Pseudoscalar flavor-singlet mixing angle and decay constants from $N_f = 2+1+1$ WtmLQCD

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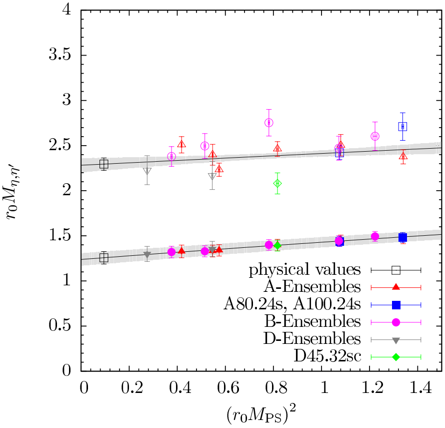
Outline

- Previous talk focused on masses; this talk on η,η' mixing
- Lattice setup
- Definition of mixing parameters
- Mixing angle(s)
- Decay constants
- Quark mass dependence, extrapolations
- Decay widths $\Gamma(\eta \to \gamma\gamma)$, $\Gamma(\eta' \to \gamma\gamma)$



Results for are still preliminary!

η,η' on the lattice

We work in the Wilson twisted mass $N_f = 2+1+1$ unitary setup:

$$\mathcal{S}_{F,l}[U,\chi_l,\bar\chi_l] = a^4 \sum_x \bar\chi_l \left(D_W + m_0 + i\mu_l\gamma_5\tau^3\right)\chi_l\,,$$

Frezzotti et. al., JHEP 0108:058 (2001)

$$\mathcal{S}_{F,h}[U,\chi_h,\bar\chi_h] = a^4 \sum_x \bar\chi_h \left(D_W + m_0 + i\mu_\sigma\gamma_5\tau^1 + \mu_\delta\tau^3\right)\chi_h\,.$$

R. Frezzotti and G.C. Rossi, Nucl. Phys. Proc. Suppl.128 (2004)

- Automatic $\mathcal{O}(a)$ improvement $\rightarrow \not{\mathcal{P}}$ and $\not{\mathcal{F}}$ at finite a
- Heavy sector not flavor-diagonal $\rightarrow$ two additional propagators G^{xy}_{cs}, G^{xy}_{sc}

$\Rightarrow$ Much more contractions for correlation functions in heavy sector

$\Rightarrow$ Cannot apply tm variance reduction trick for heavy quarks

In the physical basis 2 γ-combinations ($i\gamma_5$, $i\gamma_0\gamma_5$) available; consider only $i\gamma_5$:

phys basis: $\eta_l^{phys} = \frac{1}{\sqrt{2}}\bar\psi_l i\gamma_5\psi_l\,,\qquad \eta_{c,s}^{phys} = \bar\psi_h\left(\frac{1\pm\tau^3}{2}i\gamma_5\right)\psi_h = \begin{cases}\bar c i\gamma_5 c\\ \bar s i\gamma_5 s\end{cases},$

tm basis: $\eta_l^{tm} = \frac{1}{\sqrt{2}}\bar\chi_l\left(-\tau^3\right)\chi_l \qquad \eta_{c,s}^{tm} = \frac{1}{2}\bar\chi_h\left(-\tau^1 \pm i\gamma_5\tau^3\right)\chi_h\,.$

$\Rightarrow$ heavy operators are a sum of scalars and pseudoscalars

Considering renormalization we have

$$\eta^{tm}_{c,renormalized} = Z\left(\bar{\chi}_c i\gamma_5 \chi_c - \bar{\chi}_s i\gamma_5 \chi_s\right)/2 - \left(\bar{\chi}_s \chi_c + \bar{\chi}_c \chi_s\right)/2$$
$$\eta^{tm}_{s,renormalized} = Z\left(\bar{\chi}_s i\gamma_5 \chi_s - \bar{\chi}_c i\gamma_5 \chi_c\right)/2 - \left(\bar{\chi}_s \chi_c + \bar{\chi}_c \chi_s\right)/2 \,.$$

$\rightarrow$ Need $Z = \frac{Z_P}{Z_S}$; can avoid this for masses ...

Additional rotation of basis to disentangle „heavy“ operators

$$\eta_{S,P} = \eta^{tm}_c \pm \eta^{tm}_s = \begin{cases} \frac{1}{\sqrt{2}}\left(\bar{\chi}_c \chi_s + \bar{\chi}_s \chi_c\right) \\ \frac{1}{\sqrt{2}}\left(\bar{\chi}_c i\gamma_5 \chi_c - \bar{\chi}_s i\gamma_5 \chi_s\right) \end{cases} .$$

In tm-basis we calculate:

$$\mathcal{C}^{\eta}(t) = \begin{pmatrix} \eta_l(t)\eta_l(0) & \eta_l(t)\eta_S(0) & \eta_l(t)\eta_P(0) \\ \eta_S(t)\eta_l(0) & \eta_S(t)\eta_S(0) & \eta_S(t)\eta_P(0) \\ \eta_P(t)\eta_l(0) & \eta_P(t)\eta_S(0) & \eta_P(t)\eta_P(0) \end{pmatrix} .$$

Advantage: Number of contractions per matrix element reduced by a factor 4

Putting in Z and rotating back before solving GEVP:

$\Rightarrow$ Eigenvectors of $\mathcal{C}^{\eta}(t)$ give access to physical amplitudes $\rightarrow$ mixing parameters

Mixing (I)

Decay constants are defined from axial-vector matrix elements (amplitudes)

$$\langle 0|\, A^i_\mu \,|\mathrm{P}(p)\rangle = i f^i_{\mathrm{P}} p_\mu\,, \quad \mathrm{P} = \eta, \eta'\,,$$

either in singlet-octet (i=0,8) or quark flavor basis (i=l,s)

$$A^0_\mu = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s)\,, \qquad A^l_\mu = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)\,,$$

$$A^8_\mu = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s)\,, \qquad A^s_\mu = \bar{s}\gamma_\mu\gamma_5 s\,.$$

η and η' are not pure states in either basis; most general parametrization:

$$\begin{pmatrix} f^{8,l}_\eta & f^{0,s}_\eta \\ f^{8,l}_{\eta'} & f^{0,s}_{\eta'} \end{pmatrix} = \begin{pmatrix} f_{8,l}\cos\phi_{8,l} & -f_{0,s}\sin\phi_{0,s} \\ f_{8,l}\sin\phi_{8,l} & f_{0,s}\cos\phi_{0,s} \end{pmatrix}$$

From χPT one expects

- $|\phi_8 - \phi_0|$ is given by $\mathrm{SU}(3)_F$ breaking terms; NOT small $\frac{|\phi_8-\phi_0|}{|\phi_8+\phi_0|} \not\ll 1$
- $|\phi_l - \phi_s| \sim \mathcal{O}(1/N_C) \to$ small (?) OZI correction $\frac{|\phi_l-\phi_s|}{|\phi_l+\phi_s|} \ll 1$

Mixing (II)

On the lattice: quark flavor basis is "natural" choice

- Can check whether $|\phi_l - \phi_s|$ is small!
- Expect that only one angle $\phi \approx \phi_l \approx \phi_s$ is required:

$$\tan^2(\phi) = -\frac{f_l^{\eta'} f_s^{\eta}}{f_l^{\eta} f_s^{\eta'}} \,,$$

Singlet-octet and quark flavor angles are related

$$\begin{aligned}\phi_0 =& \phi - \arctan(\sqrt{2} f_l / f_s) + \mathcal{O}(1/N_C)\,,\\ \phi_8 =& \phi - \arctan(\sqrt{2} f_s / f_l) + \mathcal{O}(1/N_C)\,.\end{aligned}$$

- In an $\mathrm{SU}(3)_F$ symmetric world: "ideal" angle $\phi_{\mathrm{SU}(3)_F} \approx 54.7^\circ$
- Small angle difference in one basis does NOT imply small difference in other basis!

Unfortunately, the axial vector is too noisy to determine $\phi/\phi_{l,s}$ and $f_{l,s}$ directly

Mixing (III)

Pseudoscalar amplitude

$$h_{\mathrm{P}}^{i} = 2m_i < 0|P^i|\mathrm{P} > , \quad \mathrm{P} = \eta, \eta' ,$$

is related to axial vector via the anomaly equation (singlet-octet)

$$\partial^{\mu} A_{\mu}^{i} = \bar{\psi}(x) 2MT^{i} i\gamma_5 \psi(x) + \delta^{i0} \sqrt{2N_f}\, \omega(x) .$$

In the quark flavor basis this leads to *Phys.Rev. D58 (1998) 114006, Phys.Lett. B449 (1999) 339-346*

$$\begin{pmatrix} P_{l,\eta} & P_{s,\eta} \\ P_{l,\eta'} & P_{s,\eta'} \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \mathrm{diag}\left(f_l M_{PS}^2, f_s\left(2M_K^2 - M_{PS}^2\right)\right) .$$

- This expression holds to LO χPT
- Ignoring higher orders in $\mathcal{O}(1/N_C)$ (i.e. $\phi_l \approx \phi_s$) AND higher orders in masses

$\rightarrow$ some χPT-dependence compared to axial-vector approach

Two-photon decay widths

η,η'-mixing parameters are related to anomaly $\rightarrow$ relevance for several processes

- The decays $\eta,\eta' \rightarrow \gamma\gamma$ are driven by the chiral anomaly
- At LO: Wess-Zumino-Wittten term

$$\mathcal{L}_{\mathrm{WZW}}^{\mathrm{LO}} = -\frac{N_C \alpha_{\mathrm{QED}}}{4\pi} F_{\mu\nu}\tilde{F}^{\mu\nu} \mathrm{tr}[\mathrm{diag}(2/3,-1/3,-1/3)\varphi^2] \,.$$

Tree level prediction for decay widths reads

$$\Gamma[\eta \rightarrow \gamma\gamma] = \frac{\alpha_{\mathrm{QED}}^2}{576\pi^3} M_\eta^3 \left[\frac{5}{f_l}\cos\phi - \frac{\sqrt{2}}{f_s}\sin\phi\right]^2 ,$$

$$\Gamma\left[\eta' \rightarrow \gamma\gamma\right] = \frac{\alpha_{\mathrm{QED}}^2}{576\pi^3} M_{\eta'}^3 \left[\frac{5}{f_l}\sin\phi + \frac{\sqrt{2}}{f_s}\cos\phi\right]^2 .$$

Nucl.Phys.Proc.Suppl. 64 (1998) 223-231, Eur.Phys.J. C17 (2000) 623-649

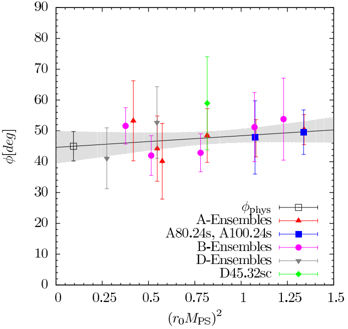
- OZI-suppressed terms are dropped $\rightarrow$ consistent with mixing scheme
- Expressions become rigorous in the chiral limit

Setup

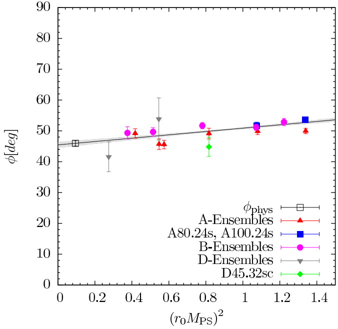
- We use almost all ETMC $N_f = 2+1+1$ ensembles (16 ensembles)
- Three lattice spacings $a_A = 0.086\,\mathrm{fm}$, $a_B = 0.078\,\mathrm{fm}$ and $a_D = 0.061\,\mathrm{fm}$
- Physical lattice size $L \geq 3\,\mathrm{fm}$ for many ensembles; $L\,M_{\mathrm{PS}} \geq 3.5$
- ~ 600 up to ~ 2500 gauge configuration per ensemble
- Charged pion masses range from $\sim 230\,\mathrm{MeV}$ to $\sim 500\,\mathrm{MeV}$
- Bare m_s, m_c fixed for each β
- We remove excited contributions in conn correlators $\rightarrow$ previous talk

Mixing angle (I)

“old method”

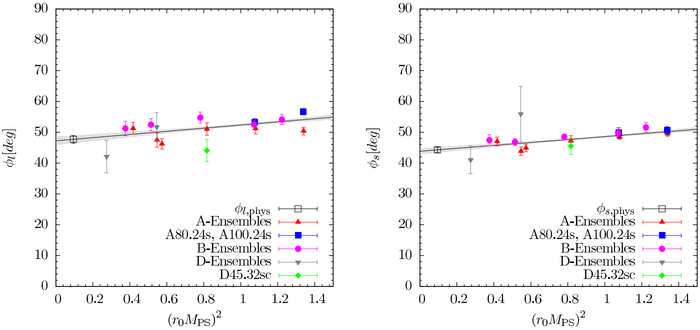


“improved method”



- Linear fit: $\phi = 46.0^\circ(0.9)_{\rm stat}(2.7)_{\rm sys}$
- Systematic error parametrizes ignorance towards m_s, a-dependence
- Compatible with old analysis $\phi = 44^\circ(5)_{\rm stat}$ *Ottnad et. al., JHEP 1211 (2012) 048*
  and other lattice and experimental results.

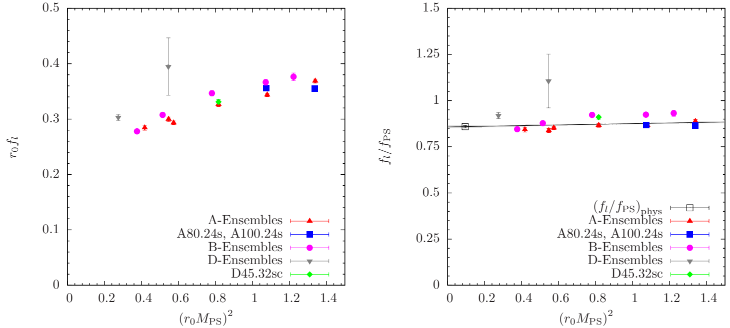
Mixing angle (II)



- Linear fits: $\phi_l = 47.7°(1.2)_{\text{stat}}(4.1)_{\text{sys}}$ and $\phi_s = 44.3°(0.9)_{\text{stat}}(3.0)_{\text{sys}}$
- Difference $\Delta\phi_{ls} = 2.8°(1.1)_{\text{stat}}(2.6)_{\text{sys}}$ confirms smallness of OZI-corrections

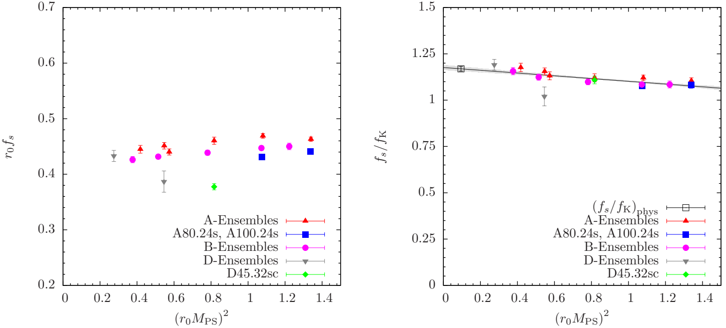
$\Rightarrow$ data well described by single angle in quark flavor basis

Decay constants - f_l



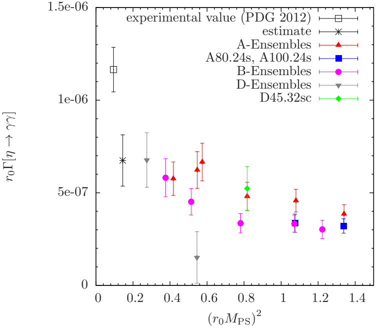
- f_l shows rather nonlinear m_l-dependence; scaling artifacts
- Most m_l-dependence cancels in the ratio f_l/f_{PS}
- m_s-dependence negligible
- Linear fit: $f_l/f_{PS} = 0.859(7)_{stat}(64)_{sys}$
- Fit to finest lattice spacing only $f_l/f_{PS}|_D = 0.924(22)_{stat}$
- Phenomenology $f_l/f_\pi = 1.07(2)$ *Th. Feldmann, Int.J.Mod.Phys. A15 (2000) 159-207*

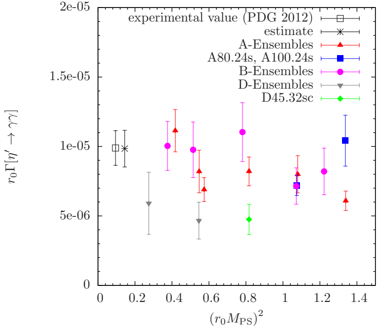
Decay constants - f_s



- f_s shows sizable m_s-dependence; possibly scaling artifacts
- f_s/f_K cancels most m_s,a-dependence; rather mild m_l-dependence
- Linear fit: $f_s/f_K = 1.166(11)_{stat}(31)_{sys}$
- Phenomenology $f_s/f_K = 1.12(6)$ *Th. Feldmann, Int.J.Mod.Phys. A15 (2000) 159-207*

Two-photon decay widths





- Very preliminary; formulae used for decay widths are tree level only
- $\Gamma[\eta \to \gamma\gamma]$ shows nonlinear m_l-dependence; additional m_s-dependence?
- $\Gamma[\eta' \to \gamma\gamma]$ rather compatible exp. value; still some a-dependence (possibly also m_s-dependence…)

Need better control of scaling artifacts and m_q-dependence for definite results!

Summary and Outlook

- First lattice determination of η,η' decay constants
- (Preliminary) results:

$$\phi = 46.0^\circ(0.9)_{\mathrm{stat}}(2.7)_{\mathrm{sys}}\,, \qquad \phi_{\mathrm{phenom}} = 39.3^\circ(1.0)$$
$$f_l/f_{\mathrm{PS}} = 0.859(07)_{\mathrm{stat}}(64)_{\mathrm{sys}}\,, \qquad (f_l/f_{\mathrm{PS}})_{\mathrm{phenom}} = 1.07(2)$$
$$f_s/f_{\mathrm{K}} = 1.166(11)_{\mathrm{stat}}(31)_{\mathrm{sys}}\,, \qquad (f_s/f_{\mathrm{K}})_{\mathrm{phenom}} = 1.12(6)$$

- Determination of ϕ, f_s/f_{K} with controlled systematics
- Our study confirms smallness of OZI corrections in quark flavor basis
- Still need better control of lattice artifacts for f_l/f_{PS}
- Decay widths for $\eta,\eta' \to 2\gamma$ accessible; need control of systematics

Further plans:

- Vary m_s for further ensembles
- Point-to-point correlators...
- ... maybe get signal for axial vector $\to$ direct access to mixing parameters

Removal of excited states (I)

Problem: (Large) disconnected contributions to η'

- Signal for η' lost at small t
- Hardly plateau for $M_{\eta'}$; impossible to extract amplitudes
- Large contamination from excited states

Possible solutions:

- Use much larger statistics $\rightarrow$ very expensive
- Increase operator basis $\rightarrow$ not easily possible, axial vector very noisy
- point-to-point correlators; stoch. distillation $\rightarrow$ will be tested
- ... or find some other method to extract quantities at small t

Removal of excited states (II)

- Ignore charm quark
- Consider $\mathcal{M}^2 = \mathrm{diag}(M_\eta^2, M_{\eta'}^2)$ in quark flavor basis

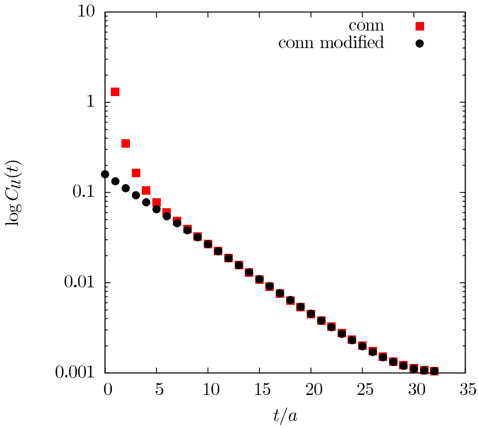
$$\mathcal{M}^2 = \begin{pmatrix} M_{ll}^2 + 2\Delta_{ll} & \sqrt{2}\Delta_{ls} \\ \sqrt{2}\Delta_{ls} & M_{ss}^2 + \Delta_{ss} \end{pmatrix}$$

- M_{ll}, M_{ss}: masses of flavor non-singlet eigenstates (connected only)
- Δ_{ll}, Δ_{ls} and Δ_{ss} give large corrections (disconnected)

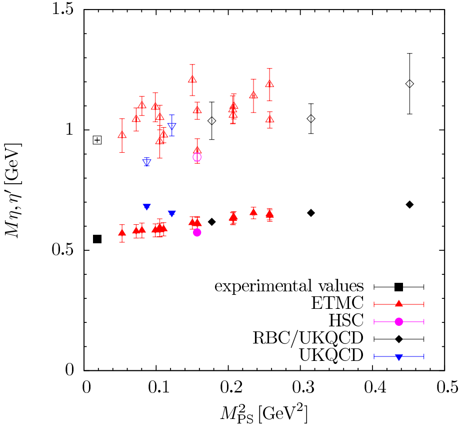
Assumption:

Disconnected diagrams couple only to η, η'

- Replace connected contributions by respective ground state contributions
- If assumption is correct we should see a plateau at very low t



Comparison of results using improved method



Two-photon decay widths (II)