Chiral Symmetry and $U(1)_A$ Symmetry in Finite Temperature QCD with Domain-Wall Fermion

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Outline

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- Observables for probing the restoration of $SU(2)_L \times SU(2)_R$ and $U(1)_A$
- Preliminary Results
- Concluding Remarks

Introduction

In QCD, the classical action of N_f massless quarks has the symmetry

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$$

In the quantum theory, at zero temperature, the chiral symmetry $SU(N_f)_L \times SU(N_f)_R$ is broken spontaneously to $SU(N_f)_V$ by the vacuum of QCD, and the $U(1)_A$ is broken by the axial anomaly.

It is expected that at high temperature, both chiral symmetry and $U(1)_A$ are restored. The question is, at what temperature T_c the chiral symmetry is restored, and whether $U(1)_A$ is also restored at $T_1 \approx T_c$

Lattice QCD with exact chiral symmetry is in a good position to answer these questions. HotQCD (DWF with not so small residual mass): extensive studies.

JLQCD (overlap fermion with fixed topology)

TWQCD simulations of Nf=2 QCD

- Chiral symmetry is preserved to a good precision with the optimal DWF [TWC, Phys. Rev. Lett., 2003].
- Topological sectors are sampled ergodically.
- Multiple-time scale integration and mass preconditioning.
- Even-Odd Preconditioning for the 4D Wilson-Dirac Matrix.
- Omelyan Integrator for the Molecular Dynamics.
- Conjugate gradient with mixed precision.
- Use a GPU cluster of 320 GPUs, with sustained 100 Tflops

Gauge Ensembles of 2-flavor QCD with ODWF

- Lattice size: 16^3 x 6 x 16
- Quark action: Optimal Domain-Wall Fermion (ODWF)
- Gluon action: Wilson plaquette (β = 5.86, 5.87, …, 5.95)
- For each β, three sea-quark masses, $m_q a$ = 0.01, 0.02, 0.03
- For each (β, m), after thermalization, 3000-5000 trajectories have been accumulated. Sampling one configuration every 10 trajectories gives 300-500 confs.
- For each conf, **zero modes** plus **200+200 conjugate pairs of low-lying eigenmodes** of the overlap operator are projected.

Statistics of HMC Trajectories for $16^3 \times 6$

Beta	m=0.01	m=0.02	m=0.03
5.86	2050	1240+	1757+
5.87	2509	2675	2693
5.88	5106	2518+	2363+
5.89	4522+	4848	3233
5.90	3235	2677	1835+
5.91	9216	2193	2267
5.92	3229	2124	1873
5.93	12998	1844	2005
5.94	2235	1797	1900
5.95	5648	2428	7252

This covers $T \sim 140 - 300$ MeV

Residual masses with ODWF at Ns=16

beta	m=0.01	m=0.02	m=0.03
5.86			
5.87			
5.88	0.00059(8)	0.00047(8)	0.00077(5)
5.89	0.00050(7)	0.00035(4)	0.00042(5)
5.90	0.00018(2)	0.00016(4)	0.00022(4)
5.91	0.00020(2)	0.00025(3)	0.00019(3)
5.92	0.00019(4)	0.00018(3)	0.00014(3)
5.93	0.00019(3)	0.00011(2)	0.00011(2)
5.94	0.00013(2)	0.00011(3)	0.00008(1)
5.95	0.00006(2)	0.00006(4)	0.00006(5)

Observables for probing the restoration of $SU(2)_L \times SU(2)_R$ and $U(1)_A$

- Chiral susceptibilities $(\chi_\pi\,,\ \chi_\delta\,, \chi_\eta\,, \chi_\sigma)$
- Eigenvalue distribution of the overlap operator

$$\chi_\pi - \chi_\delta = \int_0^\infty d\lambda \frac{4 m_q^2 \rho(\lambda)}{(m_q^2 + \lambda^2)^2}$$

Chiral Susceptibilities

Scalar mesons

$$\delta = \bar{u}d,\ \bar{d}u,\ \frac{1}{\sqrt{2}}\left(\bar{u}u - \bar{d}d\right), \quad \text{flavor non-singlet}$$

$$\sigma = \frac{1}{\sqrt{2}}\left(\bar{u}u + \bar{d}d\right), \quad \text{flavor singlet}$$

$$C_\delta(x) = \left\langle \left(\bar{u}d\right)^\dagger(x)\left(\bar{u}d\right)(0)\right\rangle$$

$$C_\sigma(x) = \left\langle \sigma^\dagger(x)\sigma(0)\right\rangle$$

$$\chi_\delta = \sum_x C_\delta(x) = -\frac{1}{L_x^3 L_t}\left\langle \mathrm{Tr}(D_c + m_q)^{-2}\right\rangle$$

$$\chi_\sigma = \sum_x C_\sigma(x) = \chi_\delta + \frac{1}{L_x^3 L_t}\left\{\left\langle\left[\mathrm{Tr}(D_c + m_q)^{-1}\right]^2\right\rangle - \left\langle \mathrm{Tr}(D_c + m_q)^{-1}\right\rangle^2\right\}$$

Chiral Susceptibilities (cont)

Pseudoscalar mesons

$$\pi = \bar{u}\gamma_5 d,\ \bar{d}\gamma_5 u,\ \frac{1}{\sqrt{2}}\left(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d\right), \quad \text{flavor non-singlet}$$

$$\eta = \frac{1}{\sqrt{2}}\left(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d\right), \quad \text{flavor singlet}$$

$$C_\pi(x) = \left\langle \left(\bar{u}\gamma_5 d\right)^\dagger (x) \left(\bar{u}\gamma_5 d\right)(0) \right\rangle$$

$$C_\eta(x) = \left\langle \eta^\dagger(x)\eta(0) \right\rangle$$

$$\chi_\pi = \sum_x C_\pi(x) = \frac{1}{L_x{}^3 L_t} \left\langle \mathrm{Tr}[\gamma_5 (D_c + m_q)]^{-2} \right\rangle$$

$$\chi_\eta = \sum_x C_\eta(x) = \chi_\pi - \frac{1}{L_x{}^3 L_t} \left\{ \left\langle \left[\mathrm{Tr}\gamma_5 (D_c + m_q)^{-1} \right]^2 \right\rangle - \left\langle \mathrm{Tr}\gamma_5 (D_c + m_q)^{-1} \right\rangle^2 \right\}$$

Chiral Susceptibilities (cont)

$$C_\delta(x) = \left\langle (\bar{u}d)^\dagger(x)(\bar{u}d)(0) \right\rangle$$

$$C_\sigma(x) = \left\langle \sigma^\dagger(x)\sigma(0) \right\rangle$$

$$\chi_\delta = \sum_x C_\delta(x)$$

$$\chi_\sigma = \sum_x C_\sigma(x)$$

$$C_\pi(x) = \left\langle (\bar{u}\gamma_5 d)^\dagger(x)(\bar{u}\gamma_5 d)(0) \right\rangle$$

$$C_\eta(x) = \left\langle \eta^\dagger(x)\eta(0) \right\rangle$$

$$\chi_\pi = \sum_x C_\pi(x)$$

$$\chi_\eta = \sum_x C_\eta(x)$$

$$\left.\begin{array}{l} \chi_\pi = \chi_\sigma \\ \chi_\eta = \chi_\delta \end{array}\right\} \Leftrightarrow \text{restoration of } SU(2)_L \times SU(2)_R$$

$$\left.\begin{array}{l} \chi_\pi = \chi_\delta \\ \chi_\sigma = \chi_\eta \end{array}\right\} \Leftrightarrow \text{restoration of } U(1)_A$$

Chiral Susceptibilities (cont)

$$\chi_\eta = \chi_\pi - \frac{1}{L_x^{\,3} L_t} \left\{ \left\langle \left[\mathrm{Tr}\gamma_5 (D_c + m_q)^{-1} \right]^2 \right\rangle - \left\langle \mathrm{Tr}\gamma_5 (D_c + m_q)^{-1} \right\rangle^2 \right\}$$

$$= \chi_\pi - \frac{\chi_t}{m_q^2}, \qquad \chi_t = \frac{1}{L_x^{\,3} L_t} \left\{ \left\langle Q_{top}^2 \right\rangle - \left\langle Q_{top} \right\rangle^2 \right\},$$ topological susceptibility

At temperature $T \geq T_c$

$$\left. \begin{array}{l} \chi_\pi = \chi_\sigma \\ \chi_\eta = \chi_\delta \end{array} \right\} \Leftrightarrow \text{ restoration of } SU(2)_L \times SU(2)_R$$

$$\chi_\pi - \chi_\eta = \frac{\chi_t}{m_q^2} \quad \Leftrightarrow \quad \chi_{disc} \equiv \chi_\sigma - \chi_\delta = \frac{\chi_t}{m_q^2}$$

Chiral Susceptibilities (cont)

At temperature $T \geq T_1$

$$\left.\begin{aligned} \chi_\pi &= \chi_\delta \\ \chi_\sigma &= \chi_\eta \end{aligned}\right\} \Leftrightarrow \text{restoration of } U(1)_A$$

If $T_1 = T_c$, then $\chi_\pi = \chi_\delta = \chi_\sigma = \chi_\eta$ for $T \geq T_c$,

restoration of $SU(2)_L \times SU(2)_R \times U(1)_A$

$$\left(\frac{\chi_t}{m_q^2}\right) \xrightarrow{m_q \to 0} \begin{cases} = 0 \,, & T \geq T_c \\ \propto \dfrac{1}{m_q} \,, & T < T_c \end{cases}$$

Chiral Susceptibilities (cont)

If $T_1 > T_c$, there exists a window $T_c \le T \le T_1$

$$\begin{Bmatrix} \chi_\pi = \chi_\sigma \\ \chi_\eta = \chi_\delta \end{Bmatrix} \text{ but } \begin{Bmatrix} \chi_\pi \neq \chi_\delta \\ \chi_\sigma \neq \chi_\eta \end{Bmatrix}$$

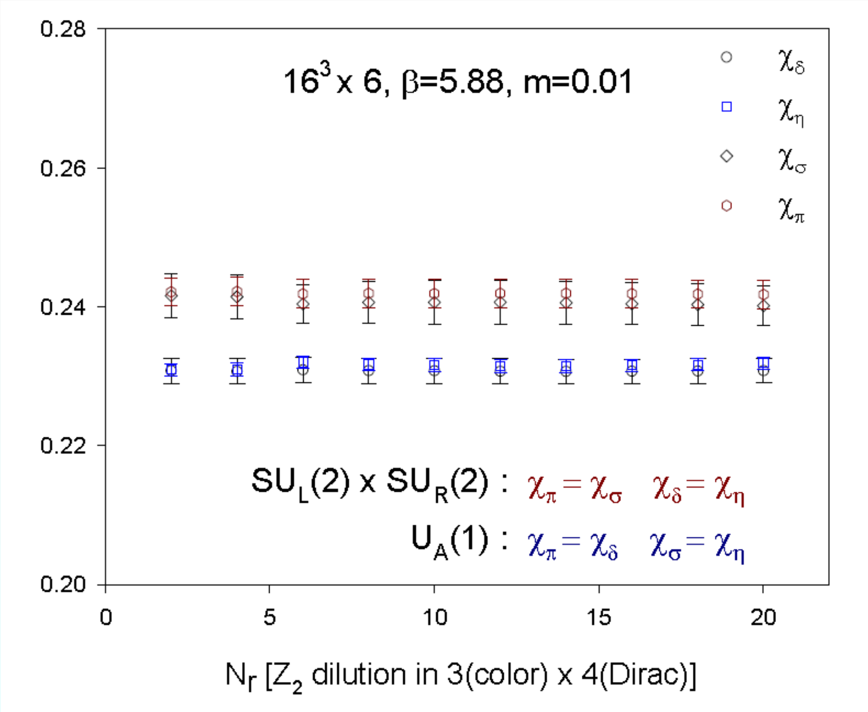
If the chiral symmetry restoration (phase transition) belongs to the $O(4)$ universality class, then we expect

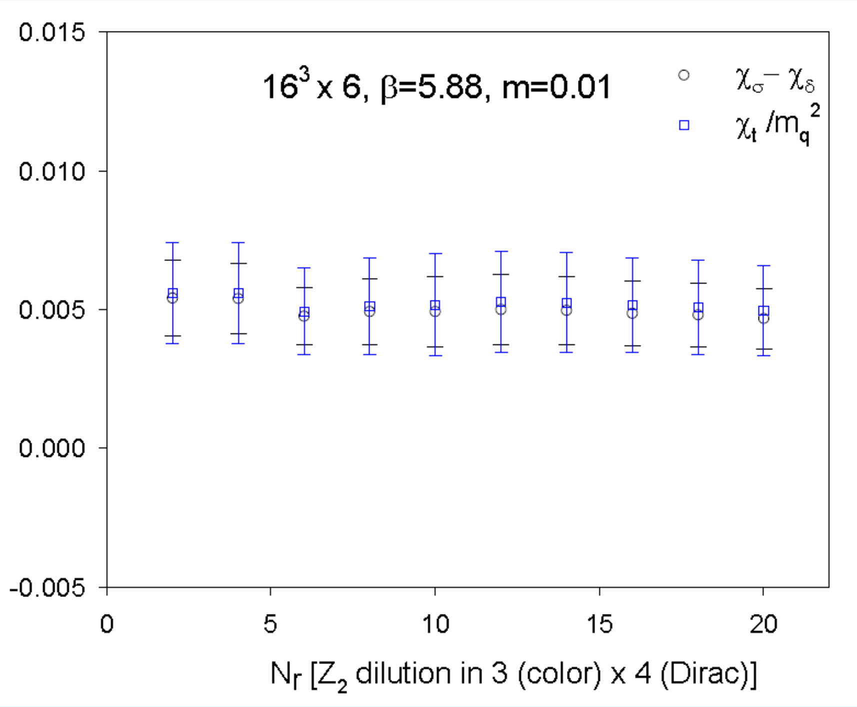
$$\left(\frac{\chi_t}{m_q^2} \right) \xrightarrow{m_q \to 0} \sim (T - T_c)^{-\gamma}, \quad T \ge T_c$$

$$\gamma = 1.453$$

Preliminary Results of Chiral Susceptibilities

The chiral susceptibilities are evaluated with the all-to-all quark propagators, which are computed using 240 Z_2 noises with dilution in the color and the Dirac indices, for each conf. The results are well saturated by the number of noise vectors.

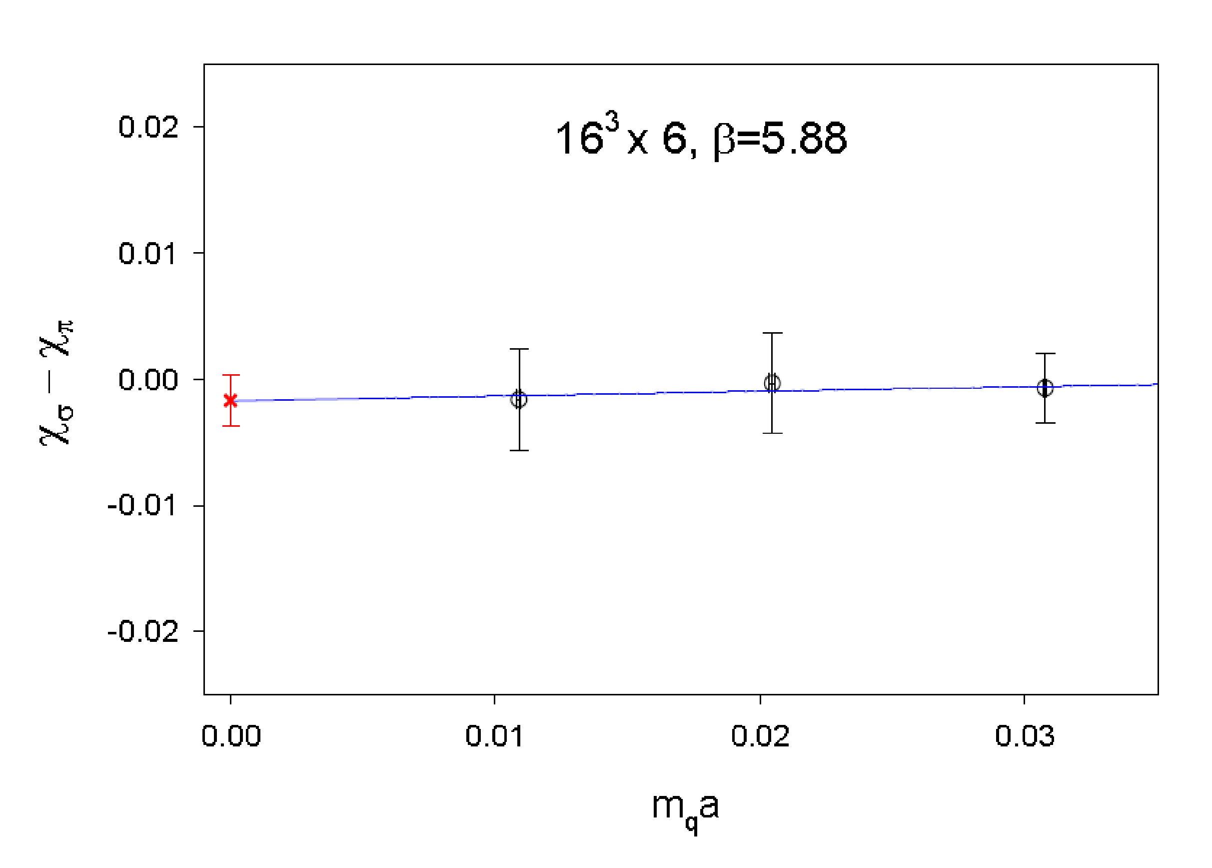




Chiral Susceptibilities (cont.)

$$\chi_\pi \simeq \chi_\sigma \simeq 0.24$$

$T \approx 180$ MeV



$\chi_\pi = \chi_\sigma \Rightarrow$ restoration of $SU(2)_L \times SU(2)_R$

Chiral Susceptibilities (cont.)

$$\chi_\eta \simeq \chi_\delta \simeq 0.23$$

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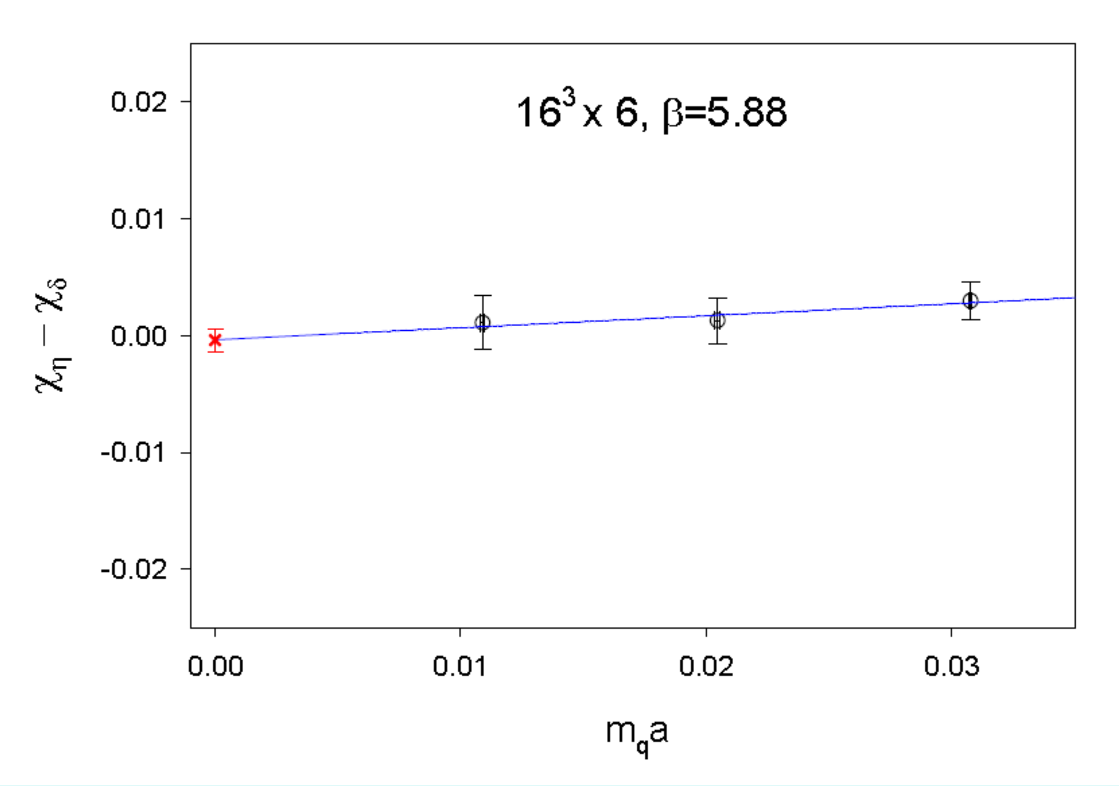

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Chiral Susceptibilities (cont.)

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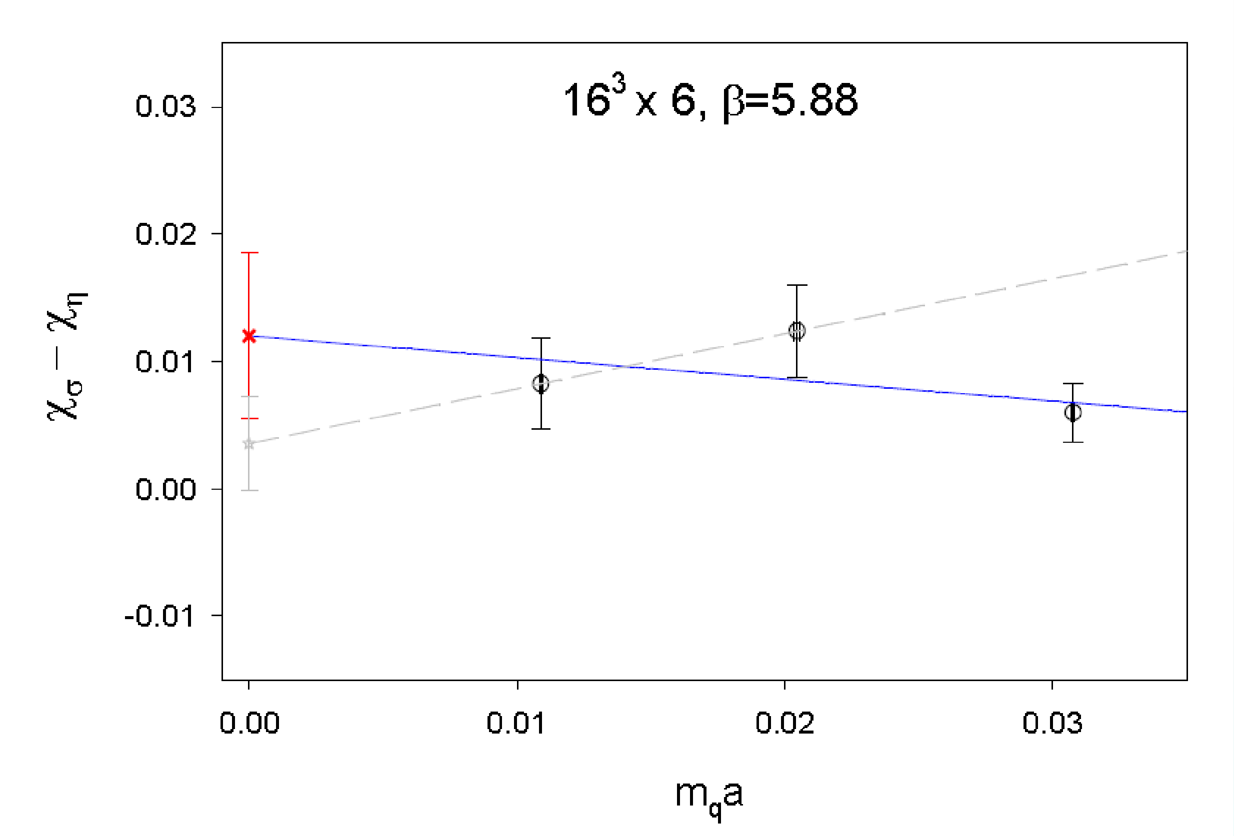
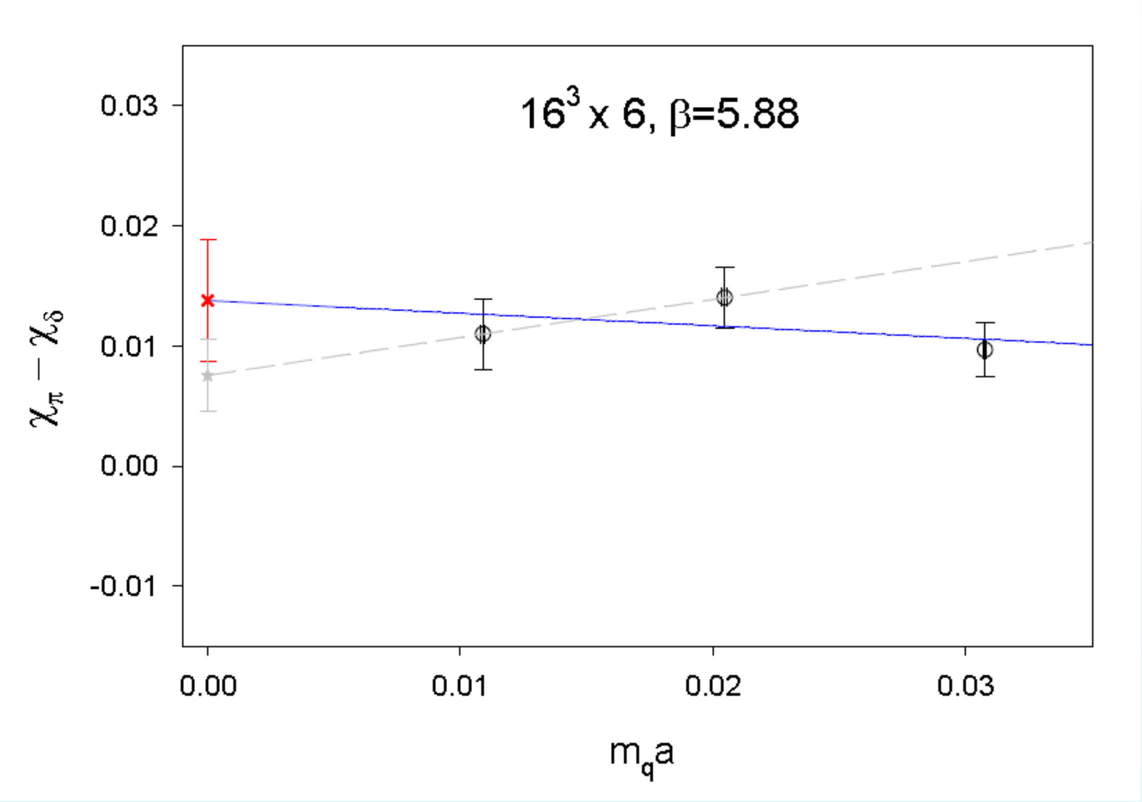


If $\chi_\sigma = \chi_\eta$ in the chiral limit, then $U(1)_A$ is restored.

Chiral Susceptibilities (cont.)

$T \approx 180$ MeV

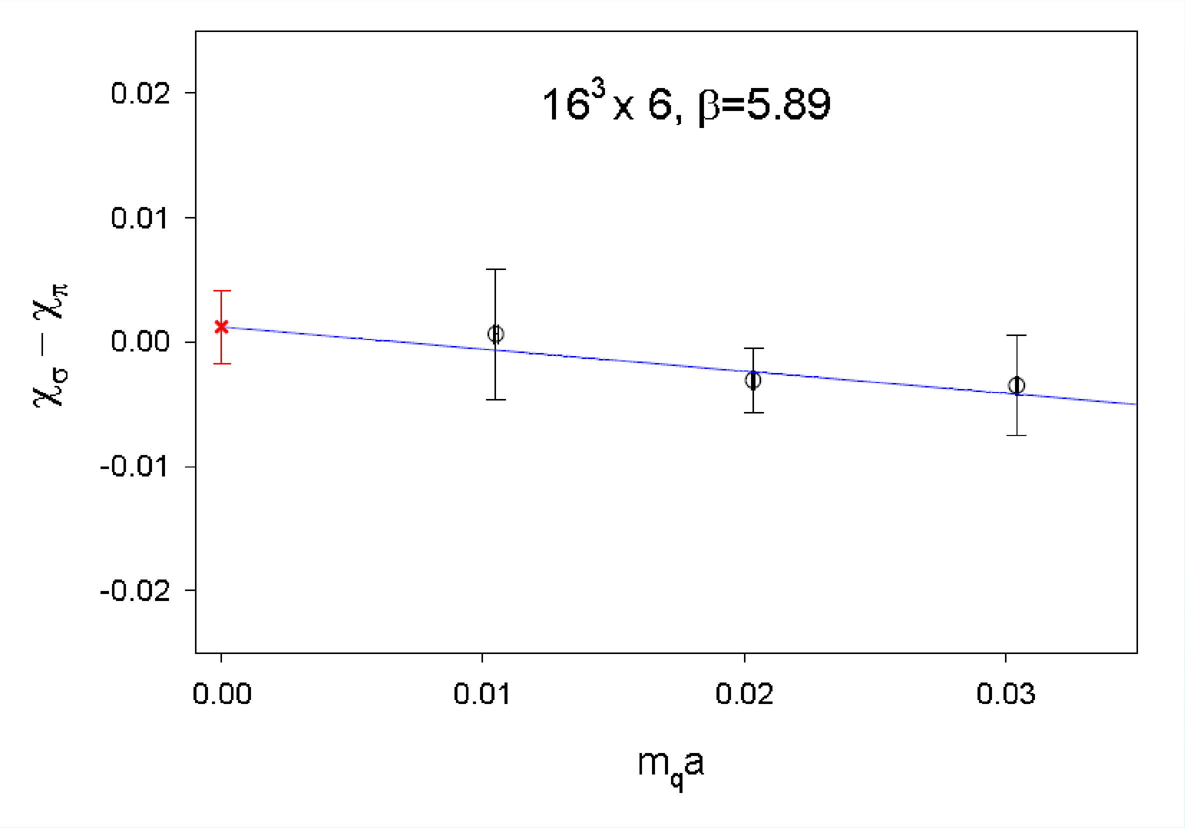


If $\chi_\pi = \chi_\delta$ in the chiral limit, then $U(1)_A$ is restored.

Chiral Susceptibilities (cont.)

$$\chi_\pi \simeq \chi_\sigma \simeq 0.24$$

$T \approx 200$ MeV



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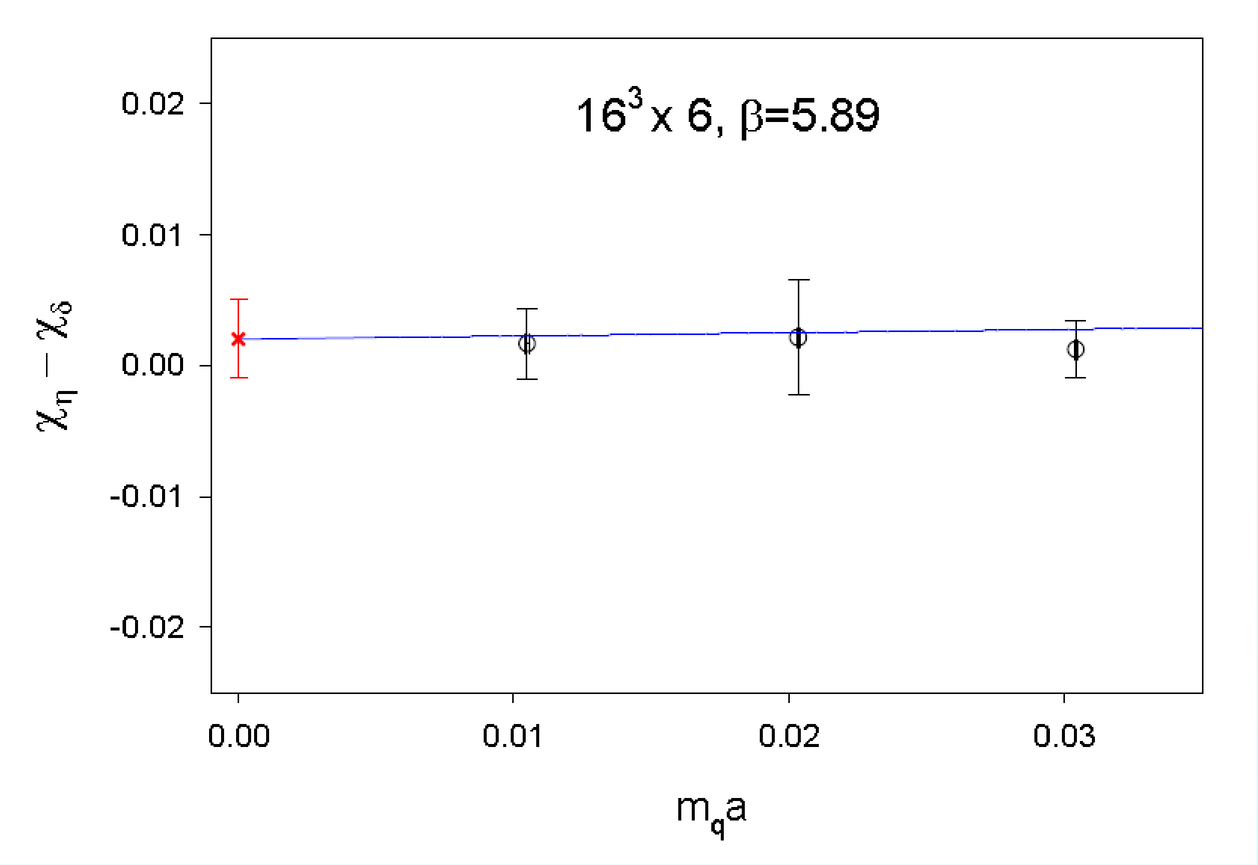

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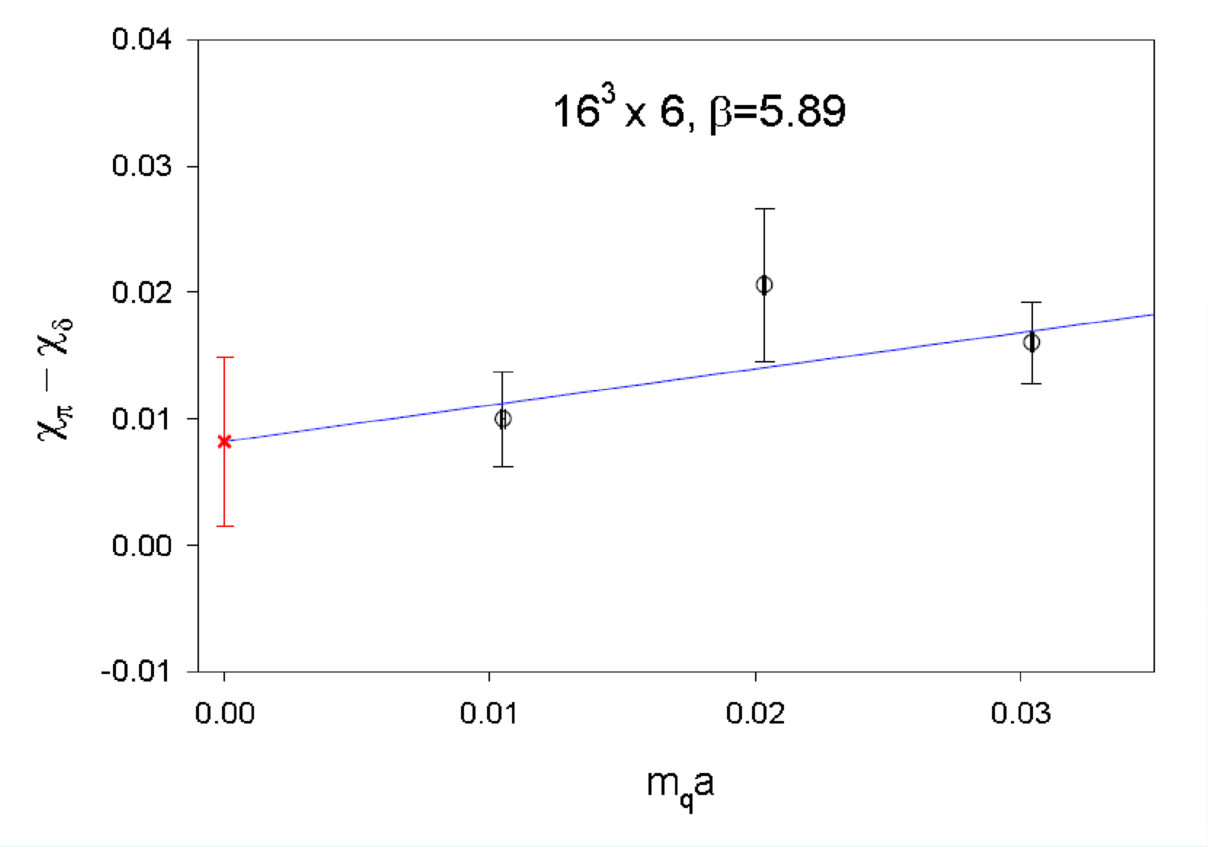

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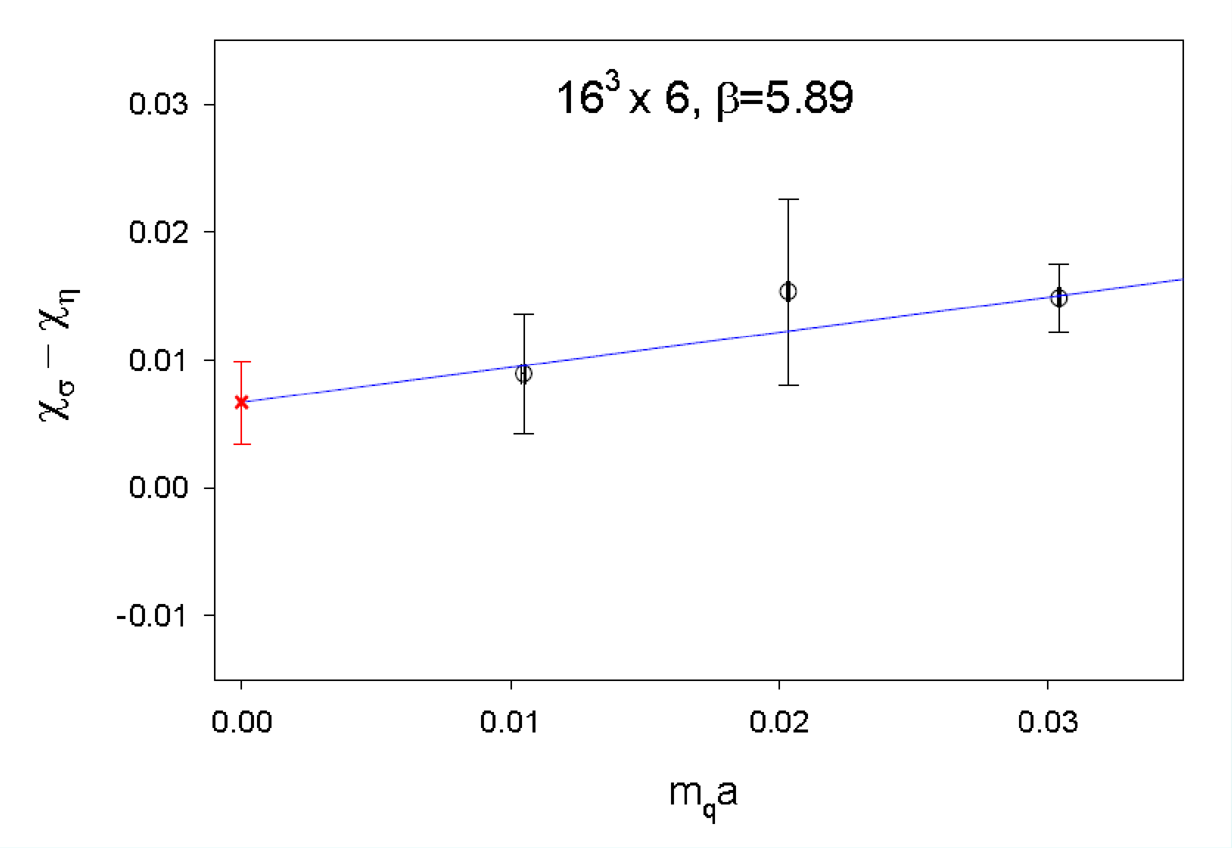
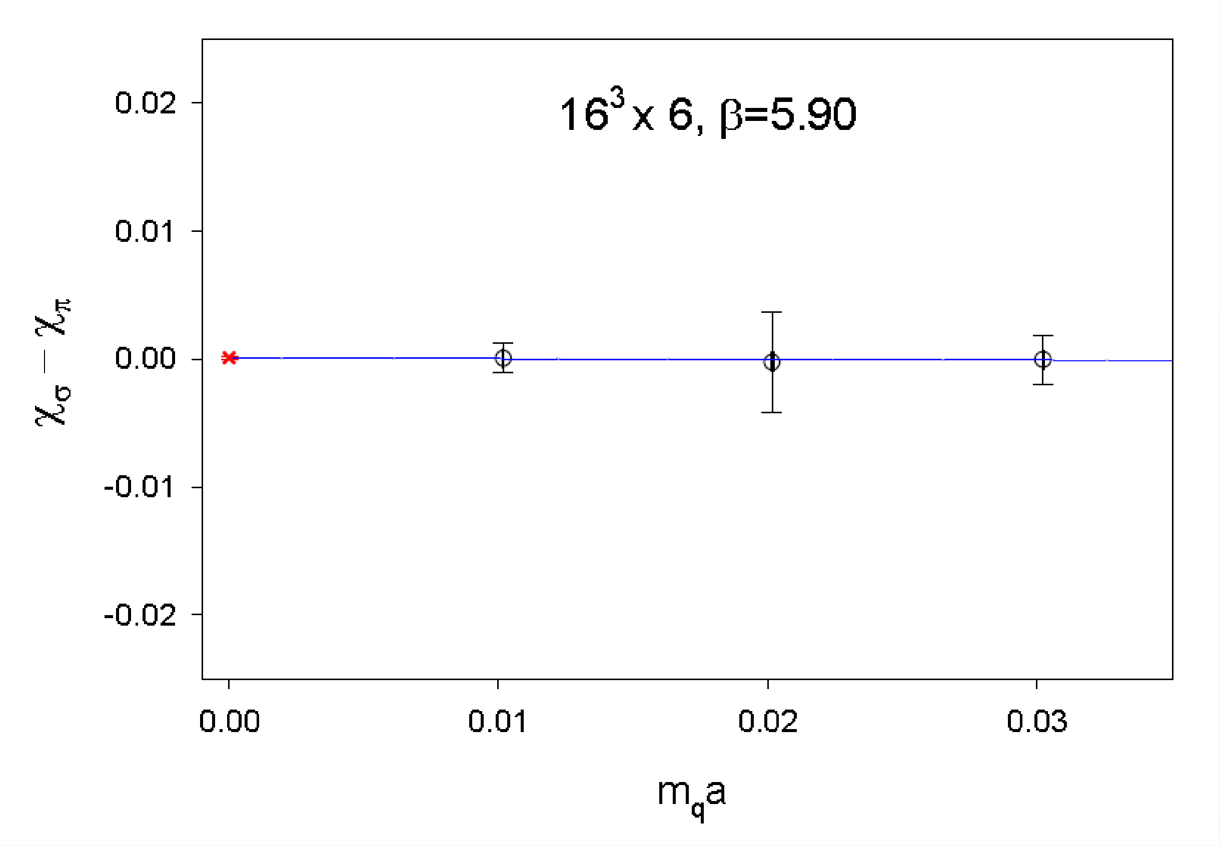


If $\chi_\sigma = \chi_\eta$ in the chiral limit, then $U(1)_A$ is restored.

Chiral Susceptibilities (cont.)

$$\chi_\pi \simeq \chi_\sigma \simeq 0.235$$

$T \approx 210$ MeV

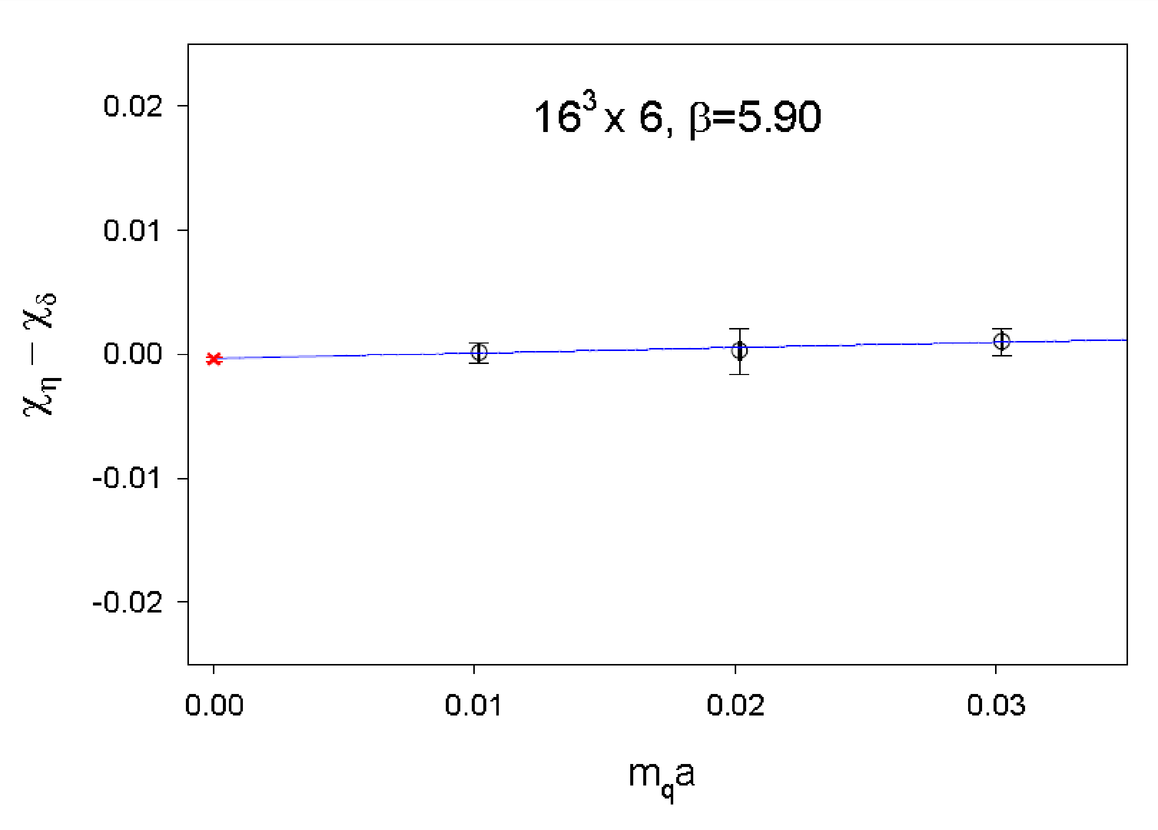


$\chi_\pi = \chi_\sigma \Rightarrow$ restoration of $SU(2)_L \times SU(2)_R$

Chiral Susceptibilities (cont.)

$$\chi_\eta \simeq \chi_\delta \simeq 0.235$$

$T \approx 210$ MeV



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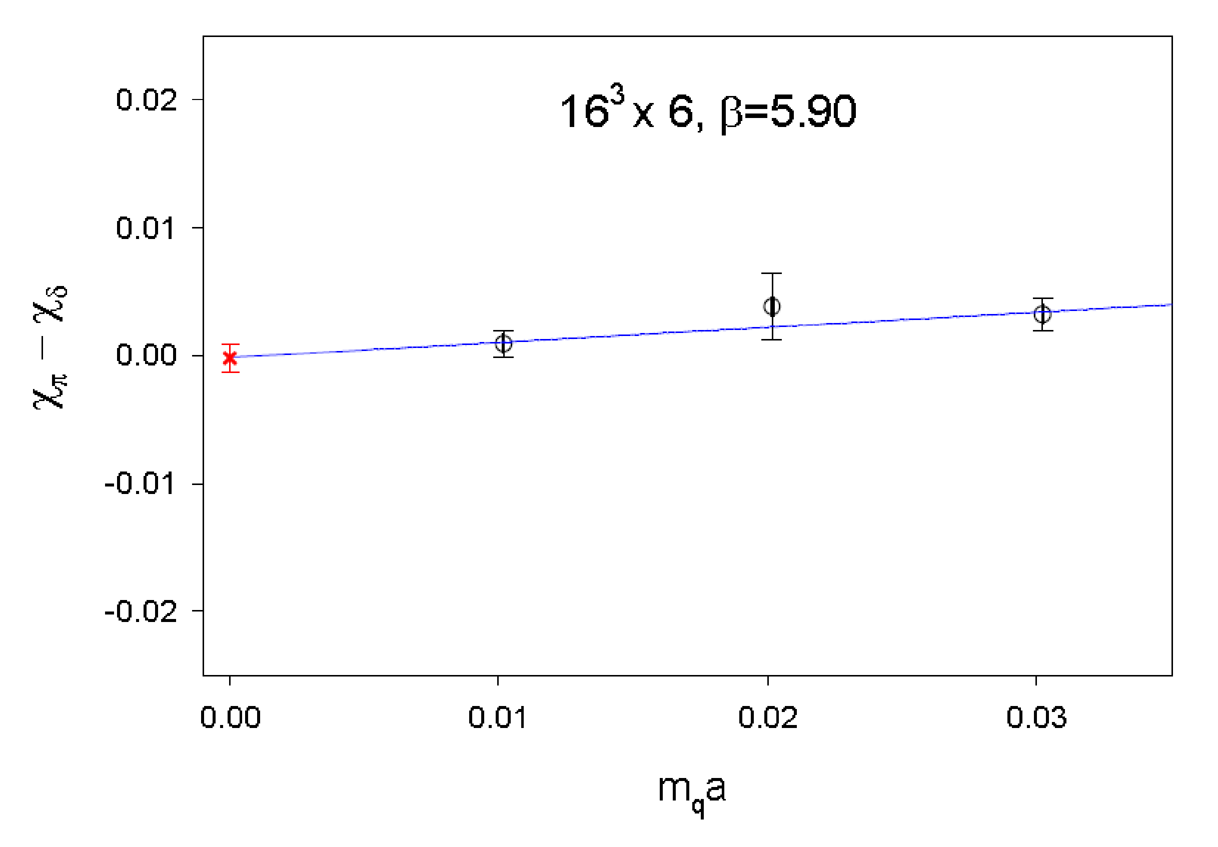

$$\chi_\pi = \chi_\delta \;\Rightarrow\; \text{restoration of } U(1)_A$$

Chiral Susceptibilities (cont.)

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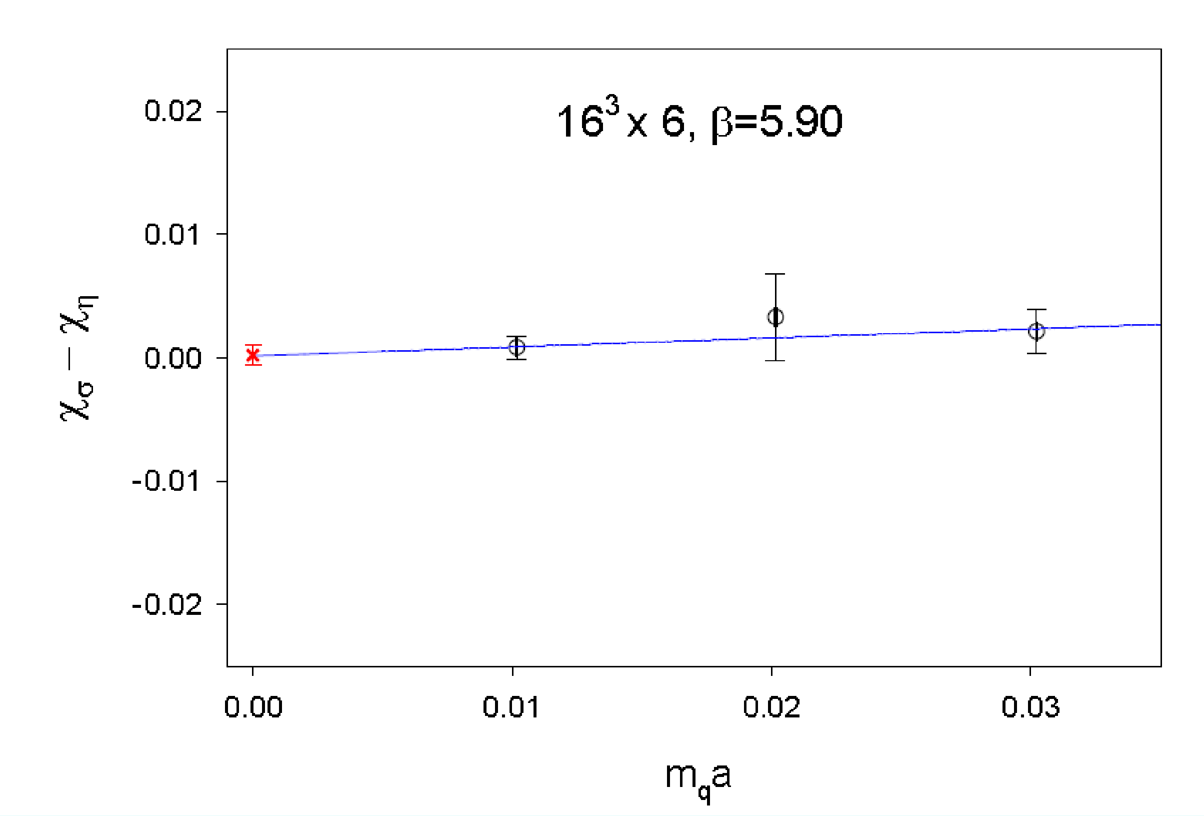
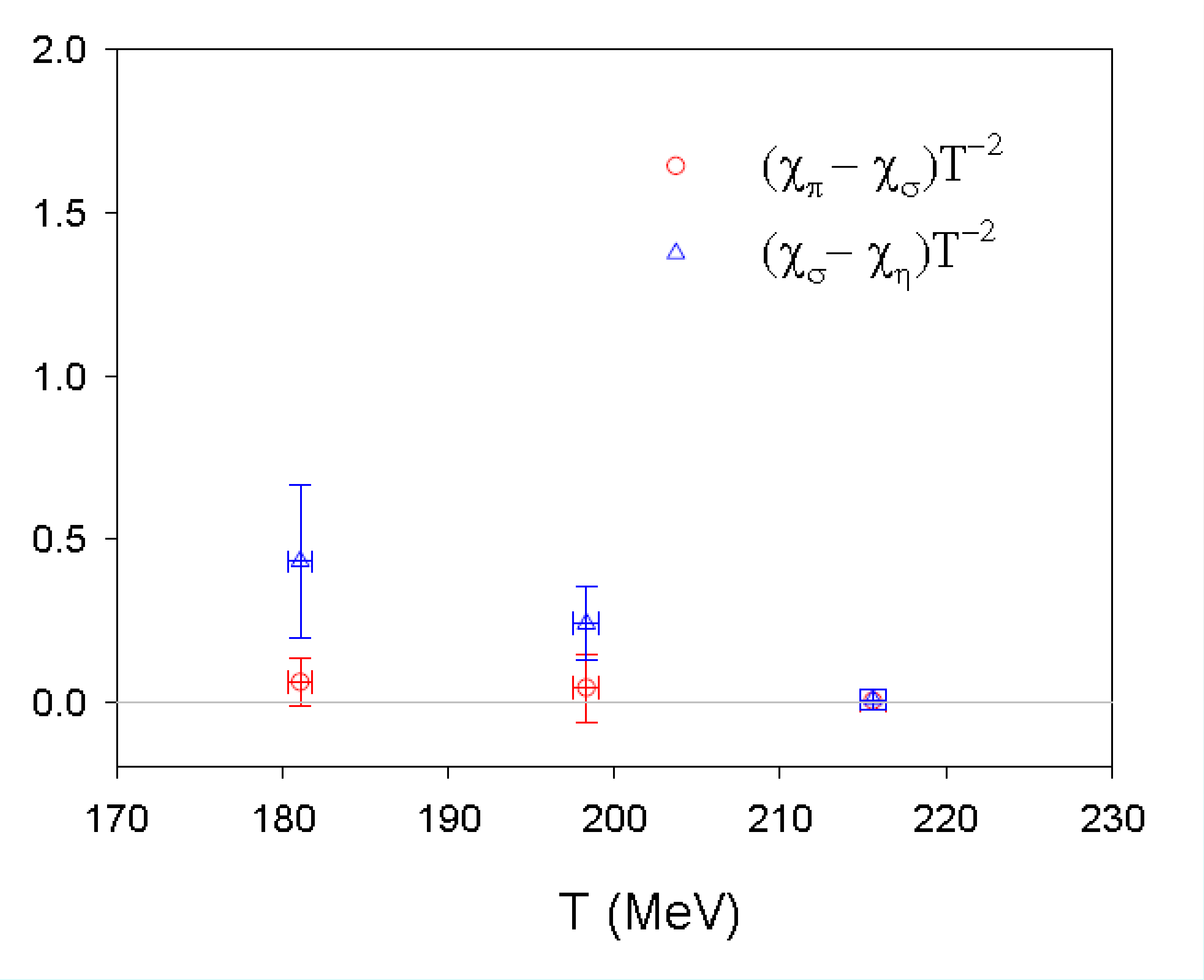


$\chi_\sigma = \chi_\eta \Rightarrow$ restoration of $U(1)_A$

Chiral Susceptibilities (cont.)



Concluding Remarks

- TWQCD preliminary results of chiral susceptibilities suggest that $U(1)_A$ symmetry is broken at T_c , and it is restored at $T_1 \approx 1.2\ T_c$
- The possibility of $U(1)_A$ restoration at $T_1 = T_c$ has not been completely ruled out, since the chiral extrapolation is nontrivial
- A more precise determination of T_c and T_1 , with a finer scan in β, and also with a larger volume, are necessary to clarify these issues.