Eigenvalue spectrum of lattice $\mathcal{N} = 4$ super Yang-Mills

(work in progress)

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Outline

1. Introduction
2. $\mathcal{N} = 4$ SYM on the lattice
3. Techniques used
4. Pfaffian
5. Eigenvalue spectrum – general results
6. Mass anomalous dimension
7. Crude attempt at Patella's calculation
8. Conclusions

Introduction

- $\mathcal{N} = 4$ SYM is of great theoretical interest
  - Very elegant theory; maximal supersymmetry
  - AdS/CFT correspondence
  - BSM physics, phenomenology
  - Connections to string theory

- Heavily studied analytically, simulations are much more recent
- Putting supersymmetry on the lattice is a fascinating problem
- Need to find a way to deal with breaking of translational invariance
  - Twisting procedure, Dirac-Kähler fermions, simulate twisted theory
  - Orbifolding procedure; produces similar action
- This is, however, not a technical talk about how to formulate SUSY on the lattice

SYM on the lattice 0903.4881, 1110.5983

- Difficult to implement (SUSY algebra is an extension of Poincaré algebra)
- Technique used here is to preserve only one of the supercharges, and recover the rest in the continuum limit
  - Anticommutator of two supercharges generates spacetime translations
  - Preserving only one supercharge sidesteps this issue
  - Resulting algebra not broken by the discretisation
- Corresponds to a topologically twisted form of the original theory
- Restriction that we work with a target theory having 2^D supercharges
- This formulation of $\mathcal{N} = 4$ SYM on the lattice was examined in 1209.5285
- Dirac-Kähler fermion action that one obtains is similar to the standard staggered action (in free field theory at least)
- We vary two parameters, the 't Hooft Coupling λ and a mass parameter μ_{L} to control fluctuations (take $\mu_{\mathrm{L}} \to 0$ later)

Questions

1. Double-check: does this theory have a sign problem?
   - Properties of the Pfaffian we know it is 'small enough', but...
   - Can we infer what will happen for larger volumes?

2. What does the eigenspectrum look like?
   - Zero modes, trace modes
   - Vary 't Hooft coupling λ and breaking parameter μ_{L}

3. Can we measure the mass anomalous dimension?
   - Scaling of the individual eigenvalues with L
   - Crudely fitting to the eigenmode density 1204.4432

Reminder – the Arnoldi/Lanczos algorithm

- Krylov subspace methods – standard way to calculate N extremal eigenvalues of $M \times M$ matrix, $M \gg N$
- The Arnoldi algorithm depends on the factorisation

$$AV = VH + fe^{\mathrm{T}}; \quad V^{\dagger} f = 0$$

  where A is our operator, V is an $M \times N$ matrix of Ritz values and H is a Hessenberg matrix; f is a M-component vector
- Lanczos: A is Hermitian, H is tridiagonal
- Given the factorisation, the eigenvalues of H are calculated; these Ritz values approximate some eigenvalues of A
- Choose the 'best' eigenvalues in H, throw away the rest, repeat
- How reliable is the algorithm?
  - Difficult to define convergence for non-Hermitian operators
  - Gershgorin circle theorem underpins Hermitian results theoretically

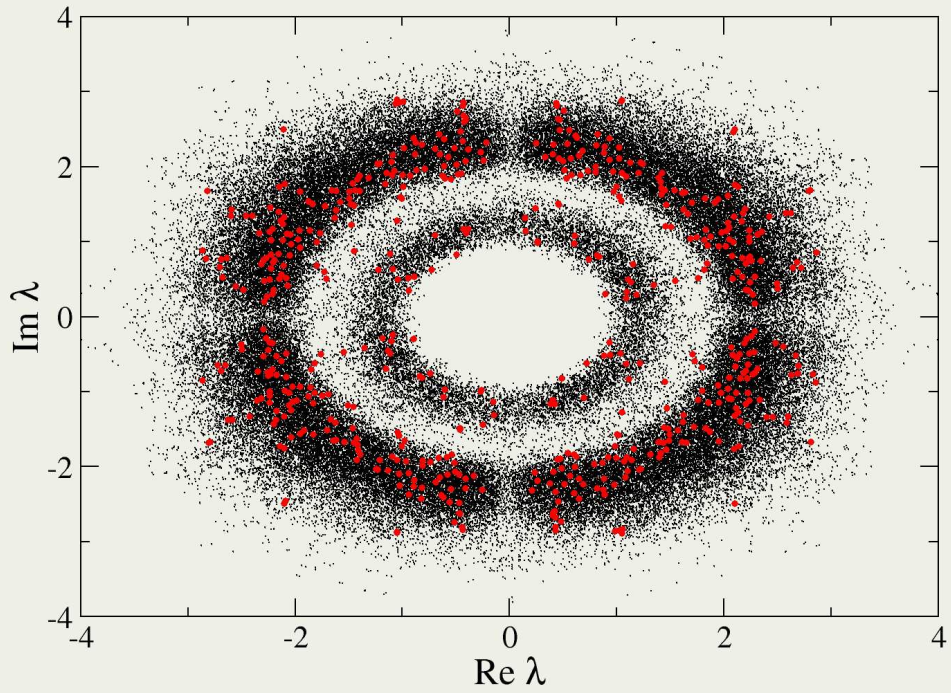
Technical details

- The simulation code is not (yet) parallelised (coming soon!) which restricts the volumes we can reach
- There is support for GPUs in the conjugate gradient algorithm but we instead make use of grid resources
  - Problem trivially parallelises; not time-critical
  - Therefore use FGI (Finnish Grid Infrastructure) rather than a traditional cluster
- Limited volumes mean we can get away with rudimentary eigenvalue calculations
  - We use ARPACK for the eigenvalue calculations[1]
  - In general, calculate 200 smallest eigenvalues, taking about 2 hours

[1] See Joni Suorsa's talk, parallel D (at 15:00, i.e. next talk) for a fully parallel, GPU-accelerated non-Hermitian eigenvalue calculation

Briefly: measuring the Pfaffian

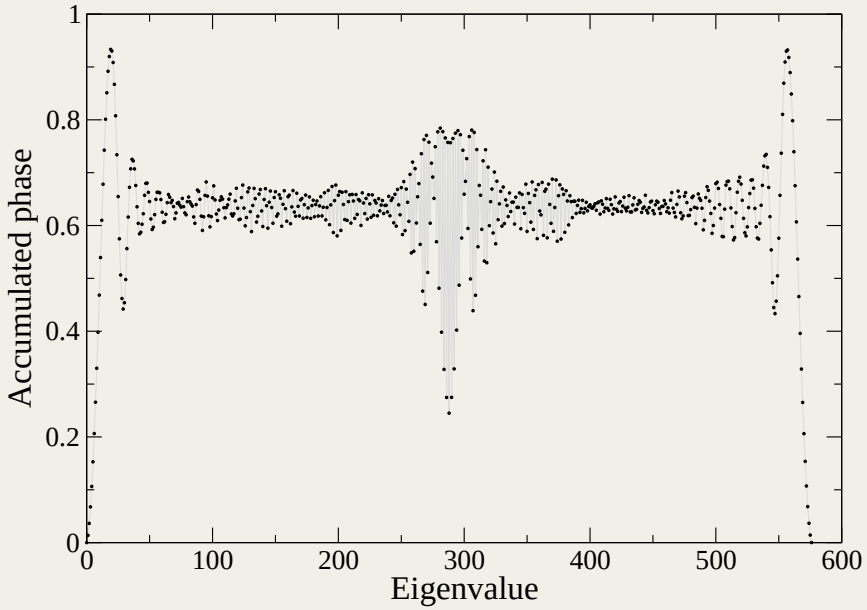
- We cannot take for granted that the theory does not have a sign problem!
- Can measure exactly in small volumes (see 1209.5285)



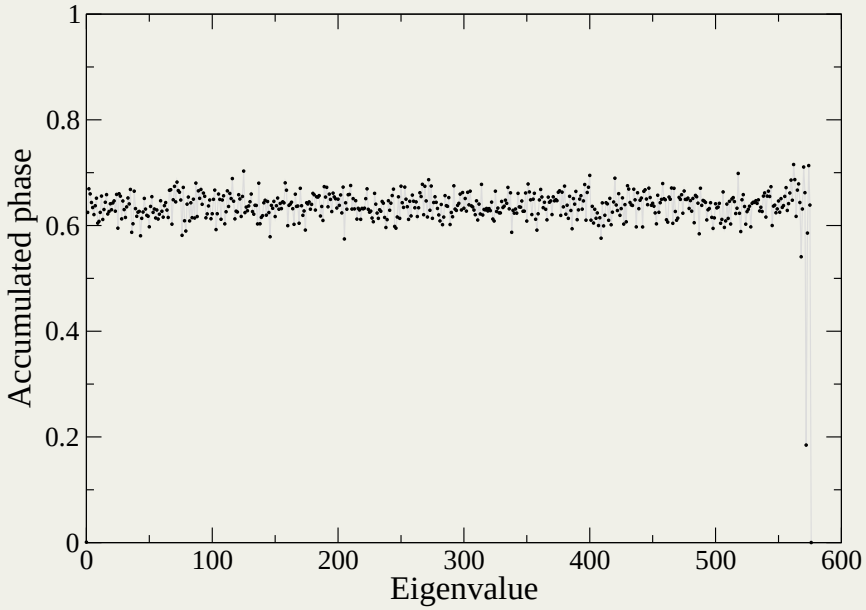
apbc, 3^4, 500 configurations, 1 configuration in red

- Looks nice, but phase only measurable if we know *full* eigenspectrum?
- Any structure at extremal eigenvalues that tells us about the sign, that we could examine at larger volumes?

Pfaffian analysis



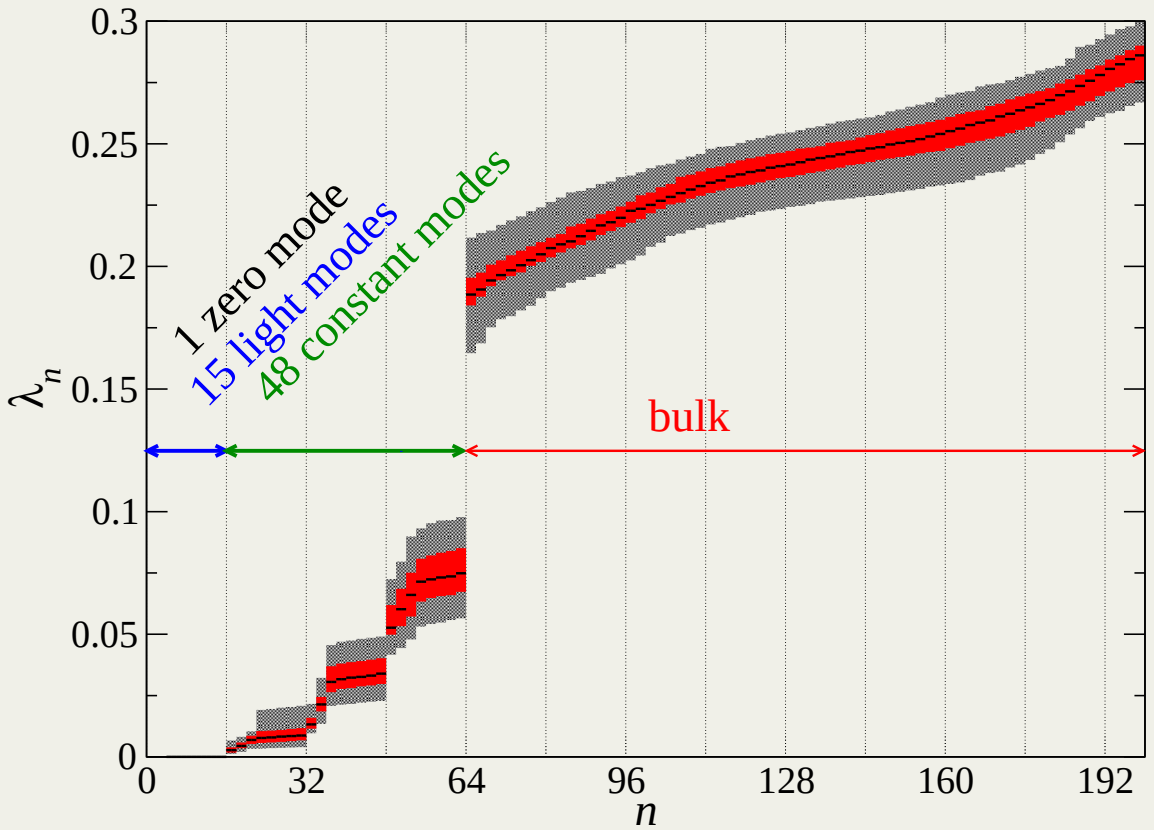
(a) by phase



(b) by magnitude

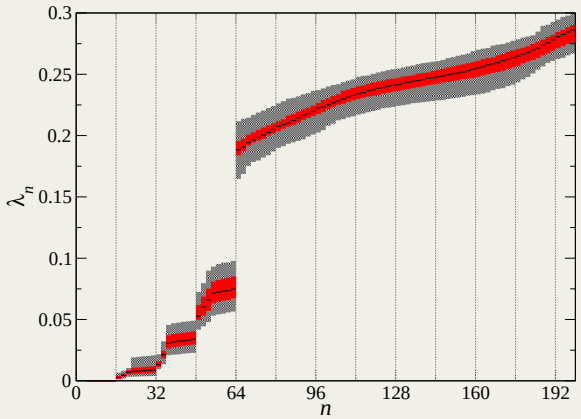
No helpful extremal structure, but at small volumes, the phase is unimportant.

Eigenvalues: what do we see?

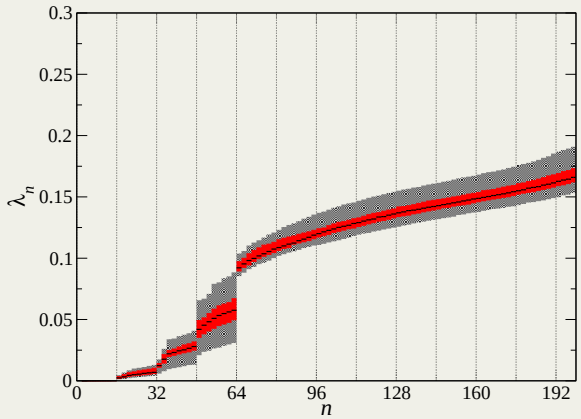


- $U(2) \to 2 \times 2 \times 16 = 48$ total would-be zero (trace) modes ($U(3)$ theory yields 144 such modes)
- We therefore discard these modes when carrying out further analysis

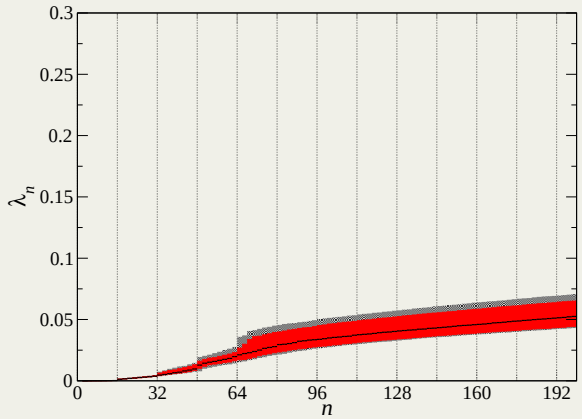
Eigenspectra



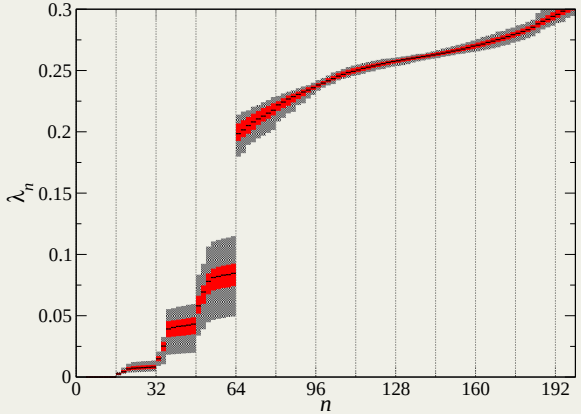
(c) $\lambda = 0.5$, $\mu_{\mathrm{L}} = 0.5$



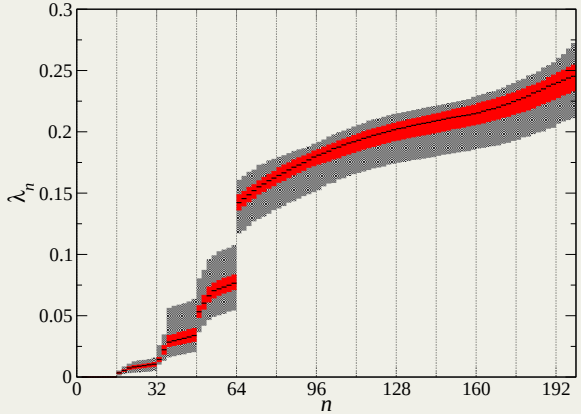
(d) $\lambda = 1.0$, $\mu_{\mathrm{L}} = 0.5$



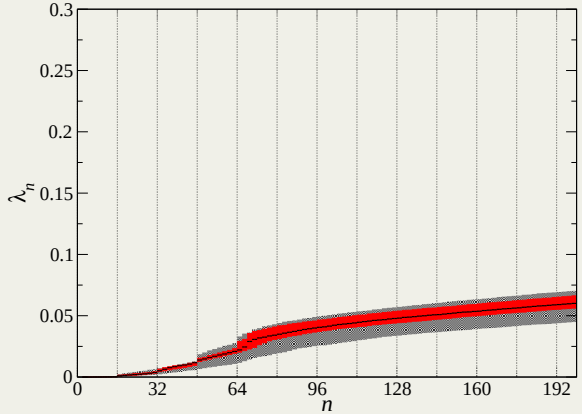
(e) $\lambda = 2.0$, $\mu_{\mathrm{L}} = 0.5$



(f) $\lambda = 0.5$, $\mu_{\mathrm{L}} = 1.0$



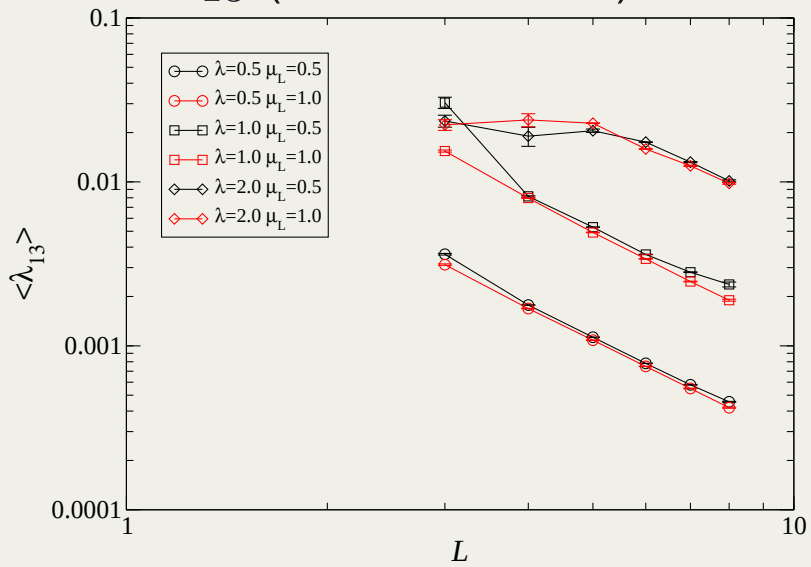
(g) $\lambda = 1.0$, $\mu_{\mathrm{L}} = 1.0$



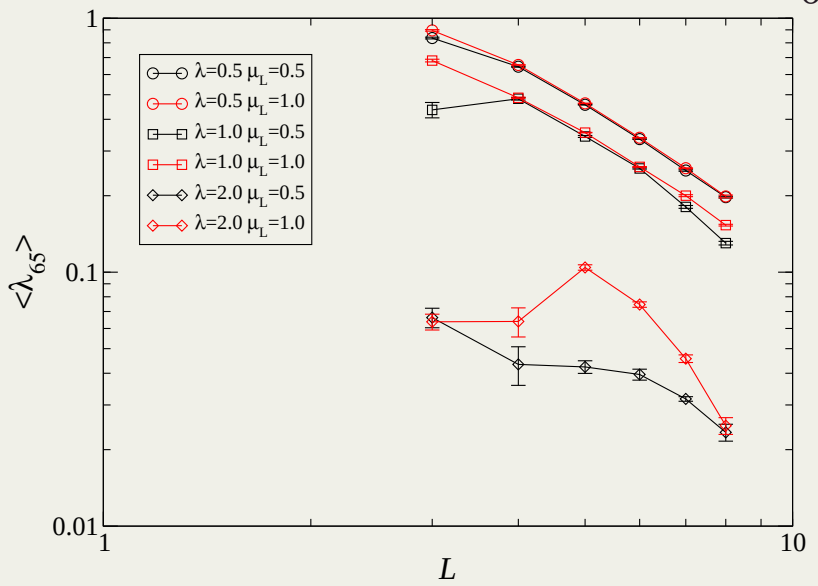
(h) $\lambda = 2.0$, $\mu_{\mathrm{L}} = 1.0$

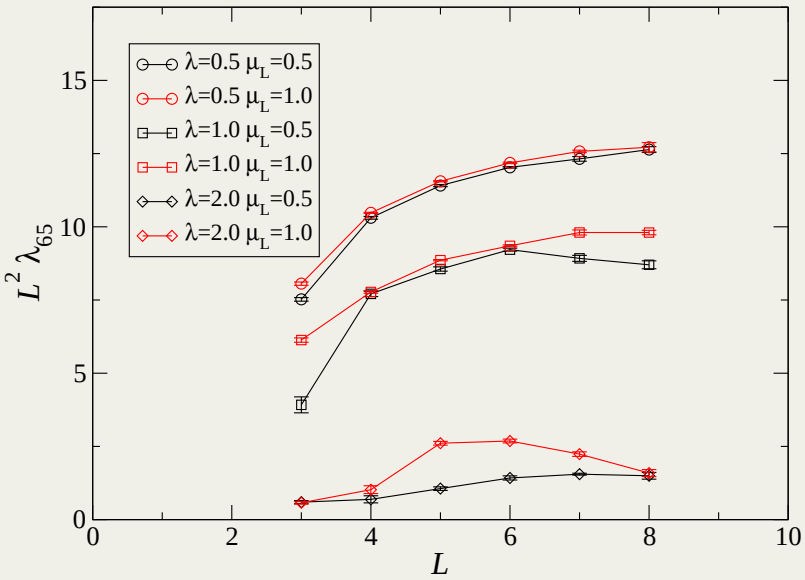
Scaling with L

λ_{13} (scales $\sim L^{-4}$):



λ_{65}:





Mass anomalous dimension

- Only observable in CFTs
- Important for phenomenology – can we reproduce the walking by perturbing the theory?
- Comparisons with continuum predictions
- Dependence of γ_* on λ?

Finite volume scaling of eigenvalues 0910.3072

- Fit $\langle \lambda_n(L) \rangle$ to $C_n L^{-y_n}$, with jackknifed errors
- Distinct change in behaviour at λ_{65}

	$\lambda = 0.5$		$\lambda = 1.0$	
n	$\mu_{\mathrm{L}} = 0.5$	$\mu_{\mathrm{L}} = 1.0$	$\mu_{\mathrm{L}} = 0.5$	$\mu_{\mathrm{L}} = 1.0$
9	3.97 ± 0.02	4.03 ± 0.01	3.94 ± 0.14	4.09 ± 0.03
17	1.57 ± 0.01	1.83 ± 0.03	1.75 ± 0.52	1.76 ± 0.24
25	1.64 ± 0.02	1.86 ± 0.03	1.83 ± 0.46	1.84 ± 0.21
33	2.33 ± 0.15	2.17 ± 0.04	2.29 ± 0.32	2.20 ± 0.06
41	2.41 ± 0.20	2.25 ± 0.03	2.36 ± 0.42	2.29 ± 0.07
49	2.27 ± 0.13	2.16 ± 0.06	2.21 ± 0.21	2.19 ± 0.02
57	2.18 ± 0.11	2.14 ± 0.05	2.13 ± 0.16	2.13 ± 0.02
65	1.74 ± 0.04	1.74 ± 0.02	1.82 ± 0.18	1.70 ± 0.01
73	1.75 ± 0.04	1.74 ± 0.03	1.84 ± 0.15	1.69 ± 0.02
81	1.75 ± 0.03	1.74 ± 0.03	1.83 ± 0.12	1.69 ± 0.03

Mass anomalous dimension from eigenmode density

- Working with the ansatz . . .

$$a^{-4}\bar{\nu}(\Omega) \approx a^{-4}\bar{\nu}_0 + A\left[(a\Omega)^2 - (am)^2\right]^{2/(1+\gamma_*)}$$

- . . . where $\bar{\nu}(\Omega)$ is the integrated spectral density . . .

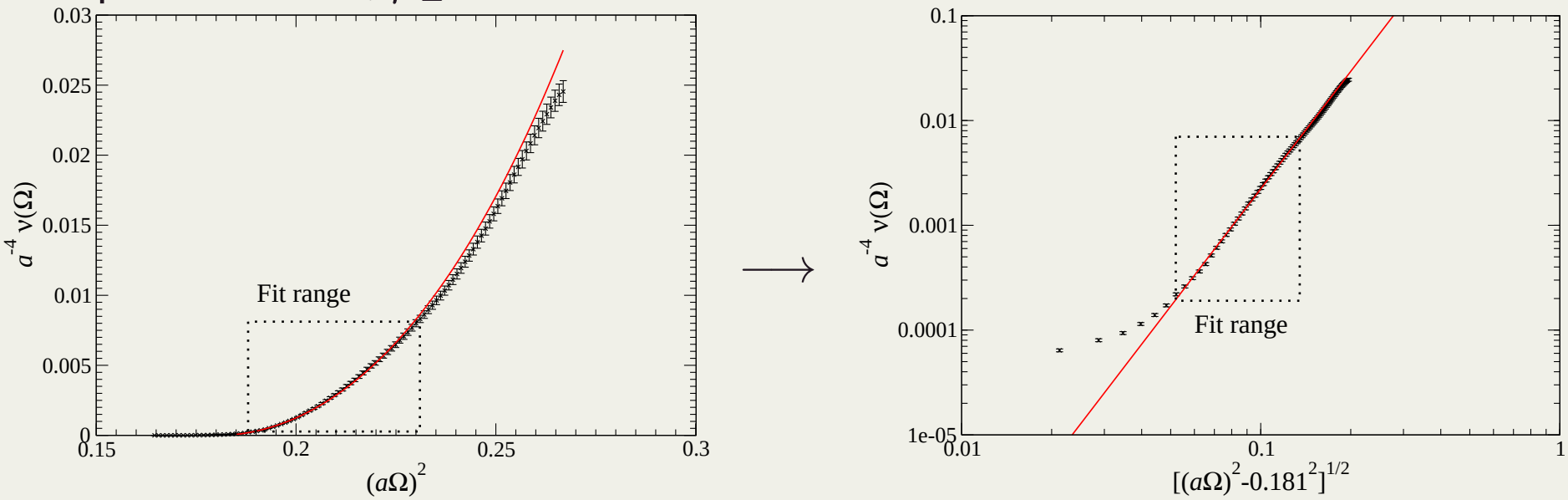
$$\bar{\nu}(\Omega) = 2\int_0^{\sqrt{\Omega^2-m^2}} \rho(\omega)\, d\omega; \qquad \rho(\omega) = \lim_{V\to\infty} \frac{1}{V} \sum_k \langle \delta(\omega - \omega_k)$$

- . . . we optimistically fit our largest lattice data ($L = 8$ – albeit ‘staggered’) to the ansatz and see if anything meaningful results
- Discard first 64 eigenvalues (zero modes, trace, constant)
- The term $a^{-4}\bar{\nu}_0$ is (therefore) consistent with zero in all our fits
- γ_* can also be obtained by fitting to our data for $\langle \lambda_n(L) \rangle$

Eigenmode density fitting – results

1. Fit to all possible ranges
2. Find the fitting range which yields χ^2/dof closest to unity
3. Examine the effect on γ_* of varying the lower and upper bounds
4. If the results lie on a ‘plateau’, we use that value of γ_*

Example – $\lambda = 0.5$, $\mu_{\mathrm{L}} = 0.5$:



- For $\lambda = 0.5$, $\mu_{\mathrm{L}} = 0.5$, we get $\gamma_* \approx 0.07$
- . . . which is consistent with fitting to $\langle \lambda_{65}(L) \rangle$, which yields $\gamma_* = 0.08 \pm 0.01$

Conclusions

- In this talk we have looked briefly at
  - The Pfaffian
  - Basic properties of the eigenspectrum; scaling with L
  - Fitting the spectral density

- As far as we can tell the phase of the Pfaffian is close to zero...
- ...unfortunately, to say for sure, we would need to measure it exactly (hard in large volumes)

- We see zero modes, trace modes (that would disappear in continuum) and then the actual physics
- We have (optimistically) attempted to measure the anomalous mass dimension by fitting the spectral density
  - Although the volume is still relatively small
  - And our understanding of the trace modes is still very rudimentary

  – typical values are around $\gamma_* \approx 0.1$.

Still to do

- Parallelised code
  - We have reached the limit of what can be done with single-core CPU code, even with the trivial parallelism used here (roughly 10k hours per parameter choice at $L = 8$, total use $>$ 100k hours)
  - Larger volumes
  - Better performance without needing GPU acceleration?
- Better control of fits to spectral density
- Stronger coupling
  - Anomalous dimensions seem to be still quite small for $\lambda = 1$
  - Results not of sufficiently high quality to draw conclusions for $\lambda = 2$