$O(a^2)$-IMPROVED ACTIONS FOR HEAVY QUARKS

~SCALING STUDIES ON QUENCHED LATTICES~

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Lattice2013@Mainz

MOTIVATION

- Precise calculation of heavy-light (D and B) and heavy-heavy (Ψ and Υ) systems. It needs good control of discretization effects of $(am_Q)^n$.
- Assume that fine-lattice simulations (a^{-1}=2.4-4.8GeV, JLQCD collaboration) are available. Still, am_Q is not very small (am_Q~0.54-0.27 for charm). Improved fermion formulations are crucial.
- We test existing and newly developed formulations
  - as a joint effort of JLQCD and UKQCD (Edinburgh-Southampton)

STRATEGY

Test for

- scaling of spectrum, decay constant
- dispersion relation

on quenched configurations of a^{-1}=2.0, 2.8. 3.4, 6.0 GeV

×

With

- $O(a^2)$-improved Brillouin fermion
- Domain-wall (or Möbius) fermion

against the standard Wilson (or clover) fermion.

PLAN OF THIS TALK

1. Lattice fermion formulation for heavy quarks

- $O(a^2)$-improved Brillouin fermion: formulation, tree-level dispersion relation, eigenvalue spectra

2. Scaling studies

- Quenched lattices
- Dispersion relation, hyperfine splitting of charmonium
- Decay constant for heavy-heavy systems

3. Outlook

1. FERMION FORMULATION FOR HEAVY QUARKS

- $O(a^2)$-improved Brillouin fermion: formulation, tree-level dispersion relation, eigenvalue spectra

LATTICE FERMIONS

- Wilson fermions
  - Discretization error is O(am). O(a)-improvement is often considered, then $O(am)^2$
- Domain-wall fermions
  - preserve chiral symmetry. Discretization error is $O(am)^2$. Limitation on the value of am.
- Improved fermions
  - We develop an $O(a^2)$-improved fermion formulation based on the Brillouin fermion. It has good properties (dispersion relation, ...) for heavy quarks.

BRILLOUIN FERMIONS

[S.Durr,G.Koutsou Phys.RevD83(2011)114512]
[M.Creutz,T.Kimura,T.Misumi JHEP 1012:041,2010]

- Brillouin operator(free)

~ improvement of Wilson fermions

-> continuum like, Ginsparg-Wilson like

$$D^{Wil}(n,m) = \sum_{\mu} \gamma_\mu \boxed{\nabla^{std}_{\mu}}(n,m) - \frac{a}{2}\boxed{\triangle^{std}}(n,m) + m_0 \delta_{n,m}$$

Derivative term

Laplacian term

$$D^{Bri}(n,m) = \sum_{\mu} \gamma_\mu \boxed{\nabla^{iso}_{\mu}}(n,m) - \frac{a}{2}\boxed{\triangle^{bri}}(n,m) + m_0 \delta_{n,m}$$

①DERIVATIVE TERM : ISOTROPIC DERIVATIVE

2 dimension

standard x-derivative

$$\nabla_x^{std}\psi_n = \frac{1}{2a}\left(\psi_{n+\hat{x}} - \psi_{n-\hat{x}}\right)$$

$$\simeq \partial_x\psi_n + \frac{a^2}{6}\partial_x^3\psi_n$$

$$= \left(1 + \boxed{\frac{a^2}{6}\partial_x^2}\right)\partial_x\psi_n \quad \Rightarrow \quad \left(1 + \boxed{\frac{a^2}{6}\triangle}\right)\partial_x\psi_n$$

unisotropic error

isotropic error

isotropic x-derivative (restore the rotational symmetry)

$$\nabla_x^{iso}\psi_n = \left(1 + \frac{a^2}{6}\partial_y^2\right)\left(1 + \frac{a^2}{6}\partial_x^2\right)\partial_x\psi_n$$

$$= \left(1 + \frac{a^2}{6}\partial_y^2\right)\nabla_x^{std}\psi_n$$

=>add a 2-hop term (in 2D)

=>add 2,3,4-hop terms (in 4D)

②LAPLACIAN TERM:(BRILLOUIN LAPLACIAN)

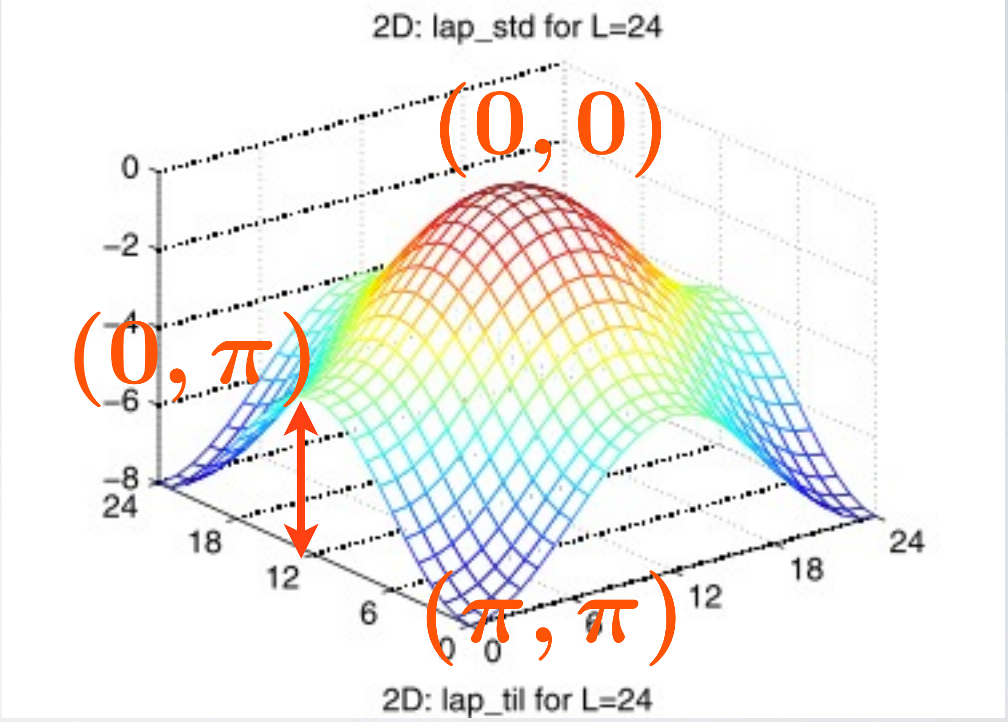
$$\triangle^{std}(p) = 2(cos(p_x) + cos(p_y) + cos(p_z) + cos(p_t) - 4)$$

$$\triangle^{bri}(p) = 4cos^2(p_x/2)cos^2(p_y/2)cos^2(p_z/2)cos^2(p_t/2) - 4$$

momentum space(2D)

[S.Durr,G.Koutsou Phys.RevD83(2011)114512]

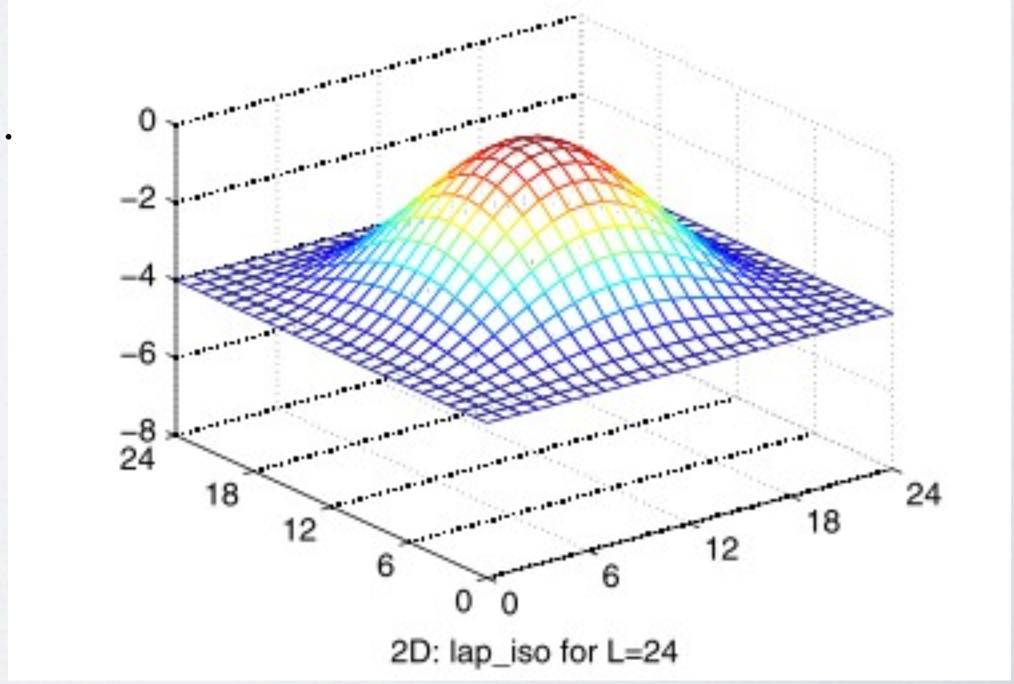
$\triangle$std



$$M(p) = M - \frac{r}{2}\triangle^{std}(p)$$
$$= M - r\left(cos(px) + cos(py) + cos(pz) + cos(pt) - 4\right)$$

$$p_\mu = (0,0,0,0) \quad \to \quad M(p) = M \qquad (\times 1)$$
$$p_\mu = (\pi,0,0,0),\cdots \to \quad M(p) = M + 2r \quad (\times 4)$$
$$p_\mu = (\pi,\pi,0,0),\cdots \to \quad M(p) = M + 4r \quad (\times 6)$$
$$p_\mu = (\pi,\pi,\pi,0),\cdots \to \quad M(p) = M + 6r \quad (\times 4)$$
$$p_\mu = (\pi,\pi,\pi,\pi),\cdots \to \quad M(p) = M + 8r \quad (\times 1)$$

$\triangle$bri



$$M(p) = M - \frac{r}{2}\triangle^{bri}(p)$$
$$= M - 2r\left(cos^2(px/2)\,cos^2(py/2)\,cos^2(pz/2)\,cos^2(pt/2) - 1\right).$$

$$p_\mu = (0,0,0,0) \quad \to \quad M(p) = M \qquad (\times 1)$$
$$p_\mu = (\pi,0,0,0),\cdots \to \quad M(p) = M + 2r \quad (\times 4)$$
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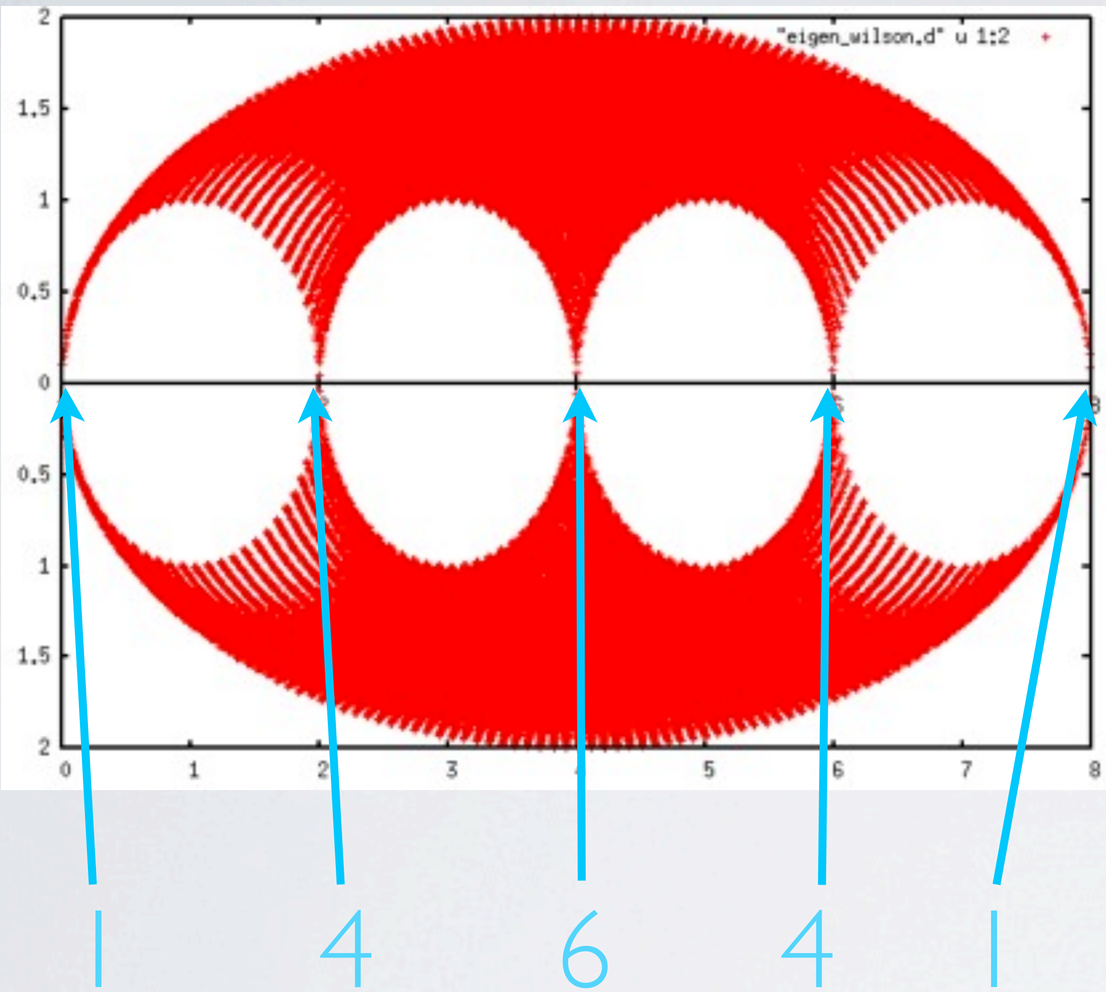
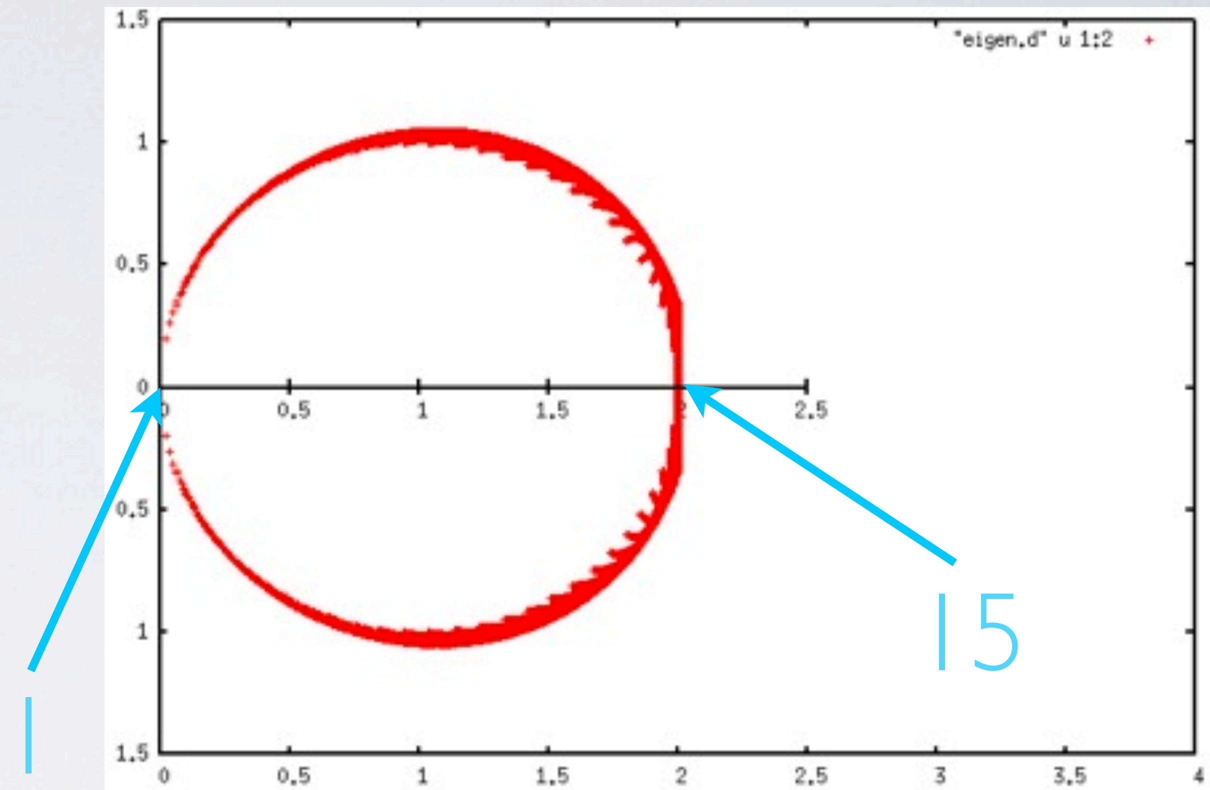
$$p_\mu = (\pi,\pi,\pi,\pi),\cdots \to \quad M(p) = M + 2r \quad (\times 1)$$

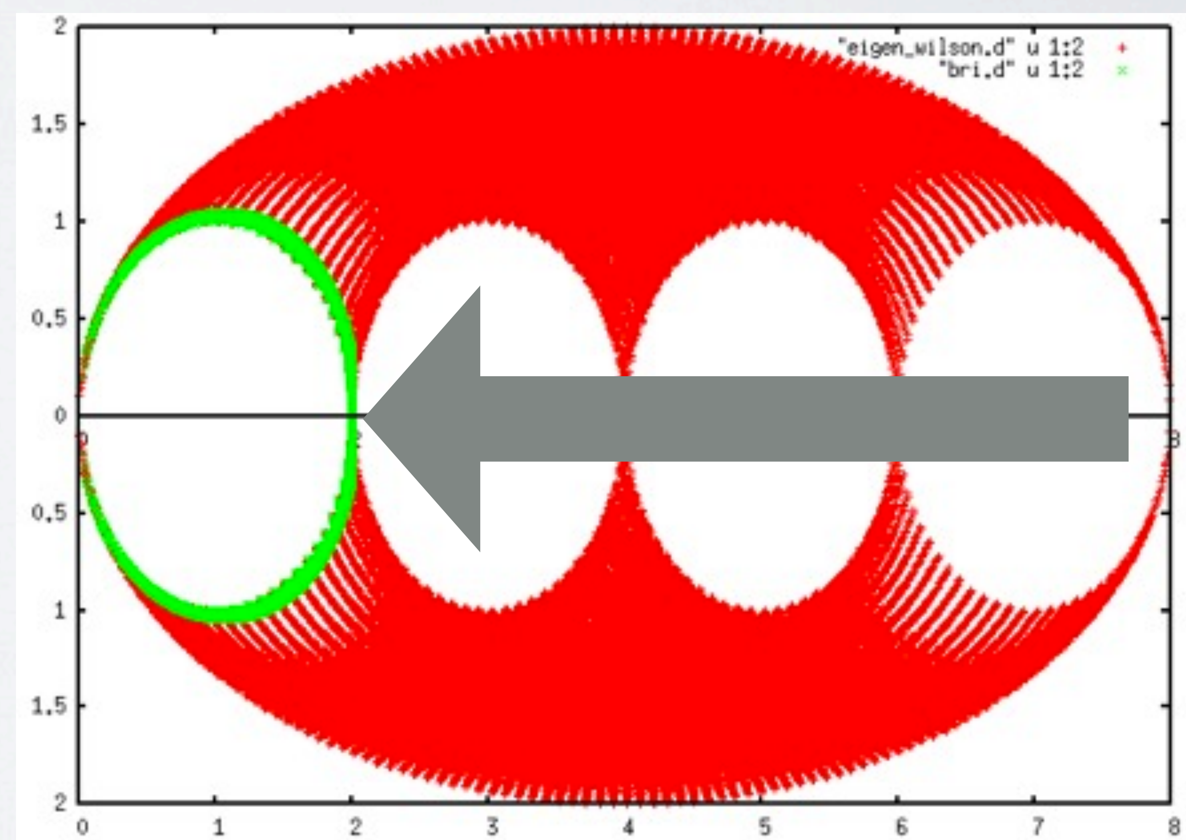
$\Rightarrow$ all doublers have a same mass.

EIGENVALUE SPECTRA (FREE)





Ginsparg-Wilson like<=

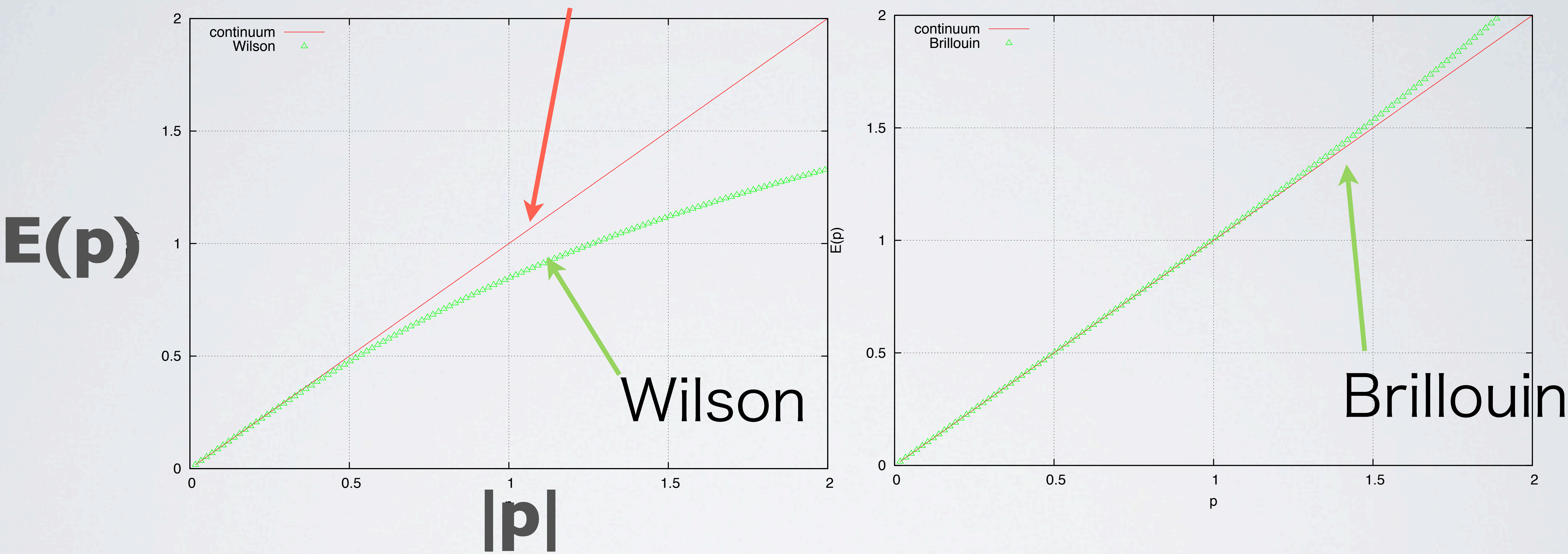


DISPERSION RELATION (FREE)

estimate the energy E(p) from the pole of $D^{-1}(p)$ in the momentum space.

m = 0.0

$E(\vec{p}, m) = \sqrt{\vec{p}^2 + m^2}$ (continuum)



- Dispersion relation of meson, baryon is good too.

  Difference of Wilson and Brillouin becomes more significant

  at heavy quarks regions. [S.Durr, G.Koutsou, T.Lippert Phys.Rev.D86(2012) 114514]

DISCRETIZATION ERRORS FOR BRILLOUIN FERMIONS

- expand the energy up to O(a^5)

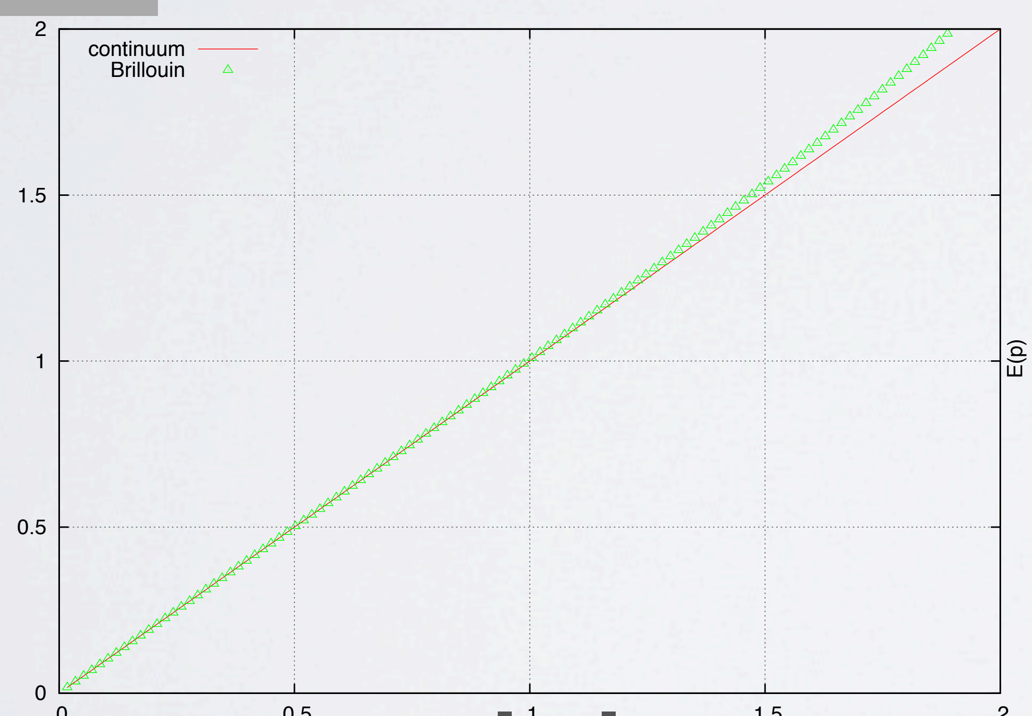
$$E\left(\vec{0}, ma\right)^2 = (ma)^2 \boxed{- (ma)^3 + \frac{11}{12}(ma)^4 - \frac{5}{6}(ma)^5}$$

=> O(a),O(a^2),O(a^3) errors

- dispersion relation for massive quarks

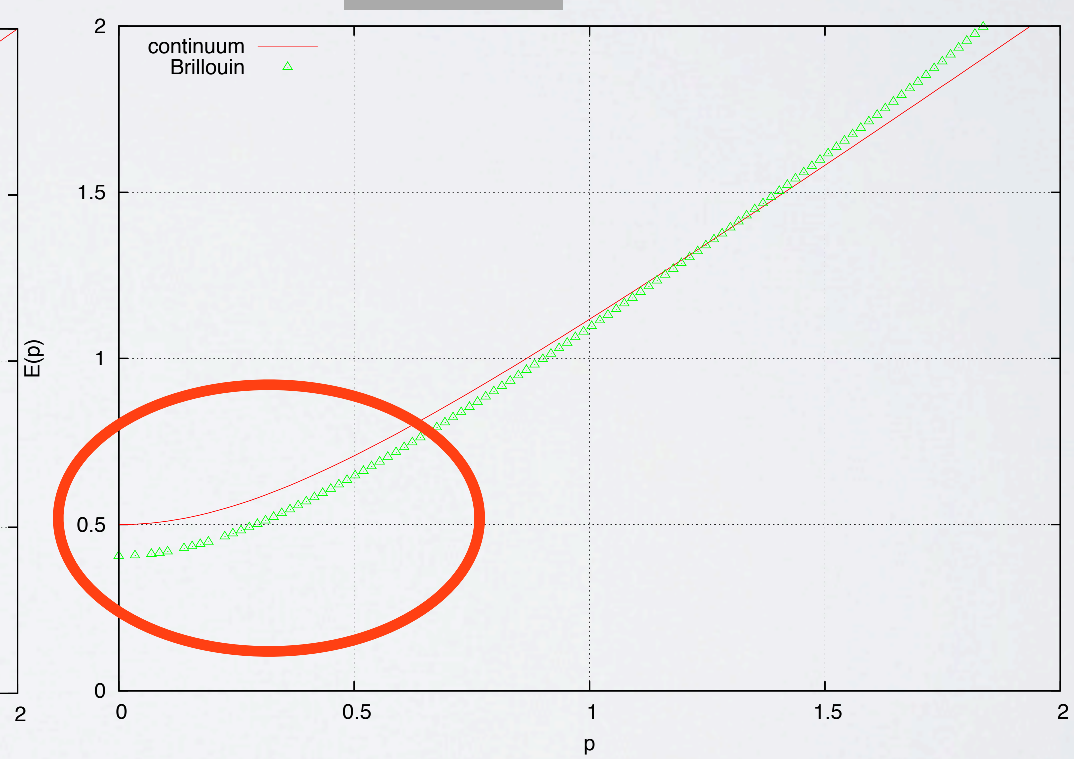
ma=0.0

E(p)



|p|

ma=0.5



=> Further improvements to decrease discretization errors of Brillouin fermions ?

O(a^2)-IMPROVED BRILLOUIN FERMIONS

- eliminate O(a^2)-errors at tree-level

- Improved Brillouin Dirac operator

$$D^{IB} = \sum_{\mu} \gamma_\mu \left(1 - \frac{a^2}{12}\triangle^{bri}\right)\nabla_\mu^{iso}\left(1 - \frac{a^2}{12}\triangle^{bri}\right) + c_{IB}a^3\left(\triangle^{bri}\right)^2 + ma$$

$$c_{IB} = 1/8$$

~~O(a),O(a^2) errors~~

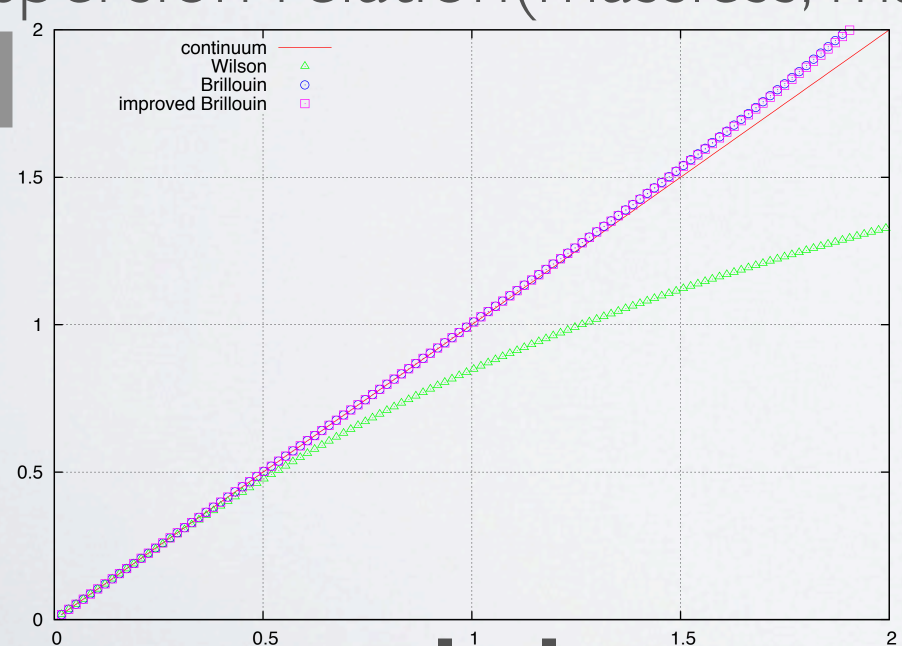
- expansion of energy up to O(a^5) $E^2\left(\vec{0}, ma\right) = (ma)^2 + \boxed{\frac{(ma)^5}{4}}$

=> start from O(a^3)

- Dispersion relation(massless, massive)

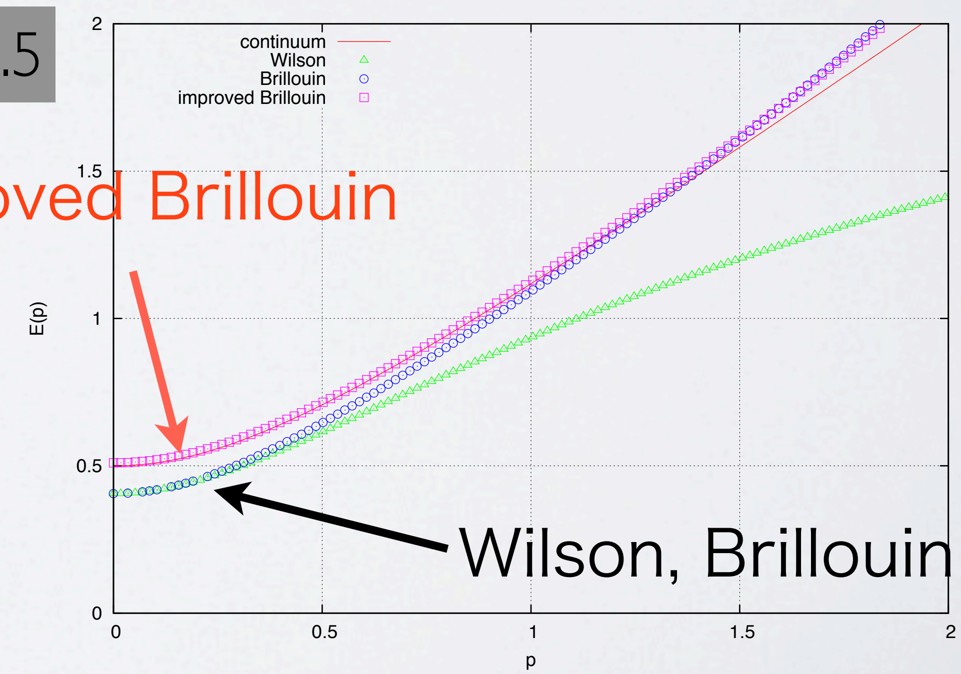
m = 0.0

E(p)

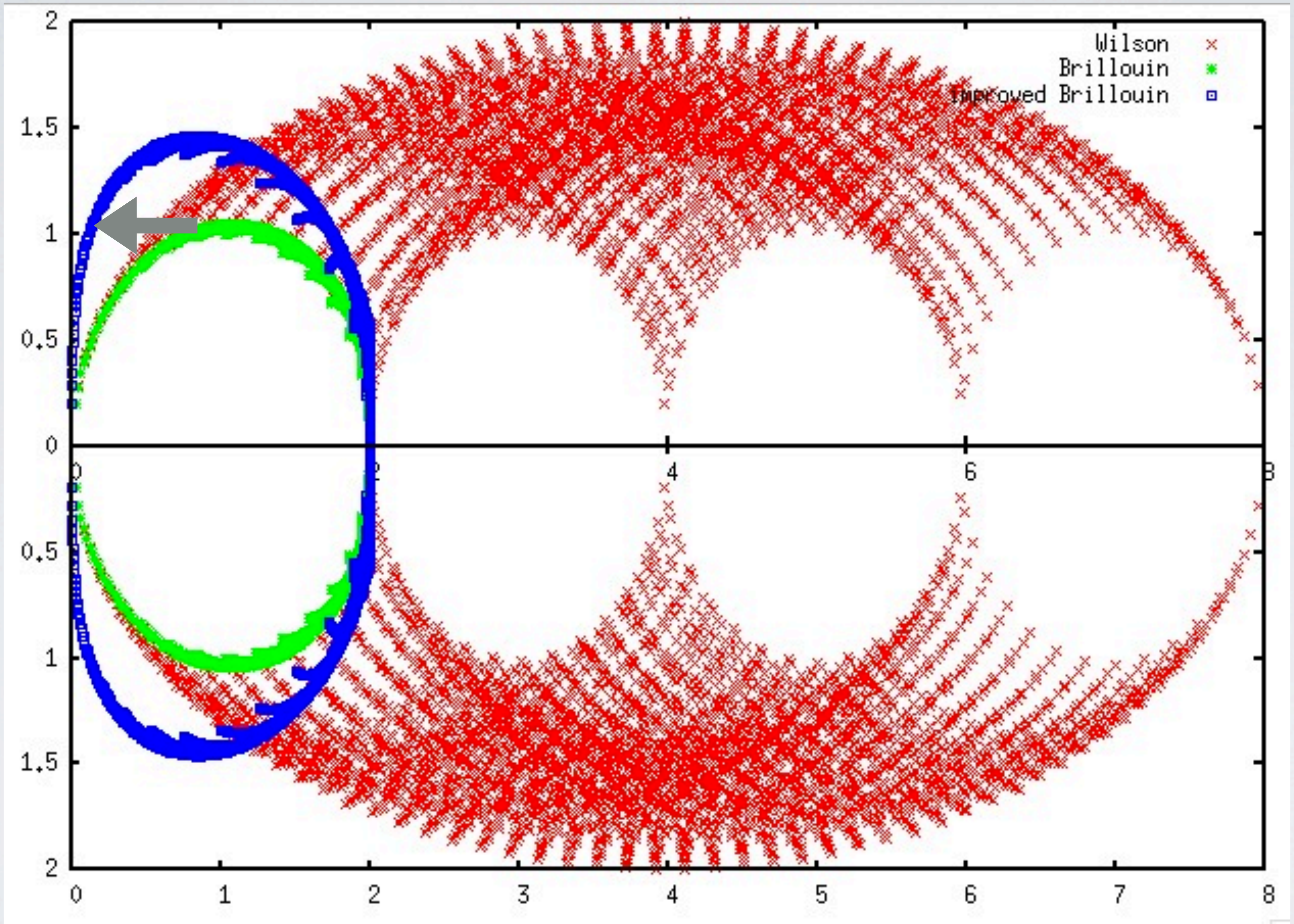


|p|

m = 0.5



EIGENVALUE SPECTRA(FREE)



Eigenvalue spectra of Improved Brillouin operator get close to the imaginary axis. (more continuum like)

3. SCALING STUDIES

- Dispersion relation, hyperfine splitting of charmonium

  ($O(a^2)$-improved Brillouin fermions vs Wilson)

- Decay constant for heavy-heavy systems

  (Domain-wall fermions (Shamir kernel))

QUENCHED CONFIGURATIONS

- Tree-level Symanzik gauge action
- Generated with CHROMA using Heat bath algorithm *(IRIDIS HPC Facility, University of Southampton)*

L/a	β	N_{conf}	w_0	a^{-1}[GeV]
16	4.41	100	1.7668(26)	1.97
24	4.66	100	2.5023(52)	2.81
32	4.80	-	-	3.39
48	4.94	-	-	5.92

- L kept fixed to ~1.6fm through the Wilson flow
  -measuring obs. w_0 introduced in [BMW-c, arXiv:1203.4469]
  -JGF code of J. Hudspith
- a^{-1} is a rough estimate, not taking into account the systematic error of quenching.

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  (O(a^2)-improved Brillouin fermions vs Wilson)

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DISPERSION RELATION, HYPERFINE SPLITTING OF CHARMONIUM

- mass tuning (set to charmonium)

1S-state spin-averaged mass

$$m_{1S} = (m_{ps} + 3m_{vec})/4 = 3.0[GeV]$$

- Dirac operator - Wilson(c_{SW}=0.0), Improved Brillouin

- effective speed of light for the pseudo scalar meson

$$c_{eff}^2\left(p^2\right) = \frac{E^2(\overrightarrow{p}) - E^2(\overrightarrow{0})}{\overrightarrow{p}^2}$$

- Hyperfine splitting

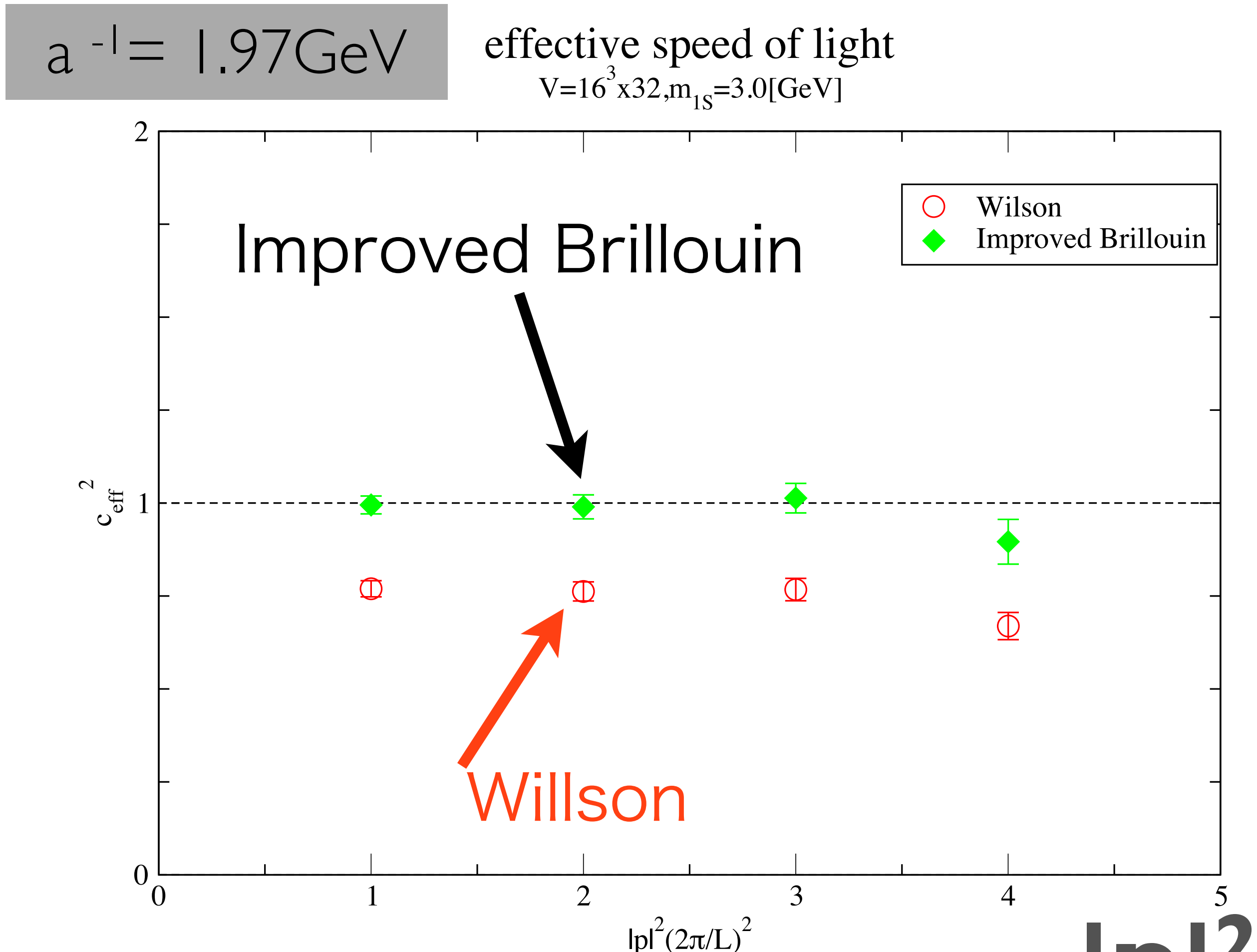
$$m_{vec} - m_{ps}$$

EFFECTIVE SPEED OF LIGHT FOR PSEUDO-SCALAR MESON

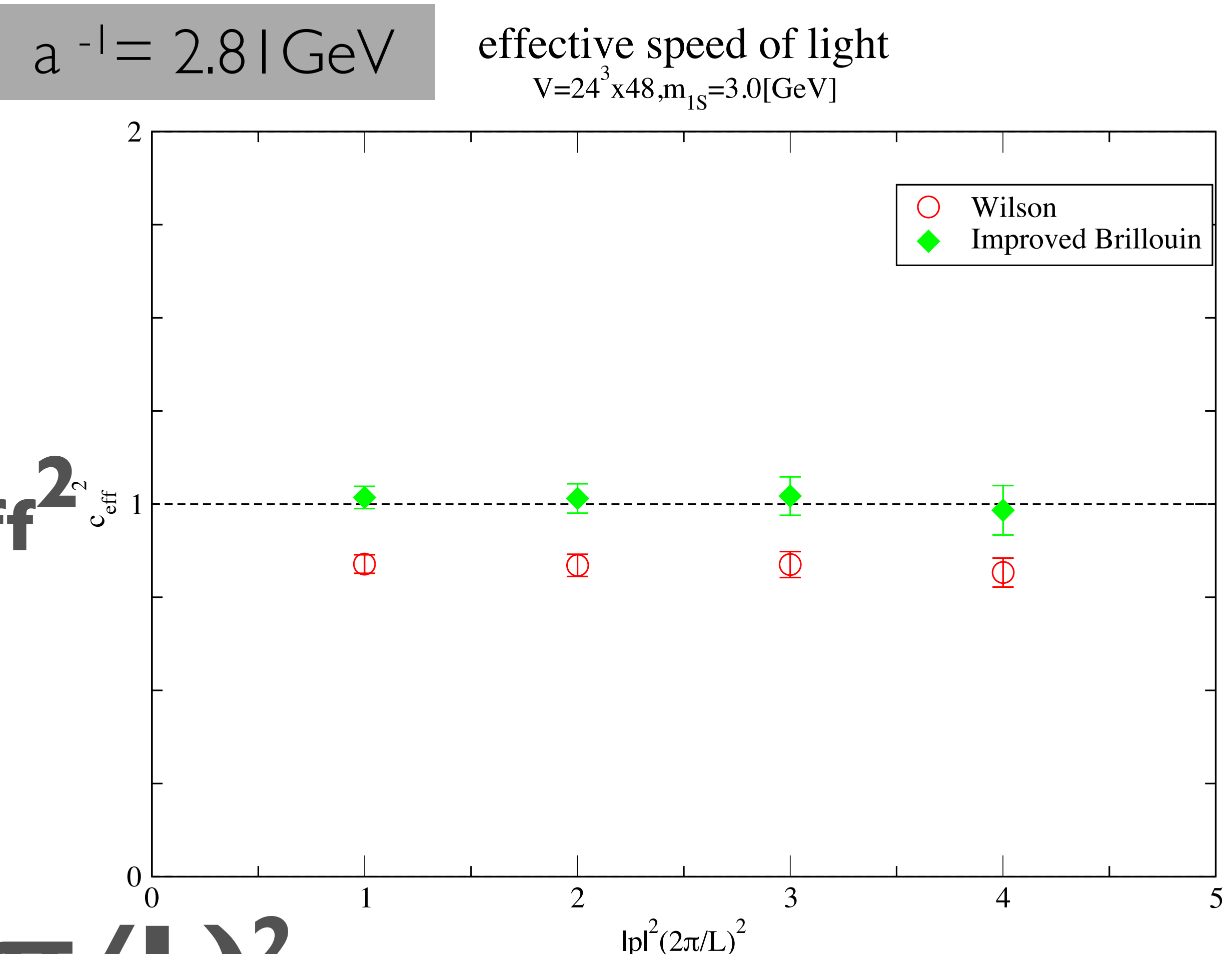
- calculate the two point correlator with momentum
- extract the energy and estimate effective speed of light

$$C\left(t, \overrightarrow{p}\right) = \sum_{\overrightarrow{x}} \left\langle J\left(\overrightarrow{x}, t\right) \bar{J}\left(\overrightarrow{0}, 0\right)\right\rangle e^{i\overrightarrow{p}\,\overrightarrow{x}} \quad \Longrightarrow \quad c_{eff}^2\left(p^2\right) = \frac{E^2(\overrightarrow{p}) - E^2(\overrightarrow{0})}{\overrightarrow{p}^{\,2}}$$

$a^{-1} = 1.97$GeV



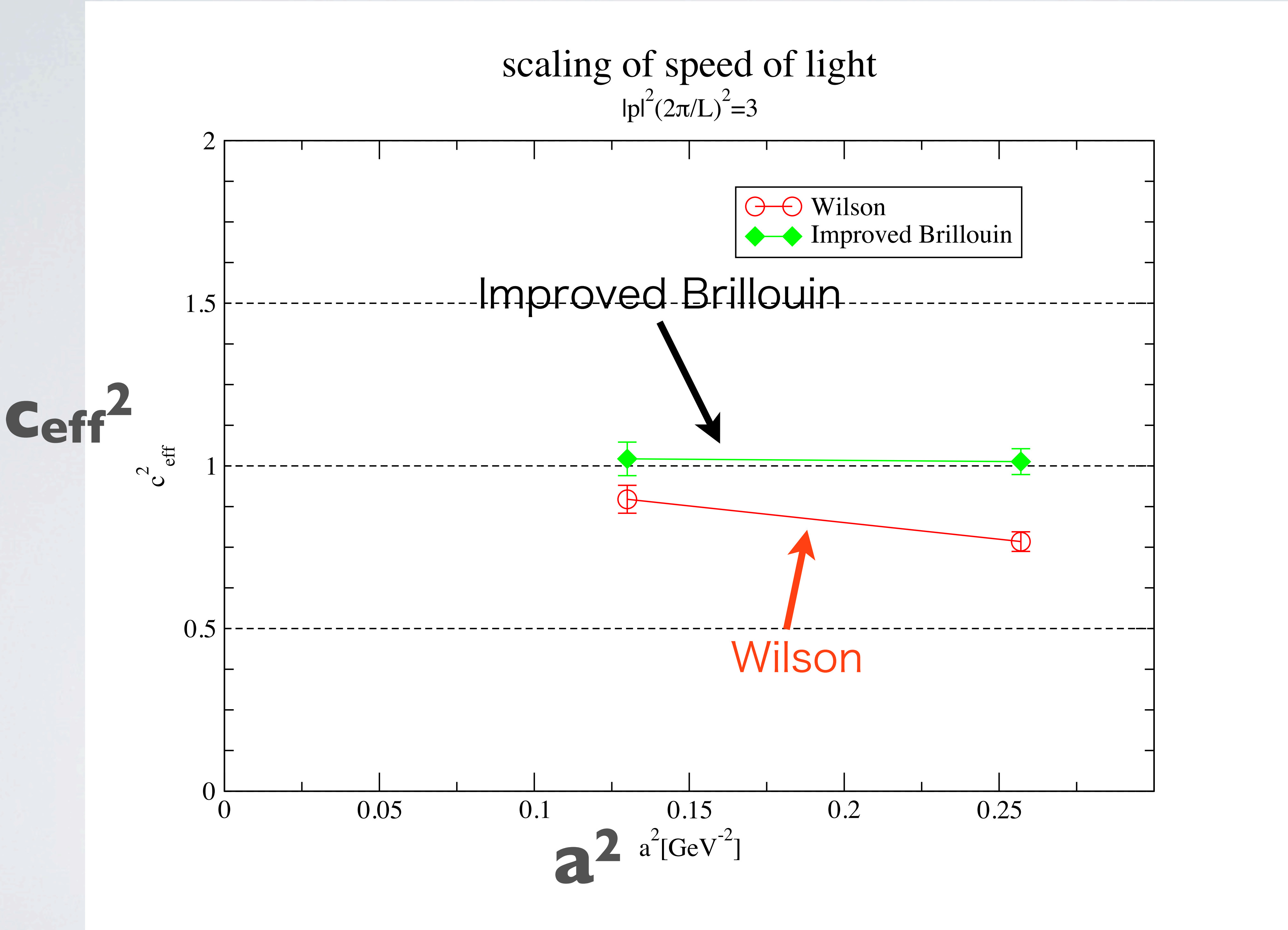
$a^{-1} = 2.81$GeV



$C_{eff}{}^2$

$|p|^2(2\pi/L)^2$

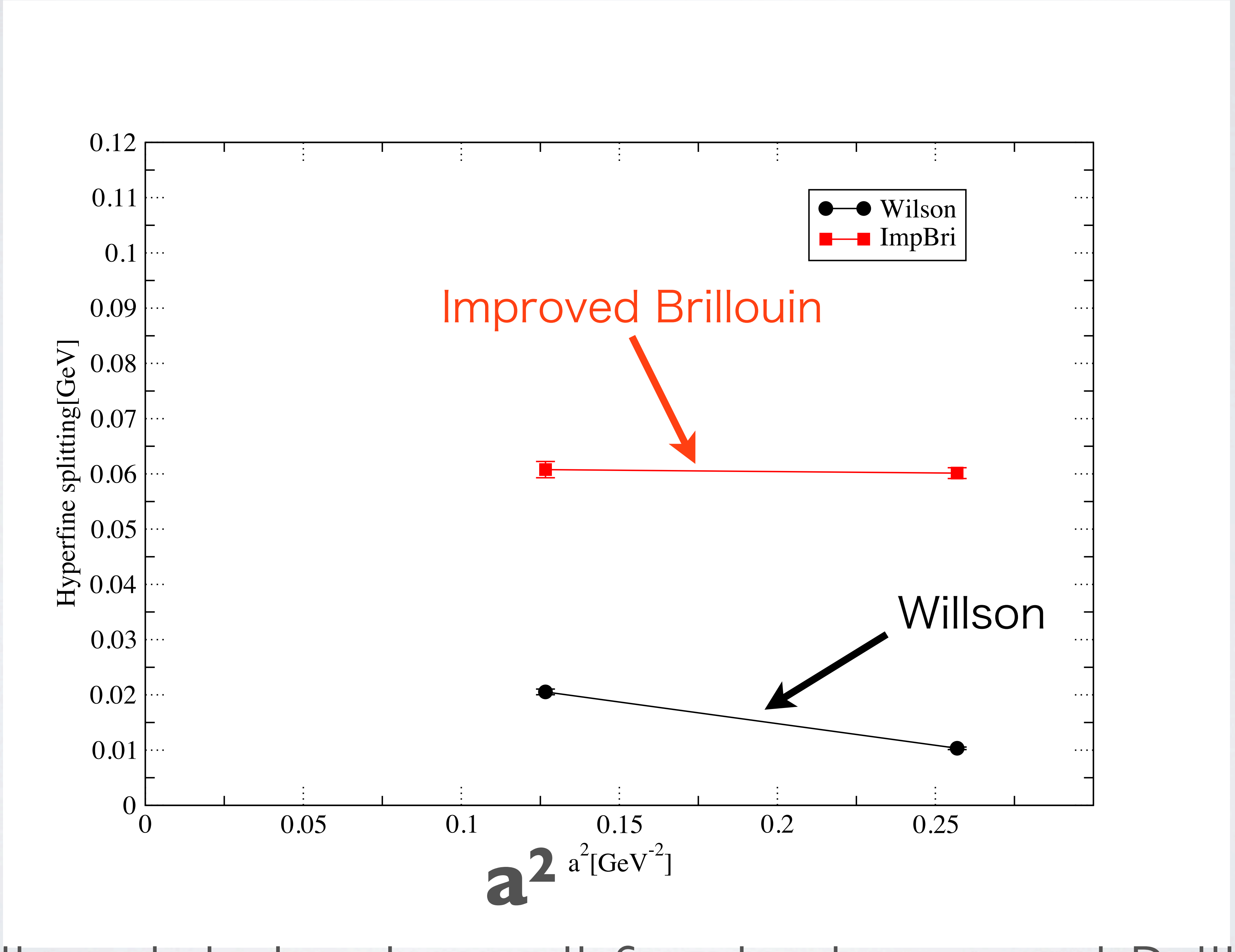
SCALING FOR SPEED OF LIGHT



scaling violation is small for the improved Brillouin.

SCALING FOR HYPERFINE SPLITTING OF 1S STATE

hyperfine splitting $= m_{vec} - m_{ps}$



scaling violation is small for the improved Brillouin.

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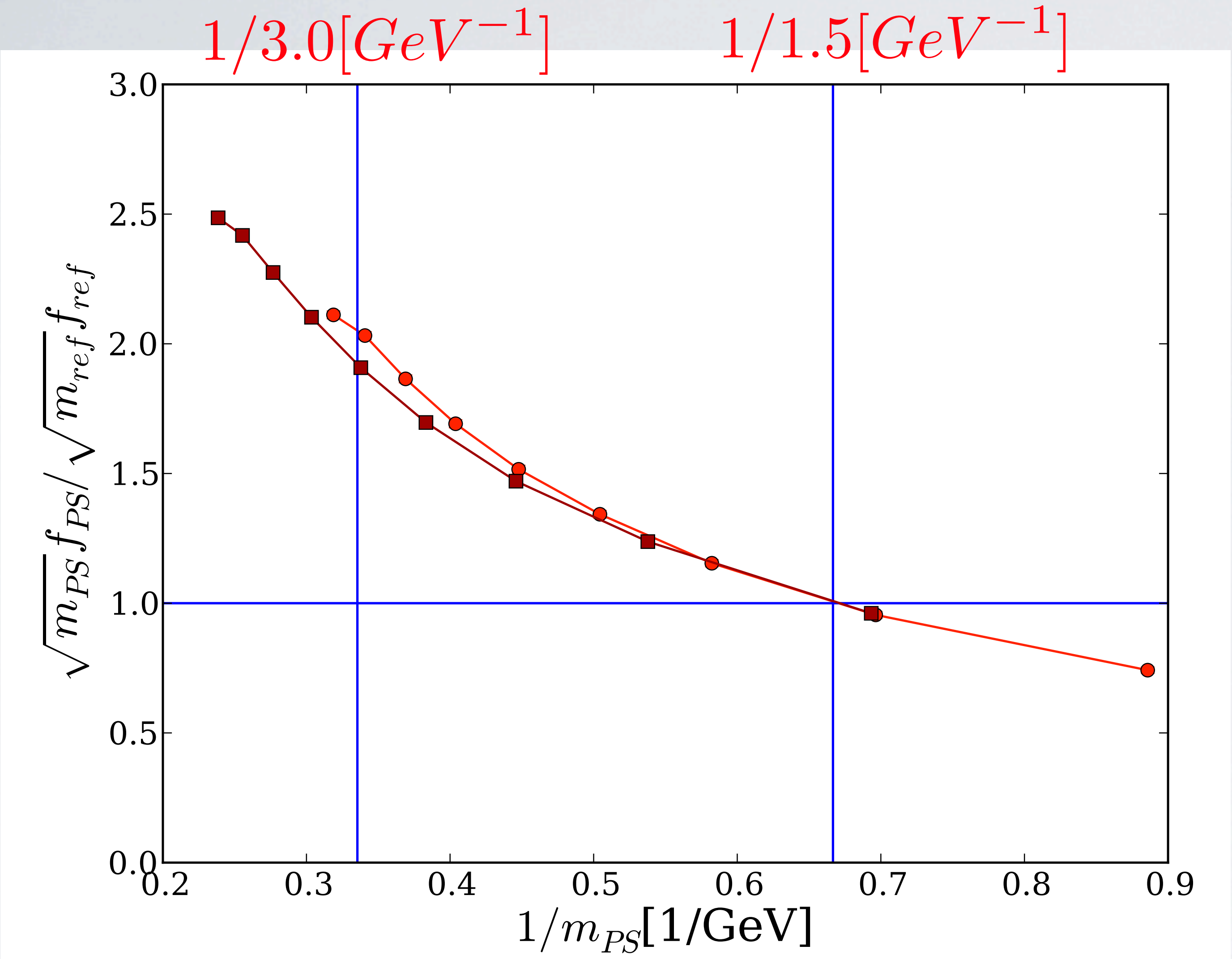
- Decay constant for heavy-heavy systems

  (Domain-wall fermions (Shamir kernel))

heavy-heavy Decay constant



- Domain-wall fermions
- Heavy quark mass varied: $am_q \in [0.1, 0.5]$
- 1/m dependence is shown with a reference point m_{ref} = 1.5 GeV.
- Data at a^{-1} = 1.97 and 2.81 GeV lattices are overlaid.

$$\frac{\sqrt{m_{PS}}f_{PS}}{\sqrt{m_{ref}}f_{ref}}$$

- Good behaviour when going towards the heavy regime, in the region $m_{PS} \in [1.5, 3]$ GeV
- 1.99 and 2.81 GeV^{-1} lattice on top of each other – indication of good scaling
- Third (finer) lattice spacing in progress

3.OUTLOOK

- scaling study for heavy quarks
  - various quantities (spectrum, decay constant, ...)
  - various formulation ($O(a^2)$-improved Brillouin fermion, domain-wall,...)
- Good scaling has so far been observed for
  - speed of light, hyperfine splitting with the $O(a^2)$-improved Brillouin fermion.
  - heavy-heavy decay constant, varying m_q, with domain-wall fermion.

Furthermore we plan to extend the studies for more (quantity x formulation) choices.

Back Up

ISOTROPIC DERIVATIVE

position space

$$
\begin{aligned}
a\nabla^{iso}_{\mu=\hat{x}}(n,m) = \frac{1}{432}[& -\delta_{n-\hat{x}-\hat{y}-\hat{z}-\hat{t},m}+\delta_{n+\hat{x}-\hat{y}-\hat{z}-\hat{t},m}-4\delta_{n-\hat{x}-\hat{z}-\hat{t},m}+4\delta_{n+\hat{x}-\hat{z}-\hat{t},m}\\
& -\delta_{n-\hat{x}+\hat{y}-\hat{z}-\hat{t},m}+\delta_{n+\hat{x}+\hat{y}-\hat{z}-\hat{t},m}-4\delta_{n-\hat{x}-\hat{y}-\hat{t},m}+4\delta_{n+\hat{x}-\hat{y}-\hat{t},m}\\
& -16\delta_{n-\hat{x}-\hat{t},m}+16\delta_{n+\hat{x}-\hat{t},m}-4\delta_{n-\hat{x}+\hat{y}-\hat{t},m}+4\delta_{n+\hat{x}+\hat{y}-\hat{t},m}\\
& -\delta_{n-\hat{x}-\hat{y}+\hat{z}-\hat{t},m}+\delta_{n+\hat{x}-\hat{y}+\hat{z}-\hat{t},m}-4\delta_{n-\hat{x}+\hat{z}-\hat{t},m}+4\delta_{n+\hat{x}+\hat{z}-\hat{t},m}\\
& -\delta_{n-\hat{x}+\hat{y}+\hat{z}-\hat{t},m}+\delta_{n+\hat{x}+\hat{y}+\hat{z}-\hat{t},m}-4\delta_{n-\hat{x}-\hat{y}-\hat{z},m}+4\delta_{n+\hat{x}-\hat{y}-\hat{z},m}\\
& -16\delta_{n-\hat{x}-\hat{z},m}+16\delta_{n+\hat{x}-\hat{z},m}-4\delta_{n-\hat{x}+\hat{y}-\hat{z},m}+4\delta_{n+\hat{x}+\hat{y}-\hat{z},m}\\
& -16\delta_{n-\hat{x}-\hat{y},m}+16\delta_{n+\hat{x}-\hat{y},m}-64\delta_{n-\hat{x},m}+64\delta_{n+\hat{x},m}\\
& -16\delta_{n-\hat{x}+\hat{y},m}+16\delta_{n+\hat{x}+\hat{y},m}-4\delta_{n-\hat{x}-\hat{y}+\hat{z},m}+4\delta_{n+\hat{x}-\hat{y}+\hat{z},m}\\
& -16\delta_{n-\hat{x}+\hat{z},m}+16\delta_{n+\hat{x}+\hat{z},m}-4\delta_{n-\hat{x}+\hat{y}+\hat{z},m}+4\delta_{n+\hat{x}+\hat{y}+\hat{z},m}\\
& -\delta_{n-\hat{x}-\hat{y}-\hat{z}+\hat{t},m}+\delta_{n+\hat{x}-\hat{y}-\hat{z}+\hat{t},m}-4\delta_{n-\hat{x}-\hat{z}+\hat{t},m}+4\delta_{n+\hat{x}-\hat{z}+\hat{t},m}\\
& -\delta_{n-\hat{x}+\hat{y}-\hat{z}+\hat{t},m}+\delta_{n+\hat{x}+\hat{y}-\hat{z}+\hat{t},m}-4\delta_{n-\hat{x}-\hat{y}+\hat{t},m}+4\delta_{n+\hat{x}-\hat{y}+\hat{t},m}\\
& -16\delta_{n-\hat{x}+\hat{t},m}+16\delta_{n+\hat{x}+\hat{t},m}-4\delta_{n-\hat{x}+\hat{y}+\hat{t},m}+4\delta_{n+\hat{x}+\hat{y}+\hat{t},m}\\
& -\delta_{n-\hat{x}-\hat{y}+\hat{z}+\hat{t},m}+\delta_{n+\hat{x}-\hat{y}+\hat{z}+\hat{t},m}-4\delta_{n-\hat{x}+\hat{z}+\hat{t},m}+4\delta_{n+\hat{x}+\hat{z}+\hat{t},m}\\
& -\delta_{n-\hat{x}+\hat{y}+\hat{z}+\hat{t},m}+\delta_{n+\hat{x}+\hat{y}+\hat{z}+\hat{t},m}]
\end{aligned}
$$

momentum space

$$
\nabla^{iso}_{\mu=\hat{x}}(p) = isinp_x(cosp_y+2)(cosp_z+2)(cosp_t+2)/27
$$

BRILLOUIN LAPLACIAN

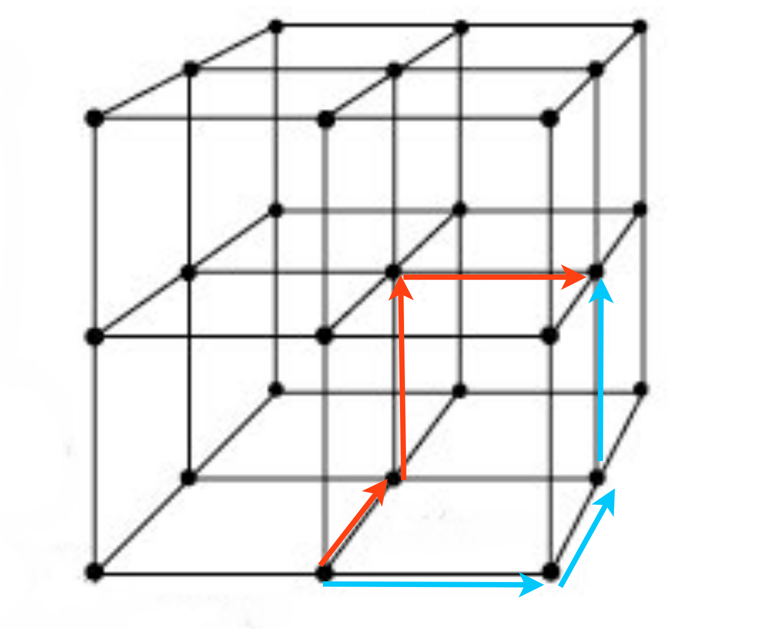
position space

$$
\begin{aligned}
a^2 \triangle^{bri}(n,m) = \frac{1}{64} [\ & \delta_{n-\hat{x}-\hat{y}-\hat{z}-\hat{t},m} + 2\delta_{n-\hat{y}-\hat{z}-\hat{t},m} + \delta_{n+\hat{x}-\hat{y}-\hat{z}-\hat{t},m} + 2\delta_{n-\hat{x}-\hat{z}-\hat{t},m} + 4\delta_{n-\hat{z}-\hat{t},m} + 2\delta_{n+\hat{x}-\hat{z}-\hat{t},n} \\
& + \delta_{n-\hat{x}+\hat{y}-\hat{z}-\hat{t},m} + 2\delta_{n+\hat{y}-\hat{z}-\hat{t},m} + \delta_{n+\hat{x}+\hat{y}-\hat{z}-\hat{t},m} + 2\delta_{n-\hat{x}-\hat{y}-\hat{t},m} + 4\delta_{n-\hat{y}-\hat{t},m} + 2\delta_{n+\hat{x}-\hat{y}-} \\
& + 4\delta_{n-\hat{x}-\hat{t},m} + 8\delta_{n-\hat{t},m} + 4\delta_{n+\hat{x}-\hat{t},m} + 2\delta_{n-\hat{x}+\hat{y}-\hat{t},m} + 4\delta_{n+\hat{y}-\hat{t},m} + 2\delta_{n+\hat{x}+\hat{y}-\hat{t},m} \\
& + \delta_{n-\hat{x}-\hat{y}+\hat{z}-\hat{t},m} + 2\delta_{n-\hat{y}+\hat{z}-\hat{t},m} + \delta_{n+\hat{x}-\hat{y}+\hat{z}-\hat{t},m} + 2\delta_{n-\hat{x}+\hat{z}-\hat{t},m} + 4\delta_{n+\hat{z}-\hat{t},m} + 2\delta_{n+\hat{x}+\hat{z}-\hat{t}} \\
& + \delta_{n-\hat{x}+\hat{y}+\hat{z}-\hat{t},m} + 2\delta_{n+\hat{y}+\hat{z}-\hat{t},m} + \delta_{n+\hat{x}+\hat{y}+\hat{z}-\hat{t},m} + 2\delta_{n-\hat{x}-\hat{y}-\hat{z},m} + 4\delta_{n-\hat{y}-\hat{z},m} + 2\delta_{n+\hat{x}-\hat{y}-} \\
& + 4\delta_{n-\hat{x}-\hat{z},m} + 8\delta_{n-\hat{z},m} + 4\delta_{n+\hat{x}-\hat{z},m} + 2\delta_{n-\hat{x}+\hat{y}-\hat{z},m} + 4\delta_{n+\hat{y}-\hat{z},m} + 2\delta_{n+\hat{x}+\hat{y}-\hat{z},m} \\
& + 4\delta_{n-\hat{x}-\hat{y},m} + 8\delta_{n-\hat{y},m} + 4\delta_{n+\hat{x}-\hat{y},m} + 8\delta_{n-\hat{x},m} - 240\delta_{n,m} + 8\delta_{n+\hat{x},m} \\
& + 4\delta_{n-\hat{x}+\hat{y},m} + 8\delta_{n+\hat{y},m} + 4\delta_{n+\hat{x}+\hat{y},m} + 2\delta_{n-\hat{x}-\hat{y}+\hat{z},m} + 4\delta_{n-\hat{y}+\hat{z},m} + 2\delta_{n+\hat{x}-\hat{y}+\hat{z},m} \\
& + 4\delta_{n-\hat{x}+\hat{z},m} + 8\delta_{n+\hat{z},m} + 4\delta_{n+\hat{x}+\hat{z},m} + 2\delta_{n-\hat{x}+\hat{y}+\hat{z},m} + 4\delta_{n+\hat{y}+\hat{z},m} + 2\delta_{n+\hat{x}+\hat{y}+\hat{z},m} \\
& + \delta_{n-\hat{x}-\hat{y}-\hat{z}+\hat{t},m} + 2\delta_{n-\hat{y}-\hat{z}+\hat{t},m} + \delta_{n+\hat{x}-\hat{y}-\hat{z}+\hat{t},m} + 2\delta_{n-\hat{x}-\hat{z}+\hat{t},m} + 4\delta_{n-\hat{z}+\hat{t},m} + 2\delta_{n+\hat{x}-\hat{z}+\hat{t}} \\
& + \delta_{n-\hat{x}+\hat{y}-\hat{z}+\hat{t},m} + 2\delta_{n+\hat{y}-\hat{z}+\hat{t},m} + \delta_{n+\hat{x}+\hat{y}-\hat{z}+\hat{t},m} + 2\delta_{n-\hat{x}-\hat{y}+\hat{t},m} + 4\delta_{n-\hat{y}+\hat{t},m} + 2\delta_{n+\hat{x}-\hat{y}+\hat{t}} \\
& + 4\delta_{n-\hat{x}+\hat{t},m} + 8\delta_{n+\hat{t},m} + 4\delta_{n+\hat{x}+\hat{t},m} + 2\delta_{n-\hat{x}+\hat{y}+\hat{t},m} + 4\delta_{n+\hat{y}+\hat{t},m} + 2\delta_{n+\hat{x}+\hat{y}+\hat{t},m} \\
& + \delta_{n-\hat{x}-\hat{y}+\hat{z}+\hat{t},m} + 2\delta_{n-\hat{y}+\hat{z}+\hat{t},m} + \delta_{n+\hat{x}-\hat{y}+\hat{z}+\hat{t},m} + 2\delta_{n-\hat{x}+\hat{z}+\hat{t},m} + 4\delta_{n+\hat{z}+\hat{t},m} + 2\delta_{n+\hat{x}+\hat{z}+\hat{t}} \\
& + \delta_{n-\hat{x}+\hat{y}+\hat{z}+\hat{t},m} + 2\delta_{n+\hat{y}+\hat{z}+\hat{t},m} + \delta_{n+\hat{x}+\hat{y}+\hat{z}+\hat{t},m}\ ]
\end{aligned}
$$

momentum space

$$\triangle^{bri}(p) = 4cos^2\left(p_x/2\right) cos^2\left(p_y/2\right) cos^2\left(p_z/2\right) cos^2\left(p_t/2\right) - 4$$

THE BRILLOUIN OPERATOR WITH GAUGE FIELDS



- take a average of all paths for every hopping term
- recursion algorithm of standard derivative and laplacian

$$a\triangle^{bri}(n,m)\,\psi_m = \frac{1}{64}\sum_{\mu} D_{\mu}^{+}\psi_n''' - \frac{15}{4}\psi_n$$

$$\psi_n''' \equiv 8\psi_n + \frac{1}{2}\sum_{\nu\neq\mu} D_{\nu}^{+}\psi_n''$$

$$\psi_n'' \equiv 4\psi_n + \frac{1}{3}\sum_{\rho\neq\mu,\nu} D_{\rho}^{+}\psi_n'$$

$$\psi_n' \equiv 2\psi_n + \frac{1}{4}\sum_{\sigma\neq\mu,\nu,\rho} D_{\sigma}^{+}\psi_n$$

$$\nabla_x^{iso}(n,m)\,\psi_m = \frac{1}{432}\left(D_x^{-}\xi_n''' + \frac{1}{2}\sum_{\nu\neq x} D_{\nu}^{+}\eta_n'''\right)$$

$$\xi_n''' \equiv 64\psi_n + \frac{1}{2}\sum_{\nu\neq x} D_{\nu}^{+}\xi_n'' \qquad \eta_n''' \equiv D_x^{-}\xi_n'' + \frac{1}{3}\sum_{\rho\neq x,\nu} D_{\rho}^{+}\eta_n''$$

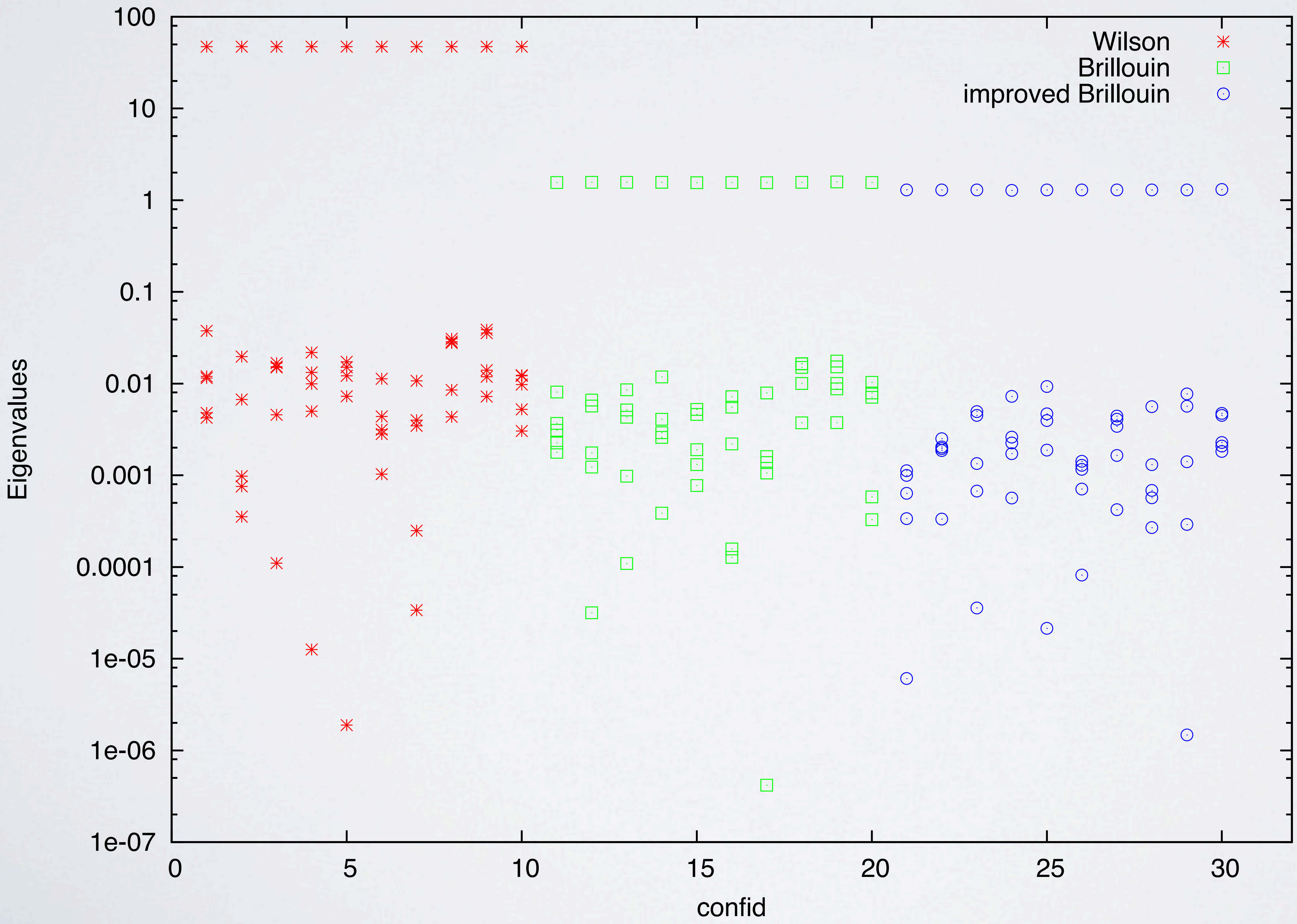
$$\xi_n'' \equiv 16\psi_n + \frac{1}{3}\sum_{\rho\neq x,\nu} D_{\rho}^{+}\xi_n' \qquad \eta_n'' \equiv D_x^{-}\xi_n' + \frac{1}{4}\sum_{\sigma\neq x,\nu,\rho} D_{\sigma}^{+}\eta_n'$$

$$\xi_n' \equiv 4\psi_n + \frac{1}{4}\sum_{\sigma\neq x,\nu,\rho} D_{\sigma}^{+}\psi_n \qquad \eta_n' \equiv D_x^{-}\psi_n$$

$$\underline{D_{\mu}^{\pm} = U_{\mu}(n)\,\psi_{n+\hat{\mu}}''' \pm U_{\mu}^{\dagger}(n-\hat{\mu})\,\psi_{n-\hat{\mu}}'''}$$

EIGENVALUES ON NON-TRIVIAL GAUGE CONFIGURATIONS

=>highest mode and lowest modes(5) of $D^\dagger D$



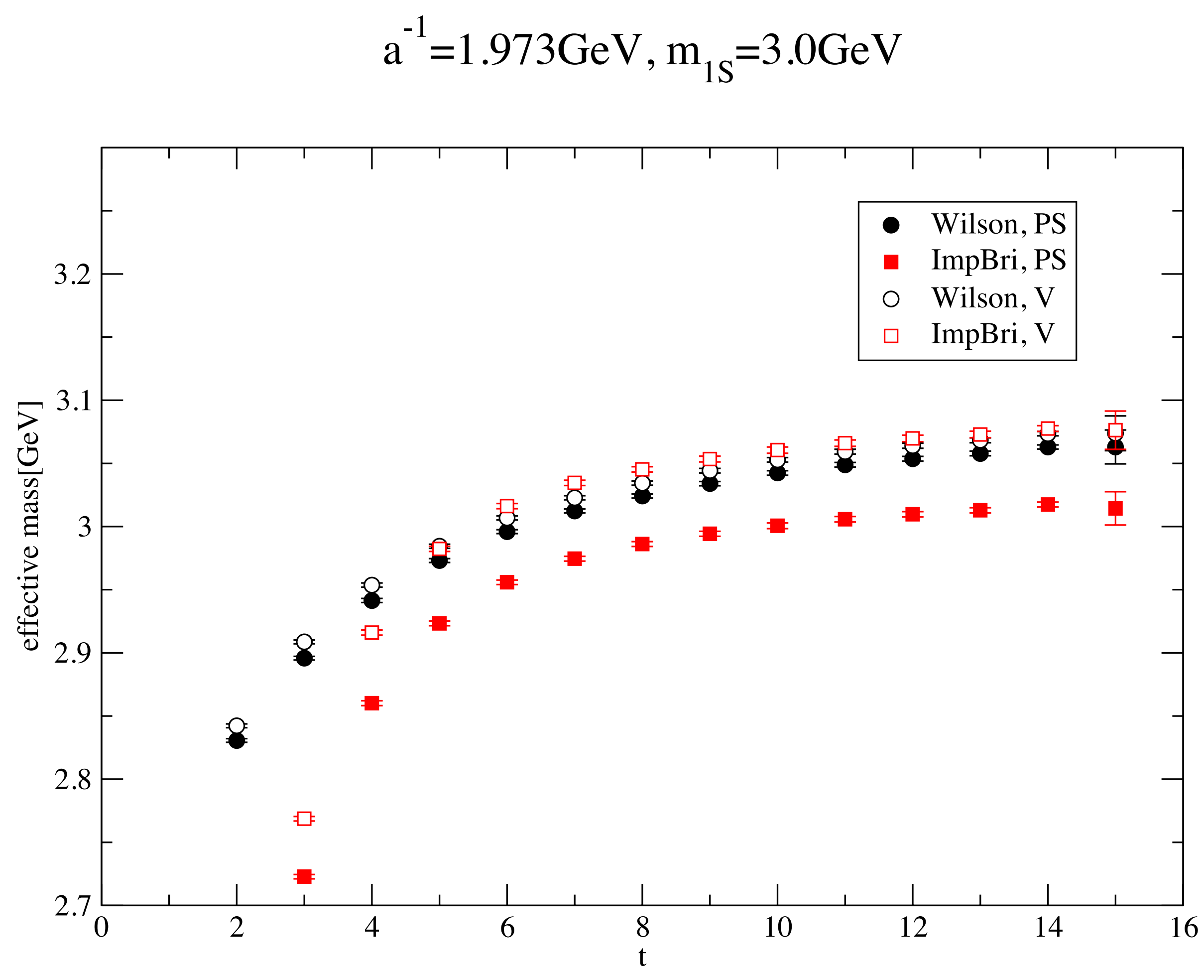
Nsmear : 3

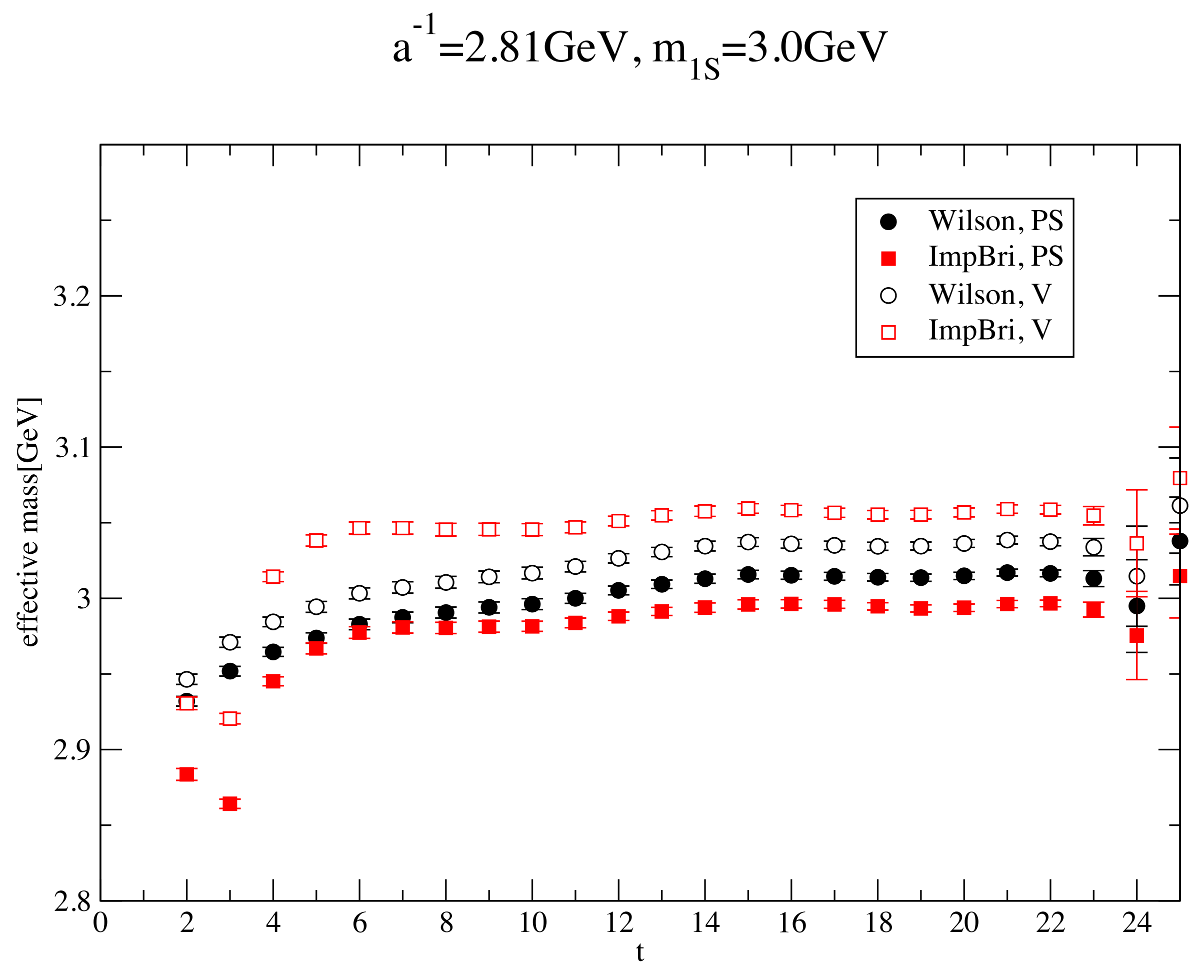
mass : -1.0

a^{-1} : 2.038(GeV)

Volume=16^3x32

EFFECTIVE MASS FOR PSEUDO SCALAR, VECTOR

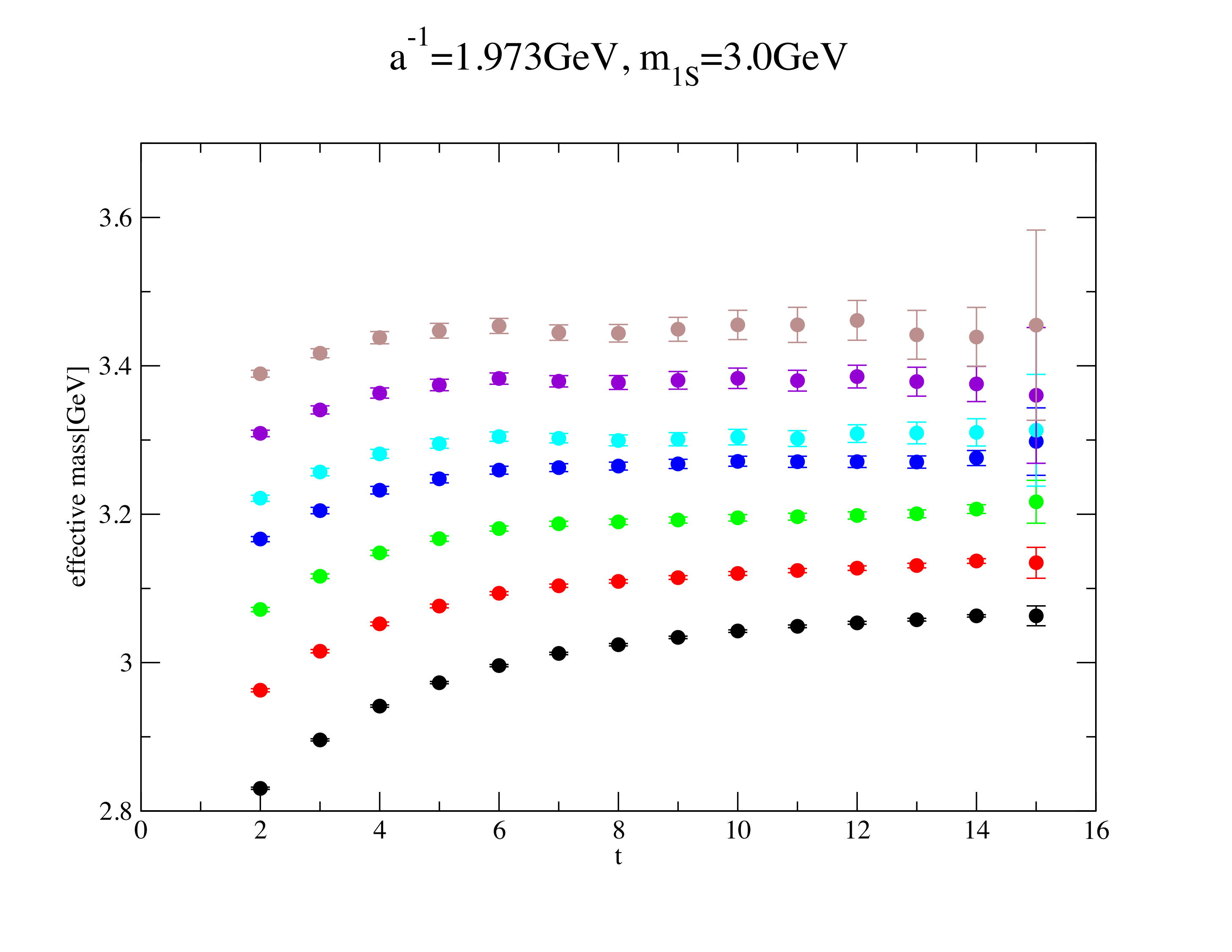




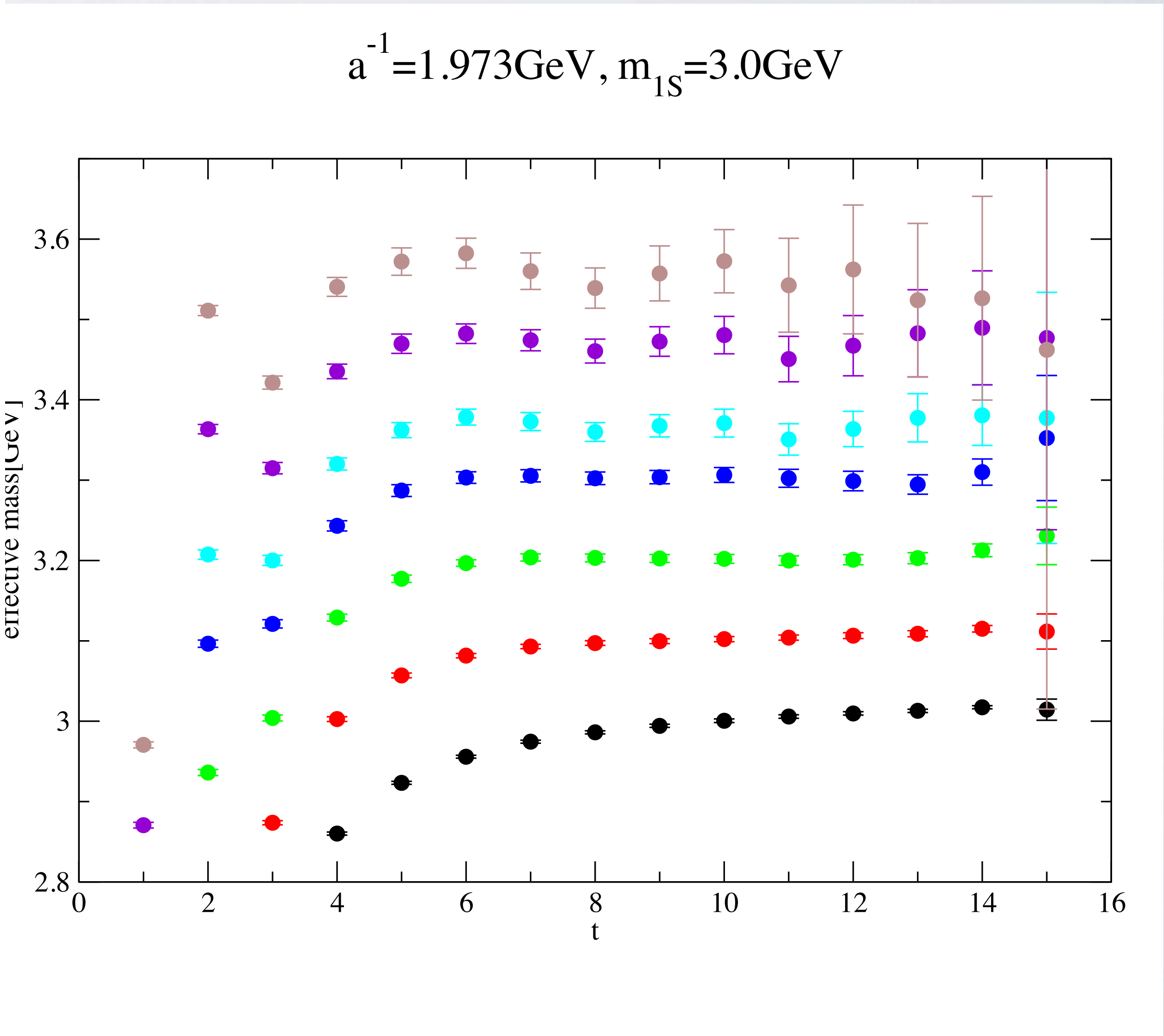
EFFECTIVE MASS WITH MOMENTUM FOR PSEUDO SCALAR PARTICLE

a^{-1}=1.973GeV

Wilson



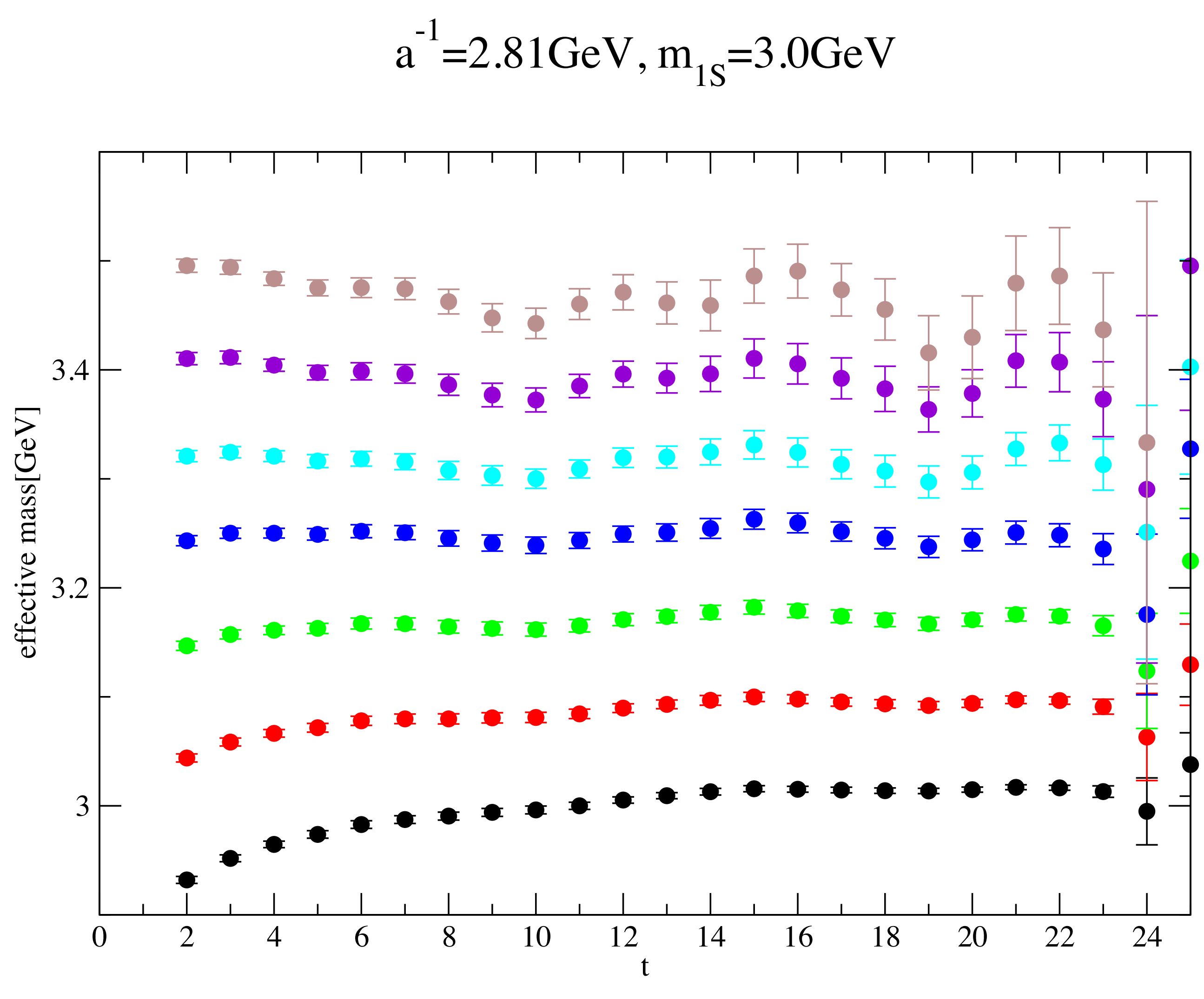
Improved Brillouin



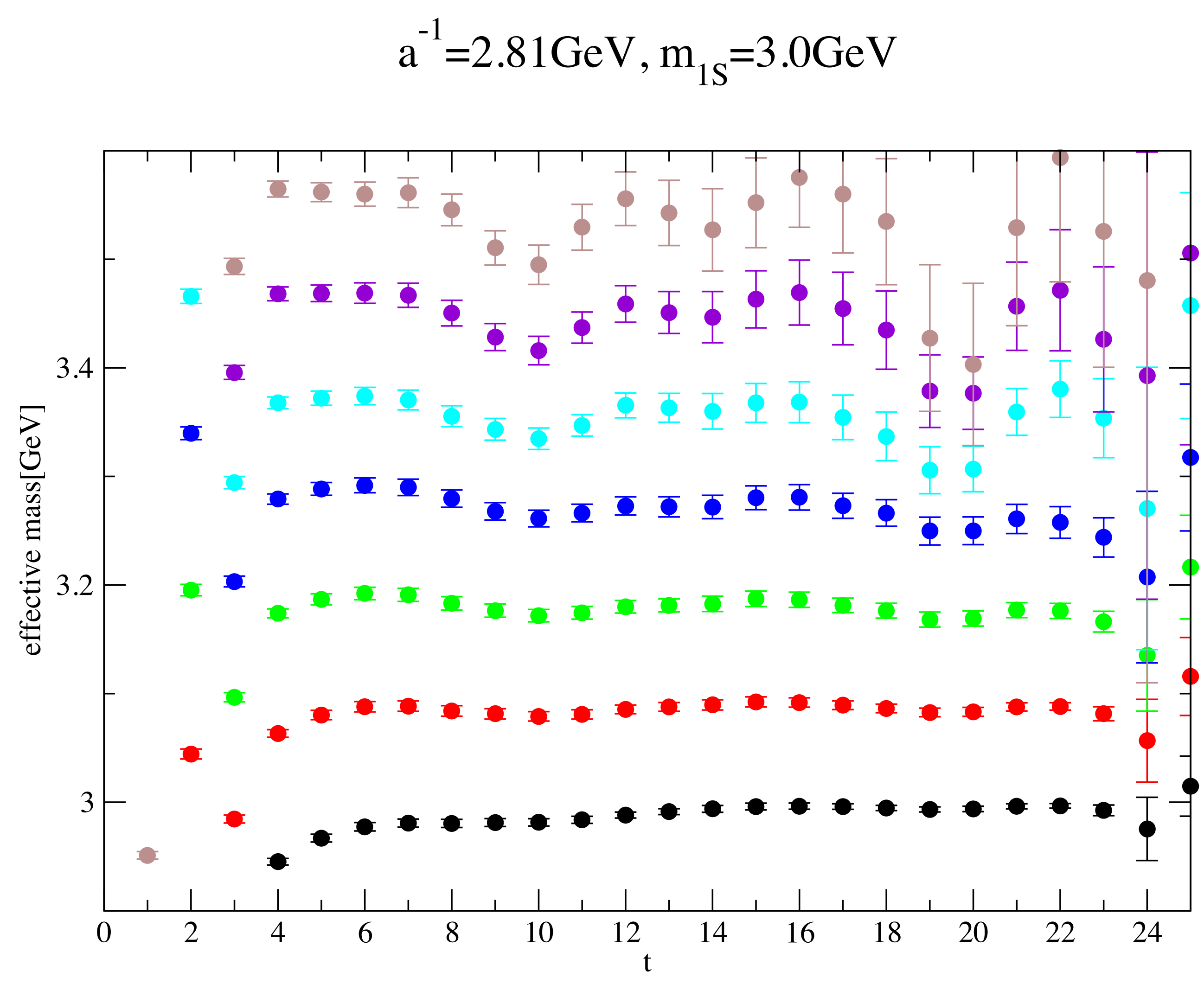
EFFECTIVE MASS WITH MOMENTUM FOR PSEUDO SCALAR PARTICLE

a^{-1}=2.81GeV

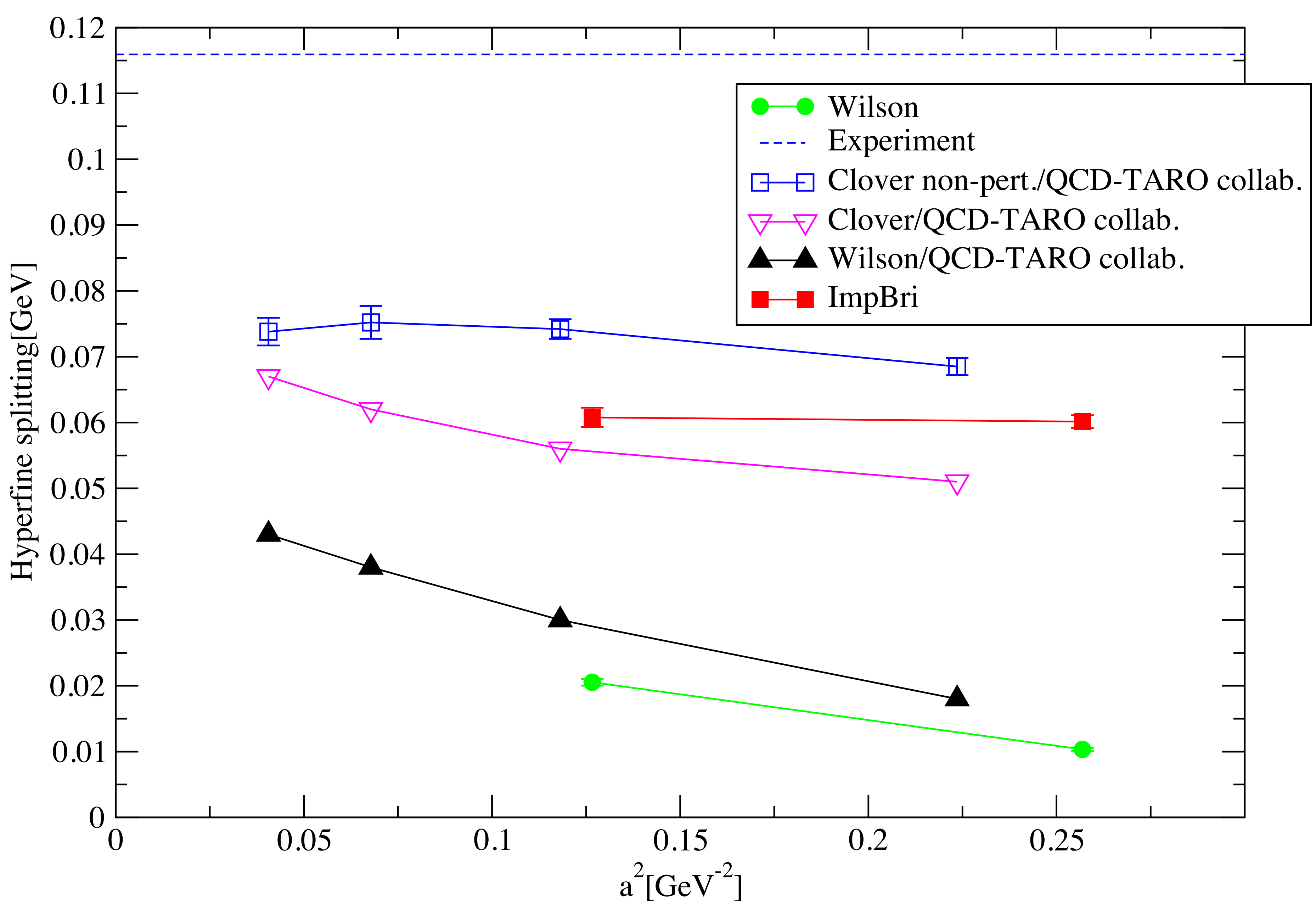
Wilson



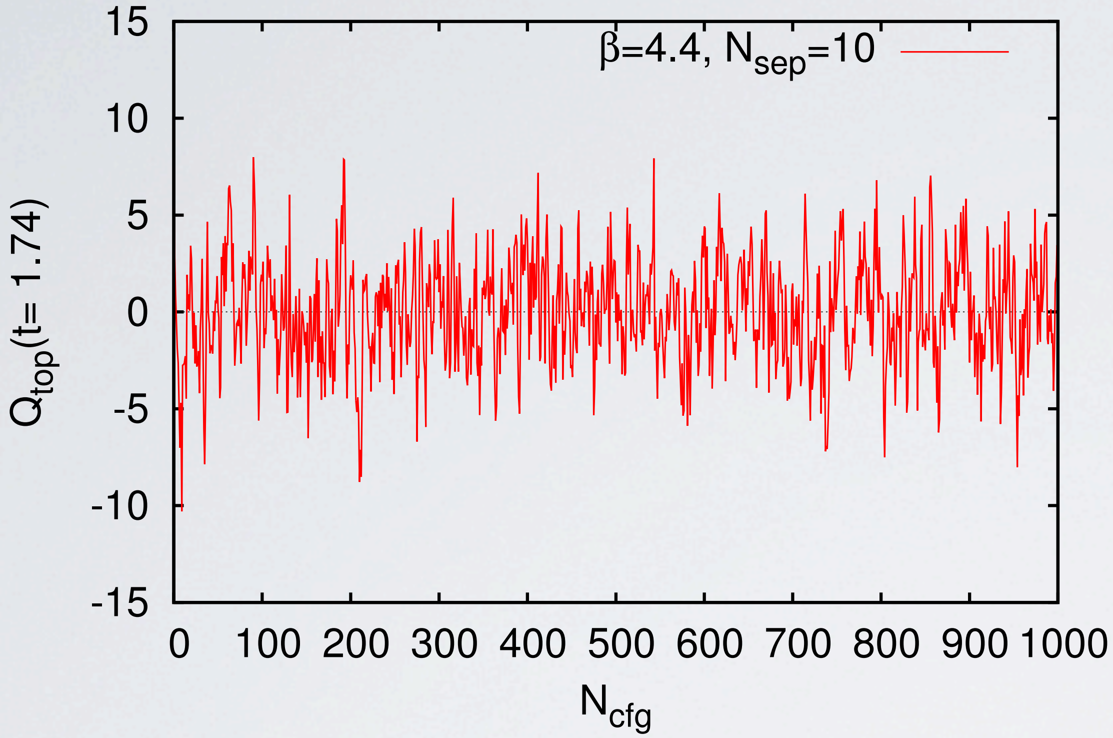
Improved Brillouin

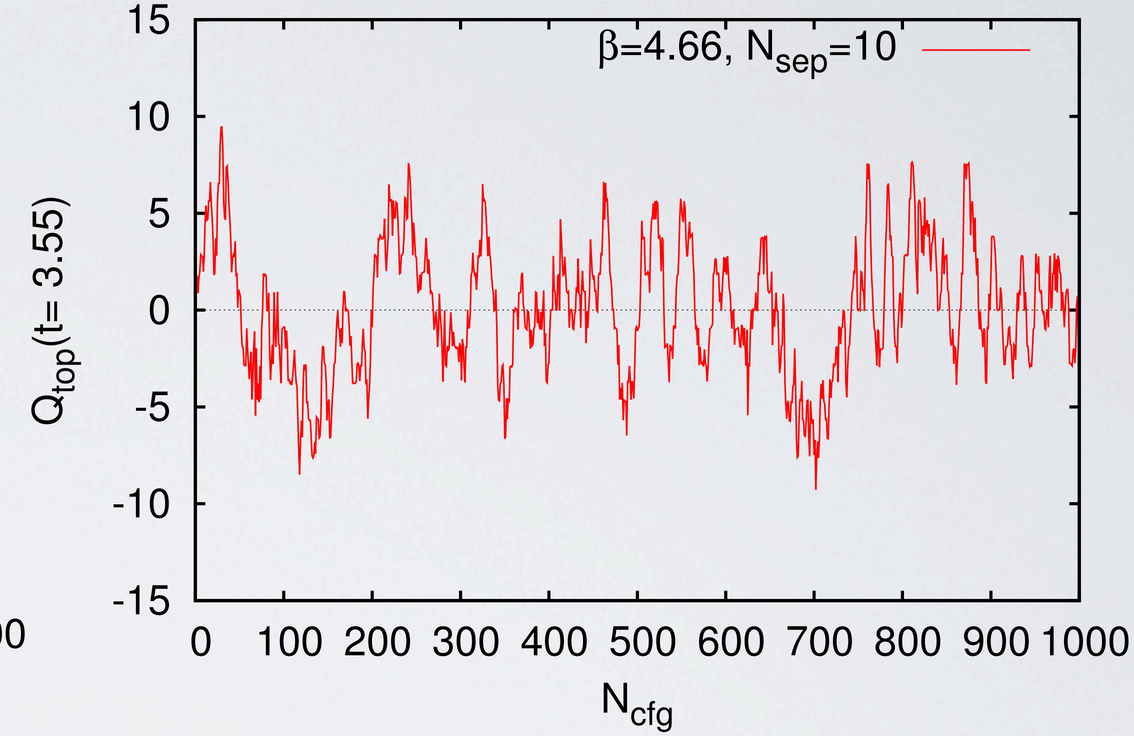


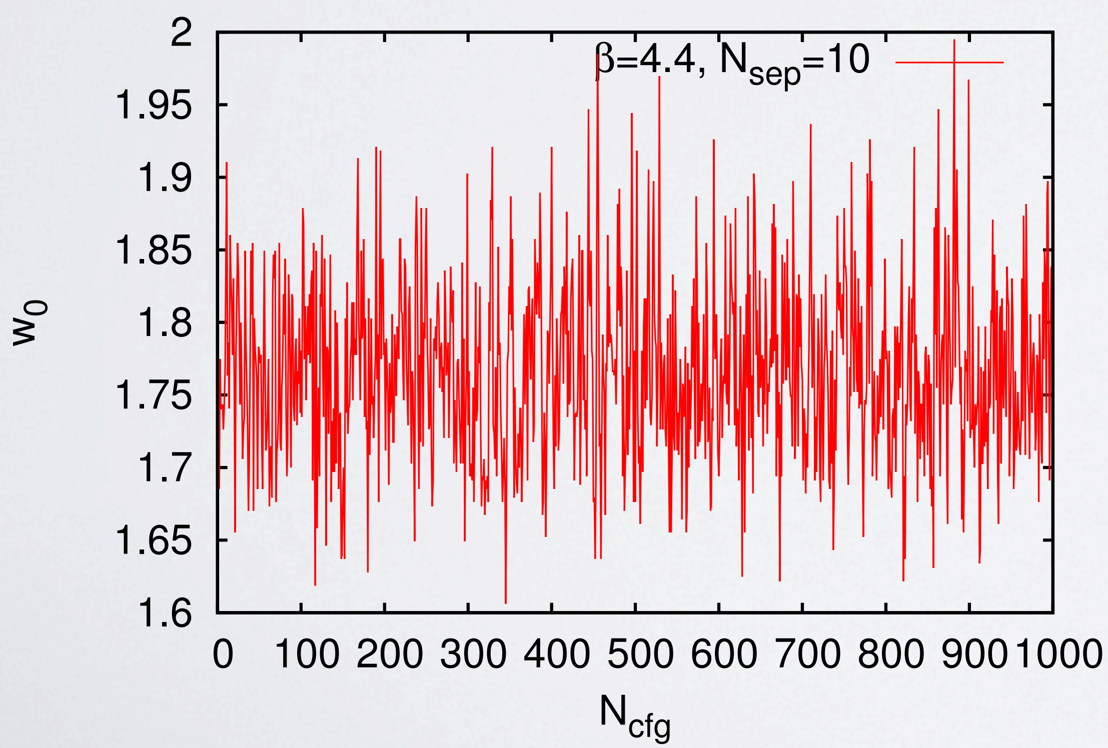
HYPERFINE SPLITTING OF 1S STATE WITH QCD-TARO DATA

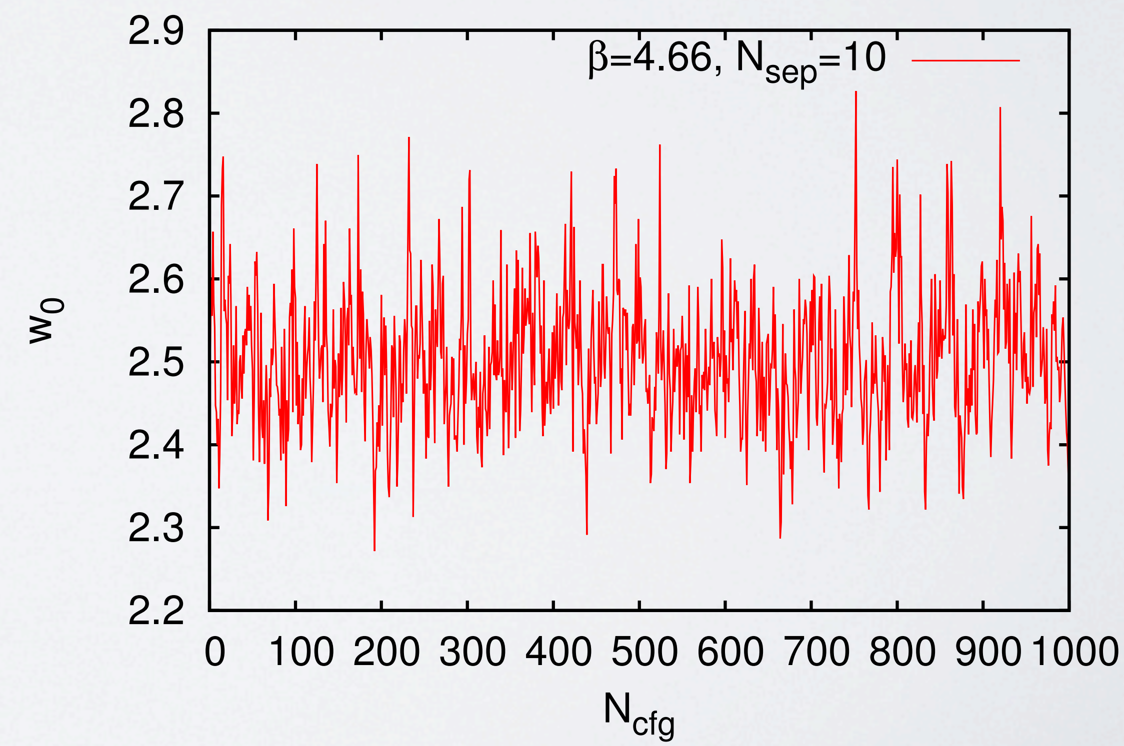


Topological charge and w_0 evolutions

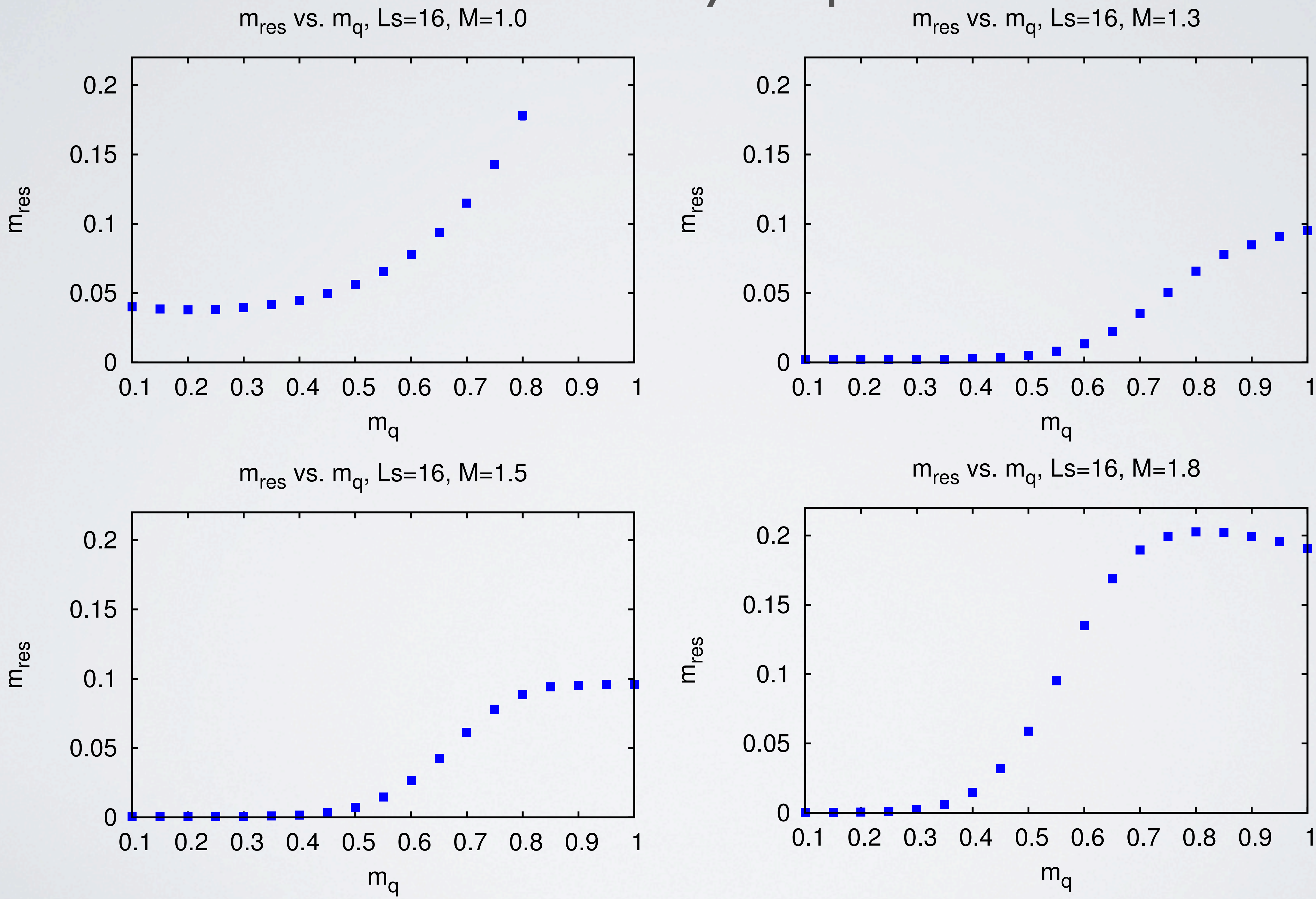








Loss of chiral symmetry for heavy quarks



- m_{res} - residual chiral symmetry breaking effects