Sigma-terms and axial charge for hyperons and charmed baryons

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with

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Outline

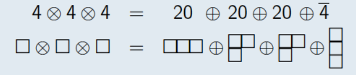
- Hyperons and charmed baryons
- Axial charge for various baryons
- Sigma terms for various baryons
- Stochastic Method for computing connected three point functions

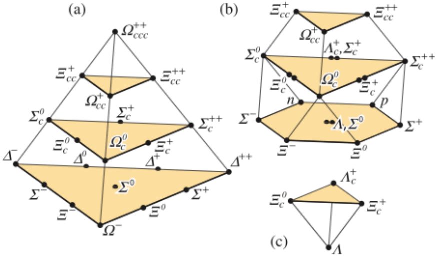
Hyperons and charmed baryons

We use $N_f = 2+1+1$ twisted mass fermions with dynamical strange and charm quark masses fixed to their physical values

$SU(4)_{flavour}$ **representations**

$4 \otimes 4 \otimes 4 = 20 \oplus 20 \oplus 20 \oplus \bar{4}$





$\Longrightarrow$ A 20-plet with $SU(3)$ octet and a 20-plet with $SU(3)$ decuplet.

Tune quark masses

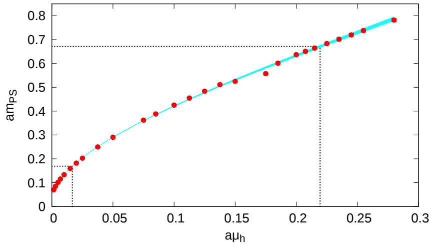
Tuning of strange and charm quarks to their physical values *B. Blossier et al arXiv:0709.4574*

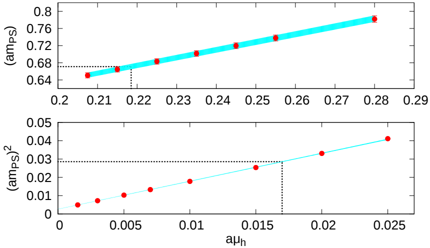
We use mixed action setup to simulate valance quarks

- For strange quark we match the kaon to its unitary mass
- For charm quark we use D-meson

$$a^2M^2_{PS}(a\mu_l, a\mu_h) = a_1(\mu_l+\mu_h)+a_2(\mu_l+\mu_h)^2+a_3(\mu_l+\mu_h)^3+a_4(\mu_l+\mu_h)(\mu_l-\mu_h)^2$$

using linear fit $a^2M^2_{PS}(a\mu_s) = c_1 + c_2 a\mu_s$, $aM_{PS}(a\mu_c) = d_1 + d_2 a\mu_s$



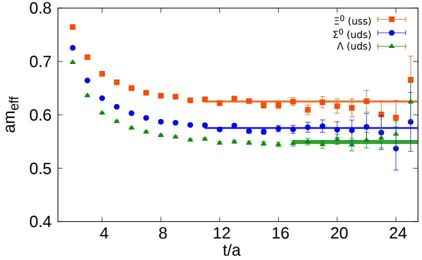


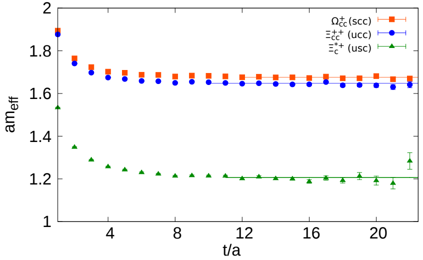
Masses of Hyperons and Charmed baryons

Two-point Correlators $$C(t) = \sum_{\vec{x}} \langle \Omega | \Gamma_0 \chi(\vec{x}, t) \bar{\chi}(\vec{0}, 0) | \Omega \rangle$$

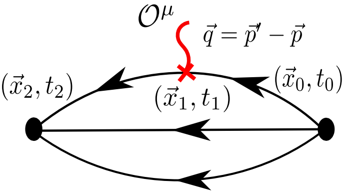
Effective mass $$m_{eff}(t) = \log\left(\frac{C(t)}{C(t+1)}\right) \approx m + \log\left(\frac{1+c_1 e^{-\Delta_1 t}}{1+c_1 e^{-\Delta_1 (t+1)}}\right) \xrightarrow{t\to\infty} m$$

Baryon	Content	Intepolating field	I	I_z
Ξ^0	uss	$\epsilon_{abc}\left(s_a^T C\gamma_5 u_b\right) s_c$	$1/2$	$+1/2$
Σ^0	uds	$\frac{1}{\sqrt{2}}\epsilon_{abc}\left((u_a^T C\gamma_5 s_b)d_c + (d_a^T C\gamma_5 s_b)u_c\right)$	1	0
Λ	uds	$\frac{1}{\sqrt{6}}\epsilon_{abc}\left(2(u_a^T C\gamma_5 d_b)s_c + (u_a^T C\gamma_5 s_b)d_c - (d_a^T C\gamma_5 s_b)u_c\right)$	0	0
Ω_{cc}^+	scc	$\epsilon_{abc}\left(c_a^T C\gamma_5 s_b\right) c_c$	0	0
Ξ_{cc}^{++}	ucc	$\epsilon_{abc}\left(c_a^T C\gamma_5 u_b\right) c_c$	$1/2$	$+1/2$
Ξ_c^{*+}	usc	$\epsilon_{abc}\left(s_a^T C\gamma_5 u_b\right) c_c$	$1/2$	$+1/2$





Evaluation of three-point functions



$$G^\mu(t_2, t_1; \vec{p}', \vec{p}; \Gamma) =$$
$$\sum_{\vec{x_2}, \vec{x_1}} \langle \Omega | \Gamma^{\beta\alpha} \chi_\alpha(\vec{x_2}, t_2) \mathcal{O}^\mu(\vec{x_1}, t_1) \bar{\chi}_\beta(\vec{0}, 0) | \Omega \rangle e^{-i\vec{x_2}\cdot\vec{p}'} e^{+i\vec{x_1}\cdot(\vec{p}' - \vec{p})}$$

In the non-relativistic basis the projectors are

$$\Gamma_0 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad \Gamma_i = \frac{1}{2}\begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix}$$

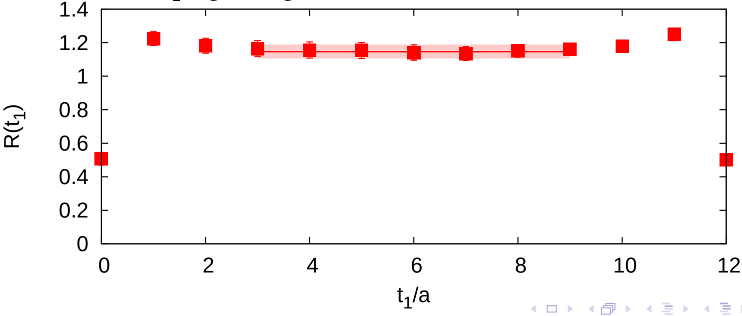
We will consider the axial charge and sigma-terms

- Consider the following ratio

$$R^{\mu}(\Gamma,\vec{q},t_2,t_1) = \frac{G^{\mu}(\Gamma,\vec{q},t_1)}{G(\vec{0},t_2)}\sqrt{\frac{G(\vec{p},t_2-t_1)G(\vec{0},t_1)G(\vec{0},t_2)}{G(\vec{0},t_2-t_1)G(\vec{p},t_1)G(\vec{p},t_2)}}$$

- For sufficiently large seperations $t_2 - t_1$ and t_1 this ratio becomes time-independent (plateau)

$$\lim_{t_2-t_1\gg 0}\ \lim_{t_1\gg 0} R^{\mu}(\Gamma,\vec{q},t_2,t_1) \longrightarrow \Pi^{\mu}(\Gamma,\vec{q})$$



Axial charge for various hadrons

Axial-Vector matrix element decomposition

$A^3_\mu \equiv \bar{\psi(x)}\gamma_\mu\gamma_5\frac{\tau^3}{2}\psi(x) \Longrightarrow$

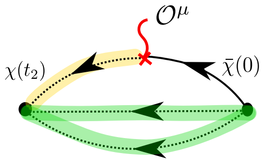
$$\langle N(\vec{p}')|A^3_\mu|N(\vec{p})\rangle = \bar{u}_N(\vec{p}')\frac{1}{2}\left[G_A(Q^2)\gamma_\mu\gamma_5 + \frac{q_\mu\gamma_5}{2m_N}G_p(Q^2)\right]u_N(\vec{p}) \Rightarrow G_A(Q^2=0) = g_A$$

Evaluation of connected diagrams using the sequential propagator

Two approaches

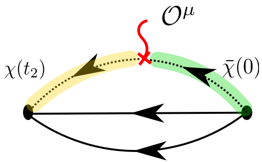
Fixed sink method

- Advantage: Calculate all Operators for a specific particle.
- Disadvantage: Fix the particle state and the projector.



Fixed insertion method (extract axial charge for all baryons)

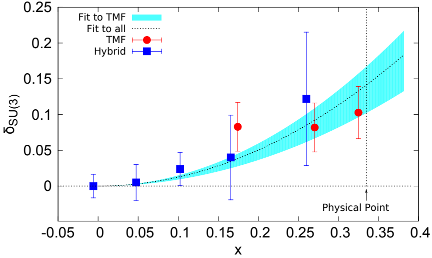
- Advantage: Calculate any particle state for any projector.
- Disadvantage: Fix the insertion operator



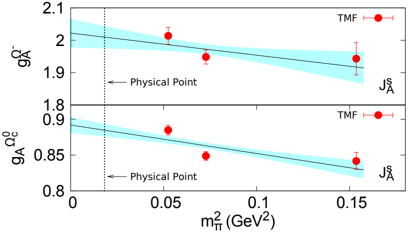
If exact SU(3) flavor symmetry: *W.Lin and K.Orginos, PRD 79,034507 (2009)*

- $g_A^N = F + D,\ g_A^\Sigma = 2F\ ,\ g_A^\Xi = -D + F \implies \boxed{g_A^N - g_A^\Sigma + g_A^\Xi = 0}$

Probe deviation: $\delta_{SU(3)} = g_A^N - g_A^\Sigma + g_A^\Xi$ versus $x = (m_K^2 - m_\pi^2)/4\pi^2 f_\pi^2$

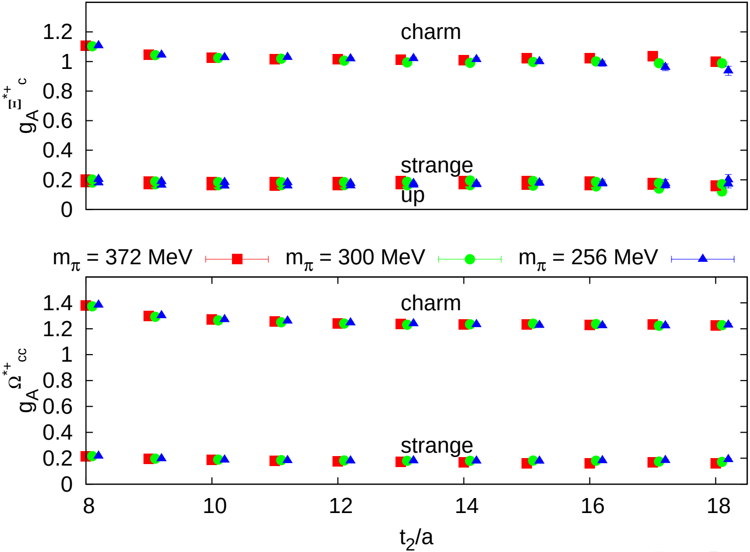


Breaking $\sim x^2$ leads to about 15% at the physical point $x_{phy} = 0.33$



Study of SU(3) breaking for the decuplet

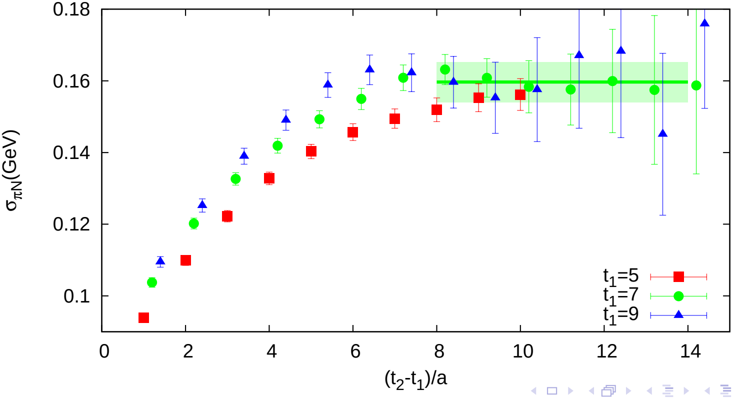
Axial charge of $(\Xi_c^{*+}(usc))$ **and** $(\Omega_{cc}^{*+}(scc))$



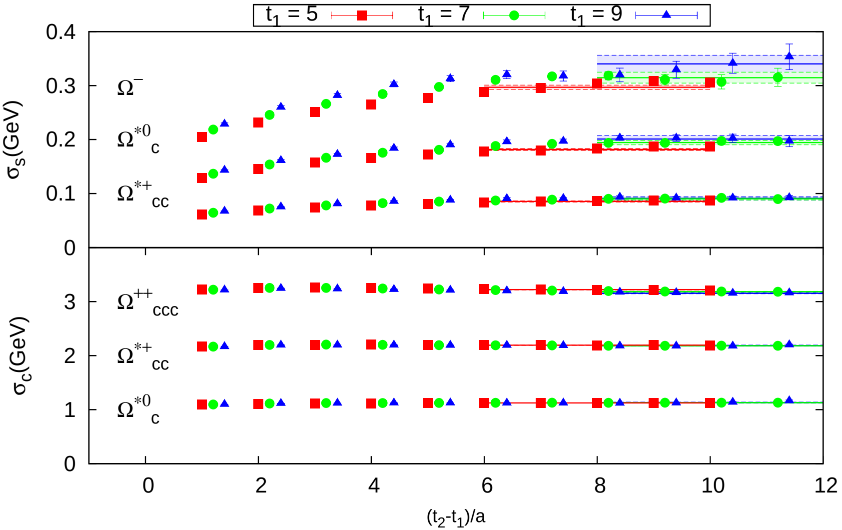
Another example using the fixed insertion are the sigma-terms (this requires a new set of inversions)

- From light sector we can extract $\sigma_{\pi N} = m_l \langle N|\bar{u}u + \bar{d}d|N\rangle$
- Similarly for the strange quark $\sigma_s = m_s \langle N|\bar{s}s|N\rangle$
- Using Feynman-Hellman theorem $\sigma_{\pi N} = m_l \frac{\partial m_N}{\partial m_l}$

$N_f = 2+1+1$ twisted mass fermions, $a = 0.082fm$ and $m_\pi \sim 372MeV$



Sigma terms for hyperons and charmed baryons

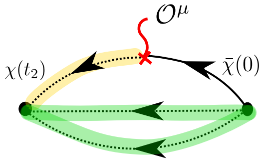


Stochastic method for connected diagrams

In order **NOT** to require new set of inversions for each operator we investigate a stochastic method

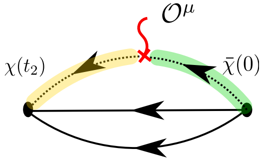
Fixed sink method

- Advantage: Calculate all Operators for a specific particle.
- Disadvantage: Fix the particle state and the projector.



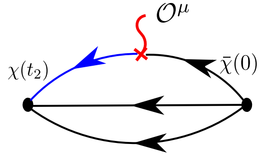
Fixed insertion method

- Advantage: Calculate any particle state for any projector.
- Disadvantage: Fix the insertion operator



Stochastic method

- Advantage: Calculate any operator for any particle state and any projector.
- Disadvantage: Introduces stochastic error.



Formulation of stochastic Method

K - F. Liu et al. PRL 74(1995)

- Noise vector using Z(4) random numbers

$$\frac{1}{N_r}\sum_{r=1}^{N_r\to\infty}\xi_r(x)^a_\mu \to 0 \qquad , \qquad \frac{1}{N_r}\sum_{r=1}^{N_r\to\infty}\xi_r(x)^a_\mu\,\xi^*_r(y)^b_\nu \to \delta(x-y)\delta_{\mu\nu}\delta^{ab}$$

- Reconstruction of all-to-all propagator

$$\phi_r(x)^a_\mu = G(x;y)^{ab}_{\mu\nu}\,\xi_r(y)^b_\nu \qquad , \qquad G(x;y)^{ab}_{\mu\nu} = \frac{1}{N_r}\sum_{r=1}^{N_r\to\infty}\phi_r(x)^a_\mu\,\xi^*_r(y)^b_\nu$$

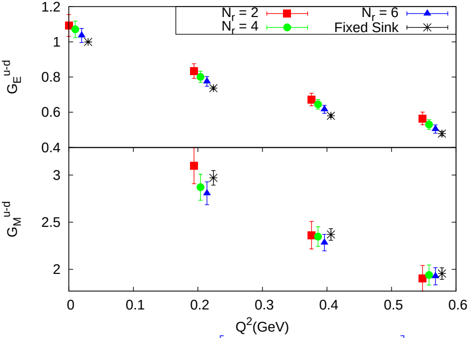
- Decomposition of double sum to two single sums

$$\sum_{\vec{y}}\sum_{\vec{x}} e^{-i\vec{p}'\cdot\vec{x}}e^{-i\vec{p}\cdot\vec{y}}G(x;y)\Gamma G(y;0) \longrightarrow$$

$$\frac{1}{N_r}\sum_{r=1}^{N_r}\left(\sum_{\vec{x}} e^{-i\vec{p}'\cdot\vec{x}}\phi_r(x)\right)\left(\sum_{\vec{y}} e^{-i\vec{p}\cdot\vec{y}}\xi^*_r(y)\Gamma G(y;0)\right)$$

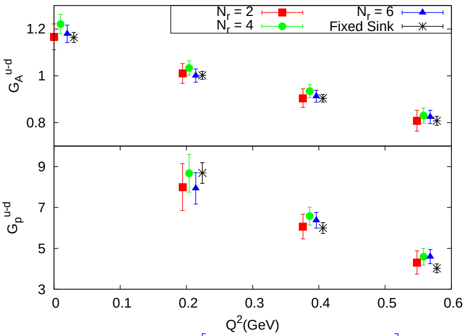
Nucleon Form Factors , 500 statistics

Electromagnetic Form Factor



$$\langle N(\vec{p}')|j^3_\mu|N(\vec{p})\rangle = \bar{u}_N(\vec{p}')\frac{1}{2}\left[\gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(Q^2)\right]u_N(\vec{p})$$

Axial Form Factor

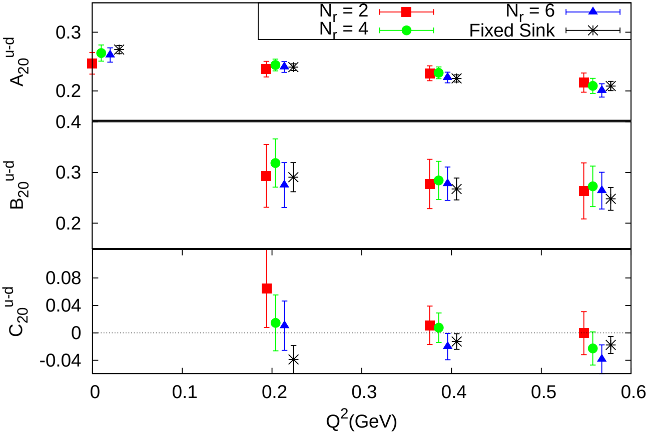


$$\langle N(\vec{p}')|A^3_\mu|N(\vec{p})\rangle = \bar{u}_N(\vec{p}')\frac{1}{2}\left[G_A(Q^2)\gamma_\mu\gamma_5 + \frac{q_\mu\gamma_5}{2m_N}G_p(Q^2)\right]u_N(\vec{p})$$

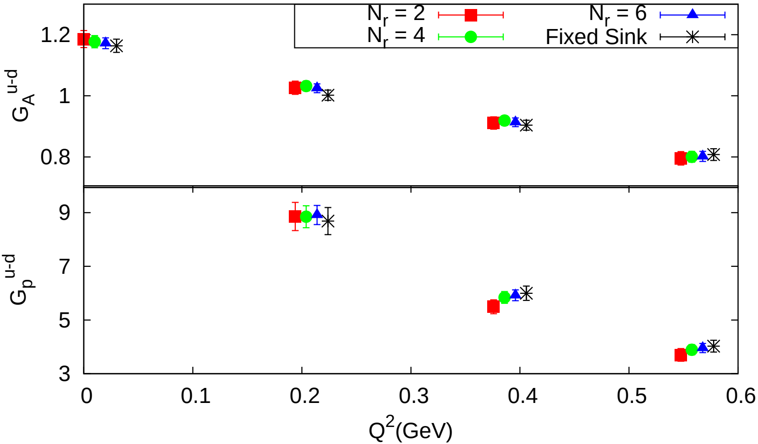
- Fully diluted (spin,color) noise vectors $\Longrightarrow$ for $N_r = 1$ we need 12 inversions
- For the fixed sink method we need also 12 inversions for each sequential source
- As you increase the statistics you need less noise vectors

One Derivative Form Factors

$$\mathcal{O}_V^{\mu\nu} \equiv \bar{\psi} i \gamma^{\{\mu} \overleftrightarrow{D}^{\nu\}} \frac{\tau^3}{2} \psi \Longrightarrow \langle N(\vec{p}\,')|\mathcal{O}_V^{\mu\nu}|N(\vec{p})\rangle = \bar{u}_N(\vec{p}\,')\frac{1}{2}\left[A_{20}(Q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}(Q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}(Q^2)\frac{1}{m}q^{\{\mu}q^{\nu\}}\right] u_N(\vec{p})$$

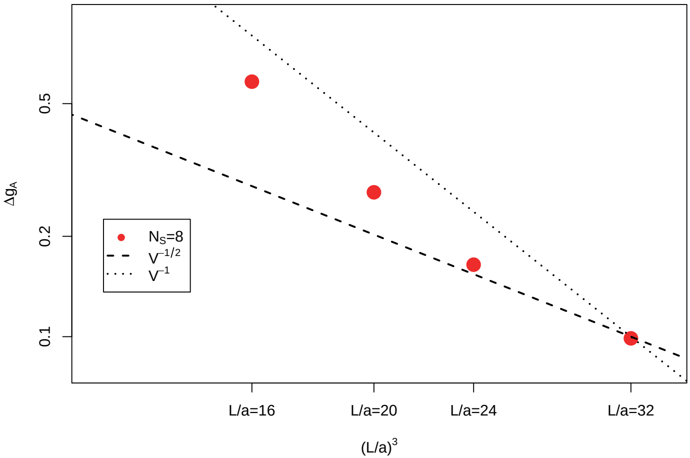


Stochastic method shows the same behavior for many operators



	Projectors	States	$\#\ N_r$	$\#$ Inversions	Err(Stoch)/Err(Fixed Sink)
Fixed Sink	$\sum_k \Gamma_k$	p	-	24	-
Stochastic	$\sum_k \Gamma_k$	p	2	2×24	~ 3
Stochastic	$\Gamma_1, \Gamma_2, \Gamma_3$	$(p+n)/2$	2	2×24	~ 1

Volume dependence of stochastic method



The combined error (gauge and stochastic) drops as $V^{-1/2}$ for large volumes. *C. Alexandrou et al arXiv:1302.2608*

$\Longrightarrow$ For larger volumes you may need less stochastic vectors

Summary

- Predictions for axial charge and sigma-terms for Hyperons and charmed baryons
- Stochastic Method can be an alternative method to compute connected three point functions
- Stochastic Method utilizes the advantages of fixed sink and insertion method
- We will use this method to evaluate many hadronic elements.

Backup slides