QCD thermodynamics with dynamical overlap fermions

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**Motivation & Aims**

- **Staggered fermions**
  - cheap, well studied
  - continuum results at physical quark masses
  - rooting: validity is still debated
  - taste breaking $\rightarrow$ large $m_{\pi,\text{rms}}$

- **Wilson fermions**
  - theoretically sound
  - no taste breaking
  - explicit chiral symmetry breaking
Motivation & Aims

- Chiral properties at finite $T \rightarrow \infty$ chiral fermions are needed

- Domain-wall fermions
  - exact chiral symmetry only in $L_5 \rightarrow \infty$ limit

- Overlap fermions
  - exact lattice chiral symmetry

- Aims of this study:
  - $a \rightarrow 0$ with dynamical overlap fermions
  - cross check of staggered fermions
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Details of action

- Tree level Symanzik improved gauge action

- \( N_f = 2 \) overlap fermions

\[
D_{ov} = \left( m_0 - \frac{m}{2} \right) \left( 1 + \gamma_5 \text{sgn} (H_W) \right) + m, \quad H_W = \gamma_5 D_W (-m_0)
\]

- multi-shift inverter, Zolotarev rational approximation
- lowest eigenvalues of \( H_W \) separately ← Krylov–Schur algorithm

- \( D_W \) Wilson kernel:
  - \(-m_0 = -1.3\)
  - 2 steps of HEX smearing, \( \alpha_1 = 0.72, \alpha_2 = 0.60, \alpha_3 = 0.44 \)
Topology fixing

- HMC trajectories $\rightarrow$ difficulties at topological sector boundaries
- fix topology:

$$S_E = \sum_x \left\{ \bar{\psi}_E(x)D_W(-m_0)\psi_E(x) + \phi^+(x)[D_W(-m_0) + im_B\gamma_5\tau_3]\phi(x) \right\}$$

- equivalent to adding $\det \left( \frac{H_W^2(-m_0)}{H_W^2(-m_0) + m_B^2} \right)$ to $S_{g,\text{eff.}}$

- $m_B = 0.54, \quad -m_0 = -1.3$
- $m_0$ and $m_B$ are fixed in lattice units $\rightarrow$ infinitely large masses in the continuum limit
- in $V \rightarrow \infty$ limit physics is topology independent
- power-like corrections at finite $V$ may arise

Line of constant physics

- Lattices:
  - $12^3 \cdot 24$ for $\beta = 3.6, 3.7, 3.8, 3.9$
  - $16^3 \cdot 32$ for $\beta = 4.0, 4.1$
  - $32^3 \cdot 32$ for $\beta = 4.2, 4.3$
- $m = 0.015 - 0.06$
- $a$ is set using $w_0 = 0.1755 \text{ fm}$
- Chiral symmetry $\rightarrow m^2_\pi \propto m$
- $m$ is set via $m_\pi \cdot w_0 = 0.312 \rightarrow m_\pi = 350 \text{ MeV}$

![Graph 1](image1)

![Graph 2](image2)
Staggered reference calculations

- Tree level Symanzik improved gauge action (same as with overlap)
- $N_f = 2$ staggered fermions
- 4 steps of stout smearing, $\rho = 0.125$
- LCP analogous to overlap
  - scale via $w_0$
  - quark mass via $m_\pi \cdot w_0 = 0.312$
  - 16 ensembles in the range $\beta = 3.8 - 4.1$

- $N_s/N_t = 2 \quad \rightarrow \quad m_\pi \cdot L \approx 3.5 - 5$ in transition regime (same as overlap)
- $N_t = 6, 8, 10$ simulations
Chiral condensate

\[ \bar{\psi} \psi = \frac{T \partial (\log Z)}{V} \frac{\partial m}{\partial m} \]

Renormalization:

\[ \frac{m_R \bar{\psi} \psi R}{m^4_\pi} = \frac{m(\bar{\psi} \psi - \bar{\psi} \psi_0)}{m^4_\pi} \]

![Graph showing the behavior of \( m_R \bar{\psi} \psi R/m^4_\pi \) against \( T/w_0 \) for different lattice sizes.](image)
Polyakov loop

- $L_0$: multiplicative divergence of the form $\exp \left[ F_0(\beta)/T \right]$.
- Renormalization condition: $L_R \left( T = 208 \text{ MeV} \right) = 1$.
- $F_0(\beta) = \frac{1}{N_t} \log L$ such that $(N_t, \beta)$ corresponds to $T = 208 \text{ MeV}$.
- $L_R = L_0 \cdot \exp \left[ -N_t \cdot F_0(\beta) \right]$. 

![Graph showing the behavior of $L_R$ vs. $T$ for different lattice sizes.](image)
Isospin susceptibility

\[ \chi_I = \frac{T}{V} \frac{\partial^2 (\log Z)}{\partial \mu_1^2} \bigg|_{\mu_1=0} \]

\[ \mu_u = \mu_1/2, \quad \mu_d = -\mu_1/2 \]
Conclusions & outlook

Conclusions

- Not conclusive yet $\rightarrow$ need more statistics
- Continuum limit looks feasible

Outlook

- Collect more statistics
- Larger volumes to check finite volume effects
- Include strange quark, reach for lower pion mass
### Stefan–Boltzmann limits of $\chi_I$

<table>
<thead>
<tr>
<th>$N_t$</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi = 2$ overlap</td>
<td>1.700</td>
<td>1.588</td>
<td>1.362</td>
<td>1.241</td>
<td>1.186</td>
</tr>
<tr>
<td>$\xi = \infty$ overlap</td>
<td>1.619</td>
<td>1.513</td>
<td>1.290</td>
<td>1.170</td>
<td>1.117</td>
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<tr>
<td>$\xi = \infty$ staggered</td>
<td>2.235</td>
<td>1.861</td>
<td>1.473</td>
<td>1.266</td>
<td>1.164</td>
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<tr>
<td>$\xi = \infty$ Wilson</td>
<td>4.168</td>
<td>2.258</td>
<td>1.521</td>
<td>1.265</td>
<td>1.161</td>
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