# Investigation of the $U_A(1)$ in high temperature QCD on the lattice

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Work done in collaboration with,

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Lattice 2013

# Outline

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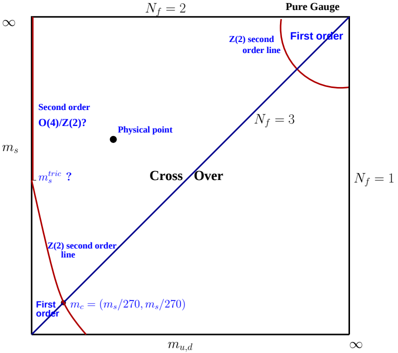
# Introduction

- Pertubative RG studies on models with same symmetries as QCD: $N_f = 2$ chiral phase transition at $\mu = 0$ is second order if magnitude of $U_A(1)$ breaking is large.
  [Pisarski & Wilczek, 84, Butti, Pelissetto & Vicari, 03]

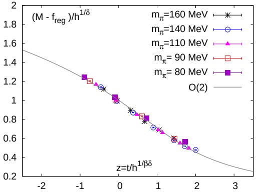
- Important to get insight from first principles Lattice study.
  [See plenary talk by Kalman Szabo]
  Recent work for QCD with two light flavours and chiral fermions
  [Cossu et. al, 2011,12; Chiu, Schroeder, Lattice, 13]




- Better understanding of $\mu = 0$ transition necessary for concluding about the existence of a critical point.

# Motivation for our work

- In our world, $m_{u,d} << \Lambda_{QCD}$.
- $2+1$-flavour QCD is sensitive to the $O(4)$ scaling due to chiral symmetry of the two massless quarks. [Bielefeld-BNL collaboration, 09, H.-T Ding's Talk Lattice, 13]




- At present improved versions of staggered fermions used extensively for QCD thermodynamics. Continuum limits for $T_c, \chi_2$ are in agreement.
- Highly improved staggered quarks(HISQ): minimal taste symmetry breaking.

# Outline

# Technique used

- We investigate the $U_A(1)$ restoration in 2+1 flavour QCD configurations generated with HISQ fermions.
- Signatures of $U_A(1)$ restoration:
  1. Gap in the low lying Dirac eigenspectrum
  2. Vanishing $\chi_\pi - \chi_\delta$
- We use the overlap operator,

$$D_{ov} = M(1 + \gamma_5 \, \mathbf{sgn}(\gamma_5 D_W(-M))) \; , \; \mathbf{sgn}(A) = A/\sqrt{A.A}.$$

  [Narayanan & Neuberger, 94, Neuberger, 98]

  and look at the eigenvalue distribution of $D_{ov}$ on the HISQ ensembles.

- $D_{ov}$ has an exact index theorem like in the continuum $\Rightarrow$ the zero modes of $D_{ov}$ related to topological structures of the underlying gauge field.

  [Hasenfratz, Laliena & Niedermeyer, 98]

# Ensembles used

- Lattice size: $32^3 \times 8$
- Volume: $m_\pi \; L > 3$
- $m_s$ physical and $m_s/m_l = 20 \Rightarrow m_\pi = 160$ MeV.
- Temperatures and configurations:

| T | # configurations | # eigenvalues/configuration |
|---|---|---|
| 1.04 $T_c$ | 100 | 100 |
| 1.23 $T_c$ | 100 | 50 |
| 1.5 $T_c$ | 100 | 50 |

- Computations were done on the GPU cluster at Bielefeld University.

# Numerical details

**Implementing the overlap operator**

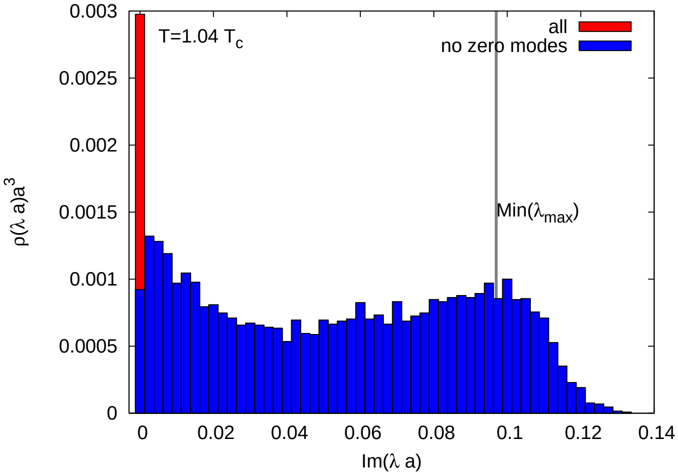
- Lowest 20 eigenvalues of $\gamma_5 D_W$ computed with $\epsilon^2 < 10^{-16}$.
- For these lowest modes sign function was computed explicitly.
- For the higher modes, sign function approximated as a Zolotarev Rational Polynomial with 15 terms.
- The sign function is computed as precise as $10^{-10}$.

**Eigenvalue computation**

- The Kalkreuter-Simma Ritz algorithm for eigenvalues of $D_{ov}^{\dagger} D_{ov}$.
- Convergence criterion: $\epsilon^2 < 10^{-8}$.

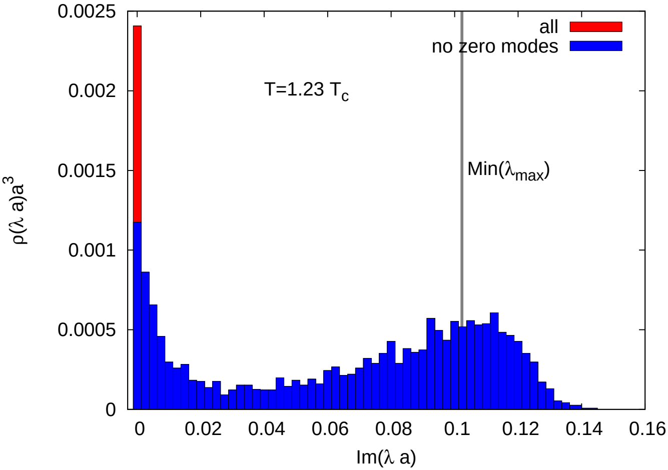
# Outline

# Eigenvalue distribution of $D_{ov}$



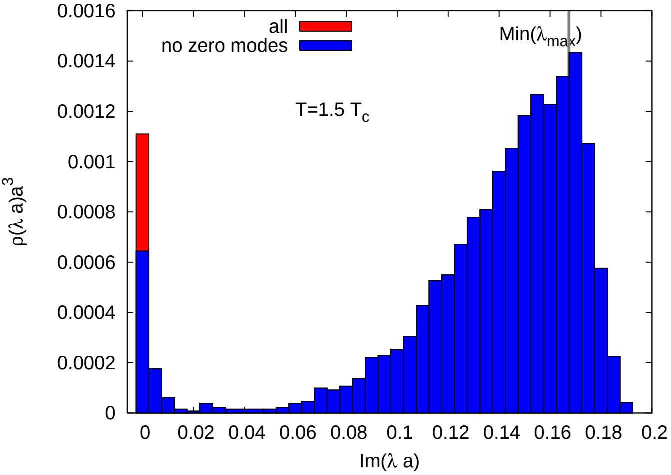

Large no of zero modes, pile up of near zero modes.

# Eigenvalue distribution of $D_{ov}$




Near zero mode contribution increases.

# Eigenvalue distribution of $D_{ov}$

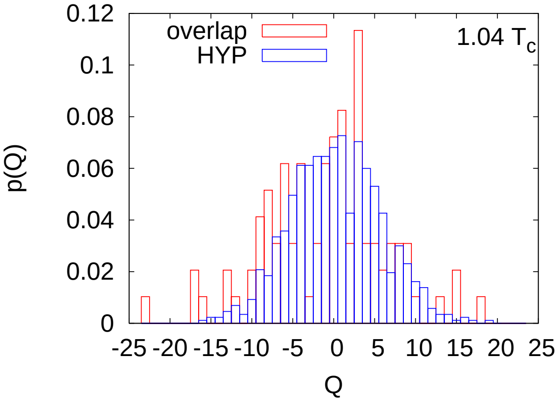



Near zero modes still contributing, no gap observed.

# Distribution of topological charge

Q computed from zero modes of $D_{ov}$ compared to from the $F\tilde{F}$ using HYP smearing on the same configurations [H. Ohno, U. M. Heller, F. Karsch and S. Mukherjee, 11]

HYP results with higher statistics!




Occurrence of higher Q modes observed in both the methods.

Q computed from zero modes of $D_{ov}$ compared to from the $F\tilde{F}$ using HYP smearing on the same configurations [H. Ohno, U. M. Heller, F. Karsch and S. Mukherjee, 11]

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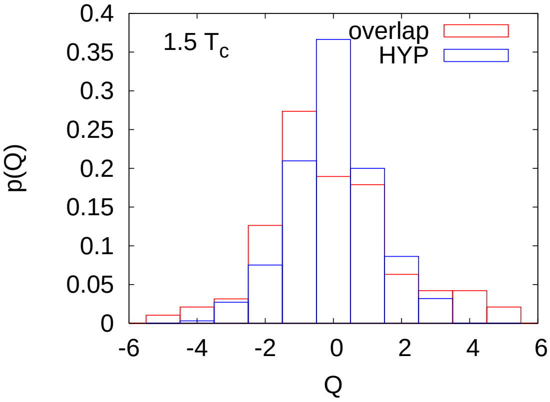


The same trend persists, Q range reduced.

Q computed from zero modes of $D_{ov}$ compared to from the $F\tilde{F}$ using HYP smearing on the same configurations [H. Ohno, U. M. Heller, F. Karsch and S. Mukherjee, 11]

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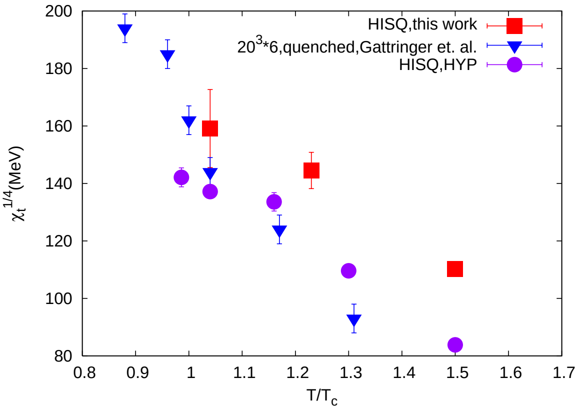



Resultant values of $\chi_t$ larger than quenched result or smearing.

# Measures of $U_A(1)$ breaking

- The quantity

$$\chi_\pi - \chi_\delta = \int d^4x \ \left[\langle i\pi^+(x) i\pi^-(0)\rangle - \langle \delta^+(x)\delta^-(x)\rangle\right]$$

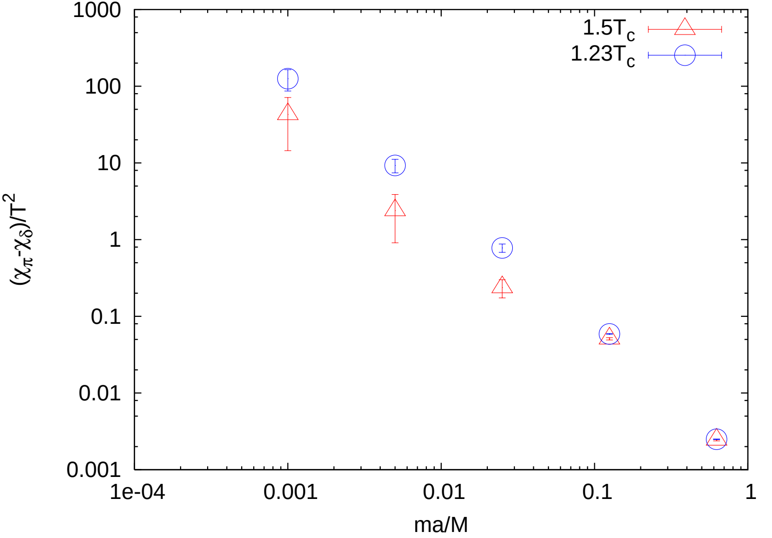
is zero as $U_A(1)$ is restored.

- In terms of the eigenvalues [Edwards, Heller & Narayanan, 98],

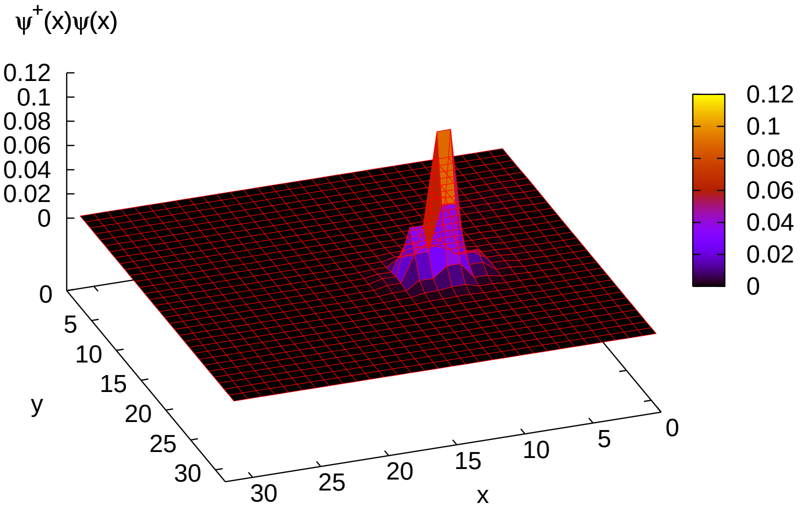
$$\chi_\pi - \chi_\delta = \frac{4}{V}\sum_{\lambda>0}\langle\frac{m^2(4-\lambda_i^2)^2}{\left[\lambda_i^2(1-m^2)+4m^2\right]^2} + 2\frac{\langle|Q|\rangle}{m^2 V}$$

- Contribution from non-zero modes $\rightarrow$ nonzero even at $1.5\ T_c$.

# Measures of $U_A(1)$ breaking



# Localization properties of zero modes at 1.5 $T_c$



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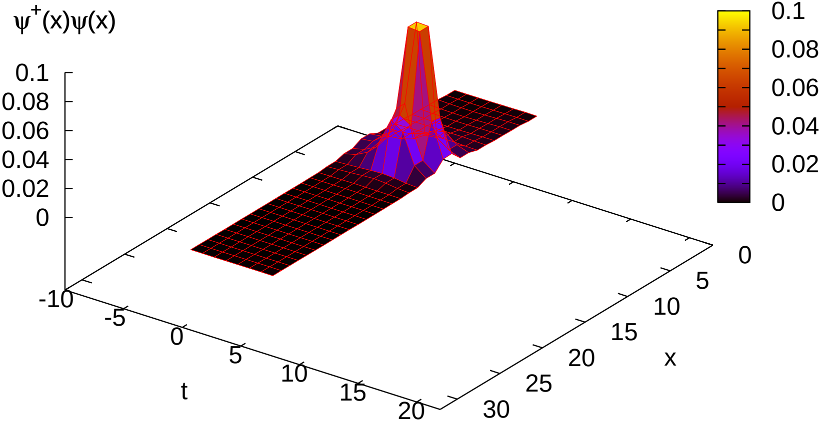
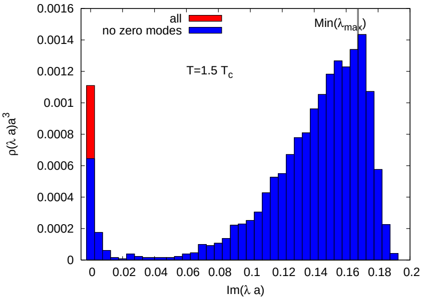


# The nature of the near zero modes at $1.5\,T_c$



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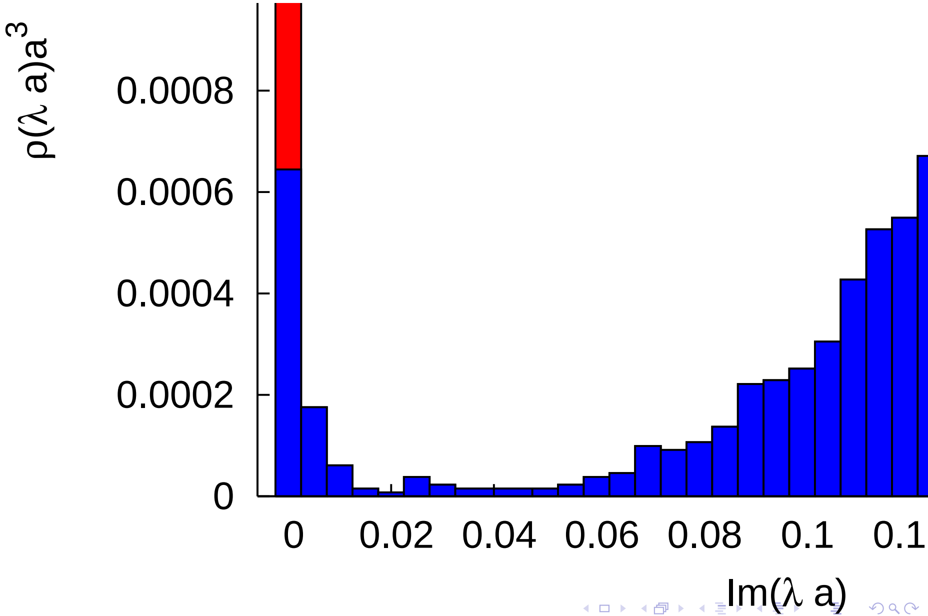
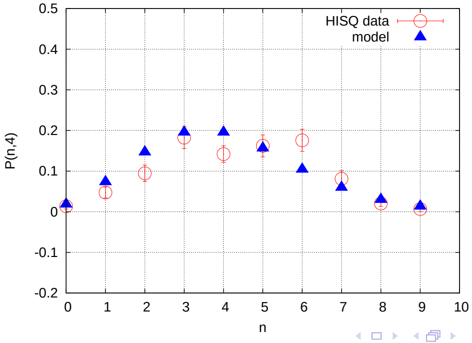


# The nature of the near zero modes at $1.5\,T_c$

- Presence expected from dilute instanton gas approximation(DIGA)? Yes for pure SU(3) gauge theory [Edwards, Heller, Kiskis & Narayanan, 99]. Observed for QCD with domain wall fermions as well [HotQCD collaboration, 12, Chris Schroeder, Lattice 2013].
- If n=total no. of instantons+anti-instantons, according to DIGA

$$P(n, \langle n \rangle) = \langle n \rangle^n \mathrm{e}^{-\langle \mathrm{n} \rangle}/\mathrm{n}!$$

- For $Im(\lambda a) < 0.036$, the value of $\langle n \rangle = 4 = \langle n^2 \rangle$.



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# Conclusions

- We have studied the topological structure of large volume HISQ fermion configurations used for QCD thermodynamics.
- All these configurations have topological structures even at $1.5\ T_c$.
- Significant presence of near zero modes and no gap even at this temperature$\Rightarrow$ $U_A(1)$ is not restored.
- These structures are localized and consistent with a dilute gas of instantons and anti-instantons.
- The observable $\chi_\pi - \chi_\delta$ is significantly different from zero at $1.5\,T_c$ further strengthens our claim.

Thank You.