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The chiral phase transition of Nf=2 QCD at zero and imaginary chemical potential

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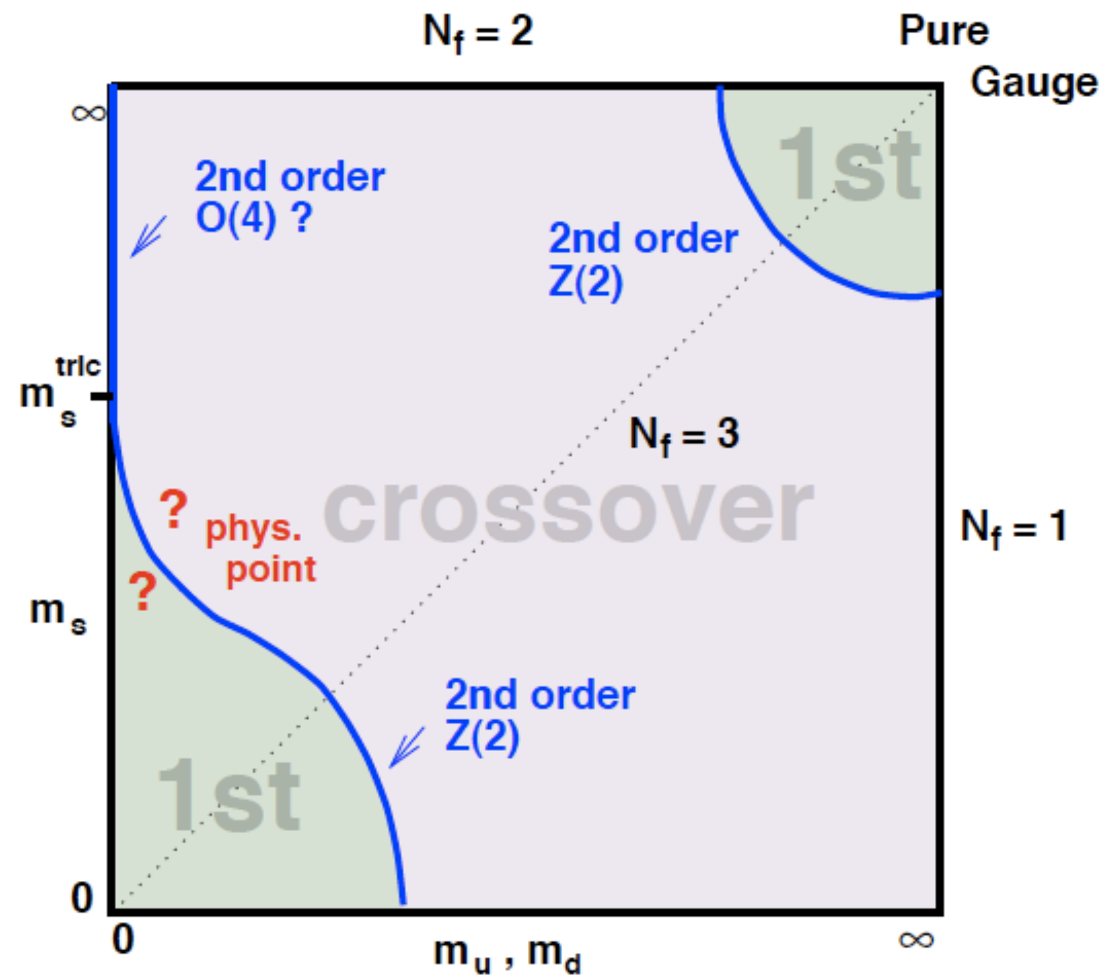
in collaboration with C. Bonati, Ph. de Forcrand, M. D'Elia, F. Sanfilippo

- A longstanding open issue
- Difficulty of the "traditional approach"
- Imaginary chemical potential as a tool to answer the question

The order of the p.t., arbitrary quark masses $\mu = 0$

chiral p.t.
restoration of global
$SU(2)_L \times SU(2)_R \times U(1)_A$
anomalous

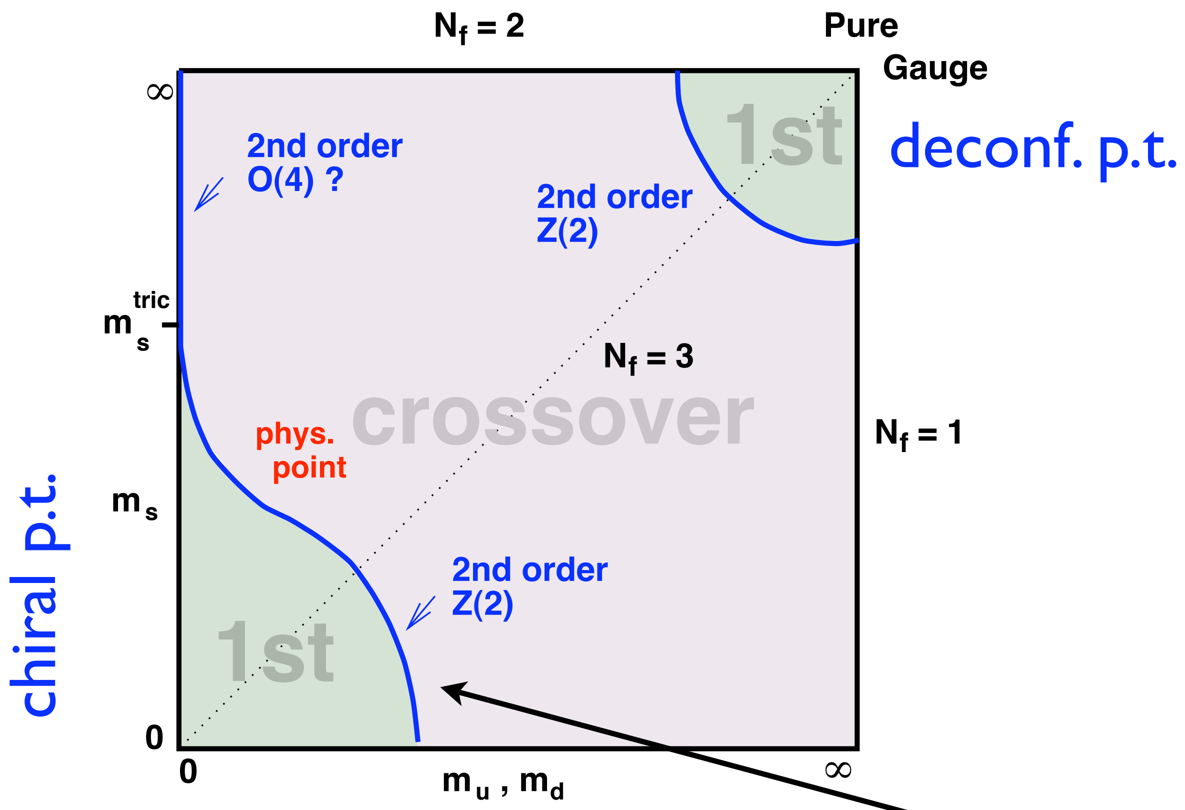
deconfinement p.t.:
breaking of global $Z(3)$

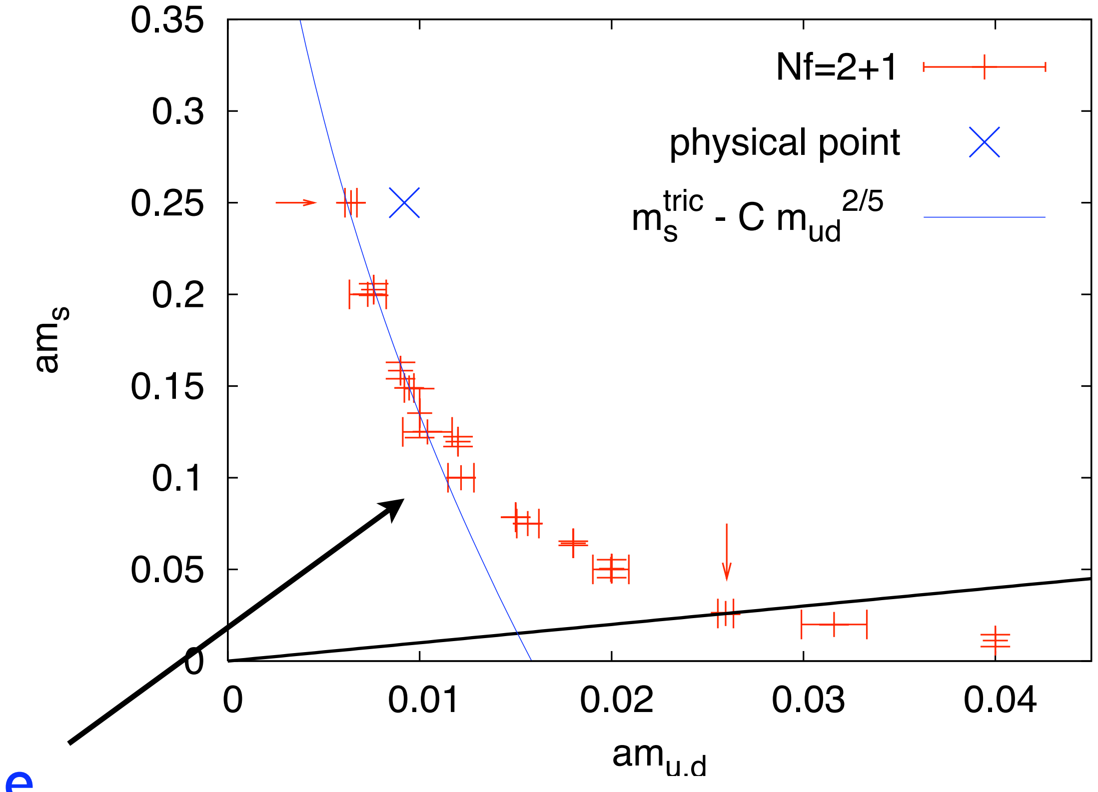


Pisarski, Wilczek 84

epsilon (=1) expansion

Order of p.t., arbitrary quark masses $\mu = 0$





chiral critical line

- physical point: crossover in the continuum — Aoki et al 06
- chiral critical line on $N_t = 4, a \sim 0.3$ fm — de Forcrand, O.P. 07
- 1st order chiral region only with coarse staggered until now
- But: $N_f = 2$ chiral O(4) vs. 1st still open — Di Giacomo et al 05, Kogut, Sinclair 07
  $U_A(1)$ anomaly! — Chandrasekharan, Mehta 07
  Cossu et al. 12, Aoki et al. 12

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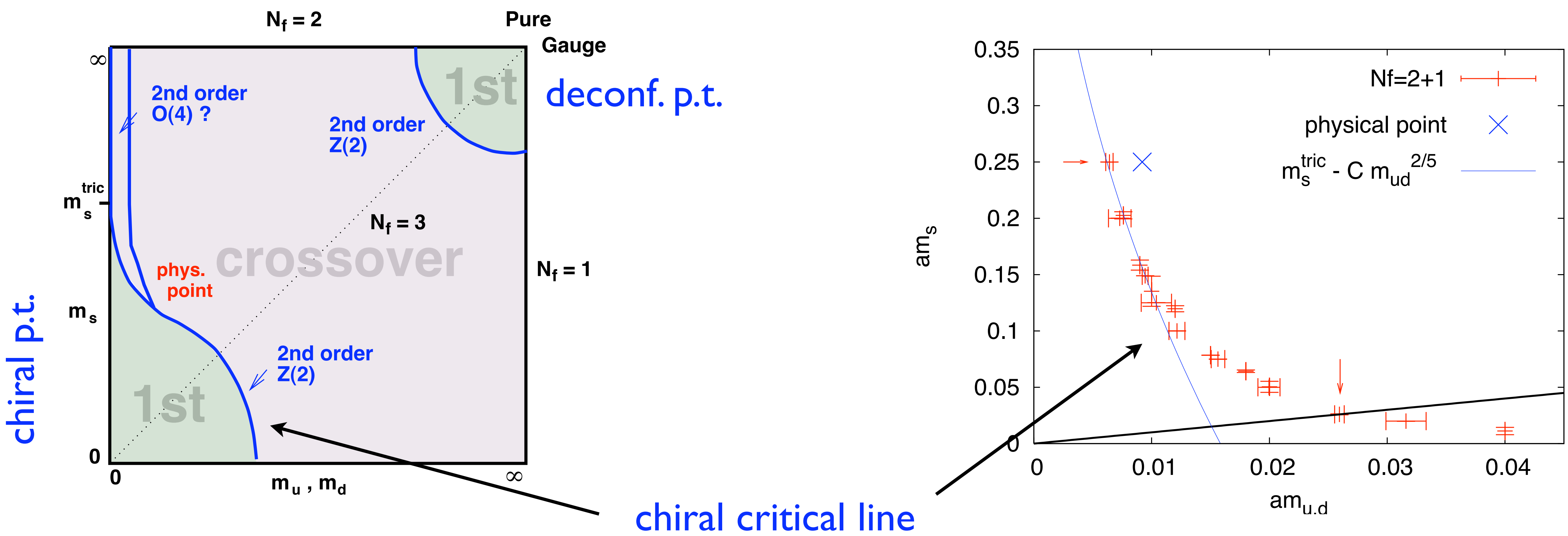

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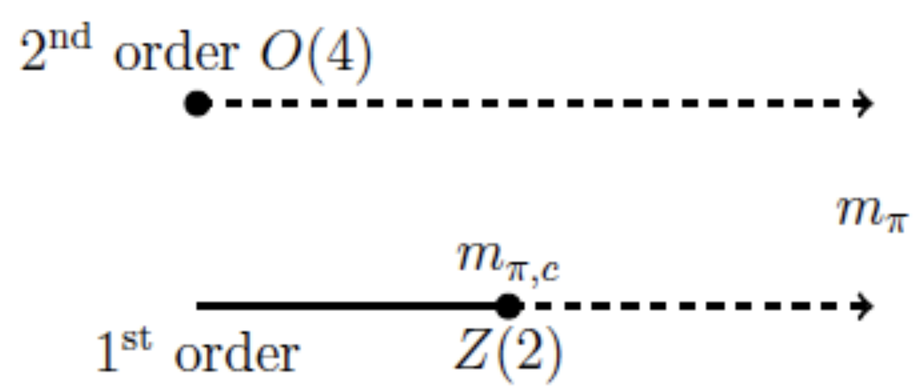
The options for Nf=2, zero density

Pisarski, Wilczek 84

Spontaneous chiral symmetry breaking + restoration:
true phase transition necessary in chiral limit!

Order depends on anomaly strength at Tc

For staggered fermions: O(4) is reduced to O(2)



anomaly still strong

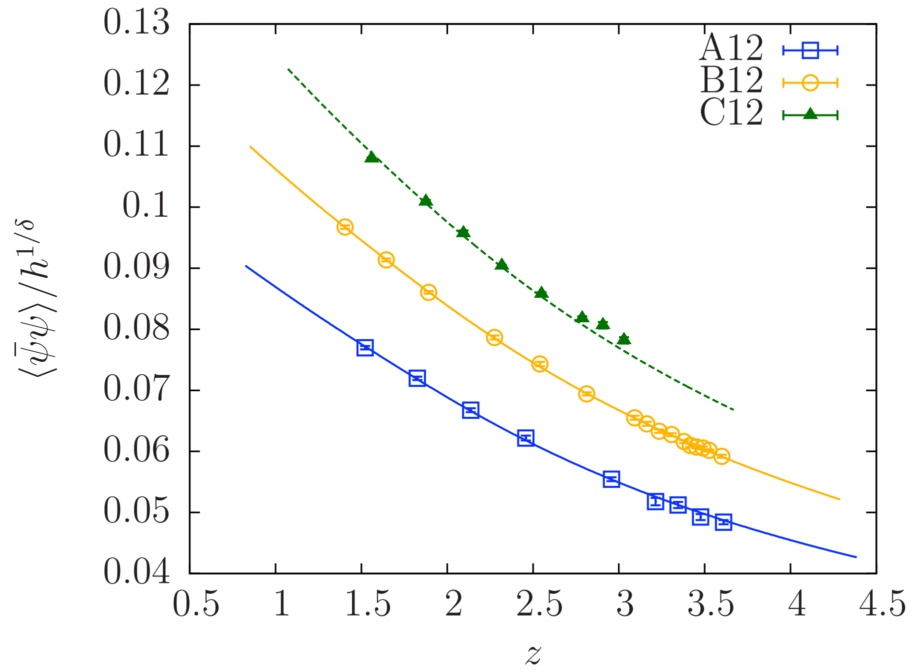
anomaly restored

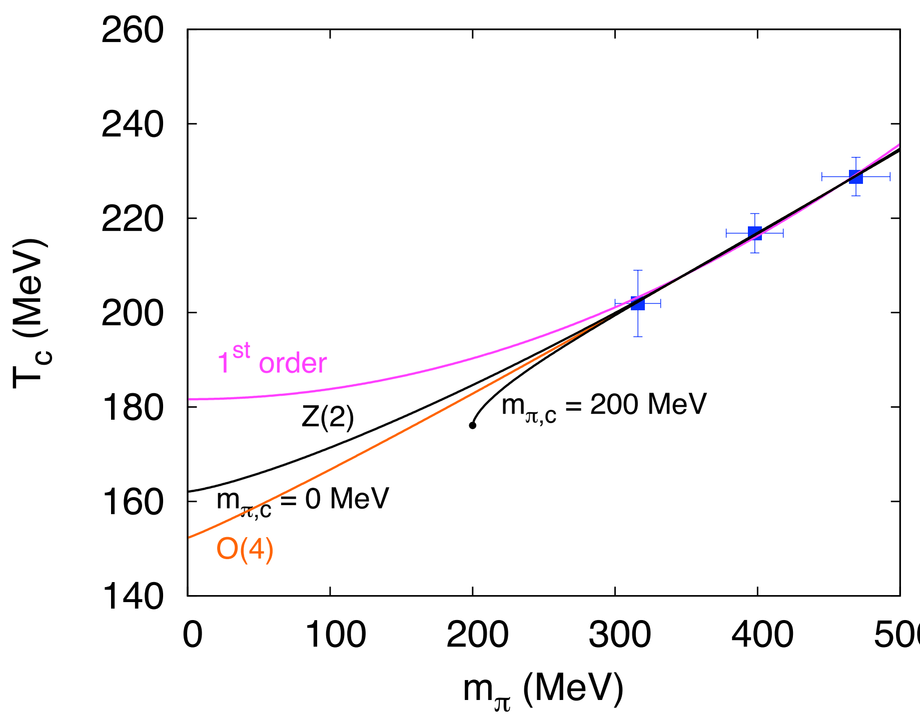
or 2nd order, U(2)_L x U(2)_R / U(2)_V Vicari LAT 07

Nf=2: chiral transition from scaling behaviour?

Traditional approach: test consistency with scaling for decreasing pion mass

Example: Wilson quarks, tmfT 13





$$\langle\overline{\psi}\psi\rangle = h^{1/\delta} c f(d\tau/h^{1/(\delta\beta)}) + a_t \tau h + b_1 h + \dots$$

$$T_c(m_\pi) = T_c(0) + A m_\pi^{2/\beta\delta}$$

Problems:
- different critical exponents indistinguishable;
- maybe too far outside scaling region;
- no unambiguous signal for criticality

$$1/(\beta\delta) = 0.537, 0.638$$

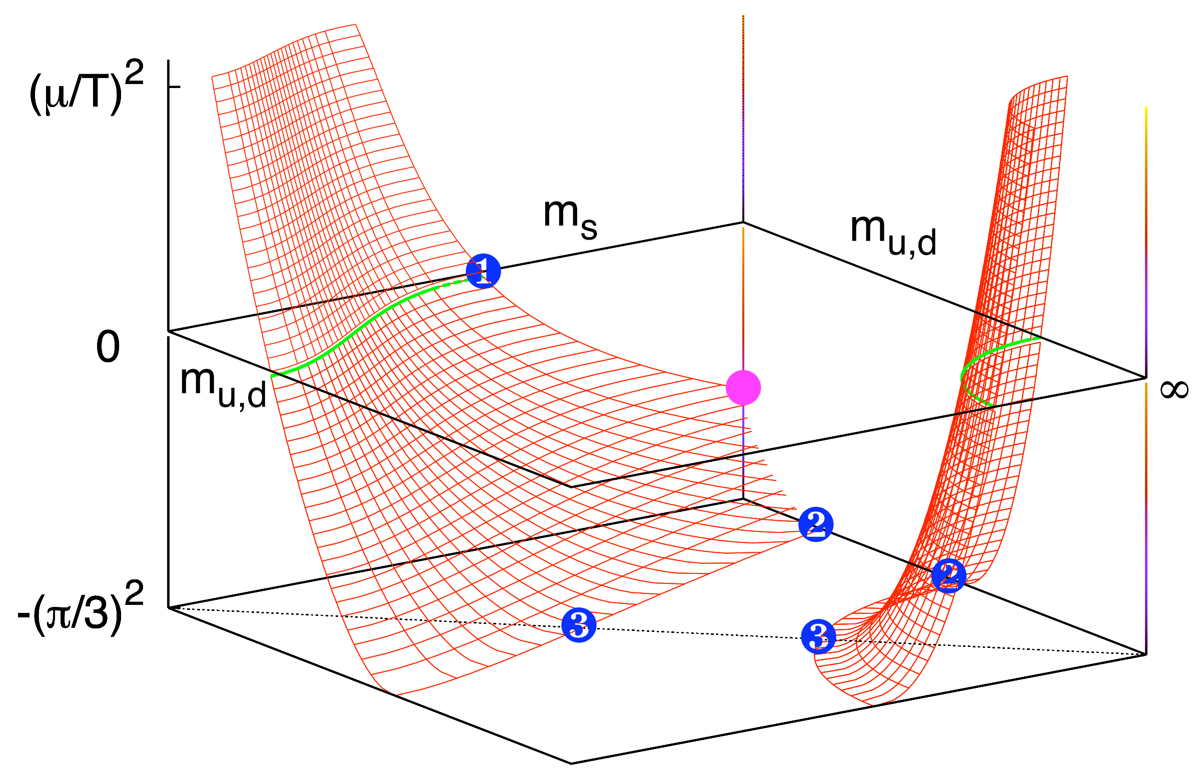
$$1/\delta = 0.21, 0.20 \text{ for } O(4) \text{ and } Z(2)$$

Different approach: use imaginary μ !

de Forcrand, O.P. LAT 11

- chiral critical surface continues to imaginary chemical potential
- no sign problem
- chiral transition stronger, i.e. visible at larger quark masses!

de Forcrand, O.P. 10

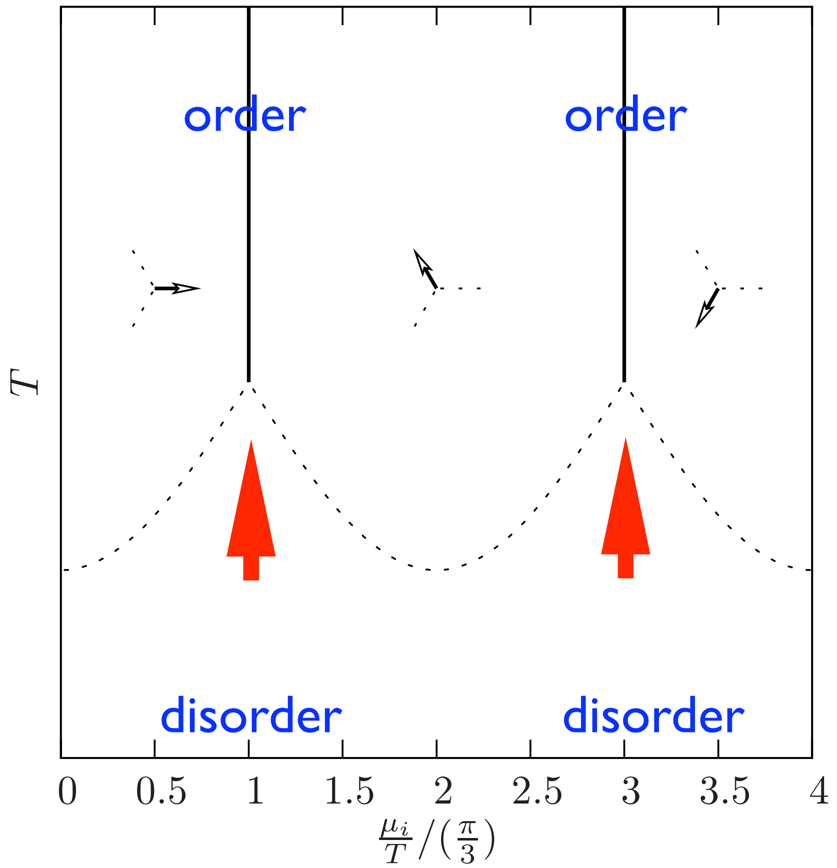


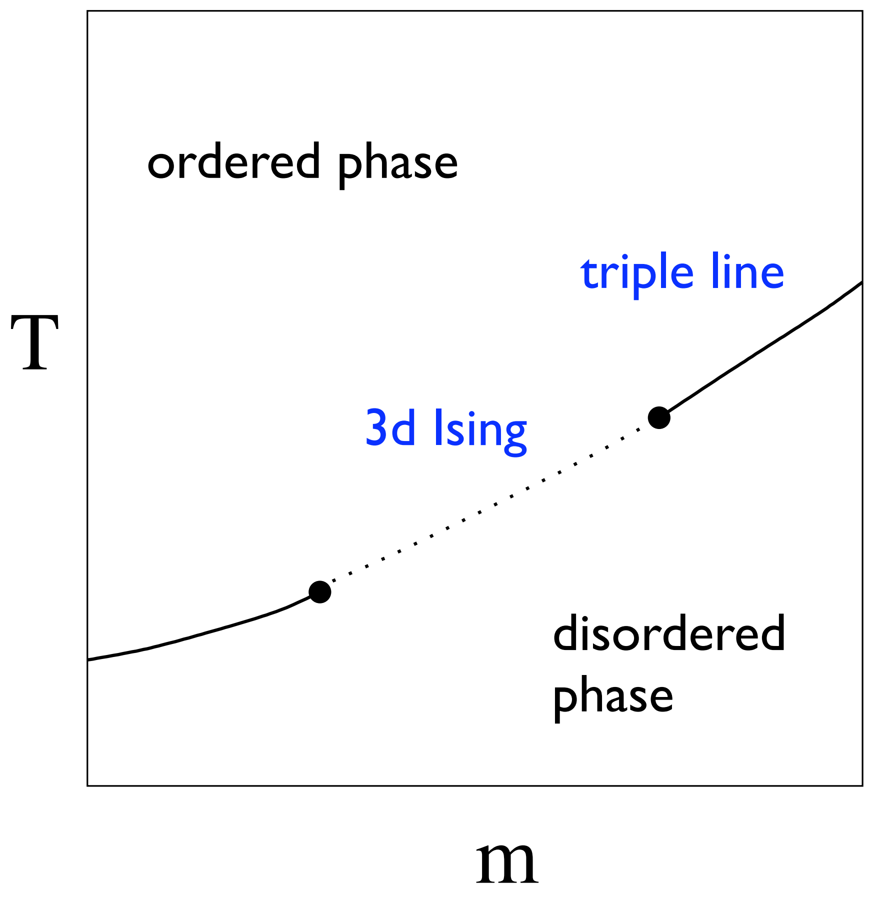
Recall Roberge-Weiss symmetry at imaginary μ

$$Z\left(i\frac{\mu_i}{T}+i\frac{2\pi n}{N}\right)=Z\left(i\frac{\mu_i}{T}\right)$$

Strategy: fix $\frac{\mu_i}{T}=\frac{\pi}{3},\pi$, measure Im(L), order parameter at $\frac{\mu_i}{T}=\pi$

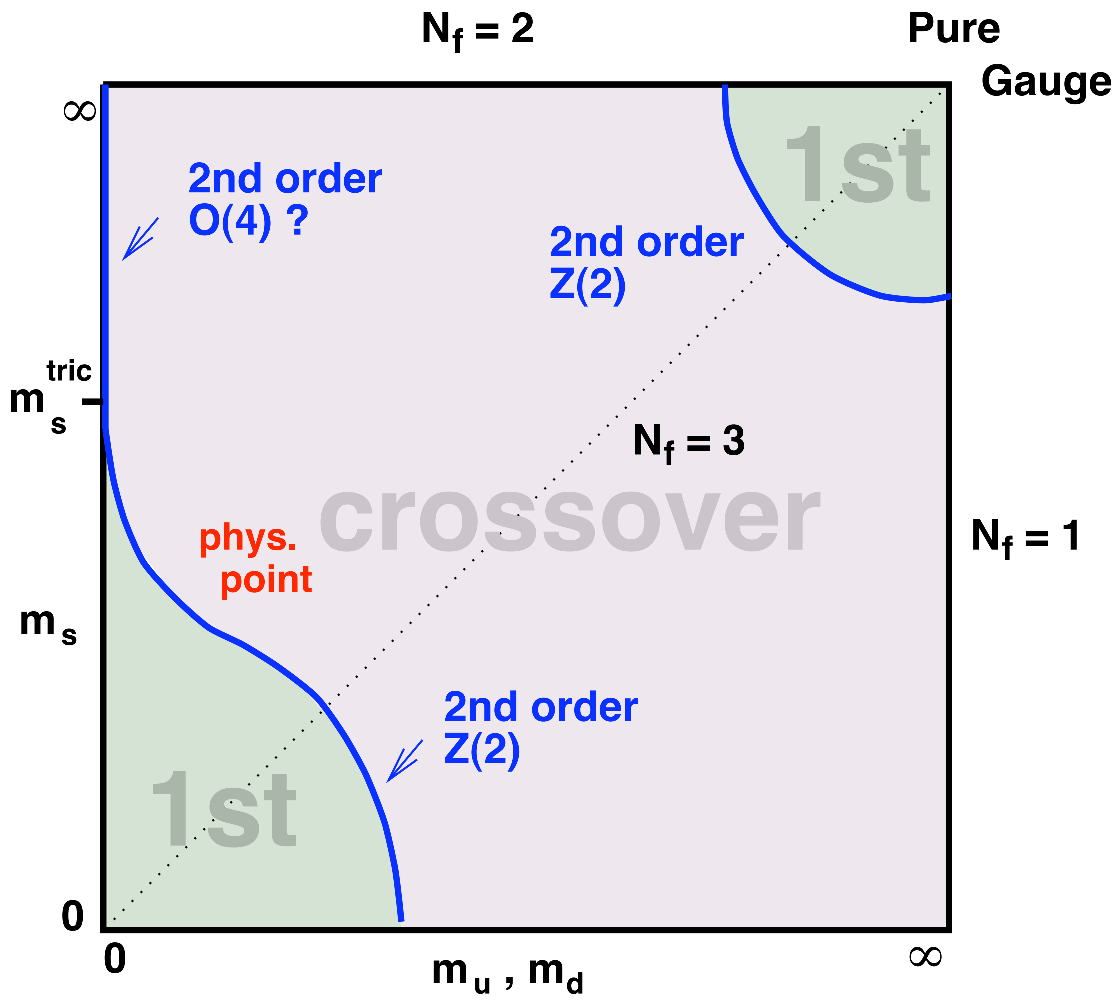
determine order of Z(3) branch/end point as function of m

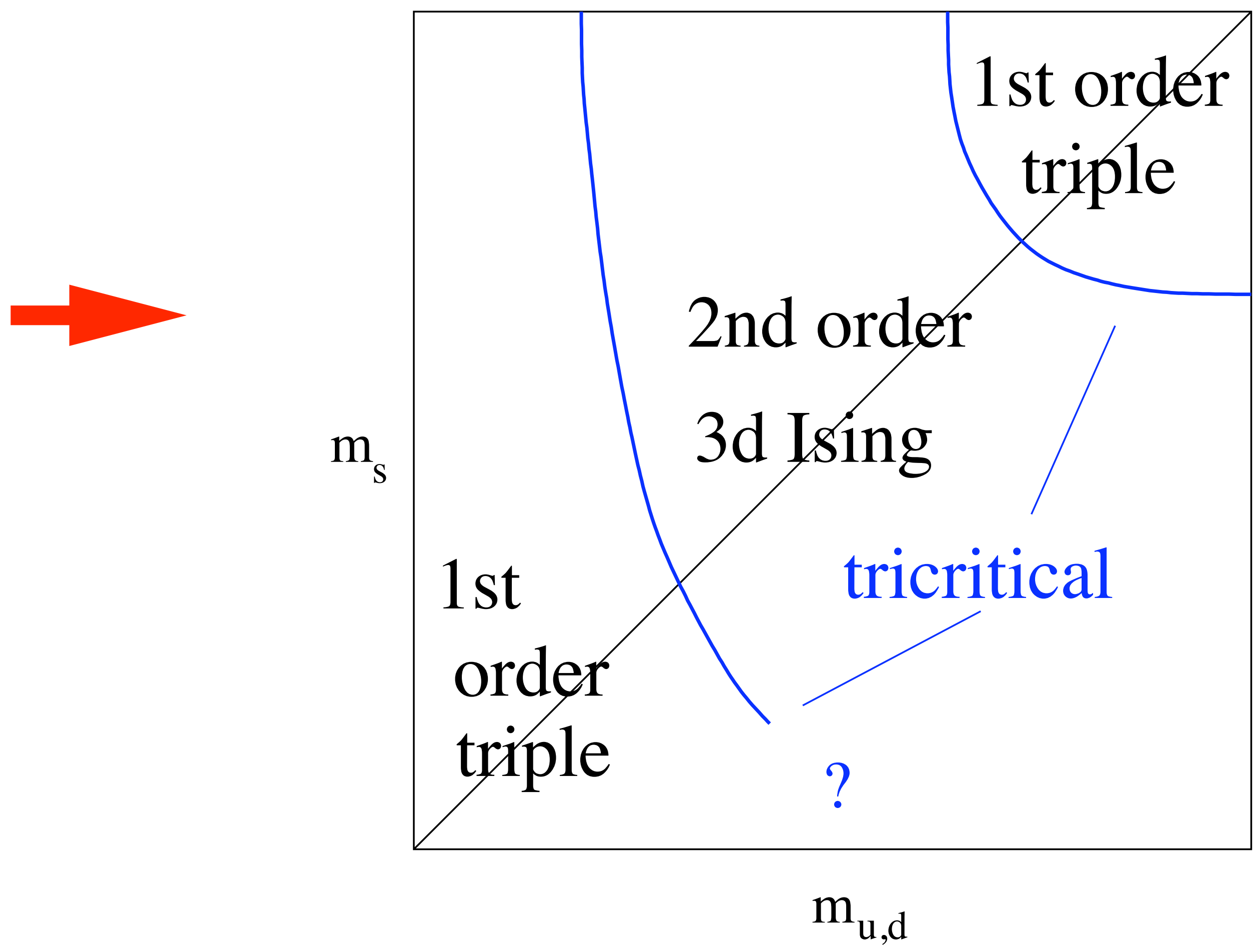




Nf=2: D'Elia, Sanfilippo 09 Nf=3: de Forcrand, O.P. 10 Talks: Pinke, Alexandru

Critical lines at imaginary μ

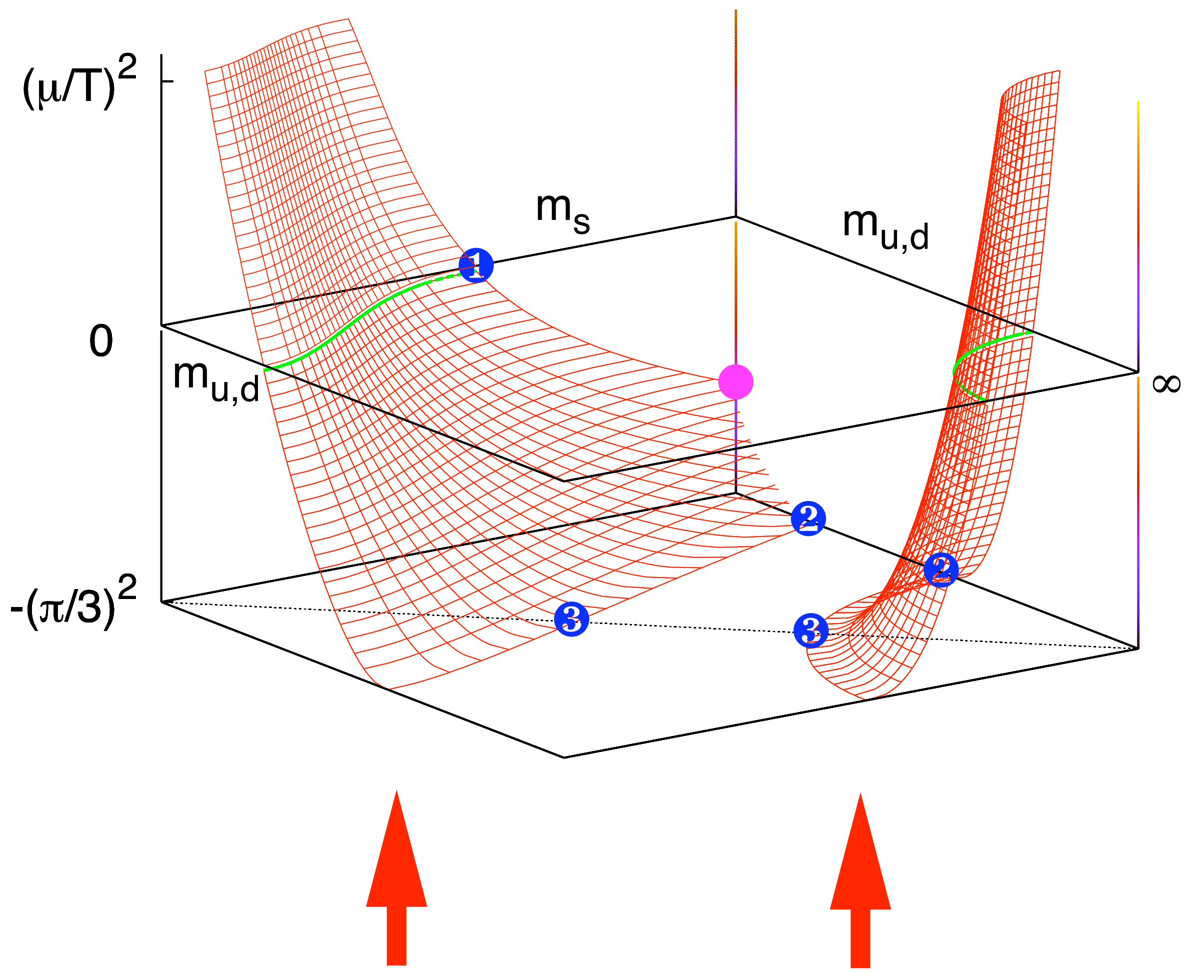




$\mu = 0$

$\mu = i\frac{\pi T}{3}$

-Connection computable with standard Monte Carlo!

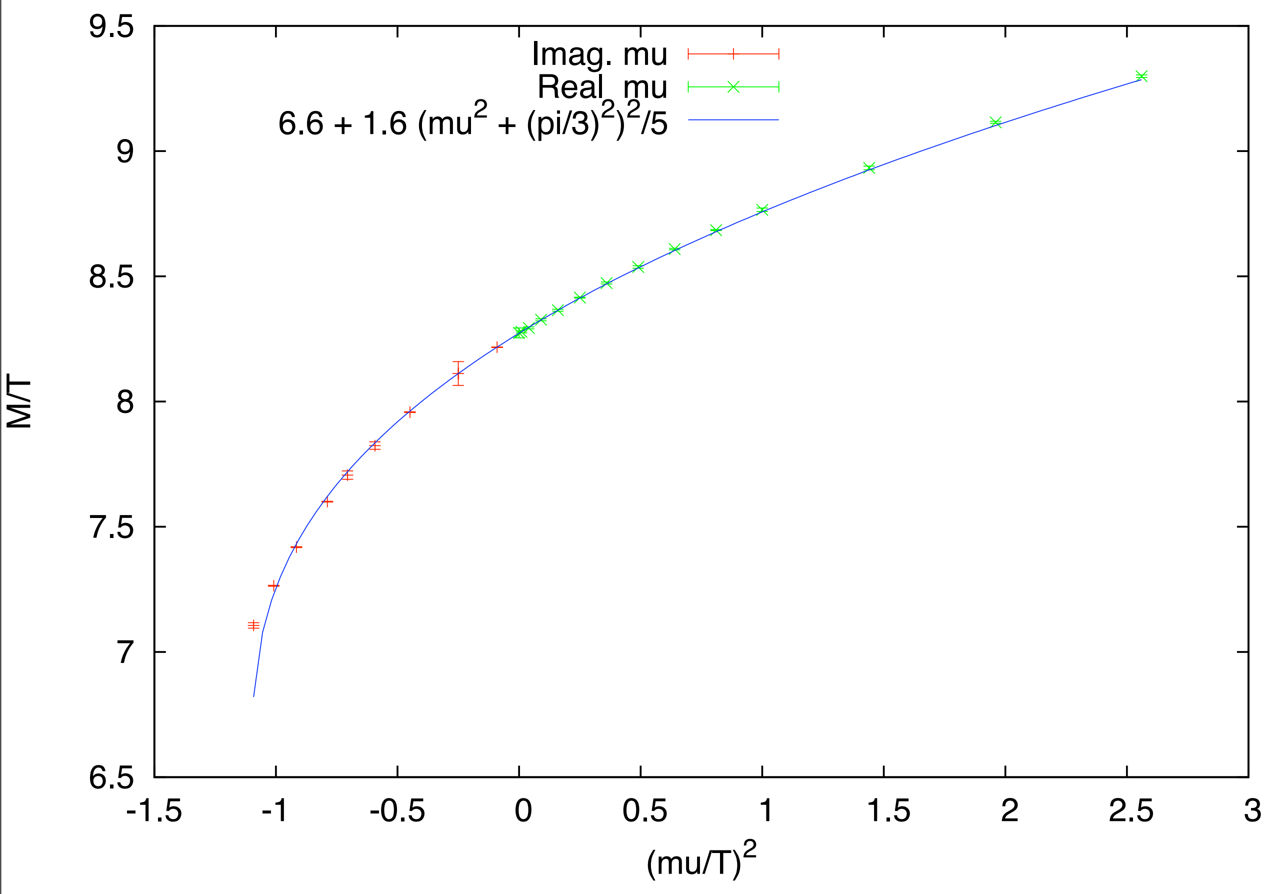


shape, sign of curvatures determined by tricritical scaling!

Heavy quarks: deconfinement critical surface

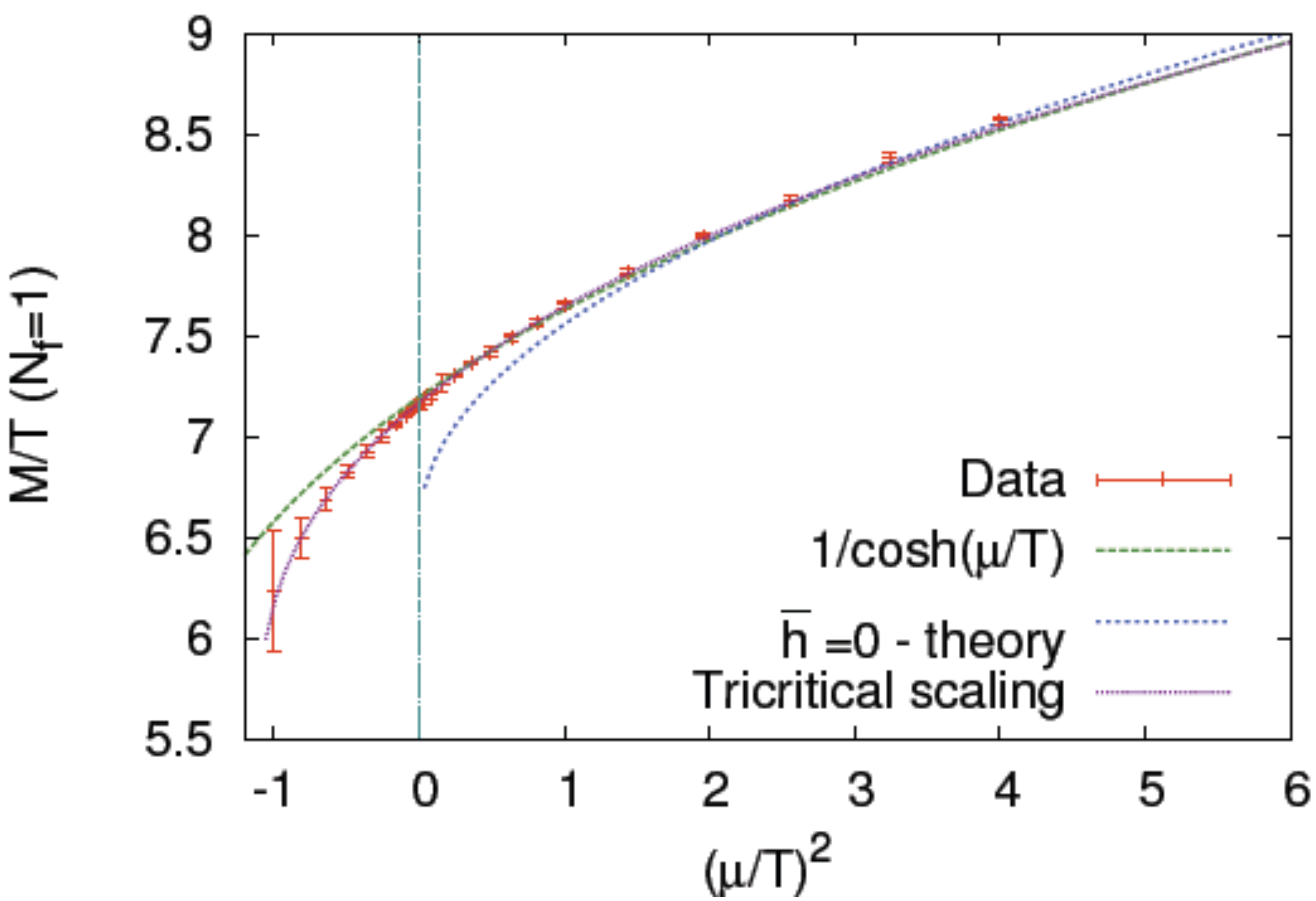
3d, 3-state Potts: same universality class!

de Forcrand, Kim, Kratochvila, Takaishi 06
de Forcrand, O.P. 10



QCD using hopping exp.

Fromm, Langelage, Lottini, O.P. 12

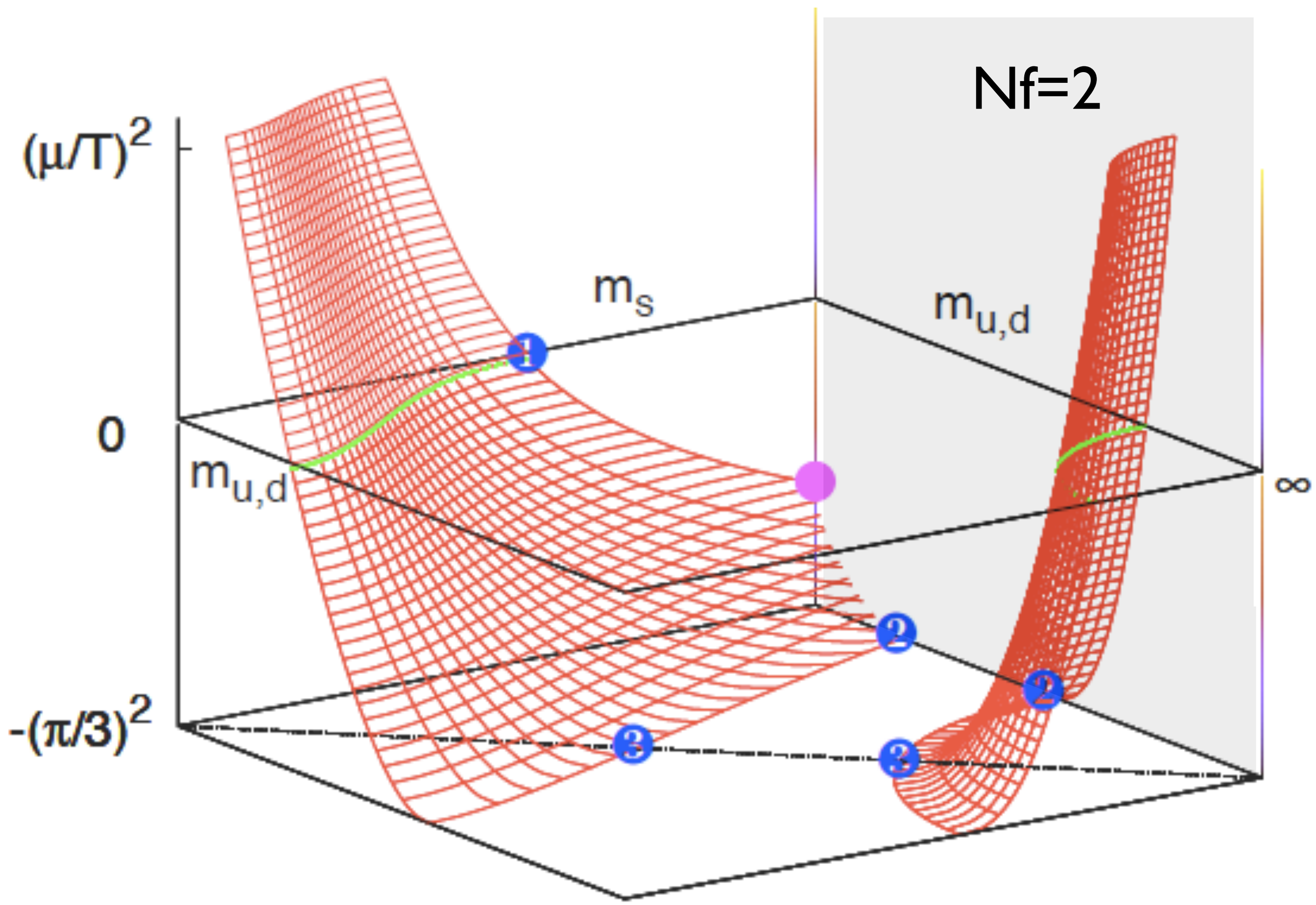


tri-critical scaling:

$$\frac{m_c}{T}(\mu^2) = \frac{m_{tric}}{T} + K\left[\left(\frac{\pi}{3}\right)^2 + \left(\frac{\mu}{T}\right)^2\right]^{2/5}$$

exponent universal

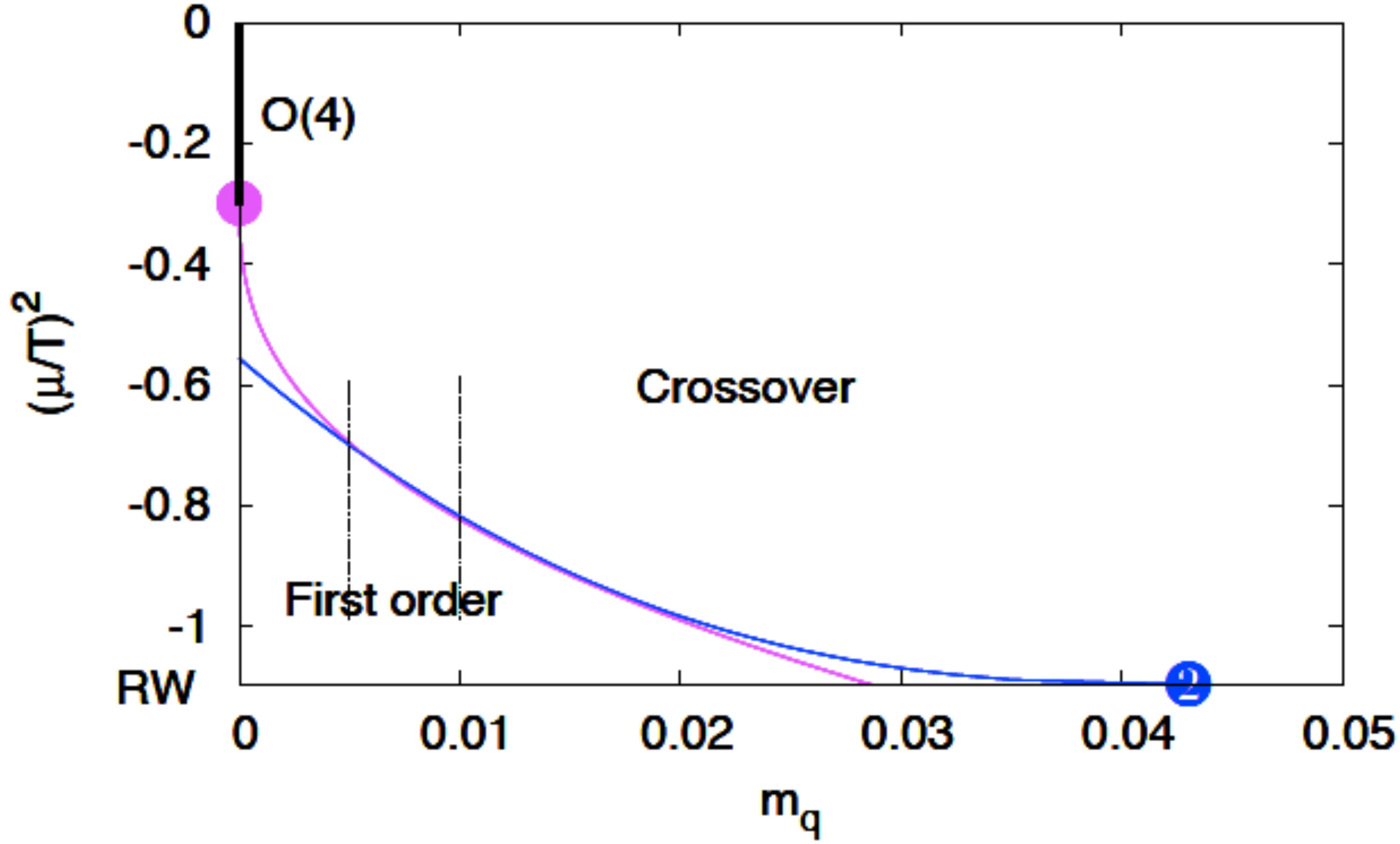
Now Nf=2 backplane



Can tricritical scaling constrain the phase diagram in the $N_f = 2$ backplane?

The Nf=2 backplane

de Forcrand, OP LAT 11

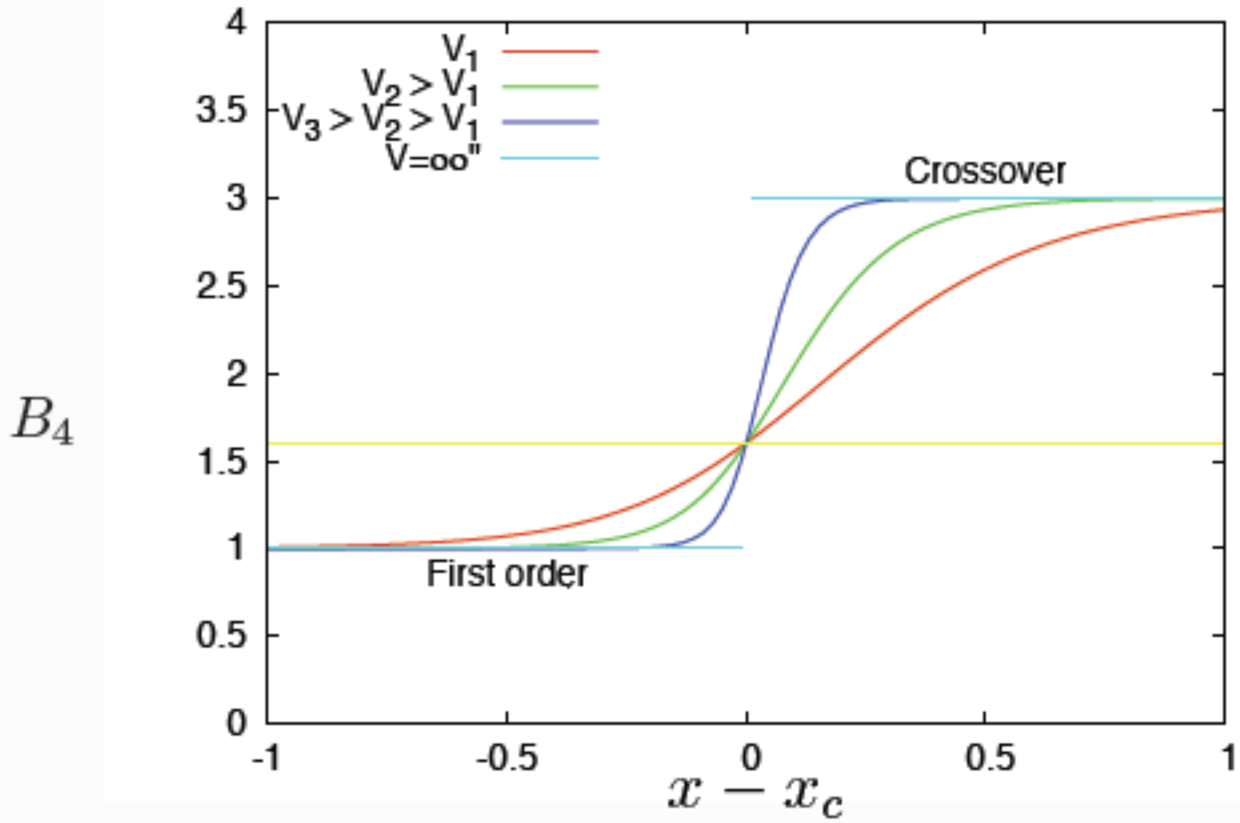


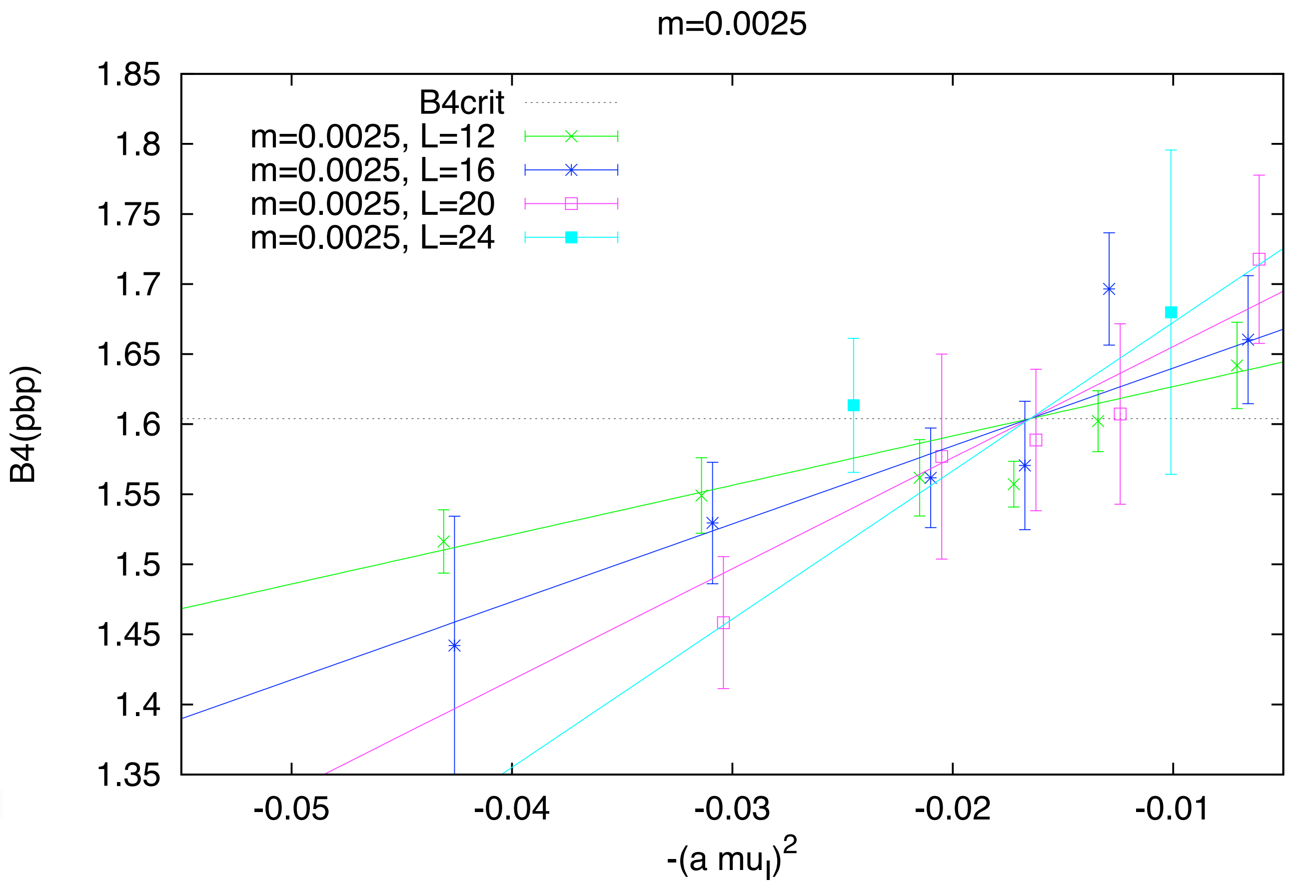
Two tricritical points joined by a critical (Ising) line

One tricritical point known – where is the other?

Observable: Binder cumulant

$$B_4(\bar\psi\psi) \equiv \frac{\langle(\delta\bar\psi\psi)^4\rangle}{\langle(\delta\bar\psi\psi)^2\rangle^2} \xrightarrow{V\to\infty} \begin{cases} 1.604 & \text{3d Ising} \\ 1 & \text{first-order} \\ 3 & \text{crossover} \end{cases}$$

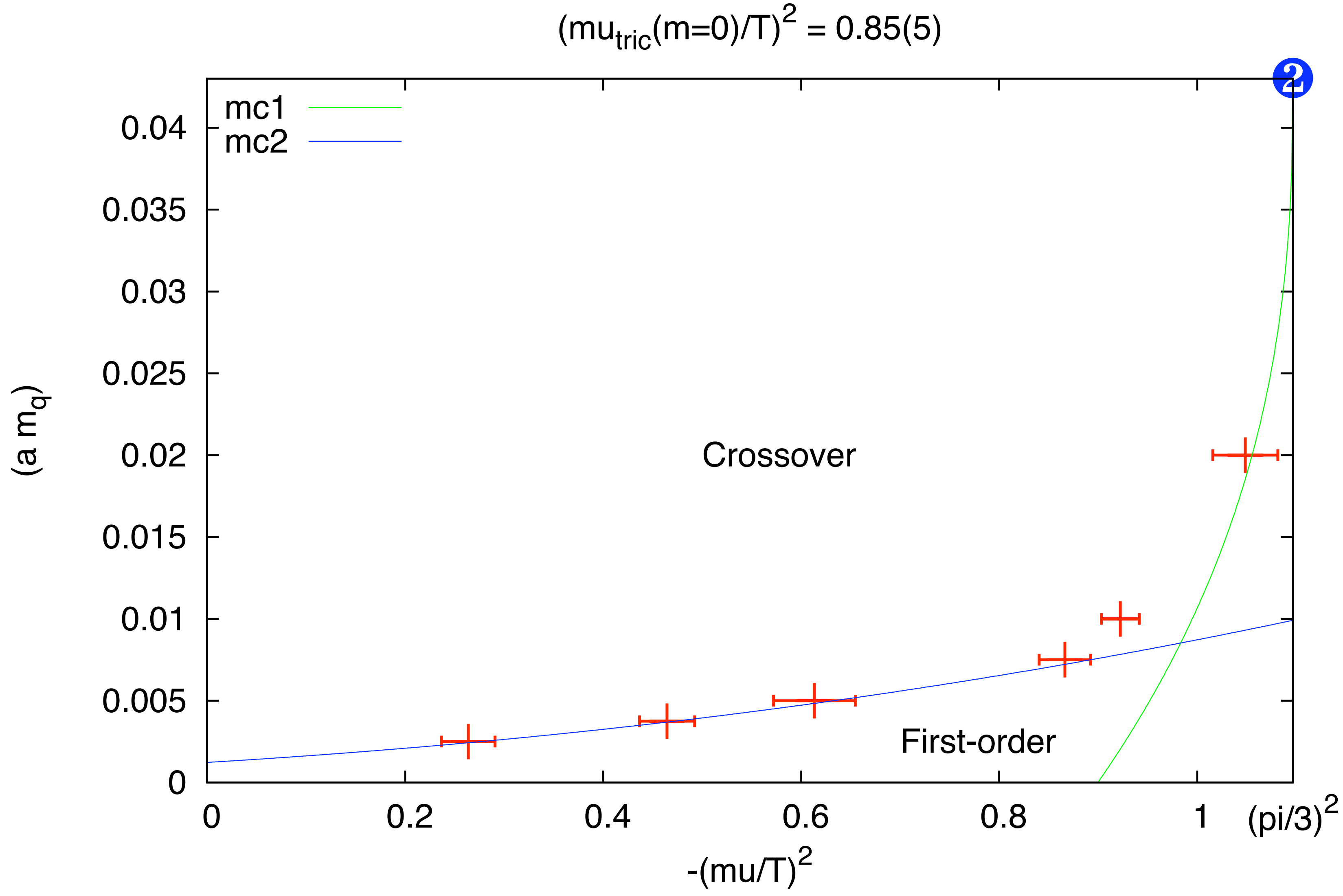




Fix a mass, then scan in chemical potential for critical point on different volumes

Result on Nt=4

Plenary Szabo



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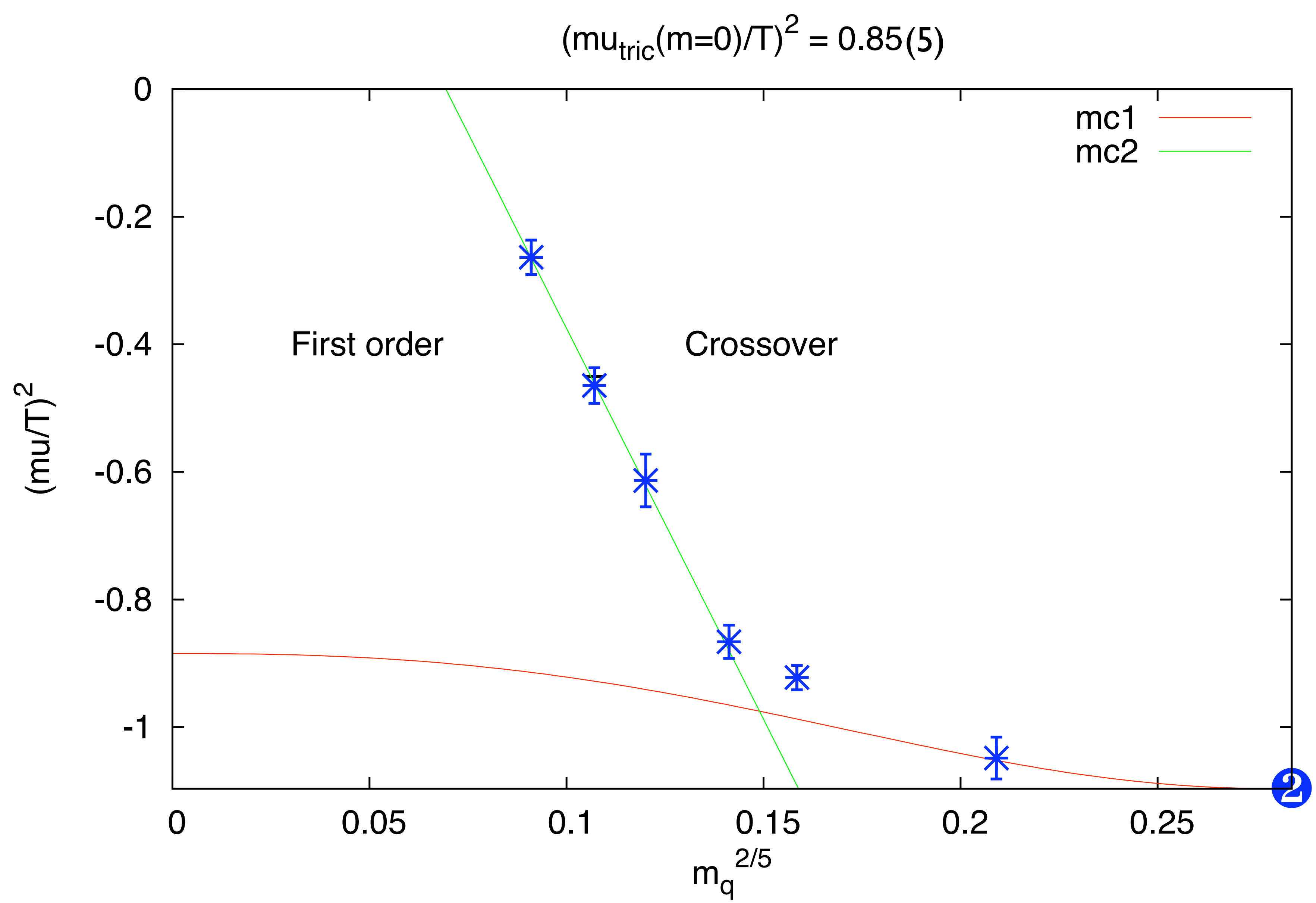
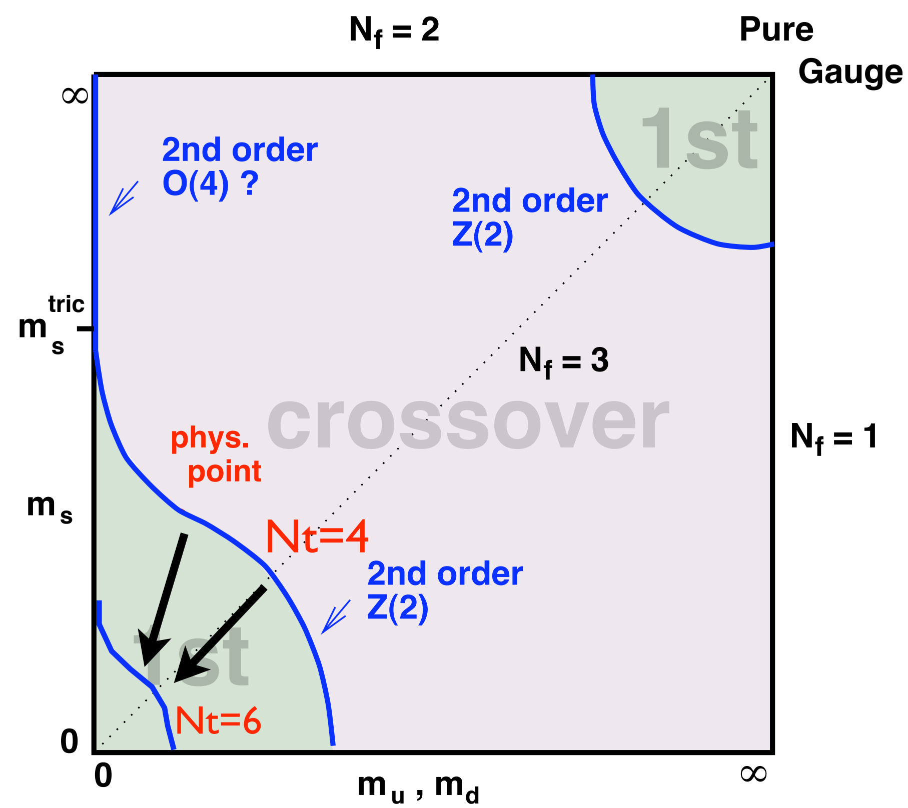


cf. D'Elia, Di Giacomo, Pica 05:
inconsistent with O(2), hints of 1st, but much larger quark masses...

Cutoff effects ?:

$N_t = 6, a \sim 0.2$ fm



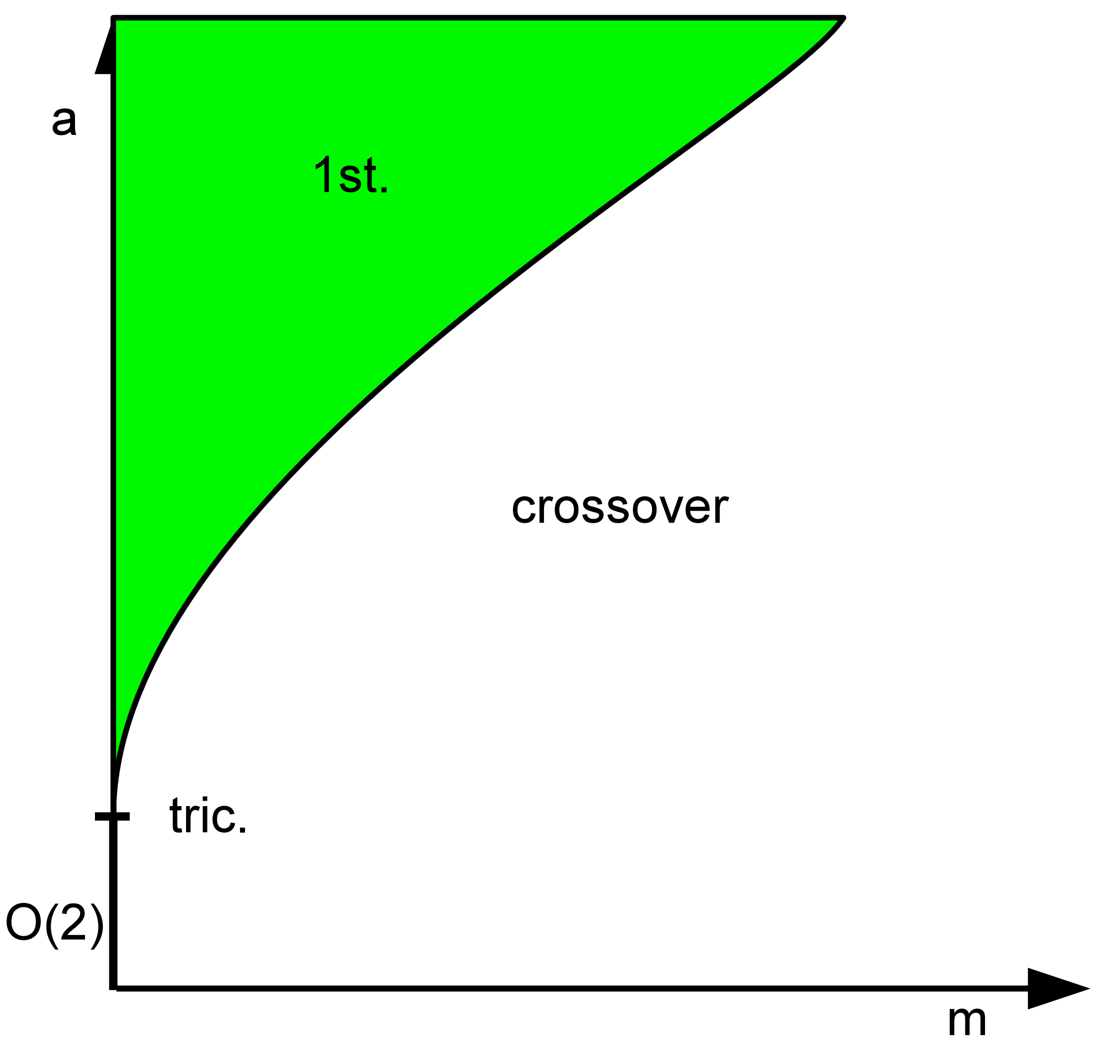
$$\frac{m_\pi^c(N_t = 4)}{m_\pi^c(N_t = 6)} \approx 1.77 \qquad N_f = 3$$

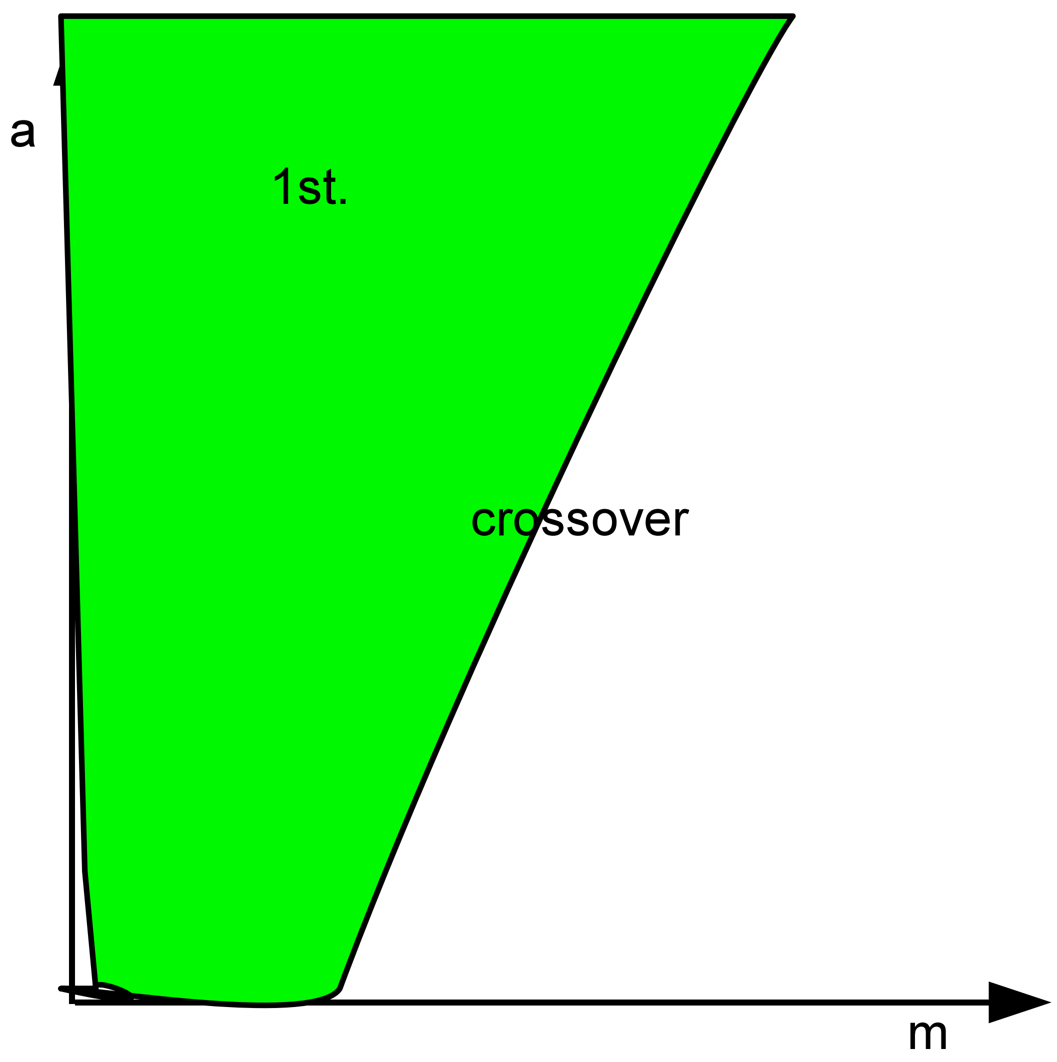
de Forcrand, Kim, O.P. 07
Endrödi et al 07

improved: BNL-Bielefeld , talk Ding

- Physical point deeper in crossover region as $a \rightarrow 0$
- Staggered chiral transition weaker in continuum than on finite lattice spacing
- Expect same for tricritical points; need continuum extrapolation

Outlook:





Conclusions

- Critical structures at imaginary chemical potential identifiable unambiguously
- 1st order chiral transition at imaginary chemical potentials for staggered quarks on Nt=4
- Z(2) boundary to crossover region has known functional form (tricriticality)
- Controlled extrapolation to zero (and positive) chemical potential possible
- Staggered Nf=2 chiral transition on Nt=4 is first order!
- Repeat with larger Nt ...