Using all-to-all propagators for $K \to \pi\pi$ decays

Daiqian Zhang

Dept. of Physics, Columbia University

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RBC & UKQCD Collaboration ($K \to \pi\pi$ subgroup)

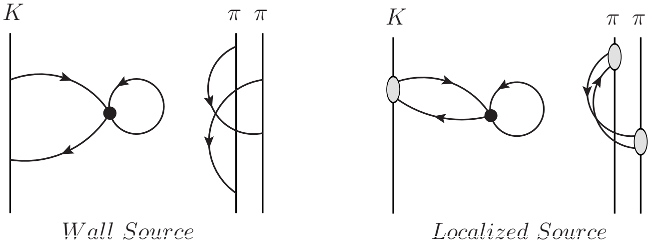
- BNL
  - Taku Izubuchi
  - Chulwoo Jung
  - Christoph Lehner
  - Amarjit Soni
- Columbia
  - Norman Christ
  - Christopher Kelly
  - Robert Mawhinney
  - Jianglei Yu
  - Daiqian Zhang
- Connecticut
  - Tom Blum
- Tata Institute of Fundamental Research
  - Andrew Lytle
- Trinity College
  - Nicholas Garron
- University of Southampton
  - Chris Sachrajda
  - Tadeusz Janowski
- University of Edinburgh
  - Peter Boyle
  - Julien Frison

Outline

1. Motivation
   - To reduce the noise from disconnected diagrams, in $K \to \pi\pi$ decay.
2. Description of all-to-all propagators
3. Optimization
   - Choose the meson field wave function.
   - Deal with the four quark operator.
4. Conclusion

Motivation

- First ab initio calculation of direct CP-violation (in $K \to \pi\pi$).
- A better choice than wall source. Better overlap with ground state and smaller overlap with vacuum.



(This figure shows an already recognized method for substantially reducing the error coming from the vacuum state: putting the source for each pion on a different time slice.)

Description

- Traditional way of calculating propagator:

$$\begin{aligned} x \to y: \quad G(y,x) &= D^{-1}_{yx'} V_{x'} \\ \textit{where } V_{x'} &= \delta(x' - x) \end{aligned}$$

  If change the source position, will need to do the inversion again.

- Propagator using stochastic source :

$$x \to y: \quad G(y,x) = (D^{-1}_{yx'} \eta_{x'}) * \eta^{\dagger}_{x}$$

  If change the source position, just use a different component of the random vector η. After doing the inversion for all the random vectors once, we get the propagator from arbitrary source to arbitrary sink.

Description

- All-to-all propagator:
(T. Kaneko et al., PoS LATTICE2010 146, 2010)
(J.Foley et al., arXiv:hep-lat/0505023)

Besides using stochastic source, we adopt a hybrid method: calculate some number of low eigen-modes in D^{-1} and use the stochastic source to estimate the deflated D^{-1}.

$$D^{-1} = \sum_{i}^{N_{ev}} \frac{h_i h_i^\dagger}{\lambda_i} + \sum_{j}^{N_{hit}} (D_{deflate}^{-1} \eta_j) \eta_j^\dagger \qquad (1)$$

The number of eigen-modes and number of stochastic sources can both be adjusted, depending on the quark mass.

Description

For simplicity, call the fermion vectors in the set $\{\frac{h_i}{\lambda_i}, D^{-1}_{deflate}\eta_j\}$ as "v", call those in the set $\{h_i^\dagger, \eta_j^\dagger\}$ as "$w^\dagger$". Both "v" and "$w^\dagger$" carry indices of mode number and space-time-spin-color.

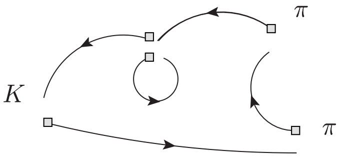
- Example: two point function (pseudo-scaler)

$$\begin{aligned}\sum_{x,y} D^{-1}_{x,y}\gamma_5 D^{-1}_{y,x}\gamma_5 &= \sum_{x,y,i,j}(v_x^i w_y^{i\,\dagger})\gamma_5(v_y^j w_x^{j\,\dagger})\gamma_5 \\ &= \sum_{i,j}(\sum_x (w_x^{j\,\dagger}\gamma_5 v_x^i)\sum_y (w_y^{i\,\dagger}\gamma_5 v_y^j)) \\ &= \sum_{i,j}\pi^{ji}_{t_x}\pi^{ij}_{t_y}\end{aligned}$$

Here the "x" and "y" have space, time, spin, and color indices in it, and the summation in second line is over its space, spin, and color indices, leaving the correlator as a function of time.

Description

- Example: $K \to \pi\pi$ decay, type3 contraction



Shaded boxes are where the random sources have been used.

$$\sum_{\vec{x}_{op}} Tr\{\gamma_\mu(1-\gamma_5)L(\vec{x}_{op}, t_{op}; t_\pi)\gamma_5 L_w(t_\pi; t_{\pi'})\gamma_5 L_w(t_{\pi'}; t_K)\gamma_5$$

$$S(t_K; \vec{x}_{op}, t_{op})\} \cdot Tr\{\gamma^\mu(1-\gamma_5)L(\vec{x}_{op}, t_{op}; \vec{x}_{op}, t_{op})\}$$

$$= \sum_{\vec{x}_{op}} \{{w'}^{m\,\dagger}_{x_{op}} \gamma_\mu(1-\gamma_5) v^i_{x_{op}}\} \cdot \{w^{j\,\dagger}_{x_{op}} \gamma^\mu(1-\gamma_5) v^j_{x_{op}}\} \cdot \pi^{ik}_{t_\pi} \pi^{kl}_{t'_\pi} K^{lm}_{t_K}$$

On the first line, L^{-1} is the propagator for up or down quark, S^{-1} is the propagator for strange quark.

On the second line, the summation is over space, and the spin-color indices in "v" and "w" are kept open until the spin-color trace is taken in each curly bracket.

Optimization: 1.Choosing meson field wave function

- The meson field in previous example contains only point-source and point-sink, the two quarks in a meson are on top of each other.

$$\pi_{t_x} = \sum_{x,a,\alpha,\beta} v_{x,a,\alpha}(\gamma_5)_{\alpha,\beta} w^{\dagger}_{x,a,\beta}$$

- but a real meson has its physical size, the two quarks may not be on top of each other. (here $S_a(x), S_b^{\dagger}(x')$ are gauge fixing matrices)

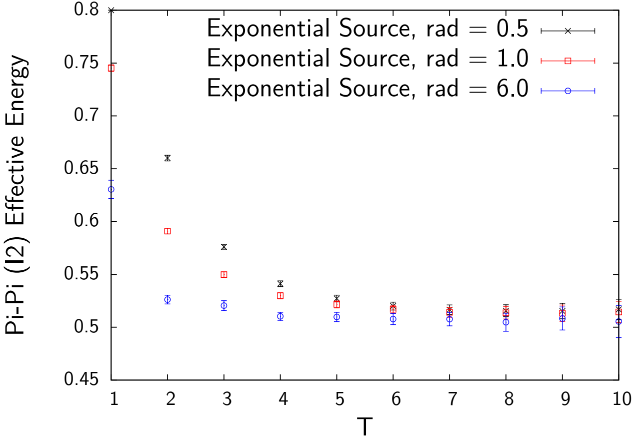
$$\pi_{t_x} = \sum_{x,x',a,b,\alpha,\beta} v_{x,a,\alpha}(\gamma_5)_{\alpha,\beta} w^{\dagger}_{x',b,\beta} \phi(|x-x'|) S_a(x) S_b^{\dagger}(x')$$

We can use a wave function for the meson, giving it a finite size and better overlap with ground state. e.g.:

$$\phi(|x-x'|) = e^{-|x-x'|/r} \text{ (exponential source)}$$

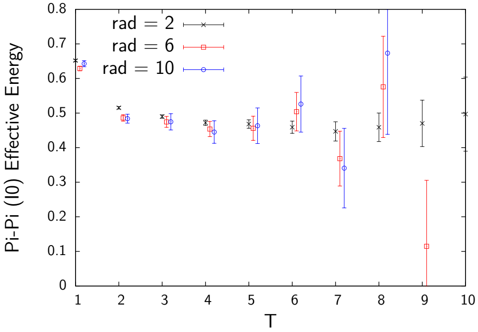
Optimization: 1.Choosing meson field wave function

- Changing the size of localized meson field will influence the plateau of effective mass. This is shown by measurement pi-pi energy (I=2) on a $16^3 \times 32$ lattice, 422 MeV pion. $a^{-1} = 1.73 GeV$ (50 configurations).



Optimization: 1.Choosing meson field wave function

- The size of meson field do affect the errorbar. As we make the radius bigger, the coupling to vacuum grows. This is shown by plotting pi-pi energy (I=0), using the same setup but 145 configurations.

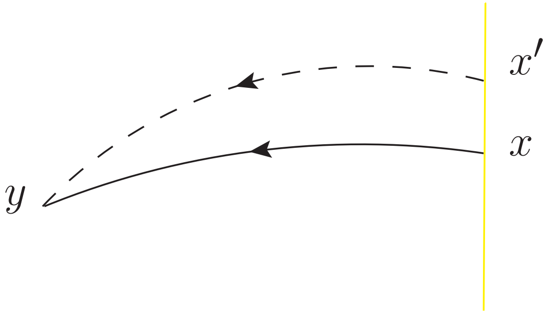


So we have to choose some appropriate meson size, not so small that excited states overwhelms ground state, not so large that the fluctuation in vacuum diagram dominates.

Optimization: 2.Deal with the four quark operator

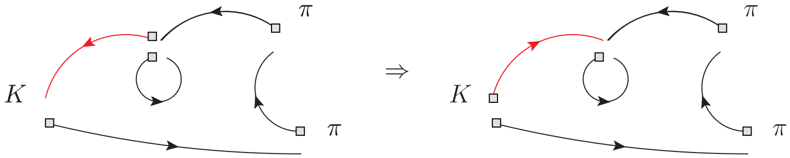
- The use of random numbers will introduce error at the source point. When calculating the propagator from x to y, we actually include other points besides x.

$$G(y,x) \;=\; (D^{-1}_{yx'}\eta_{x'}) * \eta^{\dagger}_{x}$$



Optimization: 2.Deal with the four quark operator

- the point of all-to-all propagator is to construct a better source for the meson, there is no need to use random numbers at the four-quark-operator. Because of Gamma-5 Hermiticity, we can choose which end is source and which end is sink for each propagator:

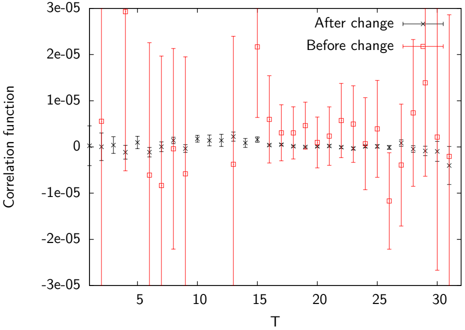


Optimization: 2.Deal with the four quark operator

The use of Gamma5 Hermiticity to change the direction of propagator is crucial in measuring some of the contractions. For example the contraction:

$$\sum_{\vec{x}_{op}} Tr\{\gamma_\mu(1-\gamma_5)L(\vec{x}_{op}, t_{op}; t_\pi)\gamma_5 L_w(t_\pi; t_{\pi'})\gamma_5 L_w(t_{\pi'}; t_K)\gamma_5$$

$$S(t_K; \vec{x}_{op}, t_{op})\gamma_5\} \cdot Tr\{\gamma^\mu(1-\gamma_5)L(\vec{x}_{op}, t_{op}; \vec{x}_{op}, t_{op})\}$$

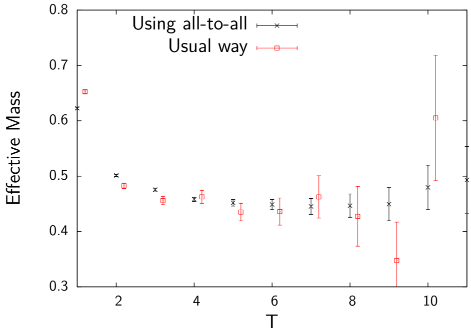


$16^3 \times 32$ lattice, 422 MeV pion. Using 100(1) eigen-modes and 1(2) set of stochastic source for light(strange) quark, 50 configurations.

Conclusion

We compared the measurement results on a $16^3 \times 32$ lattice, with 2+1 DWF fermion and Iwasaki gauge action. With $a^{-1} = 1.73 GeV, m_\pi = 422 MeV, m_K = 766 MeV$. The all-to-all propagators for light quark are using 100 low eigen-modes and 1 hit stochastic source for each time, spin and color (time-spin-color diluted).

- I=0 pi-pi phase shift. 400 configurations, 32 sources for each configuration.



	$E_{\pi\pi}^{I=0}$
All-to-all	0.449(10)
Usual way	0.438(25)

Conclusion

- $\Delta I = 1/2 \; K \to \pi\pi$ decay.

	Traditional(200 conf)		All-to-all(200 conf)	
i	$Re(A_0)(GeV)$	$Im(A_0)(GeV)$	$Re(A_0)(GeV)$	$Im(A_0)(GeV)$
1	2.6(97)e-08	0	3.8(33)e-08	0
2	4.8(17)e-07	0	2.1(15)e-07	0
3	-2.0(21)e-09	1.1(11)e-11	4.3(74)e-10	2.4(41)e-12
4	1.35(70)e-08	-4.4(23)e-11	1.6(33)e-09	5(11)e-12
5	-1.2(82)e-10	-6(43)e-13	4.7(29)e-10	2.4(15)e-12
6	-1.41(66)e-08	-8.5(40)e-11	-1.27(29)e-08	-7.6(18)e-11
7	7.0(22)e-11	1.18(36)e-13	8.11(75)e-11	1.36(13)e-13
8	-4.83(78)e-10	-2.35(38)e-12	-4.41(38)e-10	-2.15(19)e-12
9	-4.9(25)e-14	-3.2(16)e-13	9(10)e-15	-6.0(67)e-13
10	-2(17)e-12	0.6(40)e-13	1.46(68)e-11	-3.5(16)e-13
Total	5.1(16)e-07	-5.8(41)e-11	2.4(16)e-07	-6.9(22)e-11

Table : $\Delta I = 1/2$ physical amplitude

Conclusion

Performance of $\Delta I = 1/2$ $K \to \pi\pi$ decay on a 128-node IBM Blue Gene/Q machine, for each measurement:

	Traditional	All-to-all	All-to-all fully parallelized (expected)
Time	40m	70m	50m

- With a good choice of meson wave function and properly organise the way random numbers are used, the all-to-all propagator is helpful in suppressing the fluctuation in disconnected diagrams, for both I=0 pi-pi phase shift and $\Delta I = 1/2$ $K \to \pi\pi$.
- Gain in efficiency: For our 16^3 lattice $K \to \pi\pi$ example we have gained roughly a factor of four in statistics (1/2 the statistical error), for I=0 $\pi\pi$ phase shift and the imaginary part of A_0. We expect an increased computational cost of 25%, after fully parallelizing the code.

Thank you!