η and η' Masses from Lattice QCD
for the ETM collaboration

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Mainz, Lattice 2013
nine lightest pseudo-scalar mesons show a peculiar spectrum:

- 3 very light pions (140 MeV)
- kaons and the $\eta$ around 500 MeV
- $\eta'$ around 1 GeV

the large mass of the $\eta'$ meson is thought to be caused by the QCD vacuum structure and the $U_A(1)$ anomaly

$\eta'$ meson is not a (would be) Goldstone Boson

$\Rightarrow$ massive even in the SU(3) chiral limit
Noise in the $\eta'$

- $\eta'$ mainly the flavour singlet
- disconnected contributions significant
  $\Rightarrow$ hard problem
- chiral extrapolation difficult
  $\Rightarrow$ no clear picture
- need for improvement

\[ r_0 M_{\eta, \eta'} \]

Ensemble-Details

- $2 + 1 + 1$ flavour gauge configurations from ETM Collaboration
  [ETMC, R. Baron et. al., JHEP 06 111 (2010)]

- Iwasaki Gauge action
  [Iwasaki, Nucl. Phys. B258, 141]

- three lattice spacings ($A$, $B$ and $D$ ensembles):
  $a_A = 0.086$ fm, $a_B = 0.078$ fm and $a_D = 0.061$ fm

- charged pion masses range from $\approx 230$ MeV to $\approx 500$ MeV

- $L \geq 3$ fm and $M_\pi \cdot L \geq 3.5$ for most ensembles

- $\approx 600$ up to $\approx 2500$ gauge configuration per ensemble

- bare $m_s$ and $m_c$ fixed for each lattice spacing

- use $r_0 = 0.45(2)$ fm (from $f_\pi$) throughout the talk
Flavour Singlet Pseudo-Scalar Mesons

- need to estimate correlator matrix

\[
C = \begin{pmatrix}
\eta_{ll} & \eta_{ls} & \eta_{lc} \\
\eta_{sl} & \eta_{ss} & \eta_{sc} \\
\eta_{cl} & \eta_{cs} & \eta_{cc}
\end{pmatrix}
\]

- \(\eta_{XY}\) correlator of appropriate interpolating fields, e.g.

\[
\eta_{ss}(t) \equiv \langle \bar{s}i\gamma_5 s(t) \bar{s}i\gamma_5 s(0) \rangle
\]

projected to zero momentum

\(\Rightarrow\) diagonalise matrix:
masses and pseudo-scalar matrix elements

- \(\eta\): lowest state, \(\eta':\) first state, \(\eta_c\) ...
• ground state $\eta$ well determined
• $\eta'$ signal lost in noise before plateau reached
Improved $\eta'$ Extraction

- **make model assumption:**
  disconnected contributions couple only to $\eta$ and $\eta'$ states, not to higher states
  
  [Neff et al., Phys.Rev.D64 (2001)]
  

- replace connected contributions by only the ground states

- if model justified:
  there should be a plateau in the effective masses from very low times on!

![Graph showing log of $C(t)$ vs. $t/a$](image-url)
Excited State Removal

- we see a plateau from $t/a = 2$ on
- for both $\eta$ and $\eta'$
- $\eta$: good agreement with previous results
- $\eta'$: possibly much better determination
- assumption justified?
- systematic uncertainties?

\[
M(t) = \log \frac{C(t+1)}{C(t)}
\]
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Masses w/ and w/o Excited State Removal

w/ removal: only two states left $C$

$\eta$:
- masses agree well
- improved precision

$\eta'$:
- masses determined much better
- always agreement within $2\sigma$
- systematics hard to quantify
- from distribution of differences: assign 7% systematic uncertainty
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Strange Quark Mass Dependence

- $m_s$ not perfectly tuned to its physical value
- two re-tuned ensembles for $a_A$
  - can estimate $m_s$ dependence
- estimate
  \[ D_\eta \equiv \frac{d(aM_\eta)^2}{d(aM_K)^2} = 1.47(11) \]
- now assume:
  - $D_\eta$ independent of $a, m_\ell, m_s, m_c$
- ...correct $\eta$ masses
Scaling Test for $M_\eta$

- use two ensembles sets
  $(A60, B55, D45)$
  $(A40, B35, D30)$
  with $r_0 M_{PS} \approx \text{const}$

- correct $M_\eta$ using $D_\eta$ linearly in $M_K^2$
  \[ r_0 M_K = 1.34 \text{ fixed} \]

- compatible with both, constant and linear continuum extrapolation
  \[ \Rightarrow \text{assign 5\% systematic error from maximal difference} \]
Chiral Extrapolation of $M_\eta$

- more ambitious: shift all $M_\eta$ to physical strange mass
- fit $c_1$, $c_2$
  \[ g_K = c_1 + c_2 (r_0 M_{PS})^2 \]
  
  to data for $(r_0 M_K)^2$ from $A$ ensembles
- adjust $c_1$ to match physical $M_K$ for $M_{PS} = M_\pi \Rightarrow \tilde{g}_K$
- compute
  \[ \delta_K[(r_0 M_{PS})^2] = (r_0 M_K)^2 - \tilde{g}_K[(r_0 M_{PS})^2] \]
  
  for all ensembles
Chiral Extrapolation of $M_\eta$

- now correct all $(r_0 M_\eta)^2$ by corresponding

$$D_\eta \cdot \delta_K [(r_0 M_{PS})^2]$$

$$\Rightarrow (r_0 \bar{M}_\eta)^2 \propto (r_0 M_{PS})^2$$

- all $a$-values fall on the same curve!

- extrapolate $(r_0 \bar{M}_\eta)^2$ linearly in $(r_0 M_{PS})^2$ to $M_{PS} = M_\pi$

- result

$$M_\eta = 552(10)_{\text{stat}} \text{ MeV}$$

- similarly with $(\bar{M}_\eta / \bar{M}_K)^2$ or GMO relation
Chiral Extrapolation of $M_{\eta'}$

- no clear dependence on
  - lattice spacing
  - strange quark mass
- errors still significant
- include all data in extrapolation
- $(r_0 M_{\eta'})^2 \propto (r_0 M_{PS})^2$

⇒ result

$$M_{\eta'} = 1005(54)_{\text{stat}} \text{ MeV}$$

- fitting $A$, $B$ and $D$ seperately gives compatible results
• $\eta$ and $\eta'$ for three lattice spacings and various quark mass values

• presented excited state removal method

• $\eta$ can be extracted precisely

$$M_{\eta} = 552(10)_{\text{stat}}(28)_{\text{sys}} \text{ MeV}$$

• $\eta'$ from excited state removal

$$M_{\eta'} = 1005(54)_{\text{stat}}(86)_{\text{sys}} \text{ MeV}$$

→ mixing: talk by Konstantin Ottnad

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