Strange and charmed pseudoscalar meson decay constants from simulations at physical quark masses

A. Bazavov, C. Bernard, C. Bouchard, C. DeTar, D. Du, A.X. El-Khadra, J. Foley, E.D. Freeland, E. Gamiz, Steven Gottlieb, U.M. Heller, J. Kim, J. Komijani, A.S. Kronfeld, J. Laiho, L. Levkova, P.B. Mackenzie, E.T. Neil, M. Oktay, J.N. Simone, R.L. Sugar, D. Toussaint, R.S. Van de Water, R. Zhou [Fermilab lattice and MILC]

University of Arizona

July 31, 2013

Introduction

- f_K, f_D and f_{D_s}, together with experimental decay rate determinations, are the simplest, although not necessarily most accurate, ways to determine V_{us}, V_{cd} and V_{cs}.
- $f_{pseudo} = (m_A + m_B)\sqrt{\frac{3VA_{pt-pt}}{2M^3_{pseudo}}}$
- Two analyses of same data — this one with simple fitting, another using ChiPT for heavy-light correlators (C. Bernard's talk, this conference)

Introduction

- "Highly Improved Staggered Quark" (HISQ) action
- Reduced taste violations, and treat charm quark like light quarks
- Lattice spacings 0.15, 0.12, 0.09 and 0.06 fm
- Including ensembles with physical light quark masses
- $L \approx 5.5$ fm. for physical quark mass ensembles

Ensembles used

β	am_l	am_s	am_c	size	N_{lats}	a (fm)
5.80	0.013	0.065	0.838	$16^3 \times 48$	1020	0.14985(38)
5.80	0.0064	0.064	0.828	$24^3 \times 48$	1000	0.15303(19)
5.80	0.00235	0.0647	0.831	$32^3 \times 48$	1000	0.15089(17)
6.00	0.0102	0.0509	0.635	$24^3 \times 64$	1040	0.12520(22)
6.00	0.00507	0.0507	0.628	$24^3 \times 64$	1020	0.12085(28)
6.00	0.00507	0.0507	0.628	$32^3 \times 64$	1000	0.12307(16)
6.00	0.00507	0.0507	0.628	$40^3 \times 64$	1028	0.12388(10)
6.00	0.00184	0.0507	0.628	$48^3 \times 64$	999	0.12121(10)
6.30	0.0074	0.037	0.440	$32^3 \times 96$	1011	0.09242(21)
6.30	0.00363	0.0363	0.430	$48^3 \times 96$	1000	0.09030(13)
6.30	0.0012	0.0363	0.432	$64^3 \times 96$	872*	0.08773(08)
6.72	0.0048	0.024	0.286	$48^3 \times 144$	1016	0.06132(22)
6.72	0.0024	0.024	0.286	$64^3 \times 144$	836*	0.05938(12)
6.72	0.0008	0.022	0.260	$96^3 \times 192$	586*	0.05678(06)

Valence masses used

β	am_l	am_s	am_c	light masses m_A (m/m_s)	m_B (m/m_c)	ϵ_N
5.80	0.013	0.065	0.838	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.30528,-0.358197*
5.80	0.0064	0.064	0.828	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.296403,-0.348378
5.80	0.00235	0.0647	0.831	0.036,0.07,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.2995,-0.3503
6.00	0.0102	0.0509	0.635	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.191781,-0.230802*
6.00	0.00507	0.0507	0.628	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.187922,-0.224811
6.00	0.00507	0.0507	0.628	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.187922,-0.224811
6.00	0.00507	0.0507	0.628	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.187922,-0.224811
6.00	0.00507	0.0304	0.628	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.187922,-0.224811
6.00	0.00507	0.00507	0.628	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.187922,-0.224811
6.00	0.00184	0.0507	0.628	0.036,0.073,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.184938,-0.224811
6.30	0.0074	0.037	0.440	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.099067,-0.120471*
6.30	0.00363	0.0363	0.430	0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.096127,-0.1152147
6.30	0.0012	0.0363	0.432	0.033,0.066,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.096127,-0.116203
6.72	0.0048	0.024	0.286	0.05,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.043326,-0.05329
6.72	0.0024	0.024	0.286	0.05,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.043326,-0.053291
6.72	0.0008	0.022	0.260	0.036,0.068,0.1,0.15,0.2,0.3,0.4,0.6,0.8,1.0	0.9,1.0	-0.036095,-0.044314

Divide and conquer

Stage 1: Correlator masses and amplitudes

Stage 2: Decay constants on each ensemble

Stage 3: Continuum limit and sea quark mass adjustments

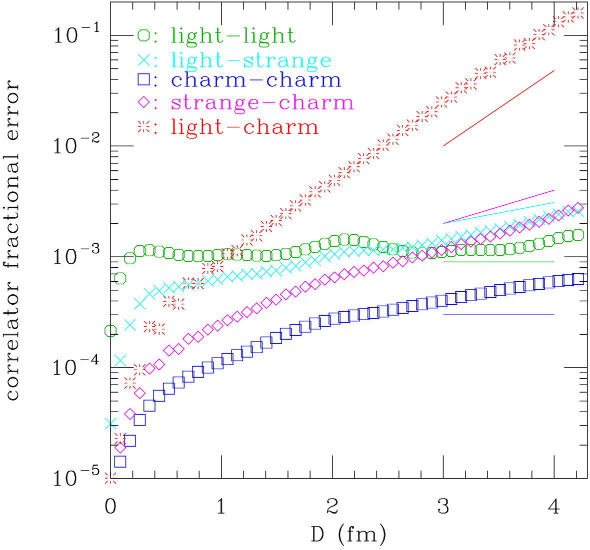
ChiPT: The heavy-light ChiPT analysis uses masses and amplitudes from stage one, and quark masses from stage two.

States dominating statistical error

State	Error	Gap(MeV)	growth length (fm)
π	2π	0 MeV	∞ fm
K	$\pi + \bar{s}s$	90 MeV	2.26 fm
η_c	$2\eta_c$	0 MeV	∞ fm
D_s	$\eta_c + \bar{s}s$	140 MeV	1.42 fm
D	$\eta_c + \pi$	310 MeV	0.64 fm

Table: States expected to control the statistical errors on the correlators, for the pseudoscalars with physical valence quark masses. The second column shows the state expected to control the growth of the statistical error on the correlator, the third column the mass gap between half the mass of the error state and the particle mass, and the fourth column the length scale for the growth of the fractional statistical error. Here $\bar{s}s$ is the unphysical flavor non-singlet state, with mass 680 MeV.

Errors on correlators



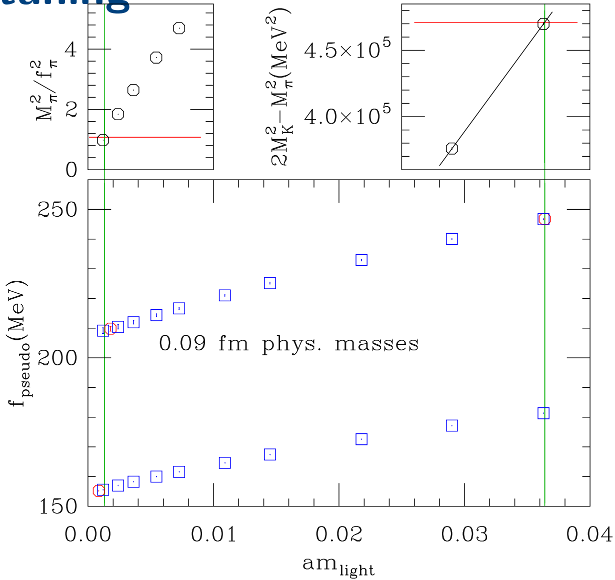
- Fractional errors for pseudoscalar correlators as a function of distance.
- These are from the 0.09 fm physical quark mass ensemble.
- The line segments show the slope expected from the states in Table 1, which give a good approximation to the observed growth

Fit types

	light-light		light-charm		charm-charm	
	form	D_{min}	form	D_{min}	form	D_{min}
$a \approx 0.15$ fm	1+1	16	2+1	8	2+0	9
$a \approx 0.12$ fm	1+1	20	2+1	10	2+0	12
$a \approx 0.09$ fm	1+1	30	2+1	15	2+0	18
$a \approx 0.06$ fm	1+1	40	2+1	20	2+0	21
$a \approx 0.045$ fm	1+1	53	2+1	26	2+0	31

Table: Fit forms and minimum distance included for the two point correlator fits. Here the fit form is the number of negative parity (i.e. pseudoscalar) states "plus" the number of positive parity states. In all cases when the valence quarks have equal masses the opposite parity states were not included. In this work the charm-charm fits are only used in computing the mass of the η_c meson, used as a check on the quality of our charm physics.

Lattice spacing and valence quark mass tuning



- Illustration of the lattice spacing and quark mass tuning
- See next two slides for details

f_D, f_{D_s} etc. on each ensemble

- Notation: m_A, m_B = valence masses, m_s, m_l, m_c = tuned valence masses.
- "Fpi_chiral tuning": Using m_A at two lightest valence masses and $M_\pi = 0$ at $m_A = 0$, interpolate/extrapolate to m_A where M_π/f_π has its physical value. Interpolation uses NLO continuum ChiPT + linear +quadratic. This fixes a using $f_\pi = 130.41$ MeV, and m_l.
- Interpolate in valence quark mass to where $2M_K^2 - M_\pi^2$ has its physical[1] value. This fixes am_s.
- Use EM adjusted K splitting to find $m_d - m_u$.
- Find charm valence mass from M_{D_s}. This fixes m_c.
- Quark masses and lattice spacings from this part go into χPT analysis.

[1]adjusted for E&M and finite size — later if I have time

f_D, f_{D_s} etc. on each ensemble

- Find (interp./extrap.) f_K at adjusted light quark mass (really f_K/f_π).
- Find (interp./extrap.) f_D and M_D (a check) at adjusted light and charm masses.
- Find (interp./extrap.) f_{D_s} at adjusted strange and charm masses.
- Find (interp./extrap.) M_{η_c} (check) at adjusted charm mass.
- Do this whole procedure inside a jackknife resampling
- Scale setting and quark mass tuning errors are then included in statistical errors.

The most important ensemble

- $a = 0.06$ fm physical quark mass ensemble, Fpi_chiral scale
- Statistical errors only!!!

$a = 0.05678(6)$ fm

$am_l = 0.000800(3)$ $am_s = 0.02188(5)$ $am_c = 0.2580(4)$

$m_s/m_l = 27.364(44)$ $m_c/m_s = 11.791(14)$

$f_K = 155.82(13)$ MeV

$M_{D_0} = 1868.1(1.0)$ MeV (cf 1864.8 - EM)

$M_{D^+} = 1870.8(0.7)$ MeV (cf 1869.6 - EM)

$M_{\eta_c} = 2982.27(29)$ MeV (cf 2980.3(1.2))

$f_D = 210.73(0.61)$ MeV $f_{D_s} = 247.89(18)$ MeV $f_{D_s}/f_D = 1.1763(32)$

Finite volume effects

- Use NLO staggered ChiPT to find f_π, M_π, f_K and M_K in a 5.5 fm box. NLO to get Φ_D and Φ_{D_s} in 5.5 fm box
- Use these values to rescale the inputs to our tuning
- Afterwards, rescale results to go back from 5.5 fm box to infinite box
- Use difference between NNLO and NLO staggered as estimate of remaining systematic error.
- Effects all come from the tuning, or f_π, M_π and M_K. Finite volume effects on Φ_D and Φ_{D_s} are small.

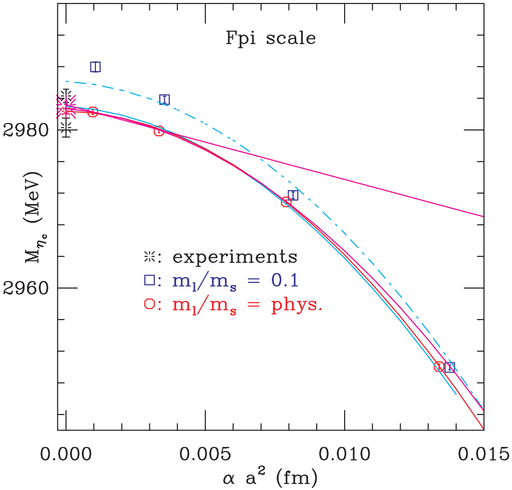
Electromagnetic effects

- From a separate calculation (Asqtad quarks), determine E&M effects on $K^+ - K^0$ mass splitting. ("EM1")
- Also determine (not quite so well defined) shift in average K mass. ("EM2")
- Use EM1 adjusted K masses in quark mass tuning procedure
- EM1 error: change Δ_{EM} by one σ, or 0.16. affects m_u/m_d
- EM2 error: subtract $901/2$ MeV^2 from average K mass^2. affects m_s
- Not included: EM effects on m_c, "direct" EM corrections to decay constants

Continuum extrapolation

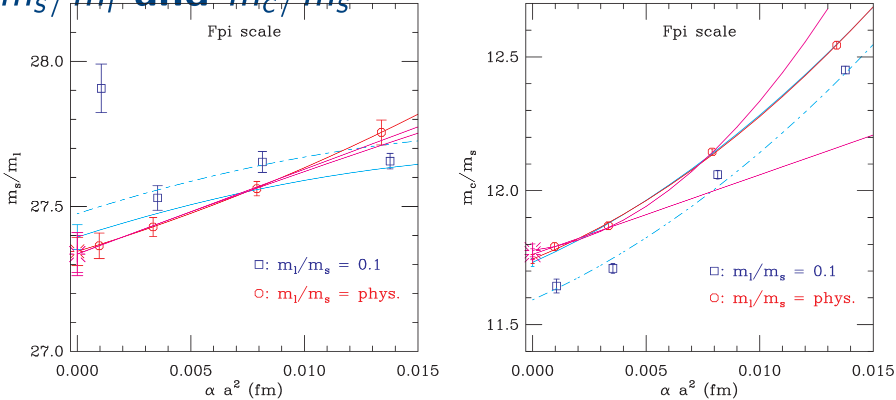
- Fitting form for continuum extrapolation makes a difference
- Quadratic in a^2, αa^2 or even $\alpha^2 a^2$ (and which α? α_V from plaquette, α_{TV} from taste violations?)?
- Include/exclude $a = 0.15$ fm? Or even linear in αa^2, 0.09 and 0.06 fm only?
- Central value is ChiPT result for f_D and f_{D_s}. For other quantities, quadratic in $\alpha_{TV} a^2$ using phys. mass ensembles.
- Use variation of extrapolated values among different fit types to estimate continuum extrapolation error.
- Note small corrections for sea quark mass mis-tuning. Use slope wrt sea quark mass from fits including $0.1\, m_s$ to shift phys. mass ensemble values slightly.
- χPT analysis uses f_{p4s} intermediate scale, this analysis uses f_π on each ensemble, which makes a^2 dependence look a little different, should agree at $a = 0$ where $f_{p4s} = 153.90(10_{stat})(34_{sys})(24_{f_\pi})$ is determined.

Continuum extrapolation: M_{η_c}



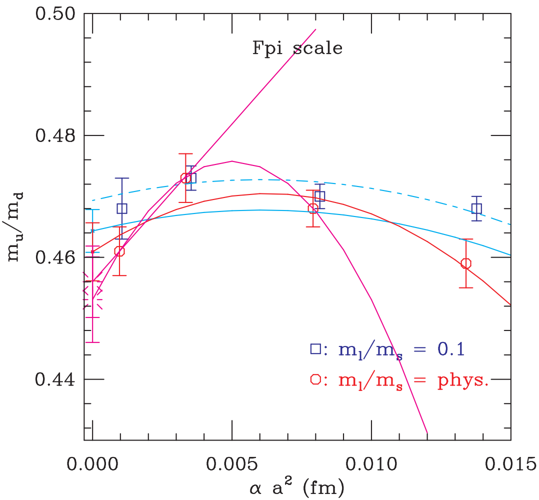
- Red: quadratic in $\alpha_{TV}a^2$, physical mass ensembles
- Cyan: quadratic in $\alpha_{TV}a^2$, physical and 0.1 m_s ensembles
- Magenta: quadratic to 3 points, linear to 2 (0 dof)
- Caveats: η_c is wide, have to decide how to define mass. Real η_c is a flavor singlet, need disconnected diagrams.
- Note: curvature, or $\sim a^4$ terms, are clearly needed

m_s/m_l and m_c/m_s



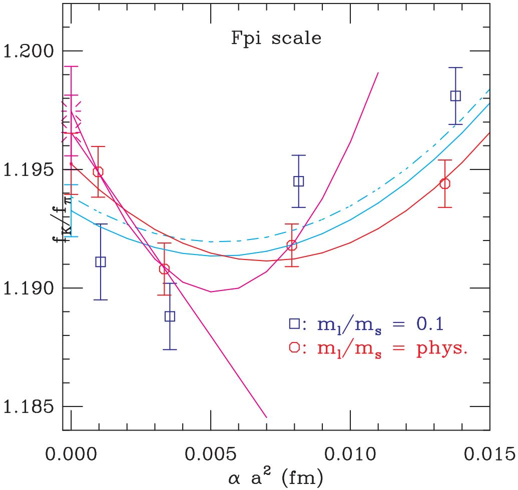
Red: quadratic in $\alpha_{TV} a^2$, physical mass ensembles.
Cyan: quadratic in $\alpha_{TV} a^2$, physical and 0.1 m_s ensembles.
Magenta: quadratic to 3 lowest points, linear to lowest 2 (0 dof)
Not plotted: quadratic in $\alpha_V a^2$ or a^2, physical mass ensembles.

Continuum extrapolation: m_u/m_d



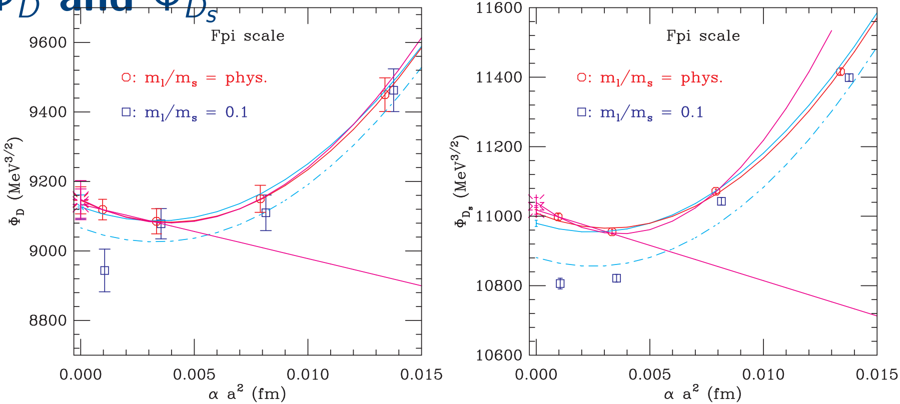
- Red: quadratic in $\alpha_{TV} a^2$, physical mass ensembles
- Cyan: quadratic in $\alpha_{TV} a^2$, physical and 0.1 m_s ensembles
- Magenta: quadratic to 3 lowest points, linear to lowest 2 (0 dof)

Continuum extrapolation: f_K/f_π



- Red: quadratic in $\alpha_{TV} a^2$, physical mass ensembles
- Cyan: quadratic in $\alpha_{TV} a^2$, physical and 0.1 m_s ensembles
- Magenta: quadratic to 3 lowest points, linear to lowest 2 (0 dof)

Φ_D and Φ_{D_s}



$$\Phi_D = \sqrt{M_D} f_D$$

Red: quadratic in $\alpha_{TV} a^2$, physical mass ensembles.

Cyan: quadratic in $\alpha_{TV} a^2$, physical and 0.1 m_s ensembles.

Magenta: quadratic to 3 lowest points, linear to lowest 2 (0 dof)

Sample worksheet: Φ_D

degree,abscissa,a_{max},masses	value(stat.)(P-value)	
Central is ChiPT	9187(22)(0.64)	
Spread of ChiPT fits		+14,-47
quad,$\alpha_{TV}a^2$,$a \leq 0.15$,$m <= .1$	9126.7(34.7)(0.36)	-60
quad,$\alpha_{TV}a^2$,$a \leq 0.15$,phys	9145.8(38.9)(0.95)	-41
quad,$\alpha_{TV}a^2$,$a \leq 0.12$,phys	9148.2(54.0)(1)	-40
lin,$\alpha_{TV}a^2$,$a \leq 0.09$,phys	9134.2(44.2)(1)	-53
quad,a^2,$a \leq 0.15$,phys	9193.3(55.3)(0.89)	+6
quad,$\alpha_V a^2$,$a \leq 0.15$,phys	9128.3(37.9)(0.59)	-59
extrap. error (asymmetric)		+14,-60
fin. size error (simple|CHiPT)		-9.3|-10.4
em error 1 (simple|CHiPT)		+1.3|+0.9
em error 2 (simple|CHiPT)		+0.7|-0.7

RESULT 9187(22 stat)($^{+18}_{-61}$ sys)

using $M_D = 1869.6$, $f_D = 212.47(0.51)(^{+0.41}_{-1.41})(0.33 f_\pi)$

cf from $f_D/f_\pi = 1.6206(65)$, $f_D = 211.34(0.85)$

Shortened worksheet: Φ_{D_s}

Central is ChiPT	11045(11)(0.64)	
Spread of ChiPT fits		+13,-55
Spread of simple fits		+16,-66
extrap. error (asymmetric)		+16,-66
fin. size error (simple|ChiPT)		-9.0|-9.3
em error 1 (simple|ChiPT)		-0.6|-0.4
em error 2 (simple|ChiPT)		-2.7|-3.7

RESULT 11045(11 stat)($^{+19}_{-67}$ sys)

using $M_{D_s} = 1968.5$, $f_{D_s} = 248.94(0.25)(^{+0.42}_{-1.50})(0.39 f_\pi)$

cf from $f_{D_s}/f_\pi = 1.9026(17)$, $f_{D_s} = 248.12(0.22)$

Shortened worksheet: f_{D_s}/f_D

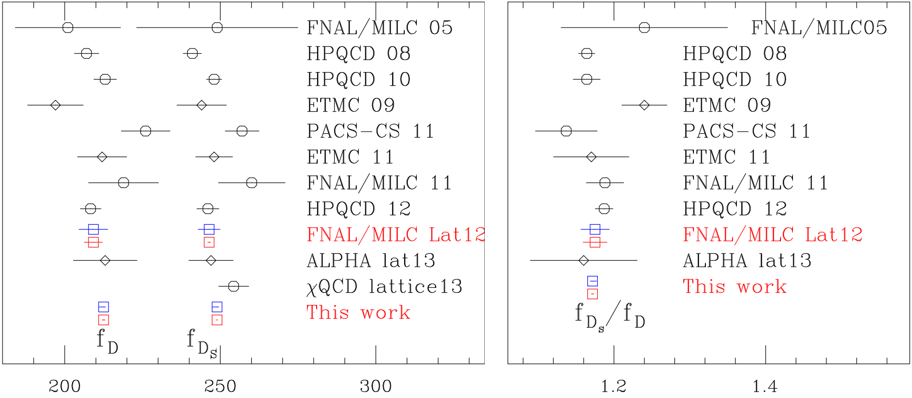
Central is ChiPT	1.1717(20)	
Spread of ChiPT fits		+0.0012,-0.0024
Spread of simple fits		-0.0052,-0.0000
extrap. error (asymmetric)		+0.0052,-0.0024
fin. size error (simple|ChiPT)		+0.0004|+0.0003
em error 1 (simple|ChiPT)		-0.0004|-0.00017
em error 2 (simple|ChiPT)		-0.0003|-0.00030

RESULT $f_{D_s}/f_D = 1.1717(20)(^{+52}_{-25})$

Results

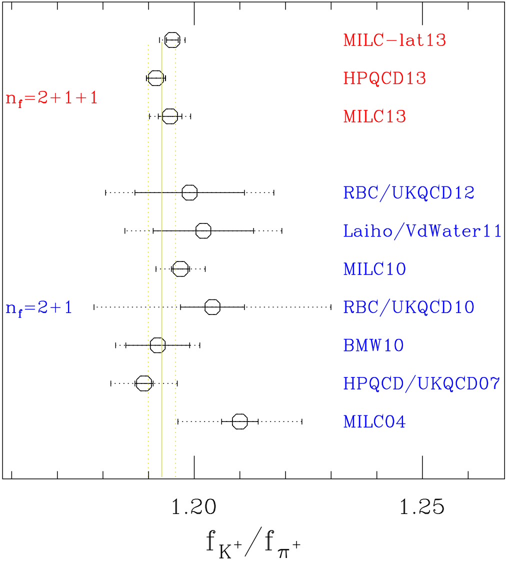
Quantity	value	stat.	systematic	largest sys.
m_c/m_s	11.746	0.017	0.059	EM2
m_s/m_l	27.345	0.049	0.122	EM2
m_u/m_d	0.4609	0.0048	0.0149	EM1
f_K/f_π	1.1952	0.0013	0.0025	cont. extrap.
f_D	212.47	0.51	$\left({}^{+0.41}_{-1.41}\right)(0.33 f_\pi)$	cont. extrap.
f_{D_s}	248.94	0.25	$\left({}^{+0.42}_{-1.50}\right)(0.39 f_\pi)$	cont. extrap.
f_{D_s}/f_D	1.1717	0.0020	${}^{+0.0052}_{-0.0025}$	cont. extrap.

Compare to previous work: f_D and f_{D_s}



Red points have statistical errors only, blue include systematic errors.

Compare to previous work: f_K/f_π



- Determinations of f_K/f_π