Kaon Mixing Beyond the Standard Model

Andrew Lytle (for RBC-UKQCD)
TIFR, Mumbai

01.08.13
Lattice 2013
Mainz, Germany

RBC-UKQCD

UKQCD

Rudy Arthur, Peter Boyle, Hei-Man Choi, Luigi Del Debbio, Shane Drury, Jonathan Flynn, Julien Frison, Nicolas Garron, Jamie Hudspith, Tadeusz Janowski, Andreas Juettner, Richard Kenway, Andrew Lytle, Marina Marinkovic, Enrico Rinaldi, Brian Pendleton, Antonin Portelli, Chris Sachrajda, Ben Samways, Karthee Sivalingam, Matthew Spraggs, Tobi Tsang.

RBC

Ziyuan Bai, Thomas Blum, Norman Christ, Tomomi Ishikawa, Taku Izubuchi, Luchang Jin, Chulwoo Jung, Taichi Kawanai, Chris Kelly, Hyung-Jin Kim, Christoph Lehner, Jasper Lin, Meifeng Lin, Robert Mawhinney, Greg McGlynn, David Murphy, Shigemi Ohta, Eigo Shintani, Amarjit Soni, Oliver Witzel, Hantao Yin, Jianglei Yu, Daiqian Zhang.

Intro/Motivation

- Lattice studies of B_K have reached few-percent level accuracy.
- Potential contributions to kaon mixing BSM are less well-studied.
- Extending the methodology used for B_K, constraints on BSM models can be improved via the kaon sector.

This work extends results from last year [Hudspith, Garron, Boyle 1206.5737] with the addition of a second lattice spacing.

Outline

- Background.
- Kaon matrix elements.
- Renormalization.
- Extrapolations.
  - Chiral.
  - Continuum.
- Future.

Background

In a generic BSM model,

$$H_{\rm BSM}^{\Delta S=2} = \sum_{i=1}^{5} C_{\rm BSM}^{i}(\mu)\, \mathcal{O}_i^{\Delta S=2}(\mu) + \sum_{i=1}^{3} \tilde{C}_{\rm BSM}^{i}(\mu)\, \tilde{\mathcal{O}}_i^{\Delta S=2}(\mu)\,,$$

with

$$\begin{aligned}
\mathcal{O}_1 &= \left[\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha\right]\left[\bar{s}_\beta \gamma^\mu (1-\gamma_5) d_\beta\right] \\
\mathcal{O}_2 &= \left[\bar{s}_\alpha (1-\gamma_5) d_\alpha\right]\left[\bar{s}_\beta (1-\gamma_5) d_\beta\right] \\
\mathcal{O}_3 &= \left[\bar{s}_\alpha (1-\gamma_5) d_\beta\right]\left[\bar{s}_\beta (1-\gamma_5) d_\alpha\right] \\
\mathcal{O}_4 &= \left[\bar{s}_\alpha (1-\gamma_5) d_\alpha\right]\left[\bar{s}_\beta (1+\gamma_5) d_\beta\right] \\
\mathcal{O}_5 &= \left[\bar{s}_\alpha (1-\gamma_5) d_\beta\right]\left[\bar{s}_\beta (1+\gamma_5) d_\alpha\right]\,,
\end{aligned}$$

and $\tilde{\mathcal{O}}_{1,2,3}$ are obtained from $\mathcal{O}_{1,2,3}$ by $(1-\gamma_5) \to (1+\gamma_5)$.

Alternative Basis.

In practice we use an alternative 'color-diagonal' basis [parity even parts],

$$
\begin{aligned}
Q_1 &= \left[\bar{s}\gamma_\mu(1-\gamma_5)d\right]\left[\bar{s}\gamma^\mu(1-\gamma_5)d\right] \\
Q_2 &= \left[\bar{s}\gamma_\mu(1-\gamma_5)d\right]\left[\bar{s}\gamma^\mu(1+\gamma_5)d\right] \\
Q_3 &= \left[\bar{s}(1-\gamma_5)d\right]\left[\bar{s}(1+\gamma_5)d\right] \\
Q_4 &= \left[\bar{s}(1-\gamma_5)d\right]\left[\bar{s}(1-\gamma_5)d\right] \\
Q_5 &= \frac{1}{2}\left[\bar{s}\sigma^{\mu\nu}d\right]\left[\bar{s}\sigma^{\mu\nu}d\right],
\end{aligned}
$$

these are simply related to the original basis as:

$$\mathcal{O}_1 = Q_1$$

$$\mathcal{O}_2 = Q_4 \qquad \mathcal{O}_3 = \frac{1}{2}(Q_5 - Q_4)$$

$$\mathcal{O}_4 = Q_3 \qquad \mathcal{O}_5 = -\frac{1}{2}Q_2$$

Definition of Bag Parameters.

Our aim is to determine the quantities

$$B_i = -\frac{\langle \overline{K^0}|\mathcal{O}_i|K^0\rangle}{N_i\langle \overline{K^0}|\bar{s}\gamma_5 d|0\rangle\langle 0|\bar{s}\gamma_5 d|K^0\rangle}\,.$$

$N_{2,3,4,5} = \frac{5}{3}, -\frac{1}{3}, -2, -\frac{2}{3}$ gives the VIA result. Alternatively,

$$R_i^{\mathrm{BSM}} \equiv \left[\frac{f_K^2}{m_K^2}\right]_{\mathrm{expt}} \left[\frac{m_P^2}{f_P^2}\frac{\langle \overline{P}|\mathcal{O}_i|P\rangle}{\langle \overline{P}|\mathcal{O}_1|P\rangle}\right]_{\mathrm{latt}}\,.$$

- f_P calculated using a ratio of two-point functions and Z_A.
- m_P is obtained from the exponential decay of two-point functions.

Definition of Correlators.

We study three-point correlation functions of the four-quark operators with K^0, $\overline{K^0}$ interpolating operators.

$$c_i(t_i, t_f, t) = \langle P(t_f)\, \mathcal{O}_i(t)\, P^\dagger(t_i) \rangle$$

The ratio R_i is determined by fitting the ratio of the correlation functions

$$r_i(t_i, t_f, t) = \frac{c_i(t_i, t_f, t)}{c_1(t_i, t_f, t)}$$

in the center region $t_i \ll t \ll t_f$ to a constant.

Simulation details.

Shamir domain-wall fermions ($N_f = 2 + 1$) and Iwasaki gauge action.

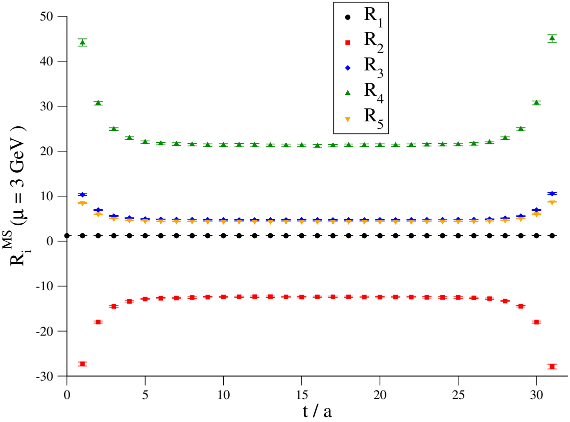
- Two lattice spacings.
- Unquenched light quarks.

extent	a^{-1} [GeV]	$am_{ud}^{\mathrm{sea}}\,(= am_{ud}^{\mathrm{val}})$	m_π [MeV]
$32^3 \times 64 \times 16$	2.310(37)	0.004, 0.006, 0.008	290, 340, 390
$24^3 \times 64 \times 16$	1.747(31)	0.005, 0.01, 0.02	330, 420, 560

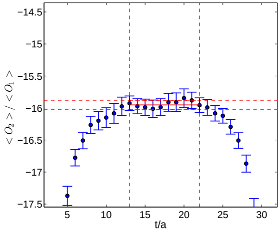
- Unquenched and partially-quenched strange sector.

a^{-1} [GeV]	am_s^{sea}	am_s^{val}	am_s^{phys}
2.310(37)	0.03	0.03, 0.025	0.0273(7)
1.747(31)	0.04	0.04,0.035, 0.03	0.0348(11)

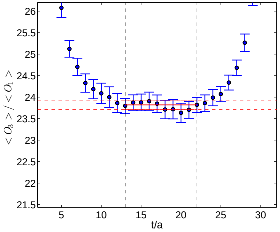
Ratio correlators.



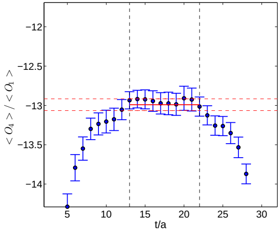
Ratio correlators - detail.



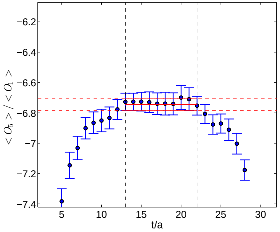
$r_2(t)$



$r_3(t)$



$r_4(t)$



$r_5(t)$

Renormalization.

Non-perturbative renormalization (NPR)

We use the non-perturbative renormalization (NPR) method to define continuum-like operators in the RI-MOM and RI-SMOM schemes. These are converted to $\overline{\mathrm{MS}}$ operators using perturbative matching factors.

$$
\begin{aligned}
Q_1 &= \left[\bar{s}\gamma_\mu(1-\gamma_5)d\right]\left[\bar{s}\gamma^\mu(1-\gamma_5)d\right] && (27,1)\\
Q_2 &= \left[\bar{s}\gamma_\mu(1-\gamma_5)d\right]\left[\bar{s}\gamma^\mu(1+\gamma_5)d\right] && (8,8)\\
Q_3 &= \left[\bar{s}(1-\gamma_5)d\right]\left[\bar{s}(1+\gamma_5)d\right] && (8,8)\\
Q_4 &= \left[\bar{s}(1-\gamma_5)d\right]\left[\bar{s}(1-\gamma_5)d\right] && (6,\bar{6})\\
Q_5 &= \frac{1}{2}\left[\bar{s}\sigma^{\mu\nu}d\right]\left[\bar{s}\sigma^{\mu\nu}d\right] && (6,\bar{6})
\end{aligned}
$$

Non-perturbative renormalization (NPR)

Require that amputated matrix elements of $\mathcal{O}_i$ with external quark states at large Euclidean p^2 take their tree-level values..

$$G_\Gamma^{ijkl} = \langle s^i \bar{d}^j \, (\bar{s}\Gamma d) \, (\bar{s}\Gamma d) \, s^k \bar{d}^l \rangle, \qquad P_\Gamma^{ijkl} = \frac{1}{\mathcal{N}} \, \Gamma^{ji} \, \Gamma^{lk}$$

$$\Lambda_{X,\Gamma} = P_\Gamma^{ijkl} \, G_{X,\ \mathrm{AMP}}^{ijkl}, \qquad Z\Lambda = F$$

Requires the condition:

$$\Lambda_{\mathrm{QCD}} \ll |p| \ll \frac{\pi}{a}$$

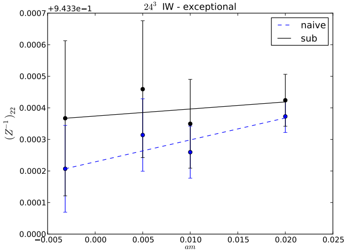
RI scheme/ mixing/ pole subtraction.

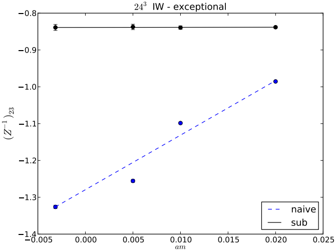
The projected matrix elements in the RI scheme have the form:

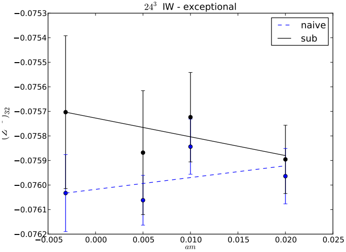
$$M_{ij} = A_{ij} + \frac{B_{ij}}{(am)} + \frac{C_{ij}}{(am)^2} + D_{ij}(am) + \mathcal{O}\big((am)^2\big)\,.$$

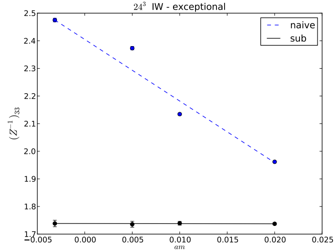
- The infrared sensitive terms $B_{ij}, C_{ij} \neq 0$ need to be subtracted from the data.
- Empirically, we find the double-pole term is benign.
- We fit $(am)M_{ij} \sim (am)A_{ij} + B_{ij}$ to determine B_{ij} and subtract this term.

Pion pole subtractions - (8,8) operators.









RI-SMOM matching factors.

We also perform the renormalization using a “non-exceptional” kinematic scheme (RI-SMOM).

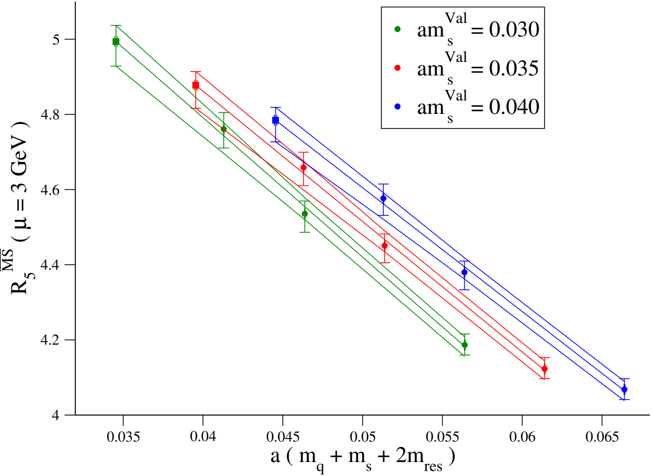
- The RI-SMOM scheme does not suffer from the unwanted IR effects present in the RI-MOM scheme.
- The SMOM $\rightarrow \overline{\mathrm{MS}}$ matching factors are not presently known for the full operator basis. In particular, the 2×2 matrix for the $(6,\bar{6})$ operators is not known.
- However, we can quote results in the RI-SMOM scheme leaving the corresponding $\overline{\mathrm{MS}}$ determination until the appropriate PT calculation is performed.

Chiral and continuum extrapolations.

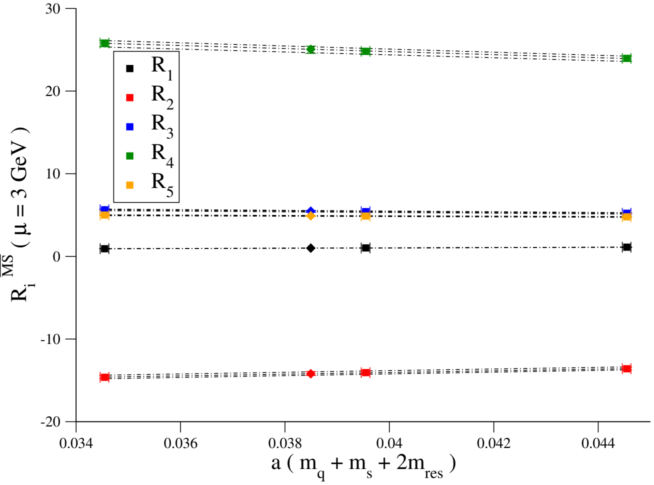
Chiral/Continuum strategy.

- We find mild quark mass dependence consistent with linear behavior in both m_{ud} and m_s.
- Extrapolate in m_{ud} and interpolate in m_s to the physical point on each ensemble.
- Linear fit in a^2 to determine the continuum result.

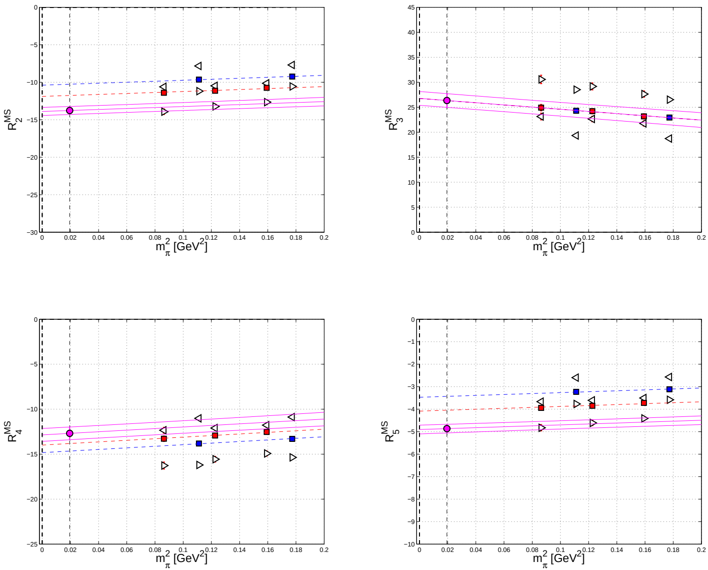
Light quark extrapolation - R_5.



Strange quark interpolation.



Continuum extrapolation – Preliminary.



Summary & Future Work.

- Improved determination of BSM kaon matrix elements improves the ability of the kaon sector to constrain new physics.
- Domain-wall fermions at two lattice spacings and unitary pions as light as 290 MeV.

Future:

- Explore extrapolations using golden ratios.
- SMOM matching reduces systematic error in NPR, also appears to improve scaling errors.
- Physical point simulation.

Thank you!

Additional Slides

Continuum extrapolation – (γ, γ) scheme– Preliminary.