$SU(3)$ flavour symmetry breaking and charmed states

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– QCDSF-UKQCD Collaboration –

Edinburgh – Regensburg – RIKEN (Kobe) – Leipzig – FZ (Jülich) – Liverpool – DESY – Hamburg – Adelaide

[Lattice 2013, Mainz, Germany]

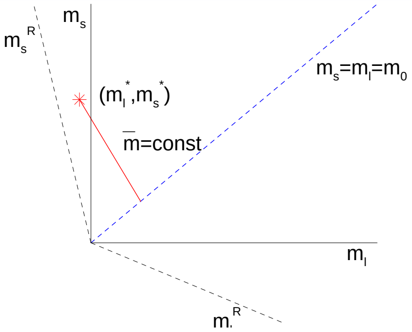
QCDSF related talks with $2+1$ flavours:

- James Zanotti — Parallels 8B (Hadron Structure)
  $SU(3)$ flavour breaking and baryon structure
- Ashley Cooke — Poster
  Flavour Symmetry Breaking in Octet Hyperon Matrix Elements
- Paul Rakow — Parallels 10C (Hadron Spectroscopy and Interactions)
  The Hadronic Decays of Decuplet Baryons
- Holger Perlt — Parallels 5C (Standard Model Parameters and Renormalization)
  Perturbatively improving renormalization constants
- Gerrit Schierholz — Poster
  Dynamical $2+1$ flavor QCD + QED

Introduction

- Background:
  - Given $2+1$ simulations (at quark masses larger than physical quark masses), how can we usefully approach the physical point?
  - Possibility: $SU(3)$ flavour expansion about flavour symmetric line
  - Mass 'fan' plots
- Extend expansion to PQ quark masses (ie valence quarks $\neq$ sea quarks)
- (quenched) charm quark
- Open charm masses
- Conclusions

Many paths to approach the physical point eg $m_u = m_d \equiv m_l$



QCDSF strategy: extrapolate
from a point on the $SU(3)_F$ flavour symmetry line to the physical point

$$(m_0, m_0) \longrightarrow (m_l^*, m_s^*)$$

Choice here: keep the singlet quark mass $\overline{m}$ constant

$$\overline{m} = m_0 = \frac{1}{3}\,(2m_l + m_s)$$

QCDSF strategy

[arXiv:1102.5300]

- develop $SU(3)$ flavour symmetry breaking expansion for hadron masses
- expansion in:

$SU(3)$ flavour symmetric point $\delta m_q = 0$

$$\delta m_q = m_q - \overline{m}\,, \quad \overline{m} = \tfrac{1}{3}(m_u + m_d + m_s) = m_0$$

  - expansion coefficients are functions of $\overline{m}$
  - trivial constraint

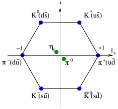
$$\delta m_u + \delta m_d + \delta m_s = 0$$

- path called 'unitary line' as expand in both sea and valence quarks

$SU(3)$ flavour symmetry breaking expansions

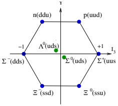
- octet pseudoscalar mesons:

$$
\begin{aligned}
M^2(a\bar{b}) \quad = \quad & M_{0\pi}^2 + \alpha(\delta m_a + \delta m_b) \\
& + \beta_0 \frac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
& + \beta_1(\delta m_a^2 + \delta m_b^2) + \beta_2(\delta m_a - \delta m_b)^2 \\
& + \ldots \qquad [a, b = u, d, s \text{ (outer ring)}]
\end{aligned}
$$



- octet baryons:

$$
\begin{aligned}
M^{N\,2}(aab) \quad = \quad & M_{0N}^2 + A_1(2\delta m_a + \delta m_b) + A_2(\delta m_b - \delta m_a) \\
& + B_0 \frac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
& + B_1(2\delta m_a^2 + \delta m_b^2) + B_2(\delta m_b^2 - \delta m_a^2) + B_3(\delta m_b - \delta m_a)^2 \\
& + \ldots \qquad [a, b = u, d, s \text{ (outer ring)}]
\end{aligned}
$$



stable under strong ints.

$$
\begin{aligned}
M^{\Lambda\,2}(aab) \quad = \quad & M_{0\Lambda}^2 + A_1(2\delta m_a + \delta m_b) - A_2(\delta m_b - \delta m_a) \\
& + B_0 \delta m_l^2 \\
& + B_1(2\delta m_a^2 + \delta m_b^2) - B_2(\delta m_b^2 - \delta m_a^2) + B_4(\delta m_b - \delta m_a)^2 \\
& + \ldots \qquad [a, b = l, s \text{ (when no } \Lambda^0 - \Sigma^0 \text{ mixing)}]
\end{aligned}
$$

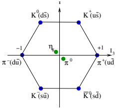
Main observation:

- Provided $\overline{m}$ kept constant, then expansion coefficients remain unaltered whether
  - $1 + 1 + 1$
  - $2 + 1$
- Opens possibility of determining quantities that depend on $1 + 1 + 1$ flavours (ie pure QCD isospin breaking effects) from just $2 + 1$ simulations

Defining the scale – using singlet quantities

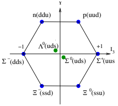
- pseudoscalar mesons (centre of mass):

$$X_\pi^2 = \frac{1}{6}(M_{K^+}^2 + M_{K^0}^2 + M_{\pi^+}^2 + M_{\pi^-}^2 + M_{\bar{K}^0}^2 + M_{K^-}^2) = (0.4116\ \mathrm{GeV})^2$$
$$= M_{0\pi}^2 + \left(\frac{1}{6}\beta_0 + \frac{2}{3}\beta_1 + \beta_2\right)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) = M_{0\pi}^2 + O(\delta m_q^2)$$



- octet baryons (centre of mass):

$$X_N^2 = \frac{1}{6}(M_p^2 + M_n^2 + M_{\Sigma^+}^2 + M_{\Sigma^-}^2 + M_{\Xi^0}^2 + M_{\Xi^-}^2) = (1.160\ \mathrm{GeV})^2$$
$$= M_0^2 + \frac{1}{6}(B_0 + B_1 + B_3)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) = M_0^2 + O(\delta m_q^2)$$



stable under strong ints.

- gluonic quantities: $X_{t_0}^2 = 1/t_0, \ldots$
- other possibilities:

$X_\Lambda^2 = \frac{1}{2}(M_\Sigma^2 + M_\Lambda^2)$, $X_\rho^2 = \frac{1}{6}(M_{K^{*+}}^2 + M_{K^{*0}}^2 + M_{\rho^+}^2 + M_{\rho^-}^2 + M_{\bar{K}^{*0}}^2 + M_{K^{*-}}^2), \ldots$

- all singlet quantities

$$X_S^2 = \# + \#(\delta m_q^2)$$

(almost) constant [$\Longrightarrow$ scale determination]

- form dimensionless ratios (within a multiplet):

$$\tilde{M}^2 \equiv \frac{M^2}{X_S^2}, \quad S = \pi, N, \ldots, \qquad \tilde{A}_i \equiv \frac{A_i}{M_0^2}, \ldots \quad \text{in expansions}$$

Lattice

- $O(a)$ NP improved clover action
  - tree level Symanzik glue
  - mildy stout smeared $2+1$ clover fermion
  - $\beta = 5.50\,[5.80]$, $32^3 \times 64$
- $$m_q = \frac{1}{2}\left(\frac{1}{\kappa_q} - \frac{1}{\kappa_{0c}}\right) \qquad \kappa_{0c} \text{ is chiral limit along symmetric line}$$

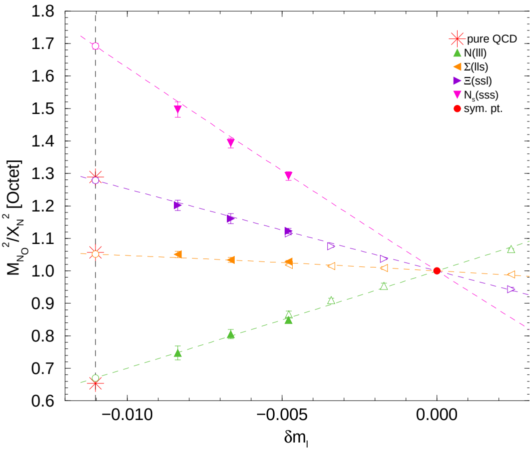
giving

$$m_0 = \frac{1}{2}\left(\frac{1}{\kappa_0} - \frac{1}{\kappa_{0c}}\right) = \overline{m} = \frac{1}{3}(2m_l + m_s) = \frac{1}{2}\left(\frac{2}{\kappa_l} + \frac{1}{\kappa_s} - \frac{1}{\kappa_{0c}}\right)$$

So $1/\kappa_{0c}$ cancels: given κ_0 and κ_l gives κ_s

$$\delta m_q = m_q - m_0 = \frac{1}{2}\left(\frac{1}{\kappa_q} - \frac{1}{\kappa_0}\right)$$

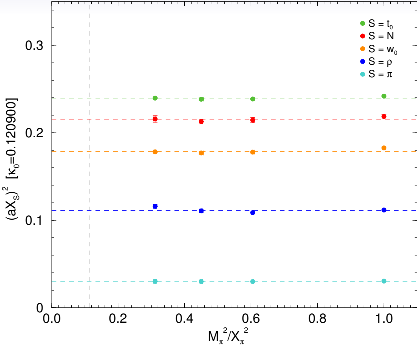
‘Fan’ plot – no visible curvature

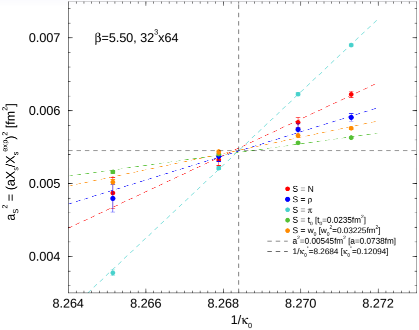


- $2+1$, $q = l, s$, $\delta m_u = \delta m_d = \delta m_l$ $\delta m_s = -2\delta m_l$
- $O(a)$-improved clover fermions; $32^3 \times 64$ lattices [fitted, filled pts]
- $\delta m_l = m_l - \overline{m}$
- $\overline{m} = \text{const.}$ [to find need to tune]
- $M_N = M^N(lll'')$, $M_\Sigma = M^N(lls)$, $M_\Xi = M^N(ssl)$, $M_{N_s} = M^N(sss'')$ [PQ]

Use the pseudoscalar fan plot to determine the physical quark mass: δm_l^*

Scale determination





- $X_{t_0}^2$, $X_{w_0}^2$, X_π^2, X_ρ^2, $X_N^2 \approx X_\Lambda^2$ along the unitary line [$M_\pi \sim 410\,\mathrm{MeV}$ - $260\,\mathrm{MeV}$]
- as constant down to physical point use X_N^{exp} to determine scale

$$a_S^2 = \frac{(aX_S)^2}{X_S^{\mathrm{exp}\ 2}}$$

- Goal: vary m_0 – when the a_S cross (ie independent of S) gives common scale a
- at this 'magic' point find

$$\begin{aligned} a &\approx 0.074\,\mathrm{fm} \\ \sqrt{t_0}^{\mathrm{exp}} &\approx 0.153\,\mathrm{fm} \\ w_0^{\mathrm{exp}} &\approx 0.179\,\mathrm{fm} \end{aligned}$$

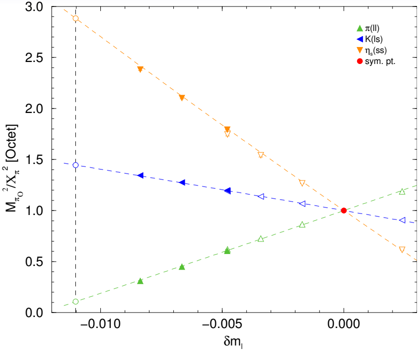
Reaching the charm quark mass range

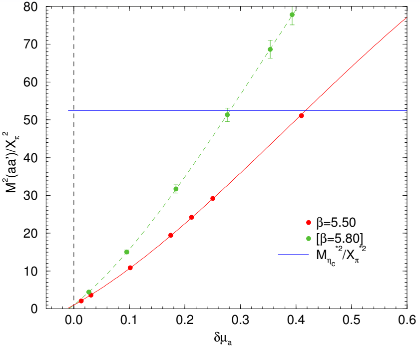
- unitary range rather small so introduce PQ partially quenching (ie valence quark masses $\neq$ sea quark masses) and NNLO
- eg pseudoscalar meson octet

$$
\begin{aligned}
M^2(a\overline{b}) \quad = \quad & M^2_{0\pi} + \alpha(\delta\mu_a + \delta\mu_b) \\
& + \beta_0 \tfrac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + \beta_1(\delta\mu_a^2 + \delta\mu_b^2) + \beta_2(\delta\mu_a - \delta\mu_b)^2 \\
& + \gamma_0 \delta m_u \delta m_d \delta m_s + \gamma_1(\delta\mu_a + \delta\mu_b)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
& + \gamma_2(\delta\mu_a + \delta\mu_b)^3 + \gamma_3(\delta\mu_a + \delta\mu_b)(\delta\mu_a - \delta\mu_b)^2
\end{aligned}
$$

- $\delta\mu_q = \mu_q - \overline{m} \quad q \in \{a, b, \ldots\}$; valence quarks of arbitrary mass, μ_q
- expansion coefficients: $M^2_{0\pi}(\overline{m})$, $\alpha(\overline{m})$, $\ldots$
- mixed sea/valence mass terms
- unitary limit: $\delta\mu_q \to \delta m_q$

$2+1$ joint fits





- unitary line data
  $[\mu_q \to m_q]$
- no visible curvature

- PQ data
  $[\delta m_l = 0]$
- illustration, to avoid 3-dim plot
  a' distinct quark but same mass as a

$$\tilde{M}^2(aa') = 1 + 2\delta\mu_a\tilde{\alpha}_1 + 2\tilde{\beta}_1\delta\mu_a^2 + 8\tilde{\gamma}_2\delta\mu_a^3$$

Very different x-scales involved

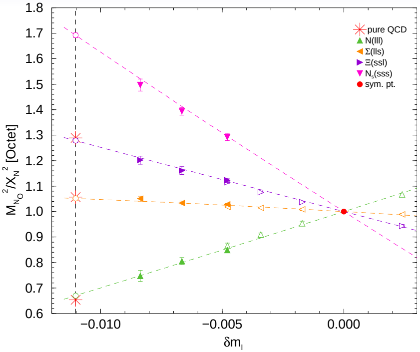
Octet baryon expansion coefficients

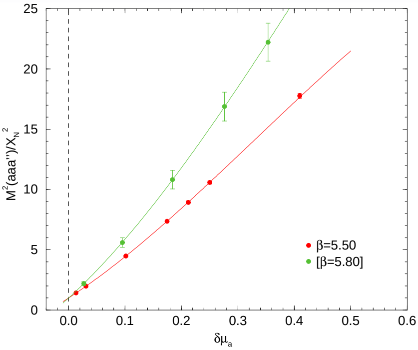
$$
\begin{aligned}
M^{N\,2}(aab) \quad = \quad & M_{0N}^2 + A_1(2\delta\mu_a + \delta\mu_b) + A_2(\delta\mu_b - \delta\mu_a) \\
& + B_0\frac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + B_1(2\delta\mu_a^2 + \delta\mu_b^2) + B_2(\delta\mu_b^2 - \delta\mu_a^2) + B_3(\delta\mu_b - \delta\mu_a)^2 \\
& + C_0\delta m_u \delta m_d \delta m_s + [C_1(2\delta\mu_a + \delta\mu_b) + C_2(\delta\mu_b - \delta\mu_a)](\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
& + C_3(\delta\mu_a + \delta\mu_b)^3 + C_4(\delta\mu_a + \delta\mu_b)^2(\delta\mu_a - \delta\mu_b) \\
& + C_5(\delta\mu_a + \delta\mu_b)(\delta\mu_a - \delta\mu_b)^2 + C_6(\delta\mu_a - \delta\mu_b)^3
\end{aligned}
$$

$$
\begin{aligned}
M^{\Lambda\,2}(aa'b) \quad = \quad & M_{0\Lambda}^2 + A_1(2\delta\mu_a + \delta\mu_b) - A_2(\delta\mu_b - \delta\mu_a) \\
& + B_0\frac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + B_1(2\delta\mu_a^2 + \delta\mu_b^2) - B_2(\delta\mu_b^2 - \delta\mu_a^2) + B_4(\delta\mu_b - \delta\mu_a)^2 \\
& + C_0\delta m_u \delta m_d \delta m_s + [C_1(2\delta\mu_a + \delta\mu_b) - C_2(\delta\mu_b - \delta\mu_a)](\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
& + C_3(\delta\mu_a + \delta\mu_b)^3 + (C_4 - 2C_3)(\delta\mu_a + \delta\mu_b)^2(\delta\mu_a - \delta\mu_b) \\
& + C_7(\delta\mu_a + \delta\mu_b)(\delta\mu_a - \delta\mu_b)^2 + C_8(\delta\mu_a - \delta\mu_b)^3
\end{aligned}
$$

- similar procedure

$2+1$ joint fits





- unitary line data $[\mu_q \to m_q]$
- no visible curvature

- PQ data (both N and Λ) $[\delta m_l = 0]$
- illustration, to avoid 3-dim plot

  a' distinct quark but same mass as a

$$\tilde{M}^2(aaa'') = 1 + 3\tilde{A}_1\,\delta\mu_a + 3\tilde{B}_1\,\delta\mu_a^2 + 8\tilde{C}_3\,\delta\mu_a^3$$

Very different x-scales involved

Method

- Use PQ data to determine expansion coefficients
  - α, β, γ – pseudoscalar octet
  - A, B, C – baryon octet
- Determine physical quark masses

$$\delta m_u^*\,,\quad \delta m_d^*\,,\quad \delta m_s^*\,,\quad \delta \mu_c^*$$

by fitting to (eg)

$$M_{\pi^+}^{\mathrm{exp}}(u\overline{d})\,,\quad M_{K^+}^{\mathrm{exp}}(u\overline{s})\,,\quad M_{\eta_c}^{\mathrm{exp}}(c\overline{c})$$

[together with κ_0, so 4 inputs]

Open Charm masses

Can describe states with same wavefunction (and hence expansion) as previously used

- pseudoscalar mesons

$$D^0(c\overline{u}), \quad D^+(c\overline{d}), \quad D_s^+(c\overline{s})$$

which all have the wavefunction

$$\mathcal{M} = \overline{q}\gamma_5 c \qquad q = u, d, s$$

- baryons
  - single open charm ($C = 1$) states

$$\Sigma_c^{++}(uuc), \quad \Sigma_c^0(ddc), \quad \Omega_c^0(ssc)$$

which all have the wavefunction

$$\mathcal{B} = \epsilon(q^T C\gamma_5 c)q \qquad q = u, d, s$$

[also if $m_u = m_d = m_l$, then in addition as no mixing $\Sigma_c^+(ll'c)$ ($= \Sigma_c^{++}(ll'c) = \Sigma_c^0(ll'c)$) and $\Lambda_c^+(ll'c)$]

  - double open charm ($C = 2$) states

$$\Xi_{cc}^{++}(ccu) \quad \Xi_{cc}^+(ccd) \quad \Omega_{cc}^+(ccs)$$

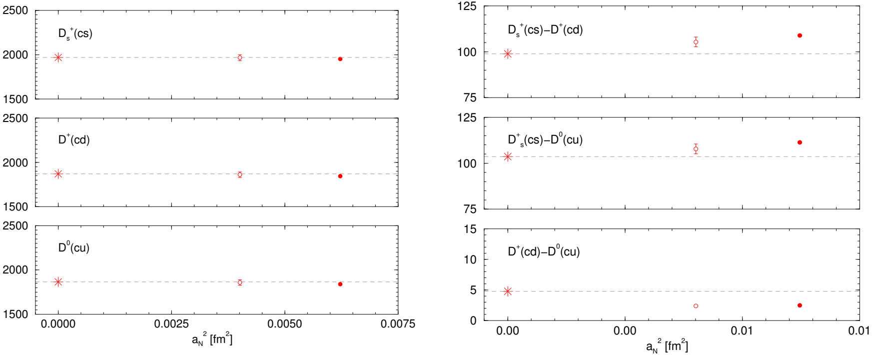
which all have the wavefunction

$$\mathcal{B} = \epsilon(c^T C\gamma_5 q)c \qquad q = u, d, s$$

$SU(4)$ 20-plet

C	S	I	I_3	baryon	wavefunction
0	0	$\frac{1}{2}$	$+\frac{1}{2}$	p	$\epsilon(u^T C\gamma_5 d)u$
0	0	$\frac{1}{2}$	$-\frac{1}{2}$	n	$\epsilon(d^T C\gamma_5 u)d$
0	1	1	$+1$	Σ^+	$\epsilon(u^T C\gamma_5 s)u$
0	1	1	0	Σ^0	$\frac{1}{\sqrt{2}}\epsilon[(u^T C\gamma_5 s)d + (d^T C\gamma_5 s)u]$
0	1	1	-1	Σ^-	$\epsilon(d^T C\gamma_5 s)d$
0	2	$\frac{1}{2}$	$+\frac{1}{2}$	Ξ^0	$\epsilon(s^T C\gamma_5 u)s$
0	2	$\frac{1}{2}$	$-\frac{1}{2}$	Ξ^-	$\epsilon(s^T C\gamma_5 d)s$
0	1	0	0	Λ^0	$\frac{1}{\sqrt{6}}\epsilon[2(u^T C\gamma_5 d)s + (u^T C\gamma_5 s)d - (d^T C\gamma_5 s)u]$
1	0	1	$+1$	Σ_c^{++}	$\epsilon(u^T C\gamma_5 c)u$
1	0	1	0	Σ_c^+	$\frac{1}{\sqrt{2}}\epsilon[(u^T C\gamma_5 c)d + (d^T C\gamma_5 c)u]$
1	0	1	-1	Σ_c^0	$\epsilon(d^T C\gamma_5 c)d$
1	1	$\frac{1}{2}$	$+\frac{1}{2}$	$\Xi_c^{\prime +}$	$\frac{1}{\sqrt{2}}\epsilon[(s^T C\gamma_5 c)u + (u^T C\gamma_5 c)s]$
1	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\Xi_c^{\prime 0}$	$\frac{1}{\sqrt{2}}\epsilon[(s^T C\gamma_5 c)d + (d^T C\gamma_5 c)s]$
1	2	0	0	Ω_c^0	$\epsilon(s^T C\gamma_5 c)s$
1	0	0	0	Λ_c^+	$\frac{1}{\sqrt{6}}\epsilon[2(u^T C\gamma_5 d)c + (u^T C\gamma_5 c)d - (d^T C\gamma_5 c)u]$
1	1	$\frac{1}{2}$	$+\frac{1}{2}$	Ξ_c^+	$\frac{1}{\sqrt{6}}\epsilon[2(s^T C\gamma_5 u)c + (s^T C\gamma_5 c)u - (u^T C\gamma_5 c)s]$
1	1	$\frac{1}{2}$	$-\frac{1}{2}$	Ξ_c^0	$\frac{1}{\sqrt{6}}\epsilon[2(s^T C\gamma_5 d)c + (s^T C\gamma_5 c)d - (d^T C\gamma_5 c)s]$
2	0	$\frac{1}{2}$	$+\frac{1}{2}$	Ξ_{cc}^{++}	$\epsilon(c^T C\gamma_5 u)c$
2	0	$\frac{1}{2}$	$+\frac{1}{2}$	Ξ_{cc}^+	$\epsilon(c^T C\gamma_5 d)c$
2	1	0	0	Ω_{cc}^+	$\epsilon(c^T C\gamma_5 s)c$

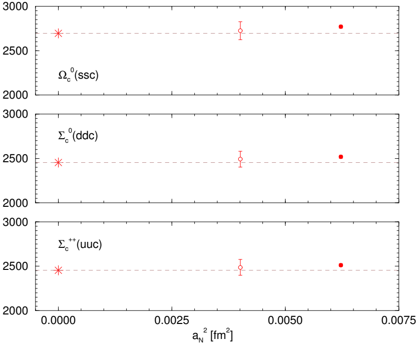
Charmed pseudoscalar mesons

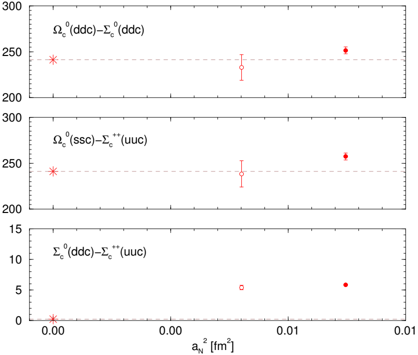


- $D^0(c\overline{u})$, $D^+(c\overline{d})$, $D_s^+(c\overline{s})$,
- small lattice artifacts

- splittings:
  $D^+(c\overline{d}) - D^0(c\overline{u})$,
  $D_s^+(c\overline{s}) - D^0(c\overline{u})$,
  $D_s^+(c\overline{s}) - D^+(c\overline{d})$

Charmed $C = 1$ baryons

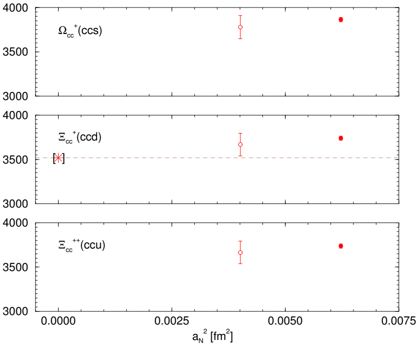


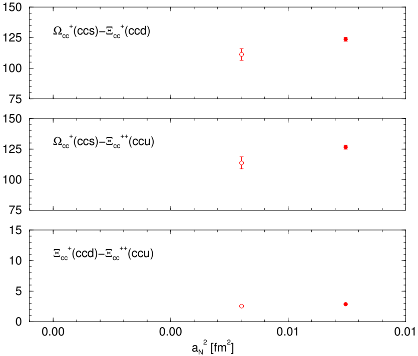


- $\Sigma_c^{++}(uuc)$, $\Sigma_c^0(ddc)$, $\Omega_c^0(ssc)$
- some lattice artifacts (?)

- splittings:
  $\Sigma_c^0(ddc) - \Sigma_c^{++}(uuc)$,
  $\Omega_c^0(ssc) - \Sigma_c^{++}(uuc)$,
  $\Omega_c^0(ssc) - \Sigma_c^0(ddc)$

Charmed $C = 2$ baryons





- $\Xi_{cc}^{++}(ccu)$, $\Xi_{cc}^{+}(ccd)$, $\Omega_{cc}^{+}(ccs)$
- some lattice artifacts (?)
- [$\star$] SELEX

- splittings:
  $\Xi_{cc}^{+}(ccd) - \Xi_{cc}^{++}(ccu)$,
  $\Omega_{cc}^{+}(ccs) - \Xi_{cc}^{++}(ccu)$,
  $\Omega_{cc}^{+}(ccs) - \Xi_{cc}^{+}(ccd)$

Conclusions

- For u, d, s quarks, have developed a method to approach the physical point
- Precise $SU(3)$ flavour symmetry breaking expansions – nothing ad-hoc
- Extend expansions – PQ (mass valence quarks $\neq$ mass sea quarks) to
    - better determine expansion coefficients
    - determine c quark mass
- Applied method to determine some open charm states
- Future:
    - need to better check $O(a^2)$ effects
    - mixing: in a $2+1$ world no Σ^0 - Λ^0 mixing, but determined coefficients can be used to determine $\Sigma^0(uds)$ - $\Lambda^0(uds)$ mixing

      work in progress

      generalise to eg Σ_c^+ - Λ_c^+, Ξ_c^0 - $\Xi_c'^0$ mixing
    - baryon decuplet
    - QED effects