QCD thermodynamics with O(a) improved Wilson fermions at $N_f = 2$

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References: chiral transition: 1008.2143 / 1011.6172 / 1210.6972

plasma properties: 1212.4200 / 1302.0675

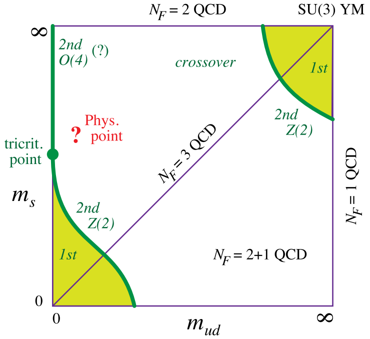
Contents

1. Introduction

- The chiral limit at $N_f = 2$
- Plasma properties near the phase transition

Directly accessible: Zero density ($\mu = 0$)

Enlarged parameter space relevant for the QCD phase diagram:

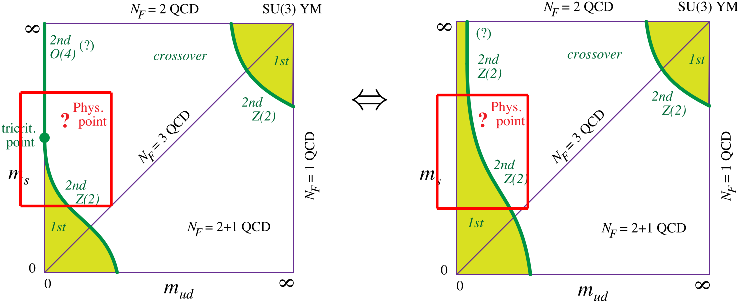


[Kanaya, PoS LAT 2010 012]

- The charm quark is to heavy to influence the transition properties. (might affect plasma properties above T_c)
- Isospin breaking effects probably also not to important.

$N_f = 2$ transition and tricritical point

Two possible scenarios:



- We know it is a true phase transition. [Pisarski, Wilczek, PRD 29, 338 (1984)]
- But it can be of first **or** second order! [Butti *et al*, JHEP 0308, 029 (2003)]

Assessing the two scenarios – Scaling

- Cannot simulate directly in (or very close to) the chiral limit.
- Only possibility:
  Simulate at larger quark masses in the crossover region and look for critical scaling in the approach to the chiral limit at constant m_s.
- What type of scaling can be expected in the two cases?
  - O(4): usual $O(4)$ scaling
    Order parameter: Chiral condensate
  - First order: $Z(2)$ scaling
    (or some remnant of first order?)
    Order parameter: ???
- How close to $m_{ud} = 0$ is necessary?
  (Probably even below physical m_{ud})
- Simulations at small quark masses are expensive!
  (especially for non-staggered fermion actions)
- There is a number of studies but no conclusive result!
  (contradicting results for staggered; no reliable chiral extrapolation for other fermion discretisations)

Assessing the two scenarios – $U_A(1)$ symmetry

Of particular importance:
Strength of the anomalous breaking of the $U_A(1)$ symmetry:

[Pisarki, Wilczek, PRD 29, 338 (1984)]
[Butti *et al*, JHEP 0308, 029 (2003)]

- If the breaking is strong:
  Transition: Second order $SU(2) \times SU(2) \simeq$ O(4) universality
- If the breaking is weak, or the symmetry restored:
  Transition: First order (or second order $\not\simeq$ O(4)).

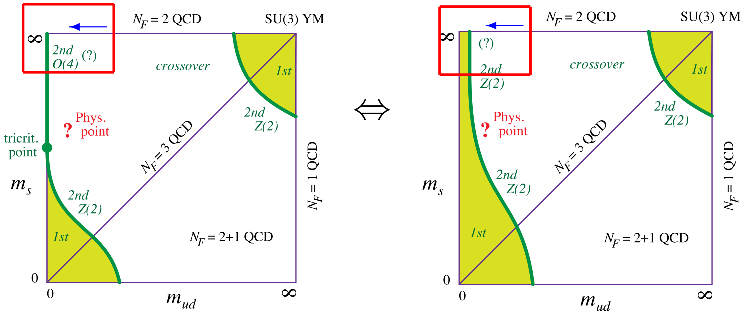
Possibilities for looking at the strength of the breaking:

- Look at suszeptibilities [cf. C. Schroeders talk on Monday, S. Datta's talk before] .
- Look at degeneracies of correlation functions and screening masses in pseudoscalar (P) and scalar channels (S).

$\Rightarrow$ Chiral extrapolation is mandatory!

Assessing the two scenarios - Our choice

Simulate at $N_f = 2$:



- Simulations are less expensive.
- Can use Wilson fermions on large lattices using the available fast algorithms and the $T = 0$ input from CLS.
- Also look at screening masses and $U_A(1)$ symmetry restoration.

Plasma properties near the phase transition

For hydrodynamic calculations and to explain phenomena observed in experiment:

Extract transport coefficients and particle production rates from the lattice!

Our study yields large lattices around T_C.
$\Rightarrow$ Can be used to study plasma properties!

Measurement of the electrical conductivity:

- Have extracted the electrical conductivity with dynamical fermions at $T \approx 250$ MeV (⇒ See end of my talk!).

  [BB *et al*, JHEP 1303, 100 (2013)]

- Crucial for this was the use of the reconstructed correlator in combination with a related sum rule.

  [Bernecker, Meyer, EPJ A47, 148 (2011)]

- We are aiming to extend this analysis over the full scan at $m_\pi \approx 290$ MeV.

Other plasma properties will be studied in the future ...

2. Setup

Action and scale setting

Action: Non-perturbatively $\mathcal{O}(a)$-improved Wilson fermions
Wilson plaquette gauge action

Algorithms: deflation accelerated DD-HMC

[Lüscher (2004-2005), e.g. CPC 165, 199 (2005)]

MP-HMC with DFL-SAP-GCR solver

[Marinkovic, Schäfer PoS LAT 2010, 031 (2010)]

$\Rightarrow$ Good scaling properties with volume and quark masses!

Scale setting: r_0 in the chiral limit as determined by CLS

[Fritzsch *et al*, NPB 865, 397 (2012)]

Mass scale: PCAC mass converted to $\overline{\mathrm{MS}}$ scheme

Renormalisation: Interpolation of ALPHA results as used within CLS.

Temperature scan setup

Basic strategy:

- Use $N_t = 16$ for all scans.
- Use 3 different volumes: 32^3, 48^3 and 64^3.
  (enables a finite volume scaling study; control FS effects)
- At least 3 different pion masses below $m_\pi \leq$300 MeV.
  (ideally even below the physical point)
- We scan in β:
  - First attempts: keep κ fixed
    $\Rightarrow$ Quark mass changes along the scan.
    (is problematic for Wilson fermions at small quark masses)
  - Now: Keep renormalised quark mass fixed!
    $\Rightarrow$ Line of constant physics (LCP)
    (conceptually much cleaner)

Observables

Chiral transition:

- Chiral condensate $\langle\bar{\psi}\psi\rangle$; (subtracted and bare)
  Order parameter of the transition in the chiral limit.
  (Problematic due to additive and multiplicative renormalisation)
- Screening masses in various channels;
  Sensitive to chiral symmetry restoration pattern.

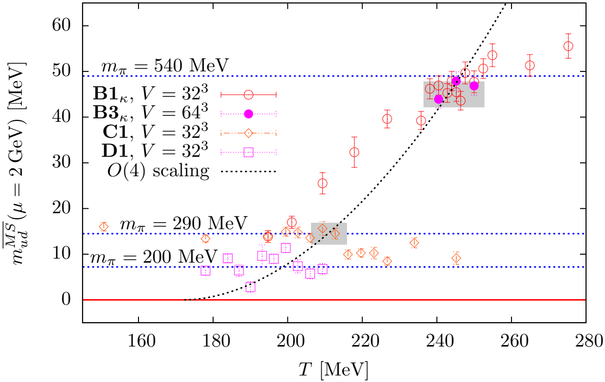
Deconfinement:

- Polyakov loop L; (APE smeared and unsmeared)
  Order parameter of the transition in the pure gauge limit.
- Quark number suszeptibility χ_q;
  Measures the net number of quarks.

Note: At the moment all quantities are not renormalised properly!
(no $T = 0$ subtractions)

3. Status of temperature scans

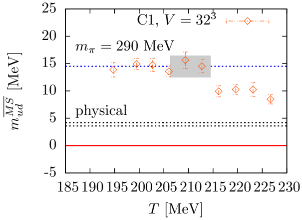
Overview over simulation points

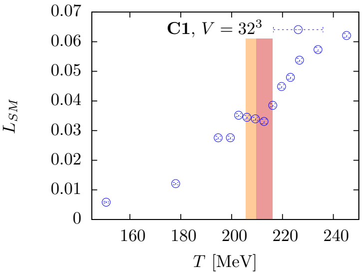


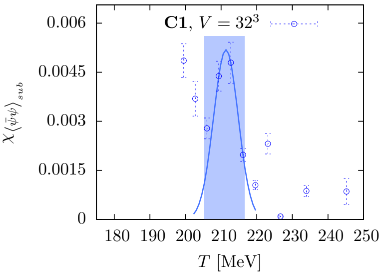
- LCP at $m_\pi \approx 290$ MeV not perfect for $T > 210$ MeV. (recent updates on $T = 0$ results)

First LCP at $m_\pi \approx 290$ MeV

- C1: 16×32^3 Lattice
  LCP at $m_\pi \approx 290$ MeV
  ($m_{ud} \approx 14.5$)
- Statistic: $\sim$12000 MD-units
- $\tau_{\text{int}}(U_P) \sim 14$ MDU
  $\Rightarrow \quad \sim 900 - 1000$ unc. meas.

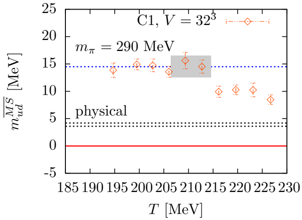






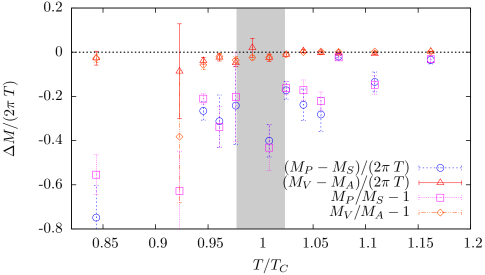
First LCP at $m_\pi \approx 290$ MeV

- C1: 16×32^3 Lattice
  LCP at $m_\pi \approx$290 MeV
  ($m_{ud} \approx 14.5$)
- Statistic: $\sim$300 configurations
  separated by 40 MDUs



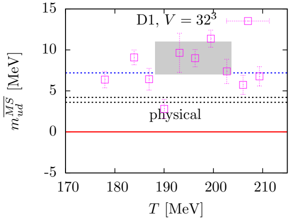
vector $\overset{SU_A(2)}{\longleftrightarrow}$ axial vec.
(V) (A)

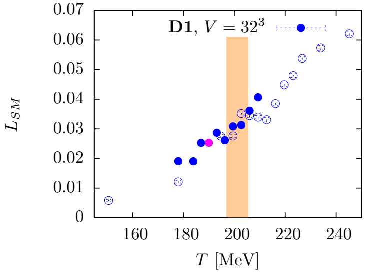
scalar $\overset{U_A(1)}{\longleftrightarrow}$ pseudosc.
(S) (P)

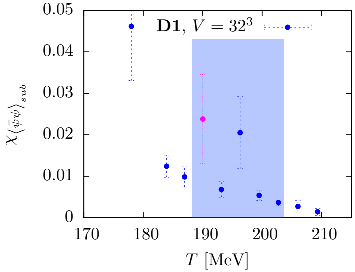


LCP at $m_\pi \approx 200$ MeV

- D1: 16×32^3 Lattice
  LCP at $m_\pi \approx$200 MeV
  ($m_{ud} \approx 7.2$)
- Statistic: ∼7000 MD-units
- $\tau_{\mathrm{int}}(U_P) \sim 7$ MDU
  (here MP-HMC - reduced τ_{int})

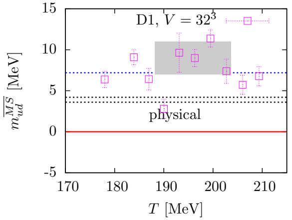


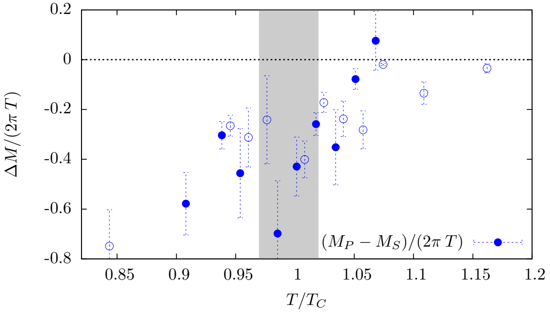




LCP at $m_\pi \approx 200$ MeV

- C1: 16×32^3 Lattice
  LCP at $m_\pi \approx$200 MeV
  ($m_{ud} \approx 7.2$)
- Statistic: ∼300 configurations
  separated by 20 MDUs





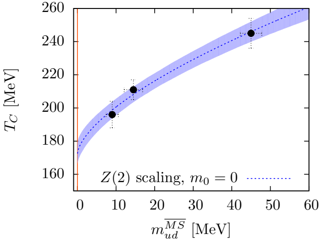
Transition temperatures and scaling

Scaling of T_C:

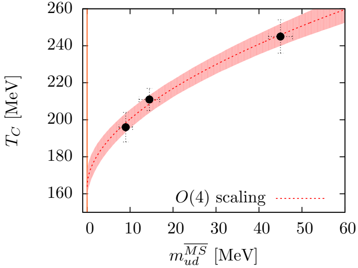
$$T_C(m_{ud}) =$$

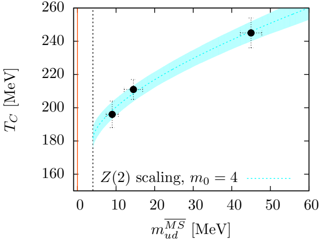
$$T_C(0)\ \left[1 + C\,(m_{ud} - m_0)^{1/(\delta\beta)}\right]$$

$Z(2)$ scaling:



$O(4)$ scaling:





4. The electrical conductivity

Vector correlator and electrical conductivity

[BB *et al*, JHEP 1303, 100 (2013)]

Kubo formula: $\dfrac{\sigma(T)}{T} = \dfrac{C_{\rm em}}{6} \lim\limits_{\omega\to 0} \dfrac{\rho_{ii}(\omega, T)}{\omega\ T}$.

$\rho_{\mu\nu}(\omega, T)$: Spectral function associated with $G_{\mu\nu}(\tau, T)$ via

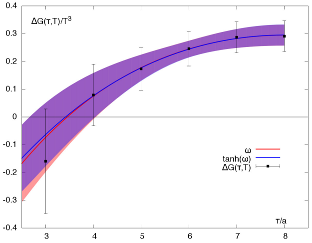
$$G_{\mu\nu}(\tau, T) = \int_0^\infty \frac{d\omega}{2\pi}\, \rho_{\mu\nu}(\omega, T)\, \frac{\cosh\left[\omega\left(1/(2T) - \tau\right)\right]}{\sinh\left(\omega/2T\right)}\ .$$

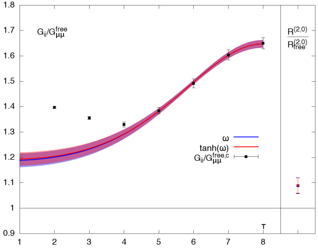
Strategy:

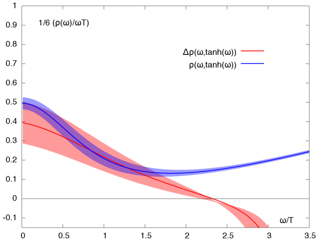
- Extract $G_{\mu\nu}(\tau, T)$ from the lattice!
- Use the reconstructed correlator $G^{\rm rec}_{\mu\nu}(\tau, T) = \sum_m G_{\mu\nu}\left(|\tau + m/T|\,, T = 0\right)$
- and the sum rule $\int_{-\infty}^{\infty} \frac{d\omega}{\omega}\left[\rho_{ii}(\omega, T) - \rho_{ii}(\omega, 0)\right] = 0$.
- Fit the difference $\Delta G_{ii}(\tau, T) = G_{ii}(\tau, T) - G^{\rm rec}_{ii}(\tau, T)$ to a phenomenologically motivated ansatz for $\Delta\rho_{ii}$ using the sum rule as a constraint.
- Extract σ from the Kubo formula.

Results are checked by an alternative fit to $G_{ii}(\tau, T)/G^{\rm free}_{\mu\mu}(\tau, T)$.

Fits and electrical conductivity







- Results at $m_\pi = 290$ MeV, $T/T_C \approx 1.2$; Lattice: $128\ /\ 16\ \times 64^3$
- Fit to $\tau \geq 5$: Very good agreement with data!
- Electrical conductivity: $\dfrac{\sigma}{C_{\mathrm{em}}\ T} = 0.40(12)$

Electrical conductivity accross the transition

Next step:

Study the temperature dependence of the conductivity.

[cf. talk by A. Amato on monday]

Problems:

- No $T = 0$ correlators available.
  $\Rightarrow$ Cannot use the reconstructed correlator and the sum rule!
- I.e. the crucial ingredient for the succesfull fits at $T/T_C \approx 1.2$ is missing at the moment.

Options:

- Measure $T = 0$ correlators.
  (along with $T = 0$ subtractions for the temp. scan)
- Find some other option to constrain the fits.

Work in progress . . .

Perspectives

- In the next couple of months we plan to accomplish the simulations at $m_\pi = 200$ MeV.
- Plan to add additional volumes.
  (This has been started to some extend)
- Long term list:
  - Simulate at lighter pion masses.
  - Calculate $T = 0$ subtractions.
    $\Rightarrow$ Accomplish renormalisation.
  - Finaly: Perform a scaling analysis!
- We also calculated the electrical conductivity at $T/T_C \approx 1.2$
- Plan to measure the conductivity accross the temperature scan and to study the fate of the ρ meson.
  (Also here the $T = 0$ subtractions are crucial!)

Thank you for your attention!

Backup slides:

Ansatz for the spectral function

$$
\begin{aligned}
\Delta\rho_{ii}^{1,2} &= \rho_{T;1,2}(\omega, T) - \rho_B(\omega, T) + \Delta\rho_F(\omega, T) \\
\rho_B(\omega, T) &= \frac{2c_B\, g_B \tanh^3(\omega/T)}{4(\omega - m_B)^2 + g_B^2} \\
\Delta\rho_F(\omega, T) &= \rho_F(\omega, T) - \rho_F(\omega, 0) \quad \text{with} \quad \rho_F(\omega, T) = \frac{3}{2\pi}\kappa\omega^2 \tanh\left(\frac{\omega}{4T}\right) \\
\rho_{T;1}(\omega, T) &= \frac{4c\omega}{(\omega/g)^2 + 1} \\
\rho_{T;2}(\omega, T) &= \frac{4cT \tanh(\omega/T)}{(\omega/g)^2 + 1}
\end{aligned}
$$

Fit parameters: $c,\ g,\ c_B$
Fixed by $T = 0$ correlator: m_B

g_B/T varied between $0.1 - 1.0$ ($g_B = 25 - 250$ MeV) $\Leftarrow$ no significant dependence