Determination of the A_2 amplitude of $K \to \pi\pi$ decays

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Introduction

In the isospin limit one can classify two kaon decay channels:

- $K \to (\pi\pi)_{I=2}$
- $K \to (\pi\pi)_{I=0}$

An experimental observation shows that the ratio of real parts of amplitudes for these decays is:

$$\frac{\mathrm{Re}(A_0)}{\mathrm{Re}(A_2)} \approx 22.5$$

This is known as '$\Delta I = 1/2$ rule' In our recent work, we calculated the A_2 amplitude at physical kinematics (arXiv:1206.5142)
This calculation was done using a single ensemble, which led to uncontrolled systematic errors related to cutoff effects.

Lattice parameters

Two new ensembles:

	$48^3 \times 96$	$64^3 \times 128$
Size[fm]	5.49	5.48
Gauge action	Iwasaki	Iwasaki
Fermion action	DWF	DWF
L_s	24	10
M	1.8	1.8
β	2.13	2.25
a^{-1}[GeV]	1.73(1)	2.30(4)
am_{ud}	0.00078	0.000678
am_s	0.0362	0.02661
am_{res}	$6.19(6) \times 10^{-4}$	$2.93(8) \times 10^{-4}$
Pion mass [MeV]	139	135
Kaon mass [MeV]	499	495.7
Number of configurations	44	21

We can get away with small number of configurations thanks to AMA procedure: arXiv:1208.4349

Operator product expansion

Weak Hamiltonian can be expanded using operator product expansion:

$$H_W = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} a^3 \sum_i C_i Q_i$$

Then the amplitudes can be written as:

$$A_{2/0} = F \langle (\pi\pi)_{I=2/0} \mid H_W \mid K \rangle$$

We are therefore interested in the following 3-point functions:

$$M_i^{I=2/0} \equiv \langle (\pi\pi)_{I=2/0} \mid Q_i^{\Delta I=(3/2)/(1/2)} \mid K \rangle$$

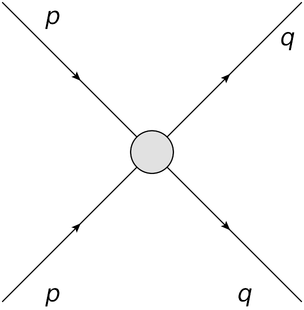
Only 3 operators contribute to $K \to (\pi\pi)_{I=2}$ process. We label them according to their chiral transformation properties (27,1), (8,8), $(8,8)_{mx}$

- (27,1) is the dominant contribution to $\mathrm{Re}(A_2)$
- (8,8) operators are the dominant contributions to $\mathrm{Im}(A_2)$

Renormalization

Wilson coefficients have been calculated in $\overline{MS}$ scheme, so we need to express our lattice results in $\overline{MS}$ scheme as well. This requires the use of an intermediate scheme, like RI-SMOM

$$M_i^{LAT} \to M_i^{RI-SMOM} \to M_i^{\overline{MS}}$$



$$p^2 = q^2 = (p-q)^2 = \mu^2$$

Only operators in the same $SU(3)_L \times SU(3)_R$ operators mix under renormalization.

Two pion momentum

In centre of mass frame, the ground state for the two pion system will correspond to each pion being at rest. To avoid this problem we use antiperiodic boundary conditions for the d quark (and periodic for the u quark). The allowed momenta for the π^+ meson become:

$$p = \pm\frac{\pi}{L}, \pm\frac{3\pi}{L}, \ldots$$

In I=2 case only, we can use the Wigner-Eckart theorem:

$$\underbrace{\langle(\pi\pi)^{I=2}_{I_3=1} \mid}_{\sqrt{2}\langle\pi^+\pi^0|} Q^{\Delta I=3/2}_{\Delta I_3=1/2} \mid K^+\rangle = \sqrt{\frac{3}{2}}\underbrace{\langle(\pi\pi)^{I=2}_{I_3=2} \mid}_{\langle\pi^+\pi^+|} Q^{\Delta I=3/2}_{\Delta I_3=3/2} \mid K^+\rangle$$

Both 48^3 and 64^3 ensembles tuned so that antiperiodic boundary conditions in 3 directions (which induce momentum $p = \frac{\sqrt{3}\pi}{L}$) correspond to physical kinematics.

Finite volume effects

Need to take into account interactions in the final state.

$$F^2 = 4\pi \left(q \frac{\partial \phi}{\partial q} + p \frac{\partial \delta}{\partial p} \right) \frac{m_K E_{\pi\pi}}{p^3}$$

with:

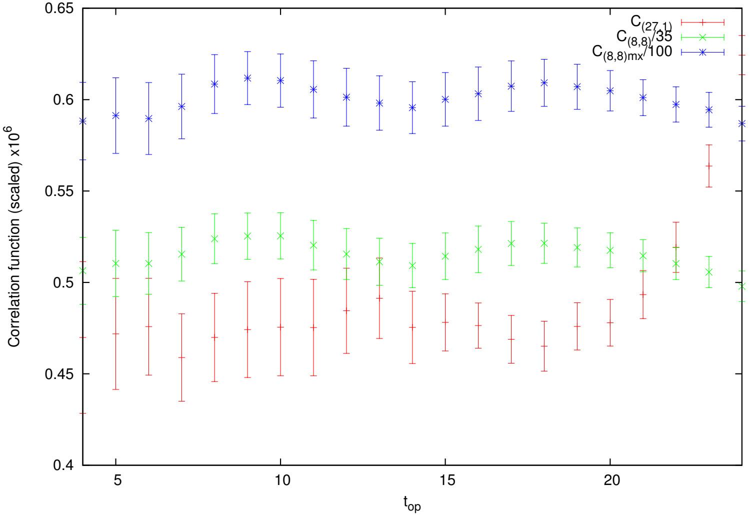
$$\tan \phi = \frac{q \pi^{3/2}}{Z_{00}(1; q)}$$

- δ is the 2-pion s-wave phase shift
- δ can be computed from the lattice using Lüscher quantization condition, but...
- $\frac{\partial \delta(p)}{\partial p}$ can not, so we have to approximate

48^3 K$\rightarrow \pi\pi$ 3-point correlation functions

Kaon - 2 pion separation 26

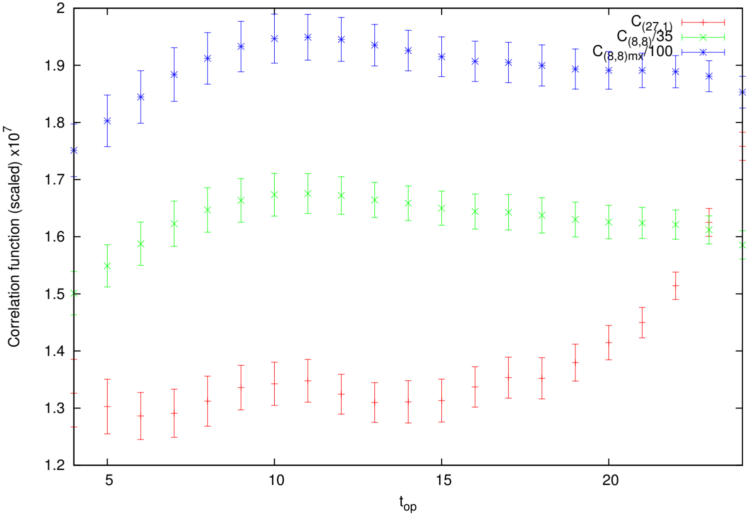
$C_i^{K\rightarrow\pi\pi}(t) = N_{\pi\pi} N_K M_i e^{-(m_K - E_{\pi\pi})t_{op}} e^{-E_{\pi\pi} t_{\pi\pi}}$



64^3 K$\to \pi\pi$ 3-point correlation functions

Kaon - 2 pion separation 26

$$C_i^{K\to\pi\pi}(t) = N_{\pi\pi} N_K M_i e^{-(m_K - E_{\pi\pi})t_{op}} e^{-E_{\pi\pi} t_{\pi\pi}}$$



Summary of results (PRELIMINARY)

	48^3	64^3
Dispersion relation (c^2)	0.999(9)	1.008(10)
Pion mass [MeV]	139.4(3)	136.0(3)
Kaon mass [MeV]	498.9(4)	495.6(5)
$(\pi\pi)_{I2}$ energy [MeV]	497.8(43)	503.7(38)
$\mathrm{Re}(A_2)$[GeV]	$1.368(41) \times 10^{-8}$	$1.358(28) \times 10^{-8}$
$\mathrm{Im}(A_2)$[GeV]	$-6.30(12) \times 10^{-13}$	$-6.31(10) \times 10^{-13}$

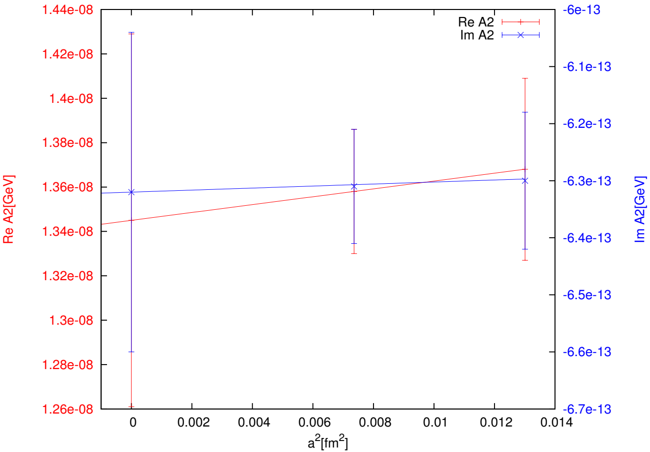
C.f. 32^3 result:

$\mathrm{Re}(A_2) = 1.381(41) \times 10^{-8}$ GeV

$\mathrm{Im}(A_2) = -6.54(46) \times 10^{-13}$ GeV

Continuum extrapolation for A_2 amplitude (PRELIMINARY)

Values shown are the amplitudes in $\overline{MS}$ scheme at 3GeV



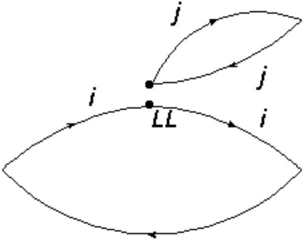
$$\mathrm{Re}(A_2) = 1.345(84) \times 10^{-8}\ \mathrm{GeV}$$
$$\mathrm{Im}(A_2) = -6.32(28) \times 10^{-13}\ \mathrm{GeV}$$

Error budget (VERY PRELIMINARY)

	$\mathrm{Re}(A_2)$	$\mathrm{Im}(A_2)$
Lattice artefacts	$15\% \searrow 6\%_{+stat}$	$15\% \searrow 5\%_{+stat}$
Finite volume corrections	$6.0\% \searrow 2\%$	$6.5\% \searrow 2\%$
Partial quenching	$3.5\% \searrow 0\%$	$1.7\% \searrow 0\%$
Renormalization	1.8% (?)	5.6% (?)
Unphysical kinematics	0.4% =	0.8% =
Derivative of phase shift	0.97% =	0.97% =
Wilson coefficients	6.6% =	6.6% =
Total	$18\% \searrow$	$19\% \searrow$

Cancellation of contractions in Re(A_2)

The dominant contribution comes from (27,1) operator. (27,1) operator is proportional to the sum of the following two contractions:



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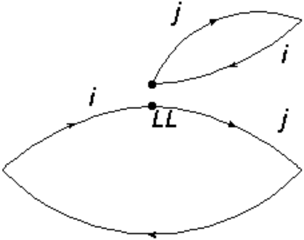

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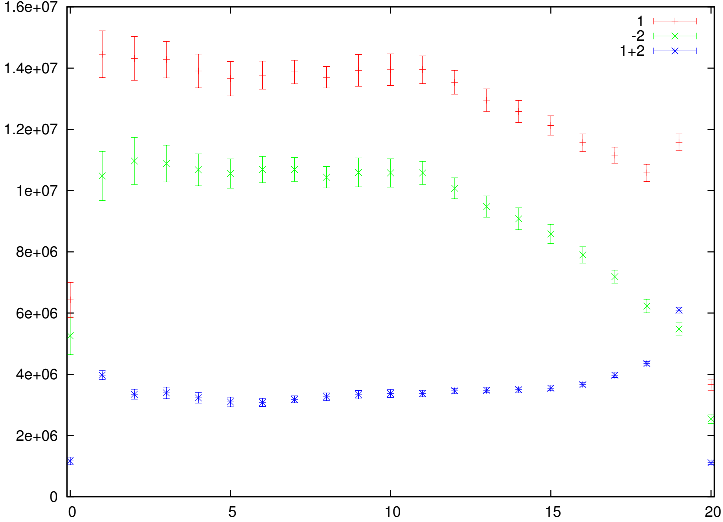
Cancellation of contractions in Re(A_2)



Cancellation of contractions in Re(A_2)

- Cancellation of contributions to Re(A_2) appears in all our simulations
- Investigation of Re(A_0) at threshold (arXiv:1212.1474) shows a that all contributions have the same sign resulting in small enhancement of Re(A_0)
- The main mechanism behind $\Delta I = 1/2$ rule seems to be a cancellation in Re(A_2)!

Conclusions

- Many lattice parameters need to be fine tuned
- (Preliminary) Results from 48^3 and 64^3 ensembles are consistent with 32^3 DSDR results
- Systematic errors due to lattice artefacts are smaller than anticipated
- Cancellation in $\mathrm{Re}(A_2)$ has been confirmed in both ensembles
- Systematic errors need to be estimated more carefully

Thank you for your attention!