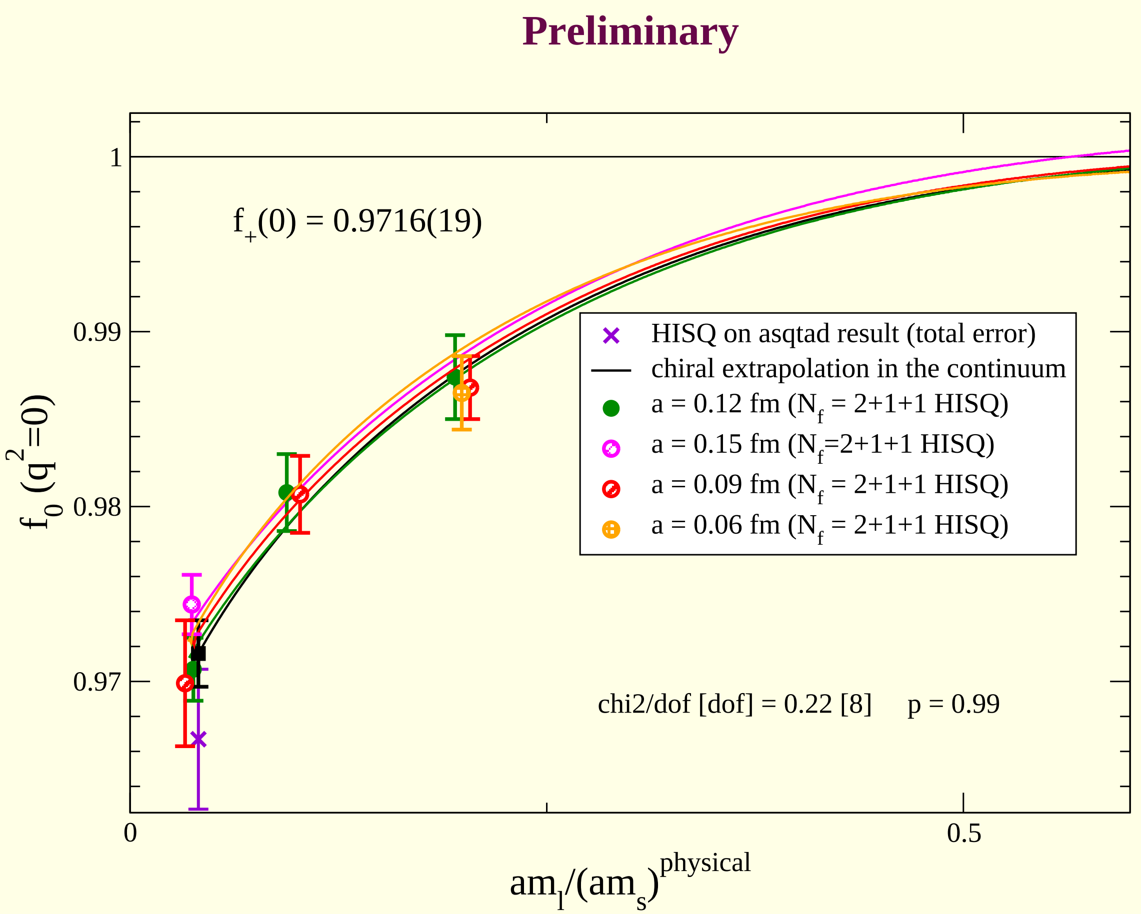
\# Statistical errors: 0.2-0.4% Still larger than in the asqtad calculation (need more statistics).

\# **No discretization effects observed except in the** $a \approx 0.15$ fm **ensem.**

\# FV effects half of statistical errors.

4.3. Chiral+continuum extrapolation

Try the same chiral+continuum extrapolation strategy: one-loop partially quenched SChPT + two loops continuum ChPT + a^2 term.



Only statistical errors included in plot

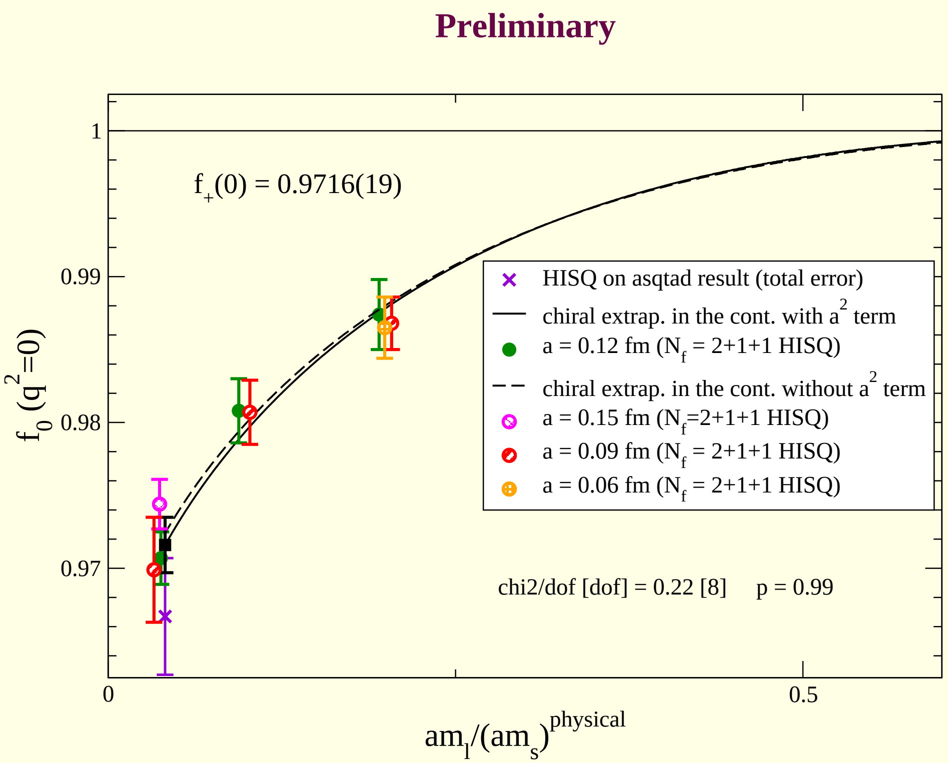
(Final extrapolation strategy still to be decided)

* The coefficient in front of the a^2 term is not well determined by data (only information from $0.15\ fm$ point) and introduces an important error in the extrapolated value $\rightarrow$ mimic most of remaining disc. effects

4.3. Chiral+continuum extrapolation

Systematic error analysis

Discretization effects



* Very stable under addition of higher order terms.

without a^2 term: $f_+(0) = 0.9724(12)$

($\sim 0.08\%$ shift)

* Other a^2 parametrizations, continuum ChPT+a^2 terms, adding an a^2 term respecting AG theorem ... give smaller shifts in the central value.

4.3. Chiral+continuum extrapolation

Systematic error analysis

Chiral extrapolation and fitting: Strongly constrained by data at physical light quark mass (including dependence on f_π).

* Adding higher order (analytical) terms in the chiral expansion: central value and error nearly unchanged.

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Chiral extrapolation and fitting: Strongly constrained by data at physical light quark mass (including dependence on f_π).

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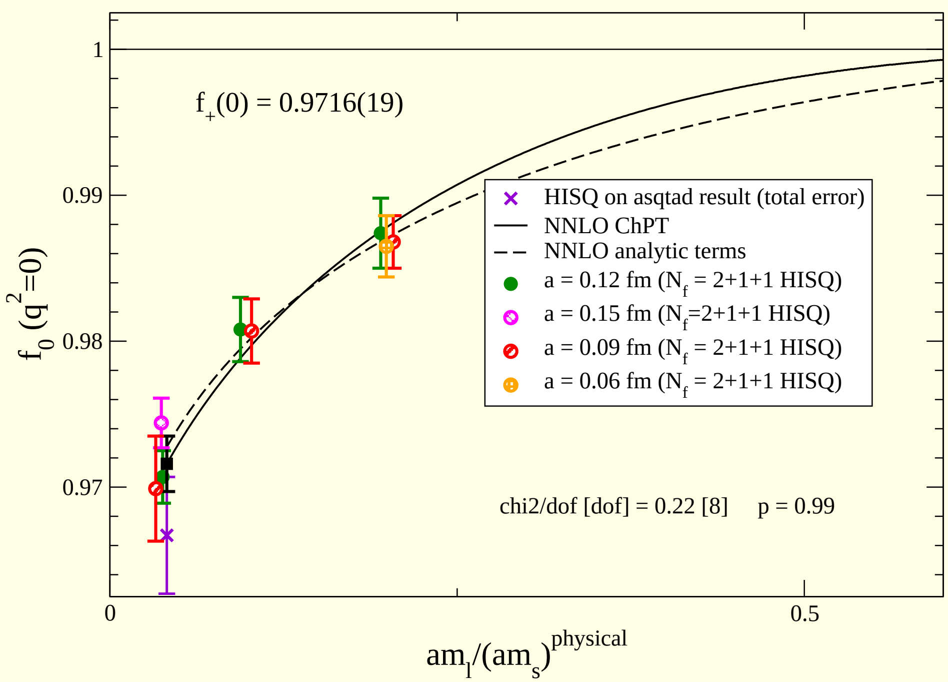
* NNLO analytical parametrization instead of two-loop ChPT:

$$f_+(0) = 0.9727(18)$$

($\sim 0.1\%$ shift)

Preliminary



* m_s^{sea} instead of m_s^{val} in NNLO ChPT: 0.07% shift

4.4. Preliminary error budget

Source of uncertainty	Error $f_+(0)$ (%)
Statistics	~ 0.2
Discretization	≤ 0.1
Chiral ext. & fitting	~ 0.1
Scale	~ 0.06
Finite volume	$\leq 0.1^*$
Total Error	~ 0.3

* In progress: Include finite volume corrections at one loop in the SChPT fit function, C. Bernard, J. Bijnens, E.G.

5. Conclusions and outlook

$N_f = 2+1$ calculation with two lattice spacings and a controlled continuum extrapolation **Phys. Rev. D. 87 (2013) 073012**

$$f_+(0) = 0.9667 \pm 0.0023 \pm 0.0033$$

(this gives $|V_{us}| = 0.2238 \pm 0.0009 \pm 0.0005$, $\sim 1.5\sigma$ lower than unitarity value)

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New determination to try to reduce dominant sources of error using **MILC** HISQ $N_f = 2+1+1$ ensembles

* Physical light quark masses: Reduce chiral extrapolation error.

* HISQ action on the sea: Smaller discretization errors.

* Better tuning of sea quark masses: Reduce chiral extrapolation error.

* Include sea charm quark effects

5. Conclusions and outlook

New determination to try to reduce dominant sources of error using MILC HISQ $N_f = 2+1+1$ ensembles

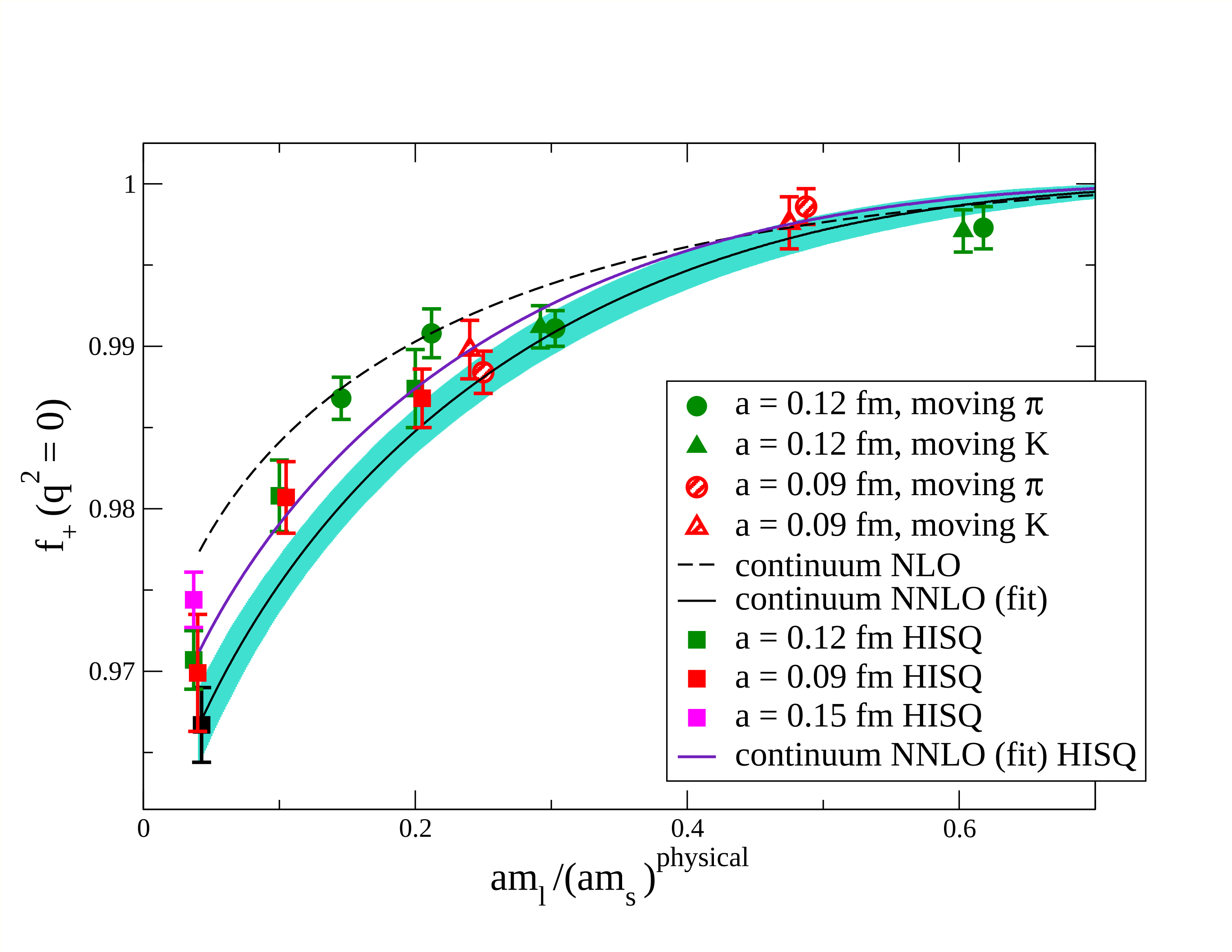
statistical error is now the dominant uncertainty

* Need more statistics on the phys. quark mass ensembles.

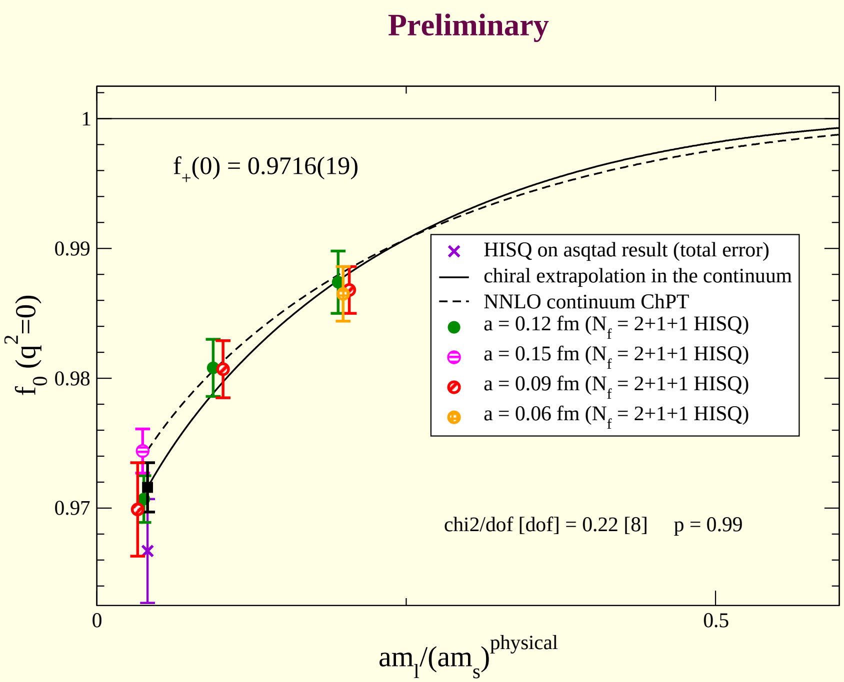
* Need $0.06\ fm$ phys. quark mass point.

* In progress: Include finite volume corrections at one loop in the SChPT fit function, C. Bernard, J. Bijnens, E.G.

asqtad and HISQ data

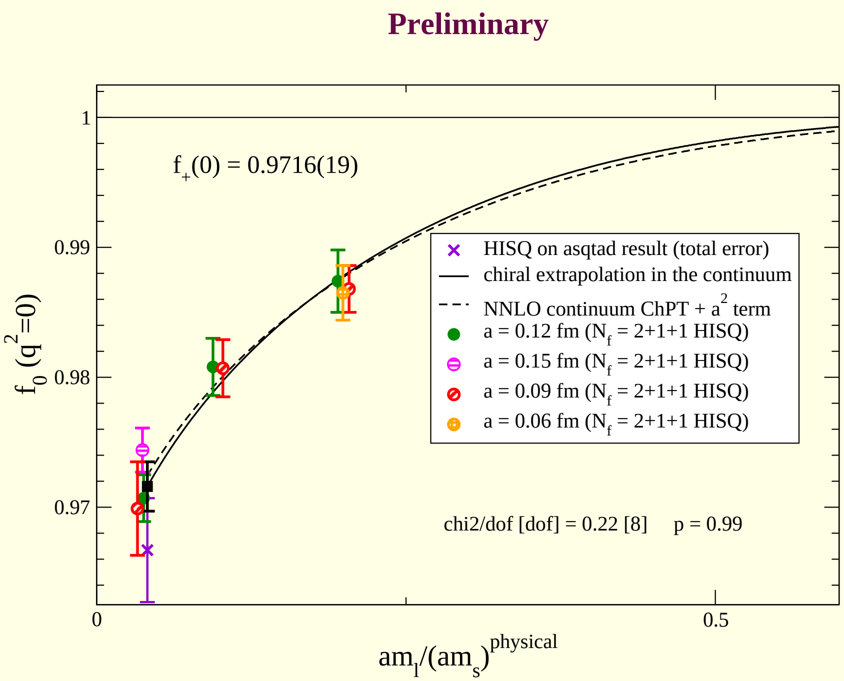


Continuum 2-loops ChPT



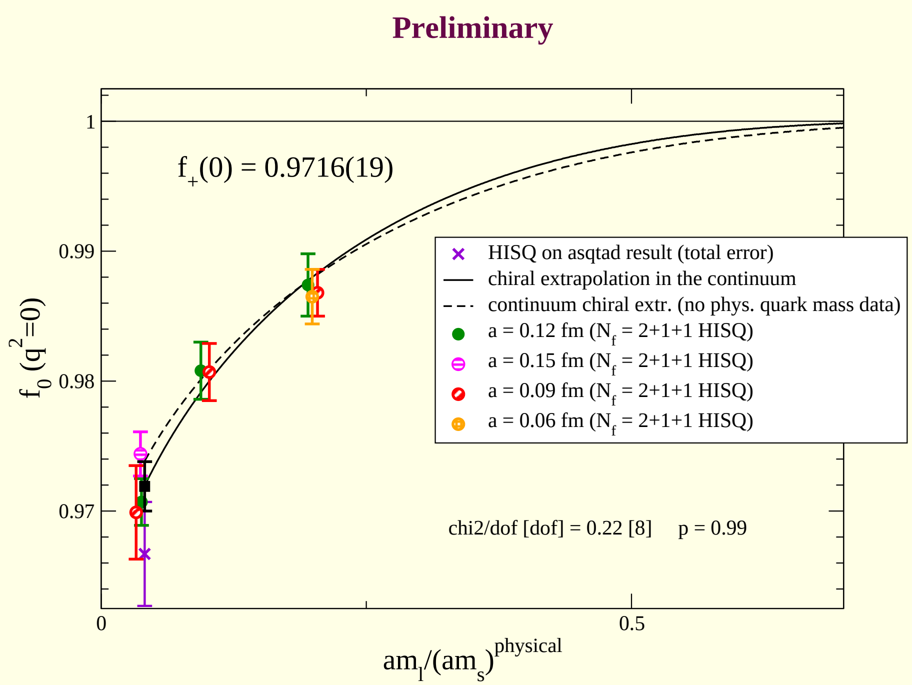
$$f_+(0) = 0.9744(12)$$

Continuum 2-loops ChPT + a^2 term



$$f_+(0) = 0.9724(19)$$

Without phys. quark mass ensemb.



$$f_+(0) = 0.9735(21)$$ (without a^2 term)