

Lattice study on octoic vector charmonium relevant to $X(4260)$

Ying Chen*, Wei-Feng Chiu, Long-Cheng Gui, Jian Liang,
Keh-Fei Liu, Zhaofeng Liu, and Yi-Bo Yang

Institute of High Energy Physics,
Chinese Academy of Sciences, China

Lattice 2013, Mainz, August 1, 2013

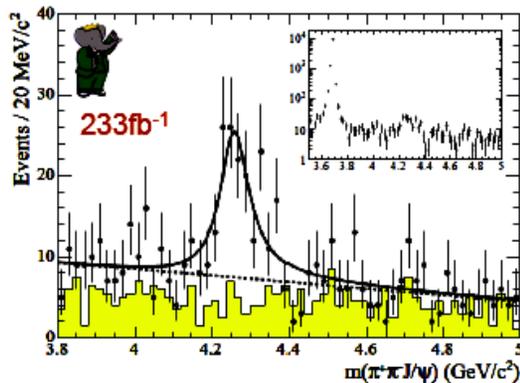
I would like to remind you of the following points :

- In the quenched approximation, even in the era of the dynamical simulation of lattice QCD
- No fancy numerical techniques
- Of phenomenological and experimental interest

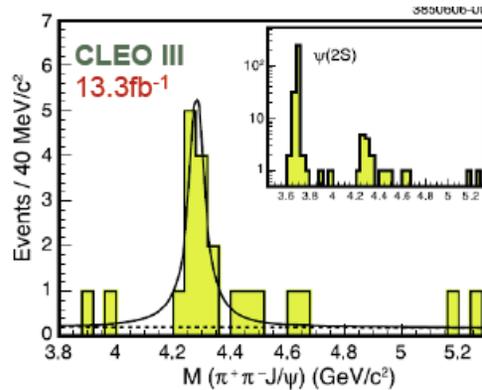
Outline

- I. Experimental status of $\Upsilon(4260)$
- II. The existence of an exotic vector charmonium in quenched LQCD
- III. The leptonic decay width of this state
- IV. Conclusion

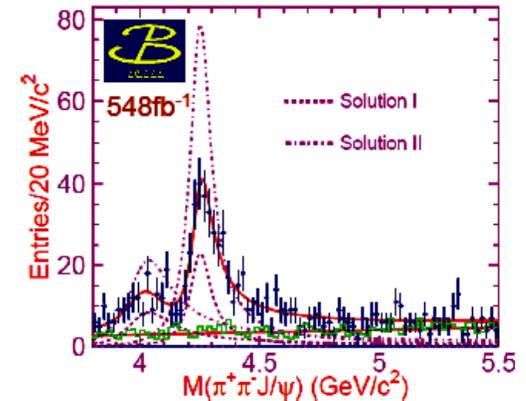
I. Experimental status of $Y(4260)$



BaBar, PRL95, 142001(05)



CLEO-III, PRD74,
091104(R)(2006)



Belle, PRL99, 182004(07)

1. Observed in the initial state radiation process
2. The resonance parameter (PDG2012)

$$M_X = 4263(8) \text{ MeV}$$

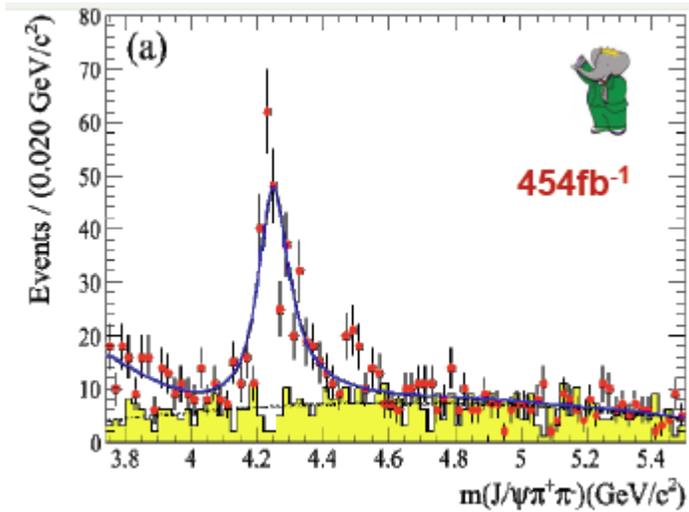
$$\Gamma_X = 95(14) \text{ MeV}$$

$$e^+ e^- \rightarrow \gamma_{ISR} X$$

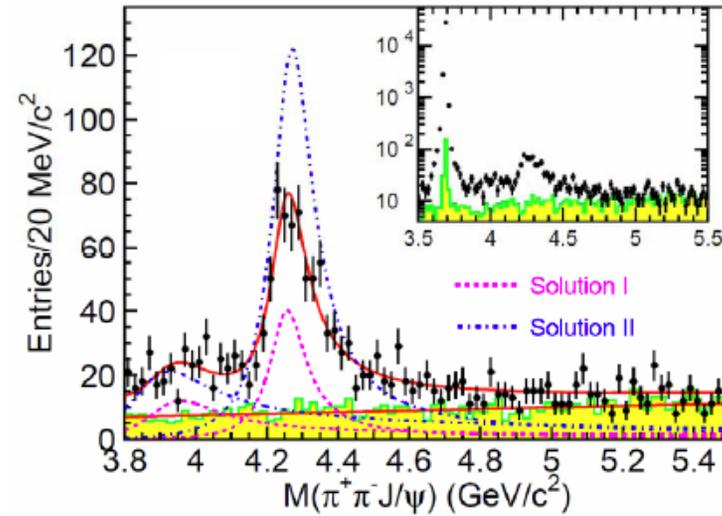
$$X \rightarrow J / \psi \pi^+ \pi^-$$

$$J / \psi \rightarrow l^+ l^-$$

3. Updated ISR $\pi^+\pi^-J/\psi$ analysis from BABAR and Belle



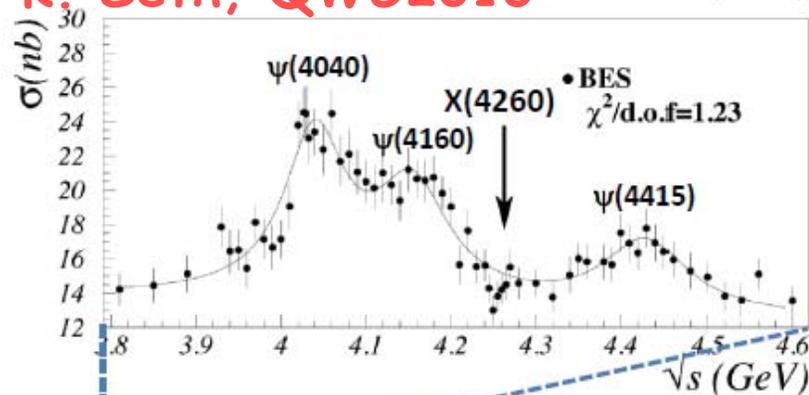
BaBar, PRD86(R), 051102(2012)



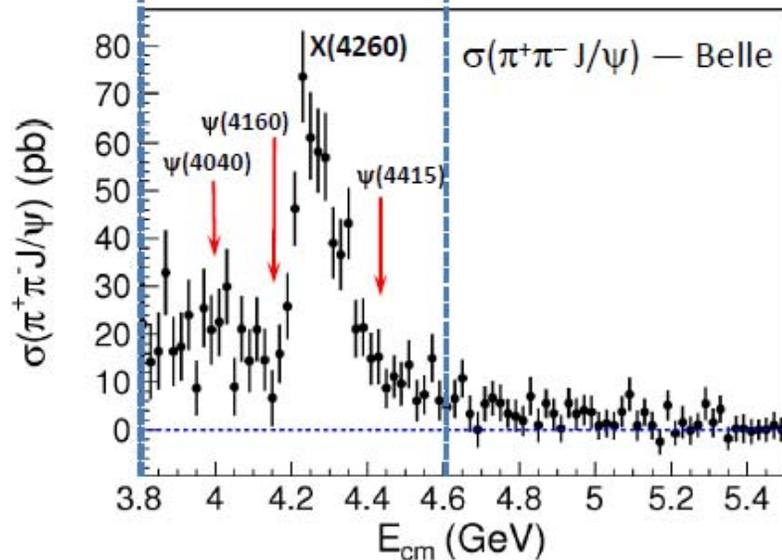
Belle, QWG2013 talk

4. The “exotic” properties of $\Upsilon(4260)$

K. Seth, QWG2013



Note that **all** 1^{--} states $\psi(4040, 4160, 4415)$ are strongly excited in $\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})$, but $X(4260) 1^{--}$ is not at all excited.



Note that **none** of the 1^{--} states $\psi(4040, 4160, 4415)$ are appreciably excited in $\sigma(\pi^+\pi^- J/\psi)$, but $X(4260)$ is strongly excited

This “orthogonality” in preferential excitation is extremely interesting and provocative

5. The possible assignment of X(4260)

A conventional charmonium $\psi(nS)$?

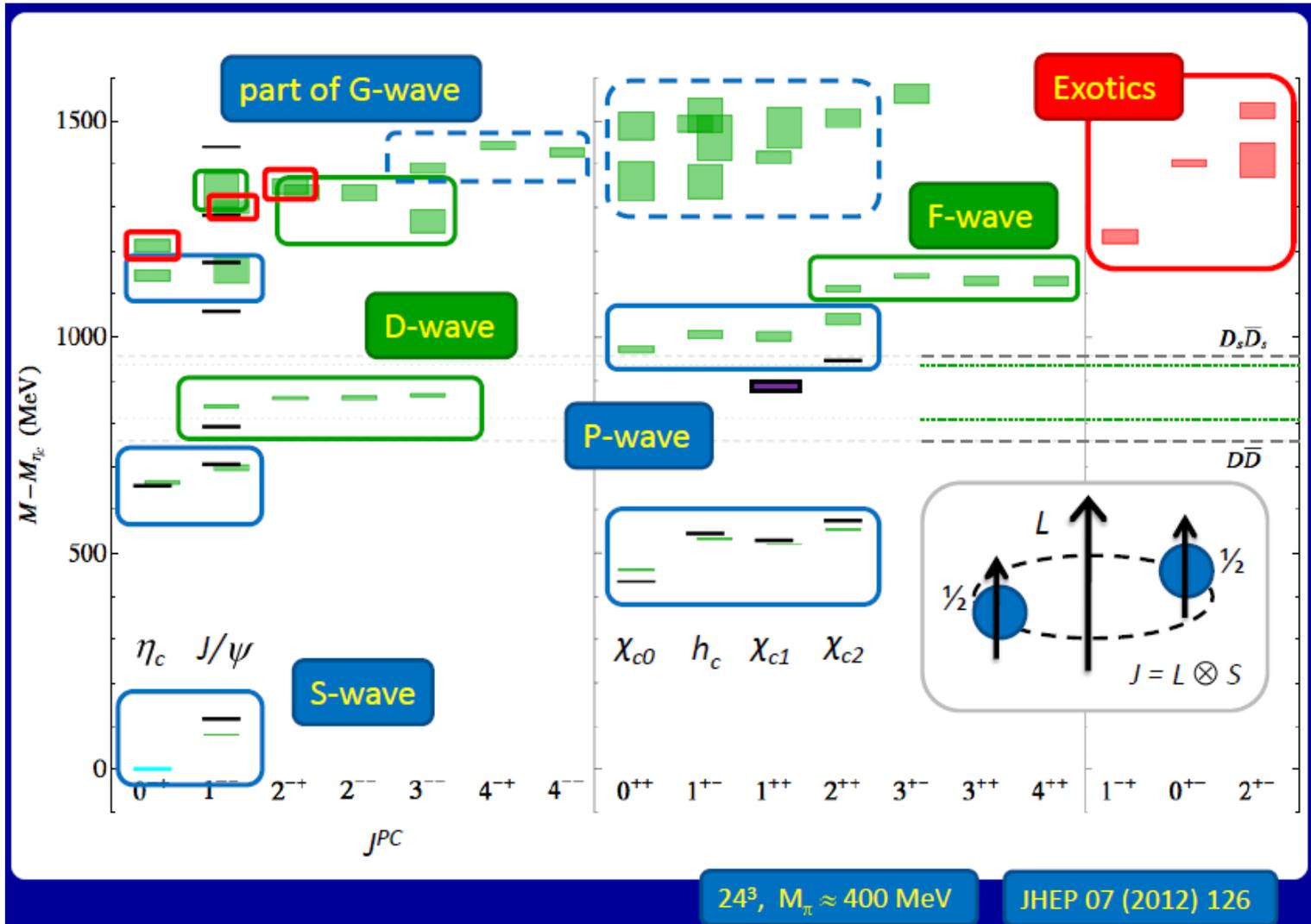
A $D\bar{D}_1$ molecule state? (near the threshold) ?

A candidate for hybrid-like charmonium?

To investigate the possible hybrid-like picture,
a not bad point of the quenched approximation——

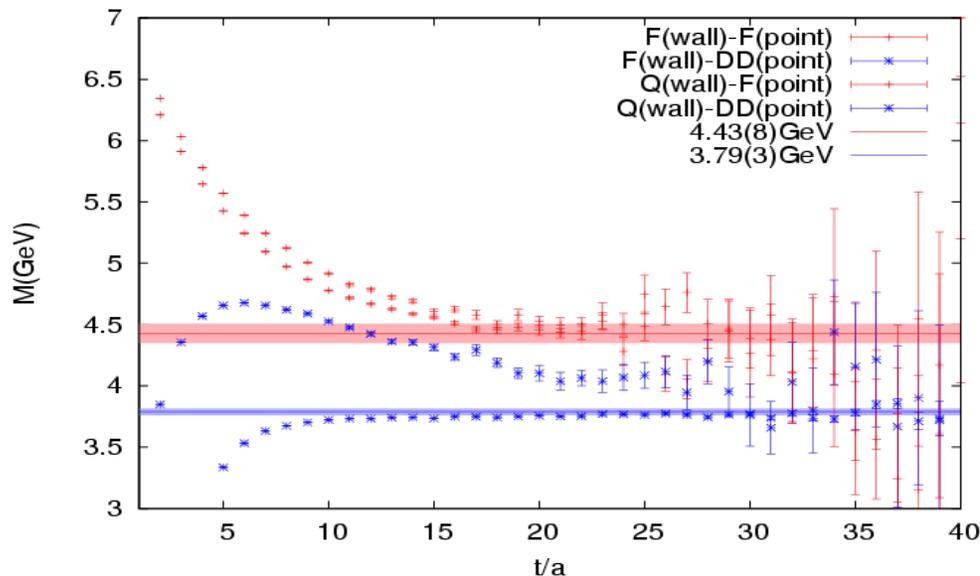
A simpler spectrum without the scattering states
for the lack of sea quarks

6. Latest charmonium spectrum from lattice QCD



II. The existence of an exotic vector charmonium in the quenched LQCD

- 1^-+ is an exotic quantum number in quark model.
- The mass of 1^-+ hybrid charmonium was predicted to be $4.1\sim 4.3$ GeV from lattice QCD study.
- In 2^-+ channel, we found a hybrid-like interpolation field operator couples most to a higher state with a mass $4.43(8)$ GeV, rather than the conventional 2^-+ charmonium η_{c2} , whose mass is predicted to be $3.80(3)$ GeV.



1. Lattice setup

Tadpole improved Symanzik's gauge action
Tadpole improved Clover's fermion action

β	ξ	a_s (fm)	La_s (fm)	$L^3 \times T$	N_{conf}
2.4	5	0.222	1.78	$8^3 \times 96$	1000
2.8	5	0.138	1.66	$12^3 \times 144$	1000

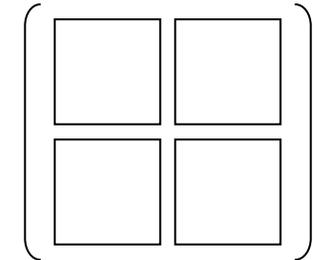
- Each configuration is fixed to the Coulomb gauge,
- N_T wall sources to calculate N_T quark propagators on each configuration
- Charm quark mass parameter is set by the physical J/psi mass.

2 1- hybrid-like interpolation field operator

$$\bar{\psi}^a \gamma_5 \psi^b B_i^{ab}$$

Nonrelativistic decomposition

$$B_i(x) = \frac{1}{2} \varepsilon_{ijk} F_{jk}(x) \rightarrow$$



$$\begin{aligned} q &= e^{\frac{\gamma \cdot D}{2m}} \begin{pmatrix} \psi \\ \chi \end{pmatrix} = \left[1 + \frac{\gamma \cdot D}{2m} + \frac{\gamma \cdot \vec{D} \gamma \cdot D}{8m^2} O(1/m^3) \right] \begin{pmatrix} \psi \\ \chi \end{pmatrix} \\ &= \begin{pmatrix} \psi \\ \chi \end{pmatrix} + \frac{i}{2m} \begin{pmatrix} -\sigma \cdot \vec{D} \chi \\ \sigma \cdot \vec{D} \psi \end{pmatrix} + \frac{(\vec{D}^2 + \sigma \cdot B)}{8m^2} \begin{pmatrix} \psi \\ \chi \end{pmatrix} + O(1/m^3), \\ \bar{q} &= \begin{pmatrix} \psi^\dagger & -\chi^\dagger \end{pmatrix} e^{-\frac{\gamma \cdot \overleftarrow{D}}{2m}} = \begin{pmatrix} \psi^\dagger & -\chi^\dagger \end{pmatrix} + \frac{i}{2m} \begin{pmatrix} \chi^\dagger \sigma \cdot \overleftarrow{D}^\dagger & \psi^\dagger \sigma \cdot \overleftarrow{D}^\dagger \end{pmatrix} \\ &\quad + \frac{(\overleftarrow{D}^2 + \sigma \cdot B)}{8m^2} \begin{pmatrix} \psi^\dagger & -\chi^\dagger \end{pmatrix} + O(1/m^3), \end{aligned}$$

0^+	$\bar{\psi} \gamma_5 \psi$	$\chi^+ \phi$
1^-	$\bar{\psi} \gamma_i \psi$	$\chi^+ \sigma_i \phi$
1^-_H	$\bar{\psi}^a \gamma_5 \psi^b B_i^{ab}$	$\chi^{a+} \phi^b B_i^{ab}$

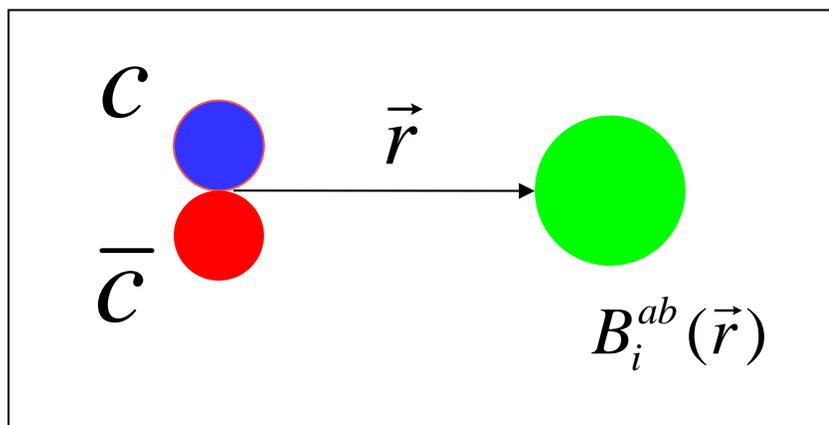
To the leading order of the nonrelativistic decomposition

$$O_i^{(H)} \equiv \bar{c}^a \gamma_5 c^b B_i^{ab} \rightarrow \chi^{a\dagger} \phi^b B_i^{ab} + O\left(\frac{1}{m_c}\right), \quad \rightarrow \text{c-cbar spin singlet}$$

$$O_i^{(M)} \equiv \bar{c}^a \gamma_i c^a \rightarrow \chi^{a\dagger} \sigma_i \phi^a + O\left(\frac{1}{m_c}\right). \quad \rightarrow \text{c-cbar spin triplet}$$

3. Spatially extended interpolation field operator for the vector charmonium-like state

In the Coulomb gauge, $O(\vec{r}) = (\bar{c}^a \gamma_5 c^b)(0) B_i^{ab}(\vec{r})$



This is equivalent to give a c - \bar{c} center of mass motion, which describes the recoil of the c - \bar{c} against additional degrees of freedom.

Intuitively, the coupling of this kind of operators to conventional vector charmonia can be suppressed from two aspects:

- spin states of the c - \bar{c} (spin flipping is suppressed by the heavy quark mass.
- center-of-mass motion (to the leading order of NR, there is no center-of-mass motions for conventional charmonia.)

3. The two-point functions calculated on the lattice

We introduce the following wall-source operators

$$O_i^{(W)}(\tau) = \sum_{\mathbf{y}, \mathbf{z}} \bar{c}^a(\mathbf{y}, \tau) \gamma_5 B_i^{ab}(\mathbf{z}, \tau) c^b(\mathbf{z}, \tau).$$

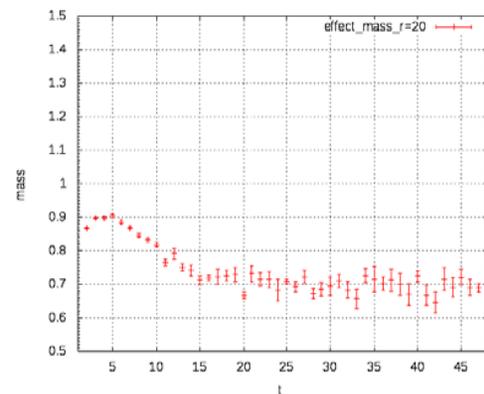
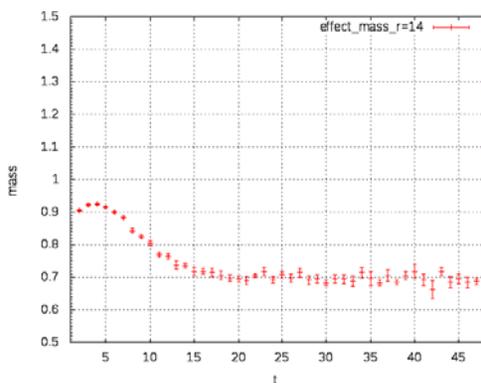
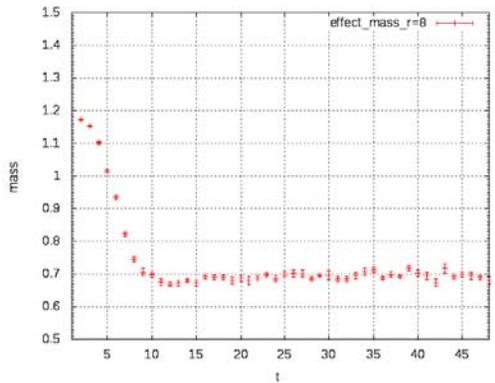
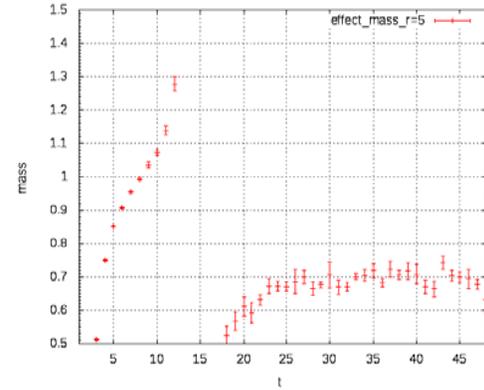
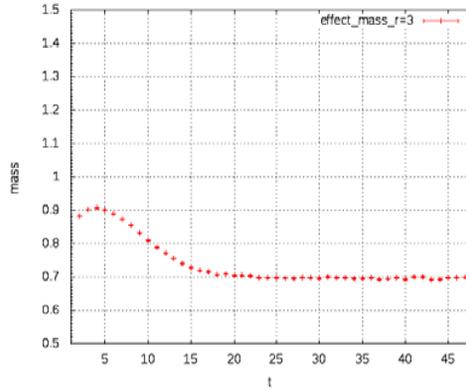
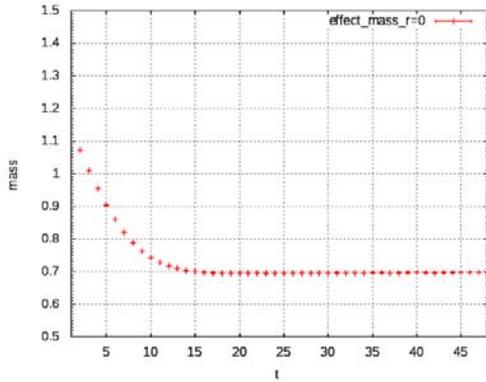
and calculate the following two-point functions

$$C(r, t) = \left\langle O(\vec{r}, t) O^{(w)+}(0) \right\rangle = \sum_i W_i(r) e^{-m_i t}$$

where $W_i(r)$ is the spectral weight of the i -th state, which varies with respect to r , and thereafter can be interpreted as the Coulomb BS wave function versus the distance between the c -bar and B field.

Effective mass plots for different r

$$m_{\text{eff}}(r, t) = \ln \frac{C(r, t)}{C(r, t+1)}$$

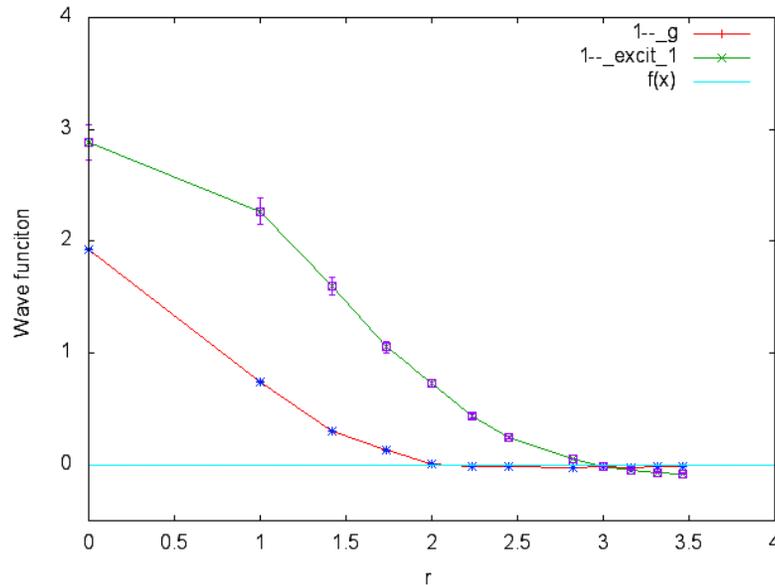


- The tails at large t show the saturation of the ground state J/ψ
- The drastic changes in the small t range imply that the operators with different r couple to the states differently.

The two-point functions with different r can be fit jointly with the same spectrum.

$$C(r,t) = \left\langle O(\vec{r},t)O^{(w)+}(0) \right\rangle = \sum_i W_i(r)e^{-m_i t}$$

2-mass term fit

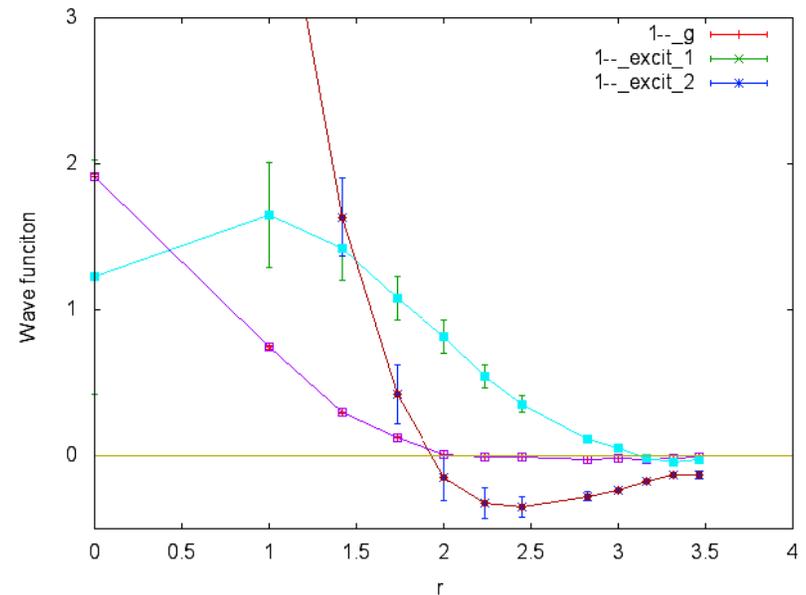


$$m_1 = 3.076(4) \text{ GeV}$$

$$m_2 = 4.275(18) \text{ GeV}$$

X(4260)???

3-mass term fit



$$m_1 = 3.072(9) \text{ GeV}$$

$$m_2 = 4.311(44) \text{ GeV}$$

$$m_3 = 4.932(88) \text{ GeV}$$

It is obviously seen that

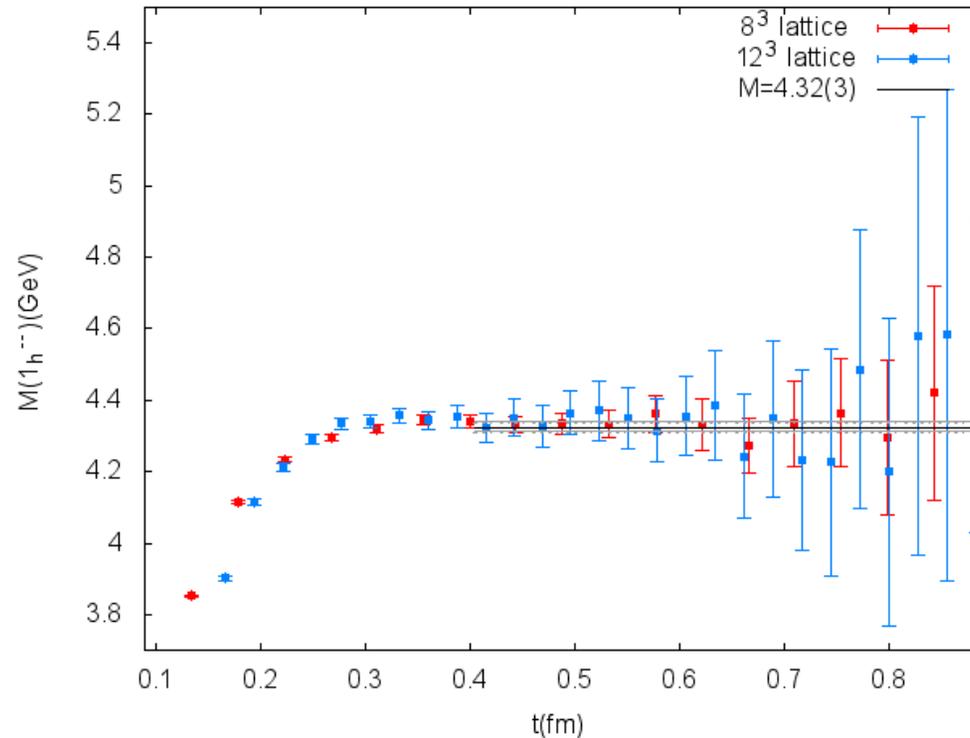
1. The spectral weight of J/psi (and actually other conventional vector charmonia) damps more rapidly with the increment of r ;
2. The small deviation of the J/psi mass from the original value 3.097 can be understood as the small contamination of higher conventional charmonia;
3. In other words, we can linearly combine them to obtain optimal two-point function

$$C_{optimal}(t) = C(r_1, t) + \omega C(r_2, t)$$

which is dominated by the exotic vector charmonium. The optimal parameter ω can be determined numerically by requesting the plateau lasting as long as enough.

Linearly combine the correlation functions with different r
we can eliminate the conventional vector charmonium and
get a relatively clean signal of the exotic vector meson.

$$C_{optimal}(t) = C(r_1, t) + \omega C(r_2, t)$$



III. The leptonic decay width of the exotic vector charmonium

1. The leptonic decay width of this exotic vector charmonium is an important quantity, which can shed light on the nature of $Y(4260)$.

$$\Gamma(Y(4260) \rightarrow e^+ e^-) \Gamma(Y(4260) \rightarrow J/\psi \pi^+ \pi^-) / \Gamma_{tot} = 5.8 eV$$

2. The leptonic decay constant of the exotic state can be calculated directly in lattice QCD.

The decay constant of a vector meson is defined as

$$\langle 0 | \bar{q} \gamma_\mu q | V(\vec{p}, r) \rangle = m_V f_V \varepsilon_\mu(\vec{p}, r)$$

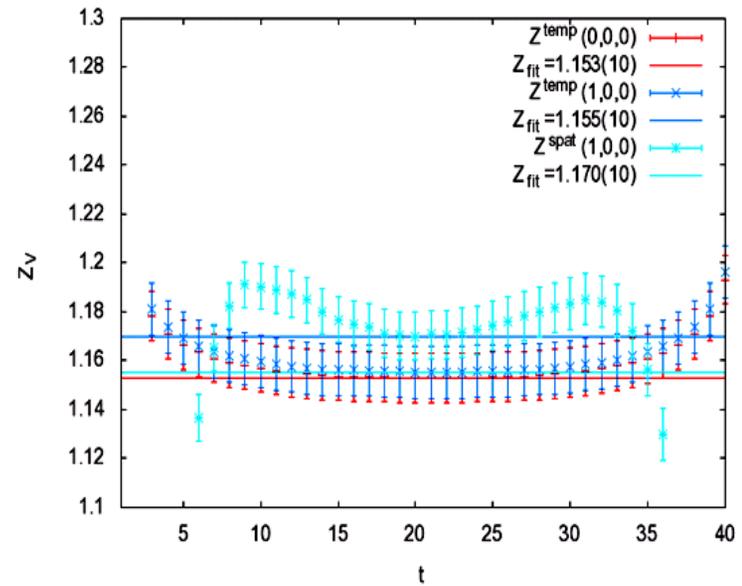
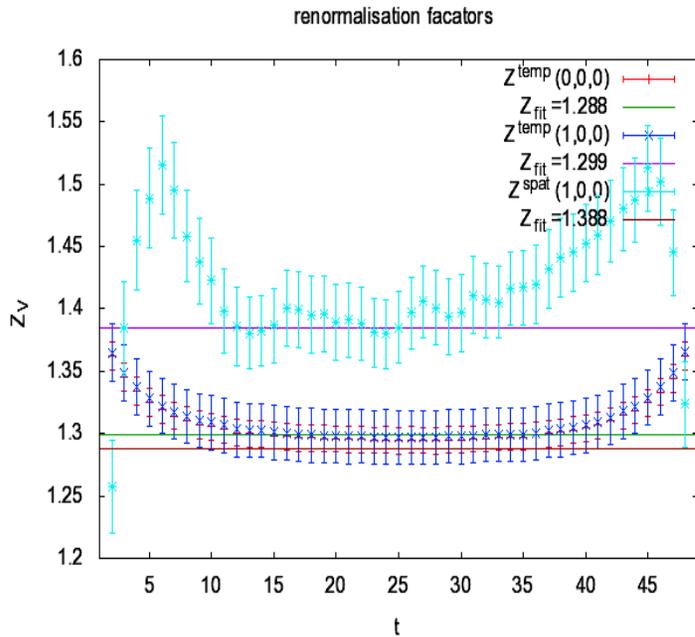
where the matrix element on the left can be derived by calculating the two point function

$$\sum_{\vec{x}} \langle 0 | \bar{q} \gamma_\mu q(\vec{x}, t) O^{(w)+}(0) | 0 \rangle = \sum_{i,r} \frac{1}{2M_i} \langle 0 | \bar{q} \gamma_\mu q | V_i, r \rangle \langle V_i, r | O^{(w)+} | 0 \rangle e^{-M_i t}$$

$$C(r, t) = \langle O(\vec{r}, t) O^{(w)+}(0) \rangle = \sum_{i,r} \frac{1}{2M_i} \langle 0 | O(\vec{r}, t) | V_i, r \rangle \langle V_i, r | O^{(w)+} | 0 \rangle e^{-M_i t}$$

$$C^{(ww)}(t) = \langle O^{(w)}(t) O^{(w)+}(0) \rangle = \sum_{i,r} \frac{1}{2M_i} \left| \langle V_i, r | O^{(w)+} | 0 \rangle \right|^2 e^{-M_i t}$$

3. The renormalization constant of the vector current



$$Z_V^\mu(t) = \frac{p^\mu}{2E(p)} \frac{\Gamma_{\eta_c \eta_c}^{(2)}(\vec{p}, t_f = T/2)}{\Gamma_{\eta_c \gamma^\mu \eta_c}^{(3)}(\vec{p}_f = \vec{p}_i = \vec{p}; t_f = T/2, t)}$$

$$\beta = 2.4 \quad Z_V \equiv Z_V^{(i)} = 1.39(1)$$

$$\beta = 2.8 \quad Z_V \equiv Z_V^{(i)} = 1.12(1)$$

β	$M(J/\psi)(\text{GeV})$	$f_{J/\psi} (\text{MeV})$	$M(Y)(\text{GeV})$	$f_Y (\text{MeV})$
2.4	3.076(4)	428(7)	4.43(7)	32(20)
2.8	3.082(1)	378(6)	4.40(7)	31(11)
Exp.	3.097	407(5)

(preliminary)

Using the formula

$$\Gamma(V_{c\bar{c}} \rightarrow e^+e^-) = \frac{16\pi}{27} \alpha_{\text{QED}}^2 \frac{f_V^2}{M_V}$$

	$\Gamma(e^+e^-)(\text{keV})$	$\Gamma(J/\psi\pi^+\pi^-) (\text{keV})$
J/ψ	5.55(14)	...
ψ'	2.35(4)	~ 100
$\psi(4040)$	0.86(7)	< 320
$\psi(4415)$	0.58(7)	...
$\psi(3770)$	0.262(18)	~ 50
$\psi(4160)$	0.83(7)	< 310

One can predict the leptonic decay width of the exotic vector charmonium

$$\Gamma(Y \rightarrow e^+e^-) \sim 23(20)eV$$

IV. Conclusion

1. Both quenched and unquenched studies on charmonium spectrum show that there may be an exotic vector charmonium state around $M \sim 4.3 \text{ GeV}$.
2. For the first time, the leptonic decay width of this state is predicted to be $23(20) \text{ eV}$, which is much smaller than that of the conventional vector charmonia.
3. If this state corresponds to $X(4260)$ observed in experiments, one can estimate the branch ratio of it decaying into $\pi^+\pi^-J/\psi$

$$\Gamma(V \rightarrow e^+e^-) \propto f_V^2$$

$$\langle 0 | \bar{q} \gamma_\mu q | V(\vec{p}, r) \rangle = m_V f_V \varepsilon_\mu(\vec{p}, r)$$

$$\Gamma(Y(4260) \rightarrow e^+e^-) \Gamma(Y(4260) \rightarrow J/\psi \pi^+ \pi^-) / \Gamma_{\text{tot}} = 5.8 \text{ eV}$$

$\sim 20 - 30\%$

Thanks!