$K_L - K_S$ mass difference from lattice QCD

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Motivation

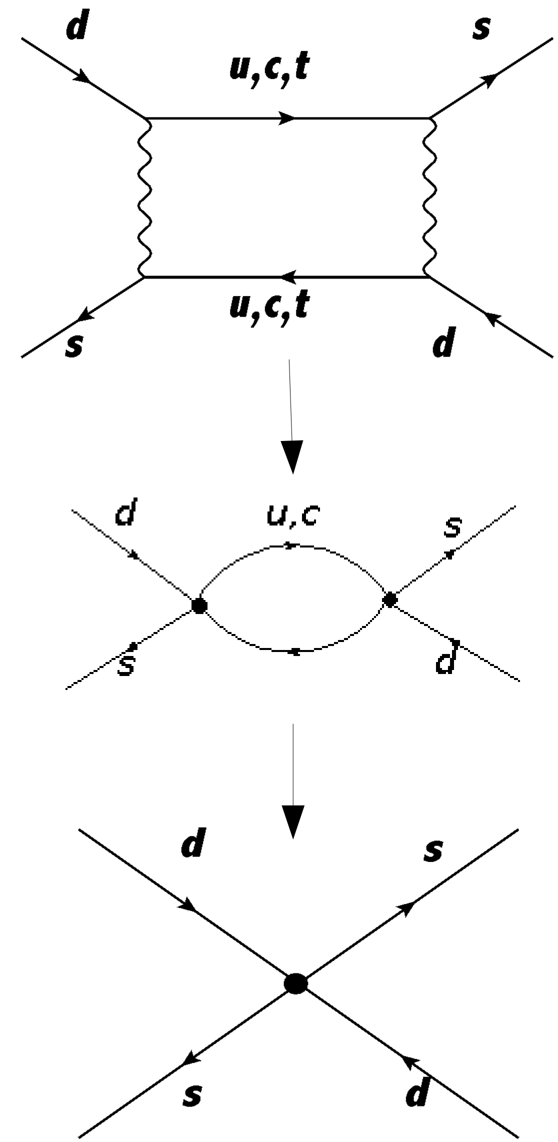
$K^0 - \overline{K}^0$ mixing

- Led to the prediction of charm quark
- Highly suppressed second-order weak process
- Important test of standard model

Perturbative calculation

- Weak convergence at charm scale
  36% difference between NLO and NNLO
- Long distance effect not included

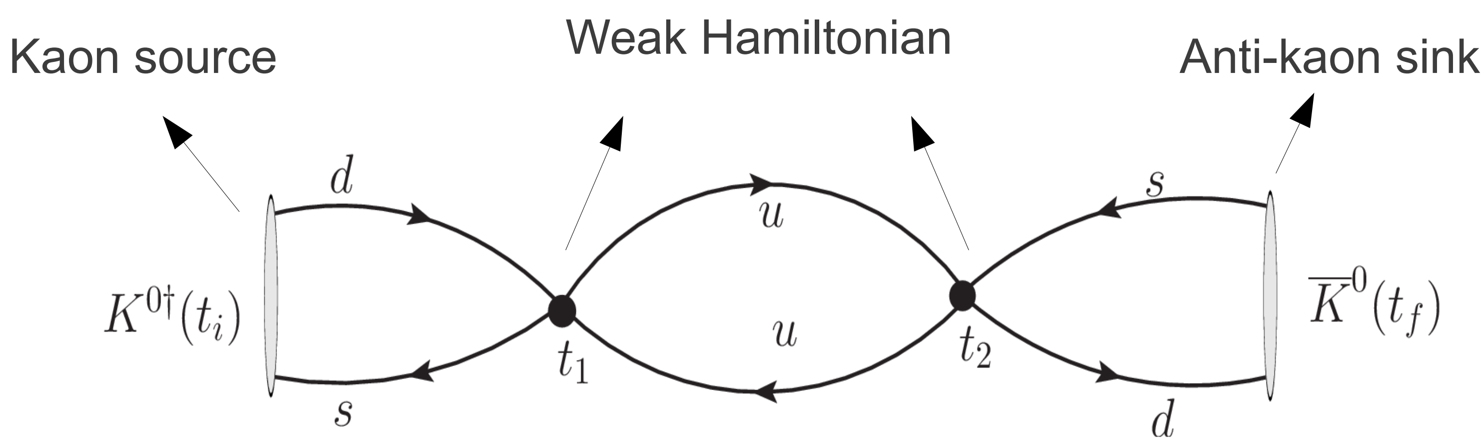
Lattice QCD is the only method to calculate the mass difference at percent level



Lattice four point function

Four point correlator :

$$G(t_i, t_1, t_2, t_f) = \langle \overline{K}^0(t_f) H_W(t_2) H_W(t_1) \overline{K}^0(t_i) \rangle$$

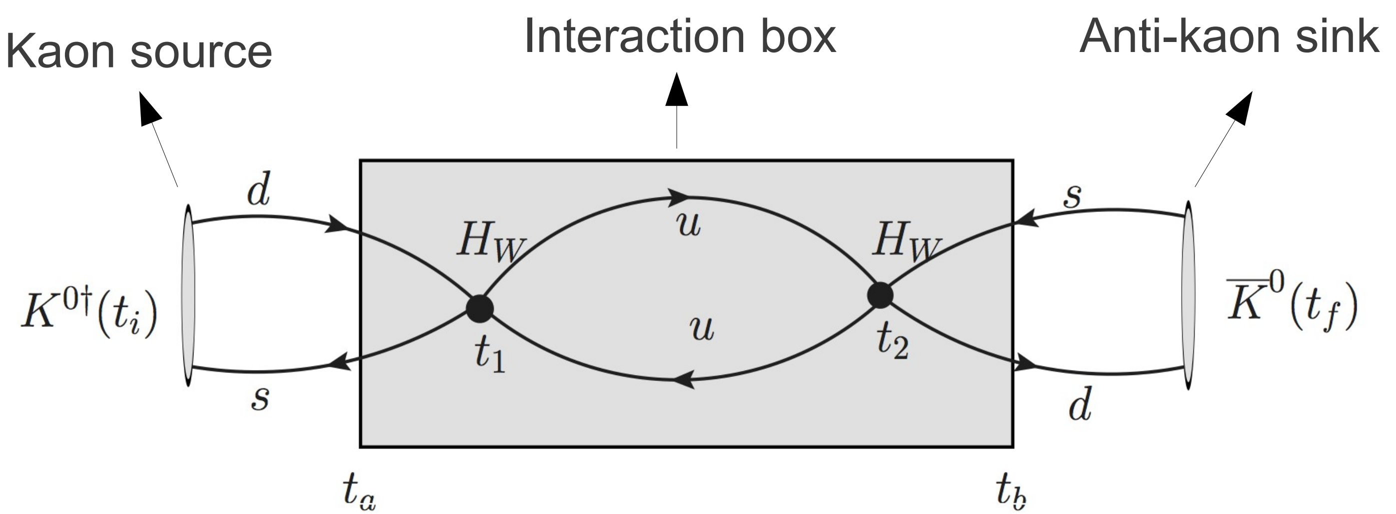


Time separation between kaon sources and weak Hamiltonian should be large enough to get a pure kaon state

Integrated correlator

Second-order integration inside a box :

$$\mathcal{A} = \frac{1}{2} \sum_{t_1=t_a}^{t_b} \sum_{t_2=t_a}^{t_b} \langle \overline{K}^0(t_f) H_W(t_2) H_W(t_1) \overline{K}^0(t_i) \rangle$$



For a given kaon separation, integrated correlator only depends on the size of interaction box $T = t_b - t_a + 1$

After inserting a sum of intermediate states one obtain :

$$\mathcal{A} = N_K^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \left(-T - \frac{1}{m_K - E_n} + \frac{e^{(m_K - E_n)T}}{m_K - E_n} \right)$$

$T=t_b-t_a+1$ is the interaction box size. Three terms inside the large parentheses :

1. Linear term, the coefficient of this term gives us ΔM_K
2. Constant term
3. Exponential term :
   - I). Exponential decreasing term, which can be neglected in large T
   - ii). Exponential increasing term come from pion and vacuum, which can be identified and subtracted

After the subtraction of exponential increasing term, a linear fit at large T will give us the K_L-K_s mass difference

Effective Hamiltonian

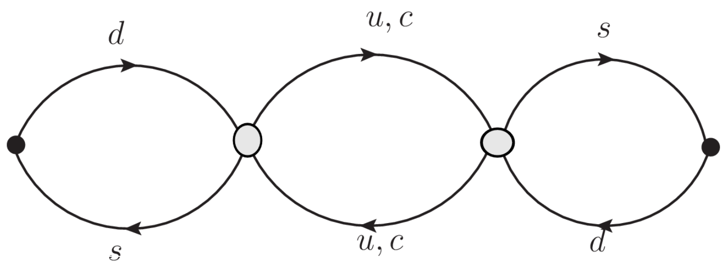
The four flavor ΔS=1 effective weak Hamiltonian :

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1^{qq'} + C_2 Q_2^{qq'})$$

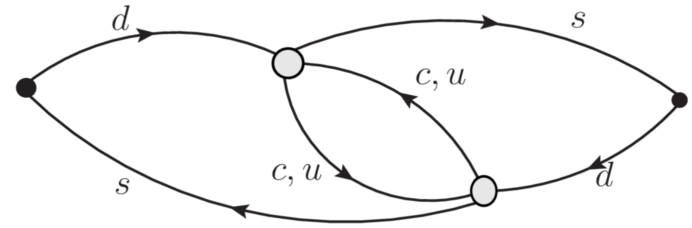
$$Q_1^{qq'} = (\bar{s}_i d_i)_L (q_j q'_j)_L$$
$$Q_2^{qq'} = (\bar{s}_i d_j)_L (q_j q'_i)_L$$

- Only include current-current operators, penguin operators are highly suppressed in four flavor theory
- Wilson coefficients are calculated using one-loop perturbation theory
- MS-bar operators and lattice operators are connected using RI/MOM non-perturbative renormalization method.

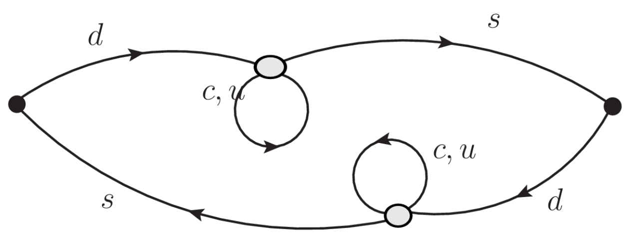
Four types of diagrams



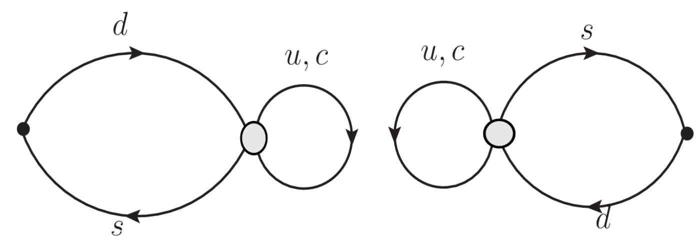
Type 1



Type 2



Type 3



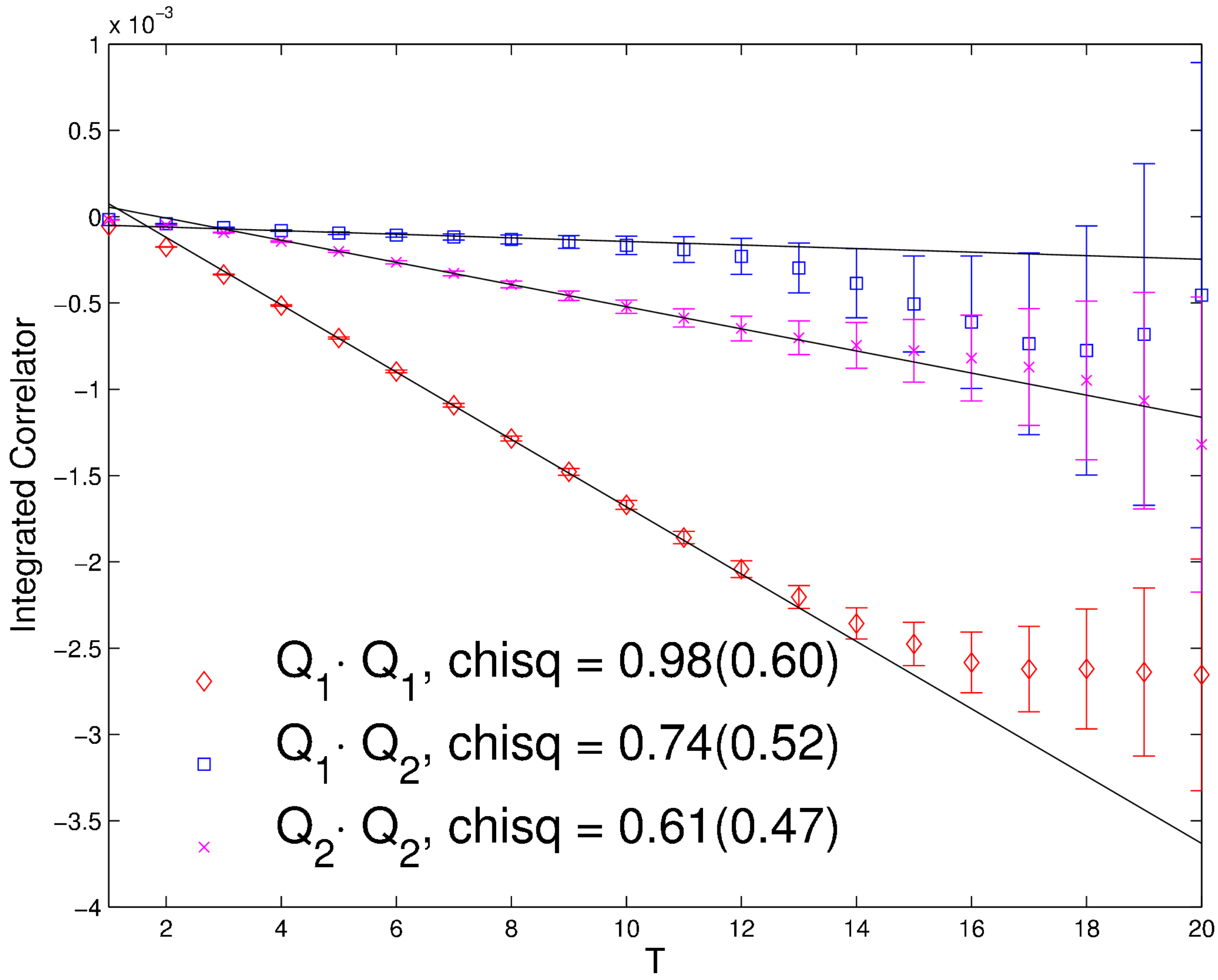
Type 4

In our previous work, only type 1 and type 2 diagrams are calculate. In this work, all the diagrams are included.

Simulation details

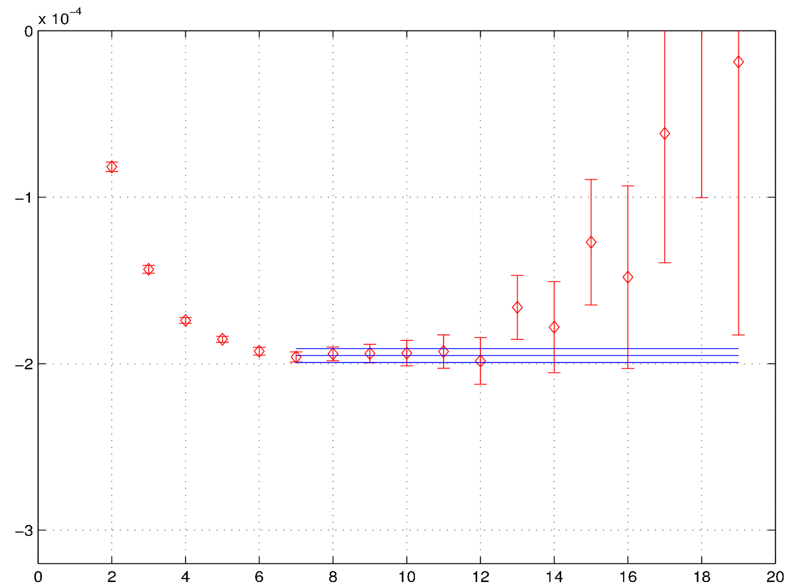
Ensemble	2+1f, DWF
L	2.74 fm
Configs	800
m_π	330 Mev
m_K	575 Mev
m_c	949 Mev
1/a	1.73 Gev

- Charm quark is quenched
- Use gauge-fixed wall sources for kaons
- The separation between two kaons is 31
- Use random source propagators to calculate the loop in type 3 and 4 diagrams
- Use low mode deflation to accelerate light quark inverter
- Take time translation average for all diagrams

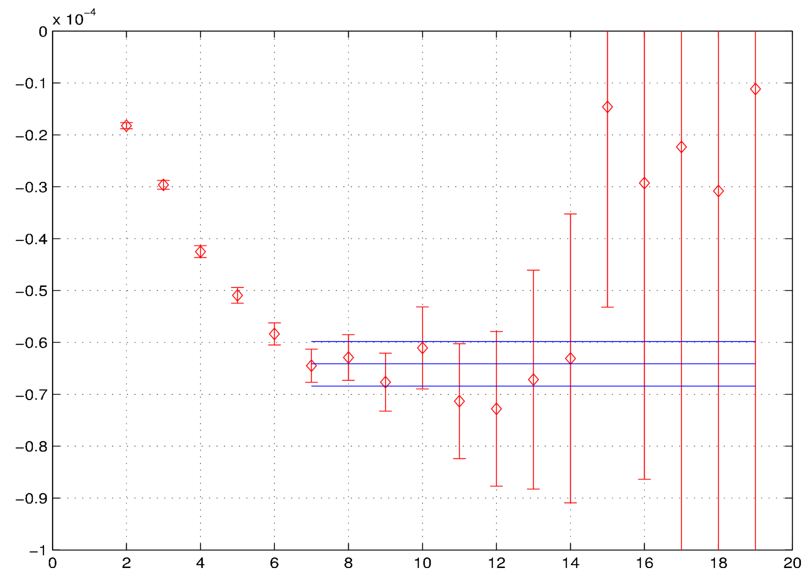


- Exponential increasing terms have been subtracted
- Straight lines show the fitting results from data points with T from 7 to 20

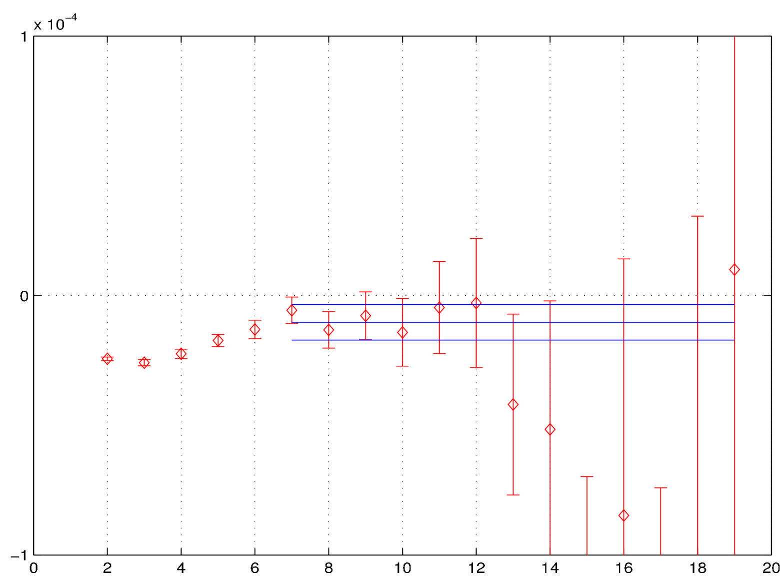
Effective slope



Q_1Q_1



Q_2Q_2



Q_1Q_2

The effective slope at time T is calculated from a correlated fit using data points at T-1, T and T+1

Fitting results

Two parameters :

- T_{min} , which is the starting fitting point
- Δ_K , which is the minimal separation between kaon sources and weak Hamiltonian

Δ_K	T_{min}	Q_1Q_1	Q_1Q_2	Q_2Q_2	Sum
6	7	0.754(42)	-0.16(15)	2.70(18)	3.30(34)
	8	0.755(45)	-0.10(17)	2.83(23)	3.49(40)
	9	0.758(53)	-0.16(22)	2.69(33)	3.28(55)

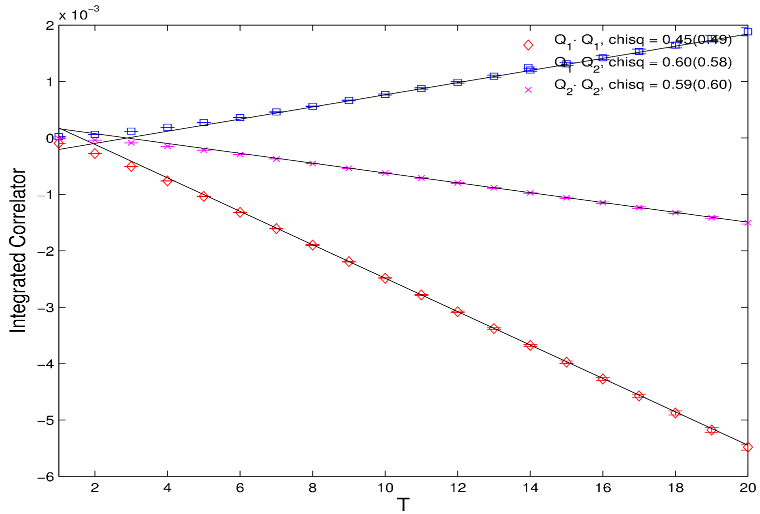
T_{min}	Δ_K	Q_1Q_1	Q_1Q_2	Q_2Q_2	Sum
7	6	0.754(42)	-0.16(15)	2.70(18)	3.30(34)
	7	0.755(42)	-0.18(15)	2.66(18)	3.23(34)
	8	0.751(42)	-0.18(15)	2.62(19)	3.18(35)

Units is 10^{-12} MeV.

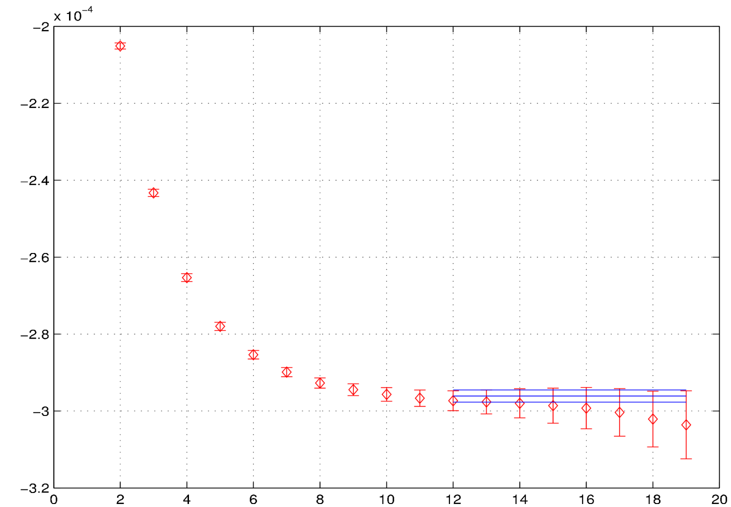
Experimental value is $3.483(6)*10^{-12}$ MeV.

Only Type 1 and 2

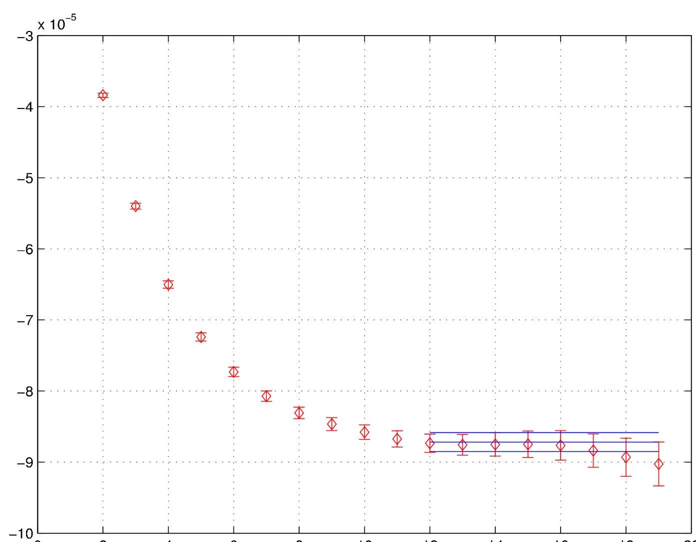
In our previous work, only type 1 and type 2 diagrams are included in the results. We also try it in this calculation to investigate the effect of type 3 and 4 diagrams



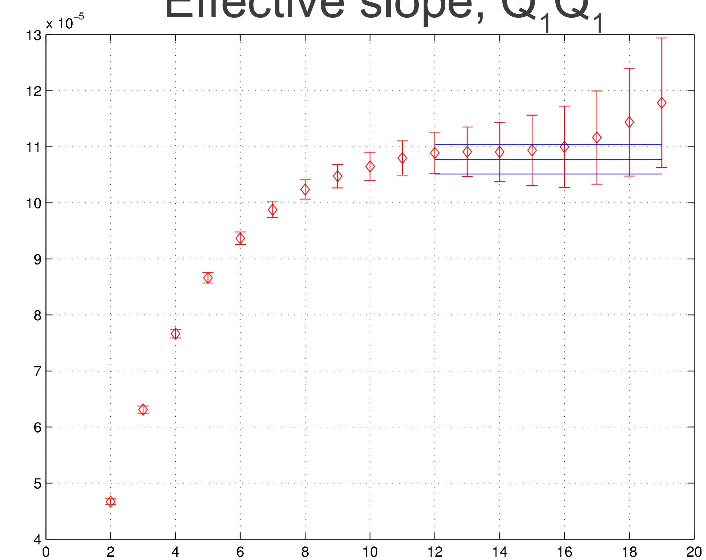
Integrated correlator



Effective slope, Q_1Q_1



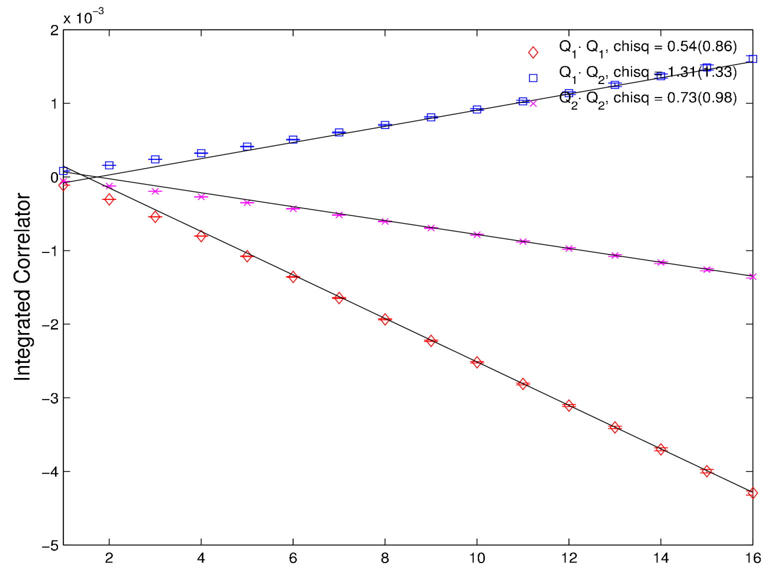
Effective slope, Q_2Q_2



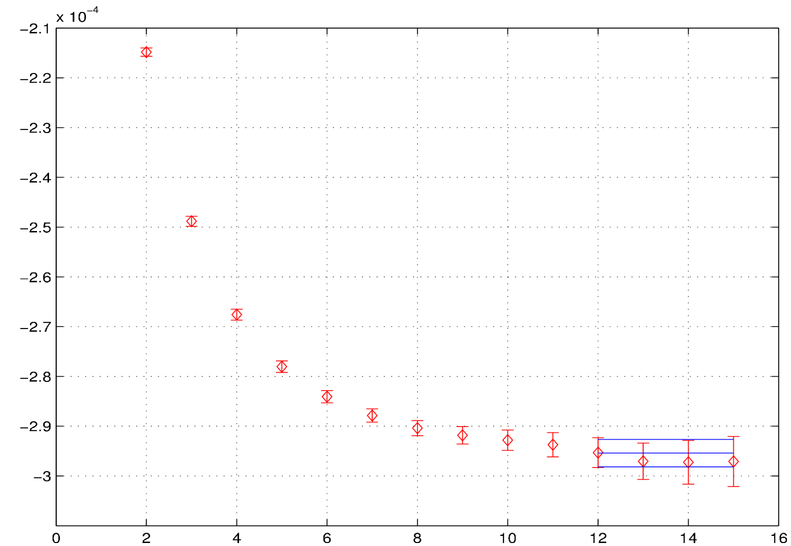
Effective slope, Q_1Q_2

Only Type 1, 2 and 3

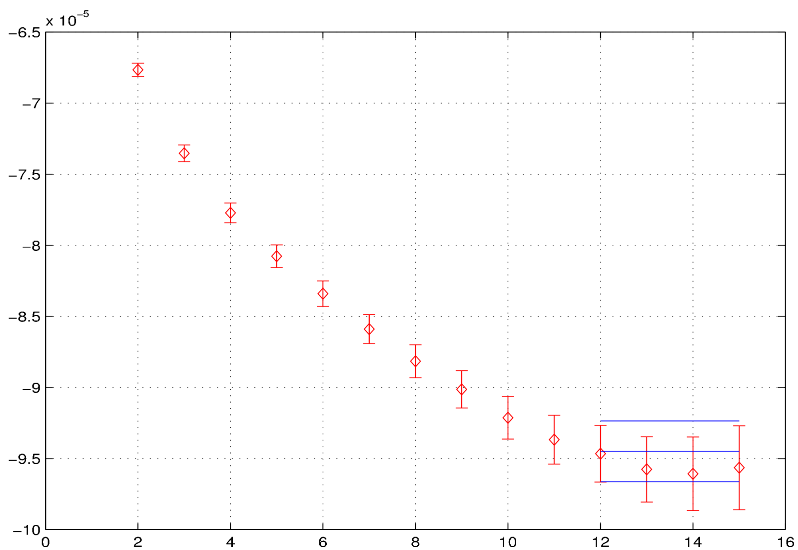
We also calculate the result without disconnected diagrams (type 4).



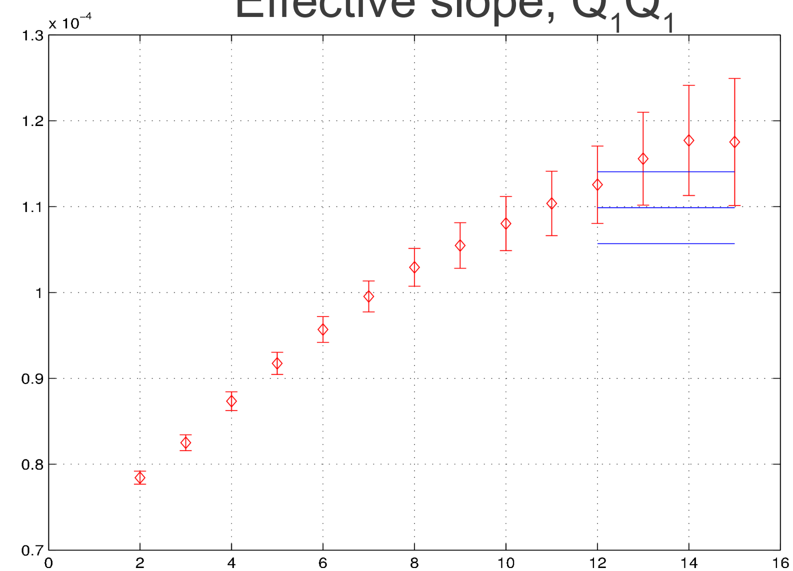
Integrated correlator



Effective slope, Q_1Q_1



Effective slope, Q_2Q_2



Effective slope, Q_1Q_2

Fitting results from different types of diagrams

Diagrams	Q_1Q_1	Q_1Q_2	Q_2Q_2	Sum
Type1,2	1.485(80)	1.567(38)	3.678(56)	6.730(96)
Type1,2,3	1.481(14)	1.598(61)	3.986(90)	7.06(15)
All	0.758(53)	-0.16(22)	2.69(33)	3.28(55)

Unit is 10^{-12} MeV.

- Type 3 diagrams doesn't change the results much
- Large cancellation between disconnected (type 4) diagrams and other type of diagrams

Conclusions

K_L-K_S mass difference has been done with unphysical kinematics.

Our result is :

$$\Delta M_K = 3.30(34)*10^{-12} \text{ MeV}$$

Experimental value is :

$$\Delta M_K = 3.483(6)*10^{-12} \text{ MeV}$$

We use 330 MeV pion and 949 MeV charm quark, so **the agreement may be just a coincidence**

Future plans :

- Physical kinematics, need to deal with the two pion intermediated states and finite volume correction
- 2+1+1 flavor fine lattice, control discretization error from charm quark