Exploring the Roper resonance in Lattice QCD

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CSSM Lattice Collaboration

Key Collaborators

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THE UNIVERSITY OF ADELAIDE AUSTRALIA

SUB CRUCE LUMEN

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

$N(1440)\ 1/2^+$

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

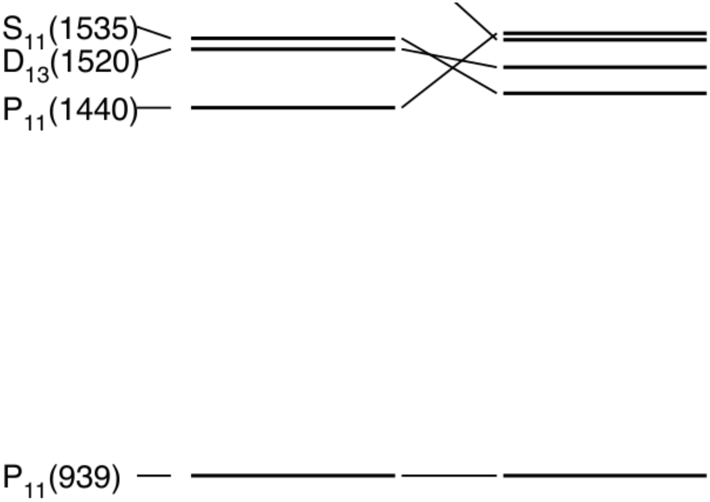
Breit-Wigner mass = 1420 to 1470 (≈ 1440) MeV
Breit-Wigner full width = 200 to 450 (≈ 300) MeV
$p_{\text{beam}} = 0.61$ GeV/c $\quad 4\pi\lambda\!\!\!^{-2} = 31.0$ mb
Re(pole position) = 1350 to 1380 (≈ 1365) MeV
−2Im(pole position) = 160 to 220 (≈ 190) MeV

N(1440) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\pi$	55–75 %	398
$N\eta$	(0.0 ± 1.0) %	†
$N\pi\pi$	30–40 %	347
$\Delta\pi$	20–30 %	147
$\Delta(1232)\pi$, P-wave	15–30 %	147
$N\rho$	<8 %	†
$N\rho$, S=1/2, P-wave	(0.0 ± 1.0) %	†
$N(\pi\pi)^{I=0}_{S\text{-wave}}$	10–20 %	–
$p\gamma$	0.035–0.048 %	414
$p\gamma$, helicity=1/2	0.035–0.048 %	414
$n\gamma$	0.02–0.04 %	413
$n\gamma$, helicity=1/2	0.02–0.04 %	413

Roper Resonance

- Quark model: $N = 2$ radial excitation of the nucleon.
- Much lower in mass than simple quark model predictions.

Roper Resonance



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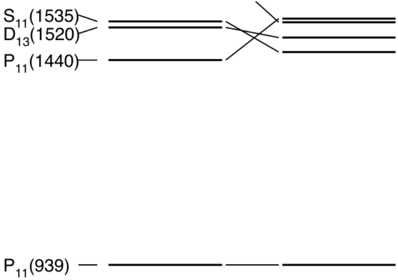

- Experiment: Lighter than $N = 1$ radial excitation of the nucleon, the negative parity $S_{11}(1535)$.
- “Exotic” in nature.

Roper Resonance

- It has proven difficult to isolate this state on the lattice.
- Consider the nucleon interpolators,

$$\chi_1(x) = \epsilon^{abc}(u^{Ta}(x)\, C\gamma_5\, d^b(x))\, u^c(x)\,,$$
$$\chi_2(x) = \epsilon^{abc}(u^{Ta}(x)\, C\, d^b(x))\, \gamma_5\, u^c(x)\,.$$

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  - We will see that this is wrong!
- Key to isolating this elusive state is an appropriate variational basis.
  - Phys.Lett. B707 (2012) 389-393, “Roper Resonance in 2+1 Flavor QCD”

Variational Method

- Construct an $n \times n$ correlation matrix,

$$G_{ij}(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle \Omega | T\{\chi_i(x)\bar{\chi}_j(0)\} | \Omega \rangle.$$

- Solve a generalised eigenproblem to find the linear combination of interpolating fields,

$$\bar{\phi}^{\alpha} = \sum_{i=1}^{N} u_i^{\alpha}\, \bar{\chi}_i, \qquad \phi^{\alpha} = \sum_{i=1}^{N} v_i^{\alpha}\, \chi_i$$

such that the correlation matrix is diagonalised,

$$v_i^{\alpha} G_{ij}(t) u_j^{\beta} = \delta^{\alpha\beta} z^{\alpha} \bar{z}^{\beta} e^{-m_{\alpha} t}.$$

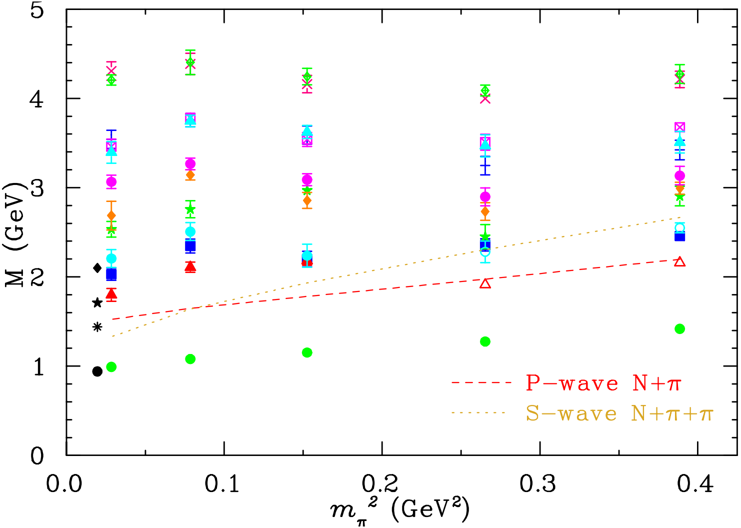
Eigenstate-Projected Correlators

- The left and right vectors are used to define the eigenstate-projected correlators

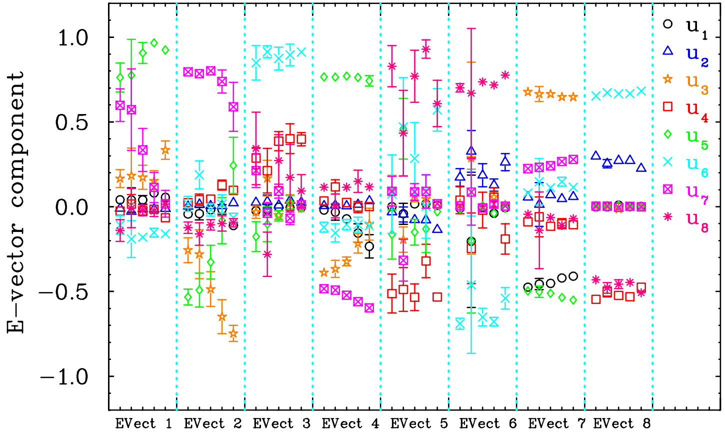
$$v_i^{\alpha} G_{ij}^{\pm}(t) u_j^{\alpha} \equiv G_{\pm}^{\alpha}(t).$$

- χ_1 and χ_2 give us two operators.
  - Not able to access the Roper using these alone.
- Solution: Use different levels of gauge-invariant quark smearing to expand the operator basis.
- PACS-CS 2+1 flavour ensembles, lightest $m_\pi = 156$ MeV.
  - S. Aoki, et al., Phys. Rev. **D79** (2009) 034503.
- 8×8 correlation matrix analysis using χ_1, χ_2 with 4 different levels ($n = 16, 35, 100, 200$) of smearing.
  - RMS radii of 2.37, 3.50, 5.92 and 8.55 lattice units.

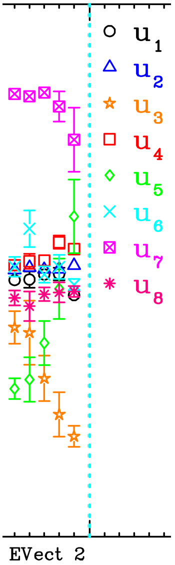
N^+ spectrum



Eigenvector analysis

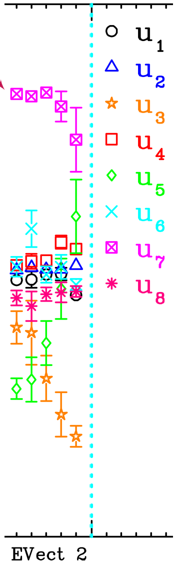


Eigenvector structure of the Roper



Eigenvector structure of the Roper

- $\chi_1, n = 200$ dominates (positive).



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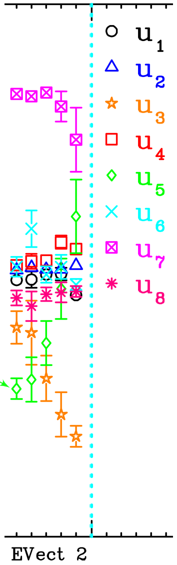
- Negative contribution from a varying mix of:
  - $\chi_1, n = 100$



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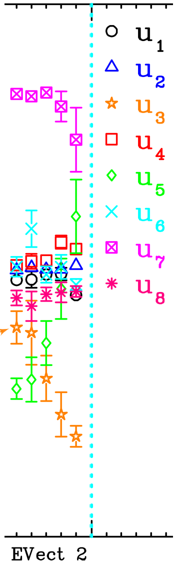

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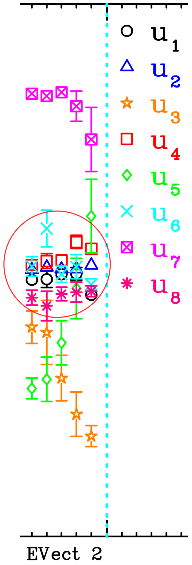
- Negative contribution from a varying mix of:
  - $\chi_1, n = 100$
  - $\chi_1, n = 35$.
- Negligible contribution from $\chi_1, n = 16$ and all χ_2 operators.



Eigenvector analysis

- First positive-parity excited state couples strongly to χ_1.
- Large smearing values are critical.
- χ_2 coupling to the Roper is negligible.
- Transition from scattering state to resonance as quark mass drops.
- At light quark mass the Roper mass is pushed up due to finite volume effects.
- How can we learn more?
  - Study multiple lattice volumes.

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    - Expensive.
  - Look at the excited state structure.

Wave function of the Roper

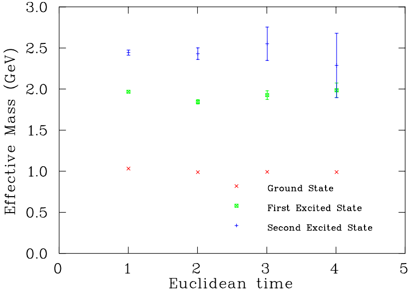
- We explore the structure of the nucleon excitations by examining the Bethe-Salpeter amplitude.
- The baryon wave function is built by giving each quark field in the annihilation operator a spatial dependence,

$$\chi_1(\vec{x},\vec{y},\vec{z},\vec{w}) = \epsilon^{abc}\,(\,u_a^T(\vec{x}+\vec{y})\,C\gamma_5\,d_b(\vec{x}+\vec{z})\,)\,u_c(\vec{x}+\vec{w}).$$

- The creation operator remains local.
- The resulting construction is gauge-dependent.
  - We choose to fix to Landau gauge.

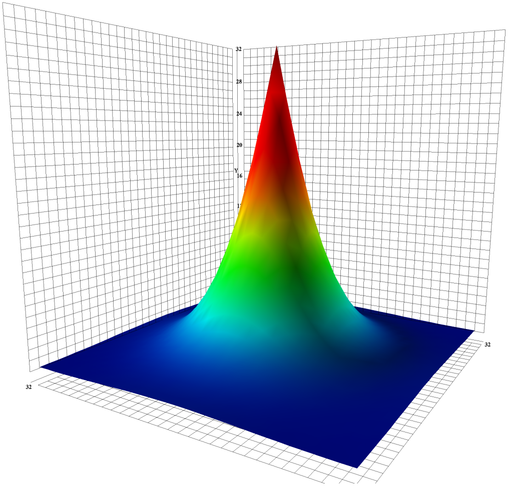
Wave function of the Roper

- Non-local sink operator cannot be smeared.
- Construct states using right eigenvector u^{α} only.

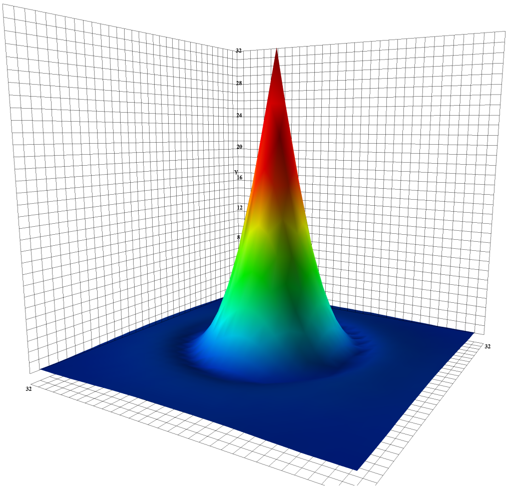


- Eigenvectors from 4×4 CM analysis using χ_1 only.
- The position of the u quarks is fixed and we measure the d quark probability distribution at $m_{\pi} = 156$ MeV.

Ground state probability distribution



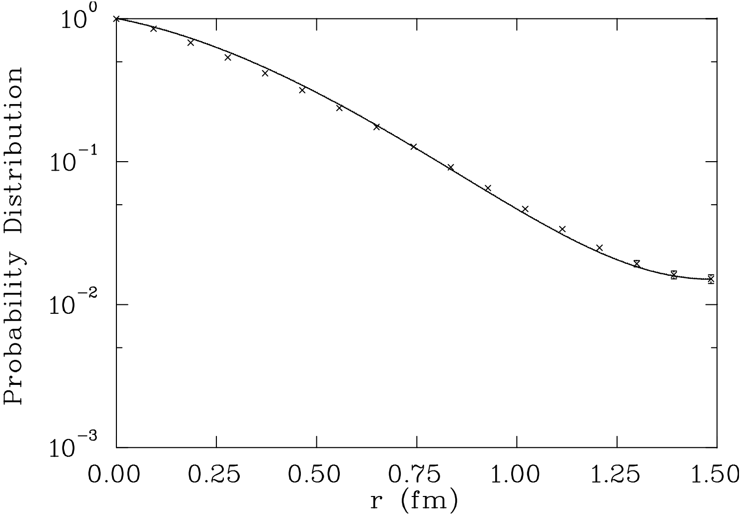
First excited state probability distribution



Quark Model comparison

- Compare to a non-relativistic constituent quark model.
  - One-gluon-exchange motivated Coulomb + ramp potential.
  - Spin dependence in R. K. Bhaduri, L. E. Cohler and Y. Nogami, Phys. Rev. Lett. 44 (1980) 1369.
- The radial Schrodinger equation is solved with boundary conditions relevant to the lattice data.
  - The derivative of the wave function is set to vanish at a distance $L_x/2$.
- Two parameter fit to the nucleon radial wave function yields:
  - String tension $\sqrt{\sigma} = 400$ MeV
  - Constituent quark mass $m_q = 360$ MeV
- These parameters are held fixed for the excited states.

Ground state comparison



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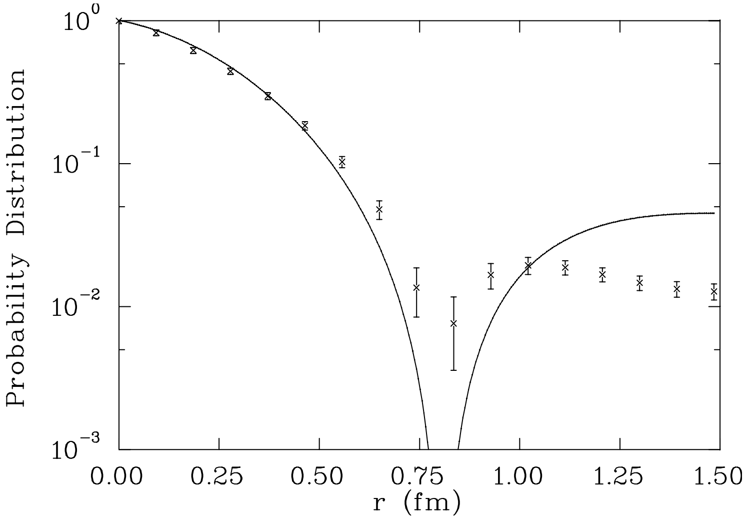
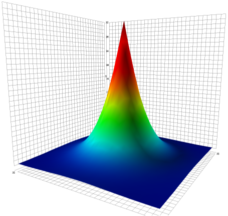
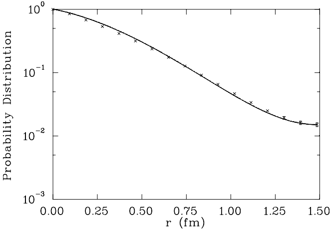


Quark Model comparison

- Ground state QM agrees well (as expected).





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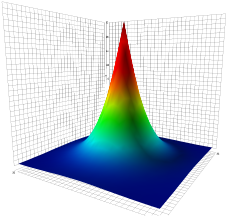
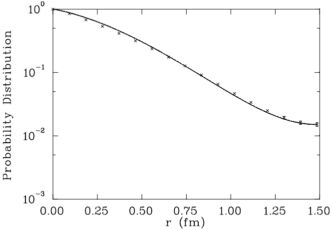
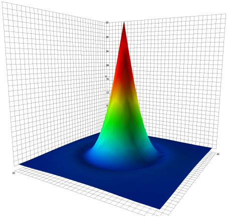
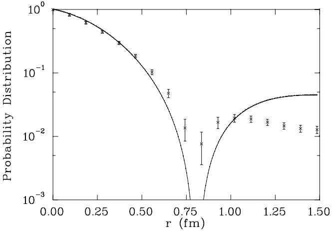




- First excited state shows a node structure.





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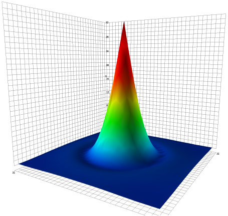

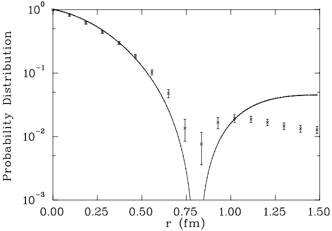

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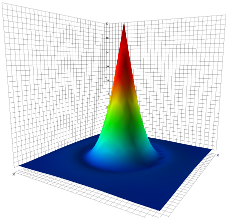

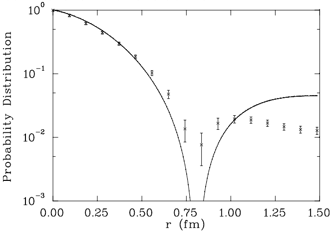

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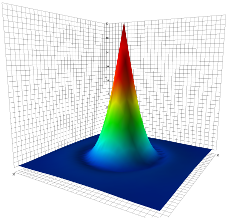

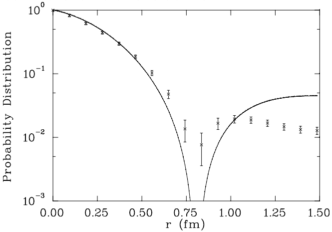

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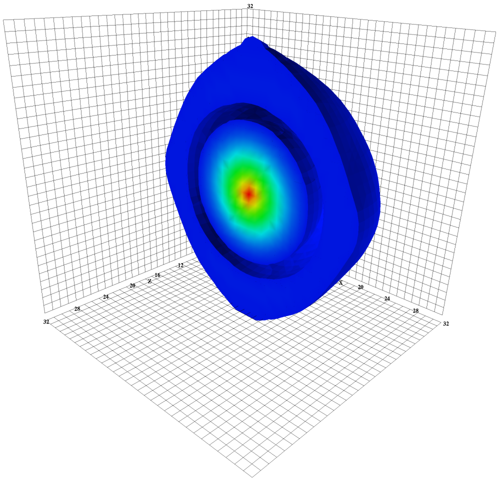
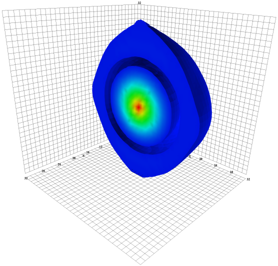
- Finite volume effects?

First excited state probability distribution



Quark Model comparison

- Wave function should be spherically symmetric.

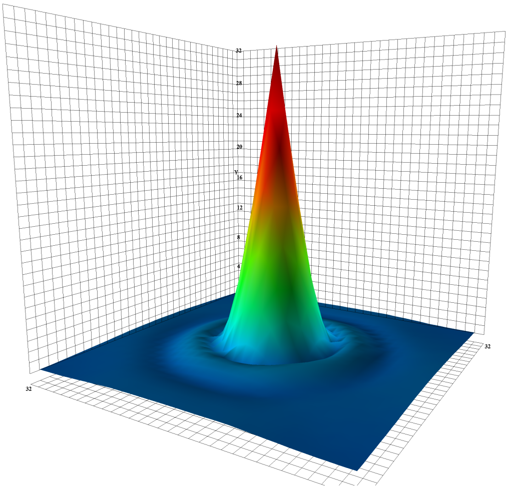


- Outer shell of Roper wave function clearly reveals distortion due to finite volume.
- Effective field theory arguments suggest the small volume will drive up the energy.

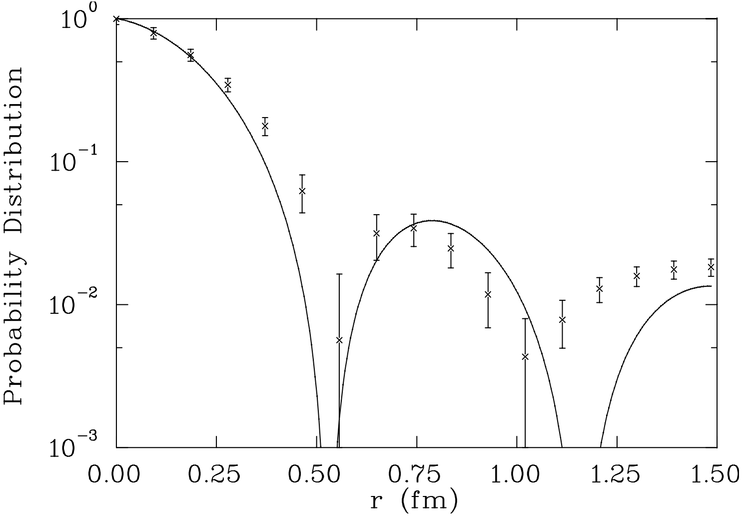
Summary

- The variational method allows us to access a state that is consistent with the Roper $N(1440)$ with standard three-quark interpolators.
- χ_2 has negligible coupling to the Roper.
- Probing the Roper wave function reveals a nodal structure.
- Multiple χ_1 operators at large smearings are critical to form the correct nodal structure.
- Qualitative agreement with QM predictions for the Roper radial wave function.
- Finite volume effects clearly evident in the Roper probability distribution.
  - Larger lattice volumes needed!

$N = 3$ excited state probability distribution



$N = 3$ excited state comparison



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