The Chromomagnetic Operator on the Lattice

Lattice 2013, Mainz — August 2, 2013

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OUTLINE

- Definition and Use of Chromomagnetic Operator (CMO)
  - Effective $\Delta S = 1$ Hamiltonian
  - Physical Processes involving the CMO (in SM / BSM)
- Lattice Setup
- Relevant Symmetries (Continuum – Lattice)
  - Possible pattern of Operator mixing
  - (Lower dimensional / Gauge non-invariant operators)
- Calculation of Mixing Matrix in the $\overline{MS}$ Scheme
  - Dimensional Regularization
  - Lattice Regularization
- Checks and Extensions
  - Non-perturbative Renormalization
  - Boosted Perturbation Theory
  - $\mathcal{O}(a^2 g^2)$ Corrections

Effective $\Delta S = 1$ ($\Delta B = 1$) Hamiltonian

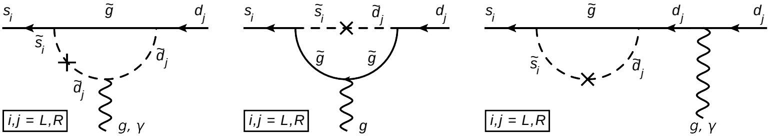
- Four magnetic operators of dimension 5: OPE:

$$H_{\rm eff}^{\Delta S=1,\ d=5} = \sum_{i=\pm}(C_\gamma^i Q_\gamma^i + C_g^i Q_g^i) + {\rm h.c.}$$

$$Q_\gamma^\pm = \frac{Q_d\, e}{16\pi^2}\,(\bar{s}_L\, \sigma^{\mu\nu}\, F_{\mu\nu}\, d_R \pm \bar{s}_R\, \sigma^{\mu\nu}\, F_{\mu\nu}\, d_L)$$

$$Q_g^\pm = \frac{g}{16\pi^2}\,(\bar{s}_L\, \sigma^{\mu\nu}\, G_{\mu\nu}\, d_R \pm \bar{s}_R\, \sigma^{\mu\nu}\, G_{\mu\nu}\, d_L)$$

- Suppressed in SM, less so beyond SM
- Penguin diagrams from SUSY, contributing to $C^i_{\gamma,g}$



[Gabbiani, Gabrielli, Masiero, Silvestrini NPB 477 (1996) 321]

Matrix Elements of the CMO and their Physical Relevance

The matrix elements of the CMO are parameterized as:

	Relevance
• $\langle \pi^0 \vert Q_g^+ \vert K^0 \rangle = \frac{-11}{32\sqrt{2}\pi^2} \frac{M_K^2 (p_\pi \cdot p_K)}{m_s + m_d} B_{g1}$	$K^0 - \bar{K}^0$mixing
• $\langle \pi^+ \pi^- \vert Q_g^- \vert K^0 \rangle = \frac{11\,\mathrm{i}}{32\pi^2} \frac{M_K^2\, M_\pi^2}{f_\pi\,(m_s + m_d)} B_{g2}$	$\epsilon'/\epsilon, \Delta I = 1/2$
• $\langle \pi^+ \pi^+ \pi^- \vert Q_g^+ \vert K^+ \rangle = \frac{-11}{16\pi^2} \frac{M_K^2\, M_\pi^2}{f_\pi^2\,(m_s + m_d)} B_{g3}$	$K \to 3\pi$

[D'Ambrosio, Isidori, Martinelli, PLB 480 (2000) 164]

To leading order in χPT, the B-parameters are all related:

$$Q_g^\pm = \frac{11}{256\pi^2} \frac{f_\pi^2\, M_K^2}{m_s + m_d} B_g \left[U (D_\mu U^\dagger)(D^\mu U) \pm (D_\mu U^\dagger)(D^\mu U) U^\dagger \right]_{23}$$

[Bertolini, Eeg, Fabbrichesi, NPB 449 (1995) 197]

Study of the EMO

- $\langle \pi^0 | Q_\gamma^+ | K^0 \rangle = \mathrm{i}\, \frac{Q_d\, e \sqrt{2}}{16\pi^2\, M_K}\, p_\pi^\mu\, p_K^\nu\, F_{\mu\nu}\, B_T\, R_T(q^2)$

  $[R_T(0) = 1]$

- $N_f = 0$ [Becirevic, Lubicz, Martinelli, Mescia, PLB 501 (2001) 98]

  $N_f = 2$ [ETMC, Baum, Lubicz, Martinelli, Orifici, Simula, PRD 84 (2011) 074503]

- B_T appears, e.g., in BR($K_L \to \pi^0\, e^+\, e^-$) in SUSY models

Operator Mixing in the Continuum

- Only operators of dimension 5
- Same flavor content as CMO
- Gauge non-invariant? (Vanish by e.o.m., BRST invariant)

$$
\begin{aligned}
\mathcal{O}_1 &= g\,\overline{\psi}_s \sigma_{\mu\nu} G_{\mu\nu} \psi_d \\
\mathcal{O}_2 &= (m_d^2 + m_s^2)\overline{\psi}_s \psi_d \\
\mathcal{O}_3 &= m_d\, m_s \overline{\psi}_s \psi_d \\
\mathcal{O}_4 &= \overline{\psi}_s \overleftarrow{D}_\mu \overrightarrow{D}_\mu \psi_d \\
\mathcal{O}_5 &= \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)(\overrightarrow{\not{D}} + m_d)\psi_d \\
\mathcal{O}_6 &= \overline{\psi}_s (\overrightarrow{\not{D}} + m_d)^2 \psi_d + \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)^2 \psi_d \\
\mathcal{O}_7 &= m_s \overline{\psi}_s (\overrightarrow{\not{D}} + m_d)\psi_d + m_d \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)\psi_d \\
\mathcal{O}_8 &= m_d \overline{\psi}_s (\overrightarrow{\not{D}} + m_d)\psi_d + m_s \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)\psi_d \\
\mathcal{O}_9 &= \overline{\psi}_s \overleftarrow{\not{\partial}} (\overrightarrow{\not{D}} + m_d)\psi_d - \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s) \overrightarrow{\not{\partial}} \psi_d \\
\mathcal{O}_{10} &= \overline{\psi}_s \overrightarrow{\not{\partial}} (\overrightarrow{\not{D}} + m_d)\psi_d - \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s) \overleftarrow{\not{\partial}} \psi_d
\end{aligned}
$$

Mixing on the Lattice?

- Operators with dimension < 5
- Broken Symmetries $\Rightarrow$
  Many more operators may mix (finite coefficients)
- Dimension 3:
  - $\overline{\psi}_s \psi_d$
  - $\overline{\psi}_s \gamma_5 \psi_d$

Mixing on the Lattice?

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- Broken Symmetries $\Rightarrow$
  Many more operators may mix (finite coefficients)
- Dimension 3:
  - $\overline{\psi}_s \psi_d$
  - $\overline{\psi}_s \gamma_5 \psi_d$
- Dimension 4:
  - $(m_d + m_s)\overline{\psi}_s \psi_d$
  - $(m_d - m_s)\overline{\psi}_s \psi_d$
  - $i\,(m_d + m_s)\overline{\psi}_s \gamma_5 \psi_d$
  - $i\,(m_d - m_s)\overline{\psi}_s \gamma_5 \psi_d$
  - $\overline{\psi}_s(\overrightarrow{\not{D}} + m_d)\psi_d + \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)\psi_d$
  - $\overline{\psi}_s(\overrightarrow{\not{D}} + m_d)\psi_d - \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)\psi_d$
  - $i\,\overline{\psi}_s \gamma_5(\overrightarrow{\not{D}} + m_d)\psi_d + i\,\overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)\gamma_5 \psi_d$
  - $i\,\overline{\psi}_s \gamma_5(\overrightarrow{\not{D}} + m_d)\psi_d - i\,\overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)\gamma_5 \psi_d$

Mixing on the Lattice?

- Dimension 5:

$$g\,\overline{\psi}_s \sigma_{\mu\nu} G_{\mu\nu} \psi_d$$
$$i\,g\,\overline{\psi}_s \gamma_5 \sigma_{\mu\nu} G_{\mu\nu} \psi_d$$
$$(m_d^2 + m_s^2)\overline{\psi}_s \psi_d$$
$$i\,(m_d^2 + m_s^2)\overline{\psi}_s \gamma_5 \psi_d$$
$$(m_d^2 - m_s^2)\overline{\psi}_s \psi_d$$
$$i\,(m_d^2 - m_s^2)\overline{\psi}_s \gamma_5 \psi_d$$
$$m_d\, m_s \overline{\psi}_s \psi_d$$
$$i\,m_d\, m_s \overline{\psi}_s \gamma_5 \psi_d$$
$$m_s \overline{\psi}_s (\overrightarrow{\not{D}} + m_d)\psi_d + m_d \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)\psi_d$$
$$m_d \overline{\psi}_s (\overrightarrow{\not{D}} + m_d)\psi_d + m_s \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)\psi_d$$
$$m_s \overline{\psi}_s (\overrightarrow{\not{D}} + m_d)\psi_d - m_d \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)\psi_d$$
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$$i\,m_s \overline{\psi}_s \gamma_5 (\overrightarrow{\not{D}} + m_d)\psi_d + i\,m_d \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)\gamma_5 \psi_d$$
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$$\overline{\psi}_s (\overrightarrow{\not{D}} + m_d)^2 \psi_d + \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)^2 \psi_d$$
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$$\overline{\psi}_s \overleftarrow{D}_\mu \overrightarrow{D}_\mu \psi_d$$
$$i\,\overline{\psi}_s \gamma_5 \overleftarrow{D}_\mu \overrightarrow{D}_\mu \psi_d$$
$$\overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)\,(\overrightarrow{\not{D}} + m_d)\psi_d$$
$$i\,\overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)\,\gamma_5\,(\overrightarrow{\not{D}} + m_d)\psi_d$$
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[Eigenfunctions of discrete transformations]

Lattice Action – Fermions

- Need a large set of discrete symmetries, to exclude mixing candidates
- Fermion action [R. Frezzotti, G. Rossi]:
  - Twisted mass for valence quarks
    (Necessitates compensating ghost action)
  - Osterwalder - Seiler for sea quarks
- Valence part in physical basis:

$$
\begin{aligned}
S_F[\psi_f, \bar{\psi}_f, U] &= a^4 \sum_f \sum_x \bar{\psi}_f(x) \Big[\gamma \cdot \widetilde{\nabla} - i\gamma_5 W_{\rm cr}(r_f) + m_f \Big] \psi_f(x) \\
\gamma \cdot \widetilde{\nabla} &\equiv \frac{1}{2} \sum_\mu \gamma_\mu (\nabla^\star_\mu + \nabla_\mu) \\
W_{\rm cr}(r_f) &\equiv -a\frac{r_f}{2} \sum_\mu \nabla^\star_\mu \nabla_\mu + M_{\rm cr}(r_f)
\end{aligned}
$$

r_f: Wilson parameter for the flavour $f = u,\ d,\ s$
$M_{\rm cr}(r_f)$: corresponding critical mass ($M_{\rm cr}(-r_f) = -M_{\rm cr}(r_f)$).

Lattice Action – Gluons

- Gluon action: Symanzik improved

$$S_G = \frac{2}{g_0^2}\Big[c_0 \sum_{\text{plaq.}} \mathrm{Re\,Tr}\,\{1 - U_{\text{plaq.}}\} + c_1 \sum_{\text{rect.}} \mathrm{Re\,Tr}\,\{1 - U_{\text{rect.}}\} + c_2 \sum_{\text{chair}} \mathrm{Re\,Tr}\,\{1 - U_{\text{chair}}\} + c_3 \sum_{\text{paral.}} \mathrm{Re\,Tr}\,\{1 - U_{\text{paral.}}\}\Big]$$

c_0, c_1, c_2, c_3 : arbitrary, subject to: $c_0 + 8c_1 + 16c_2 + 8c_3 = 1$

Symmetries of the Lattice Action

Defining: [Frezzotti and Rossi, JHEP 0410 (2004) 70]

$$\mathcal{P} \quad : \quad \begin{cases} U_0(x) \to U_0(x_{\mathcal{P}})\,, \quad U_k(x) \to U_k^\dagger(x_{\mathcal{P}} - a\hat{k})\,, & k = 1,2,3 \\ \psi_f(x) \to \gamma_0 \psi_f(x_{\mathcal{P}}) & x_{\mathcal{P}} = (-\mathbf{x}, x_0) \\ \bar{\psi}_f(x) \to \bar{\psi}_f(x_{\mathcal{P}})\gamma_0 & \end{cases}$$

$$\mathcal{D}_d \quad : \quad \begin{cases} U_\mu(x) & \to U_\mu^\dagger(-x - a\hat{\mu}) \\ \psi_f(x) & \to e^{3i\pi/2}\psi_f(-x) \\ \bar{\psi}_f(x) & \to e^{3i\pi/2}\bar{\psi}_f(-x) \end{cases}$$

$$\mathcal{C} \quad : \quad \begin{cases} \psi(x) \to i\gamma_0\gamma_2 \overline{\psi}(x)^T \\ \bar{\psi}(x) \to -\psi(x)^T i\gamma_0\gamma_2 \\ U_\mu(x) \to U_\mu^\star(x) \end{cases}$$

$$\mathcal{S} \quad : \quad \psi_s \leftrightarrow \psi_d\,, \quad m_s \leftrightarrow m_d$$

$$\mathcal{R}_5 \quad = \quad \prod_f \mathcal{R}_{f\,5}\,, \qquad \mathcal{R}_{f\,5} : \begin{cases} \psi_f & \to \gamma_5 \psi_f \\ \bar{\psi}_f & \to -\bar{\psi}_f \gamma_5 \end{cases}$$

Symmetries of the Lattice Action

Defining: [Frezzotti and Rossi, JHEP 0410 (2004) 70]

$$\mathcal{P} \;:\; \begin{cases} U_0(x) \to U_0(x_\mathcal{P})\,, \quad U_k(x) \to U_k^\dagger(x_\mathcal{P} - a\hat{k})\,, & k = 1,2,3 \\ \psi_f(x) \to \gamma_0 \psi_f(x_\mathcal{P}) & x_\mathcal{P} = (-\mathbf{x}, x_0) \\ \bar{\psi}_f(x) \to \bar{\psi}_f(x_\mathcal{P})\gamma_0 \end{cases}$$

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$$\mathcal{R}_5 = \prod_f \mathcal{R}_{f\,5}\,, \qquad \mathcal{R}_{f\,5} : \begin{cases} \psi_f \to \gamma_5\psi_f \\ \bar{\psi}_f \to -\bar{\psi}_f\gamma_5 \end{cases}$$

Symmetries

- $\mathcal{P} \times \mathcal{D}_d \times (m \to -m)$, ($m$: all masses except $M_{\rm cr}$)
- $\mathcal{D}_d \times \mathcal{R}_5$
- $\mathcal{C} \times \mathcal{S}$, if $r_s = r_d$
- $\mathcal{C} \times \mathcal{P} \times \mathcal{S}$, if $r_s = -r_d$

Operator Mixing on the Lattice – Twisted Mass

Operators with same eigenvalues as CMO under the symmetries of the action

$$
\begin{aligned}
& \mathcal{O}_1 &=& \; g\,\overline{\psi}_s \sigma_{\mu\nu} G_{\mu\nu} \psi_d \\
& \mathcal{O}_2 &=& \; (m_d^2 + m_s^2)\overline{\psi}_s \psi_d \\
& \mathcal{O}_3 &=& \; m_d\, m_s \overline{\psi}_s \psi_d \\
& \mathcal{O}_4 &=& \; \overline{\psi}_s \overleftarrow{D}_\mu \overrightarrow{D}_\mu \psi_d \\
& \mathcal{O}_5 &=& \; \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)(\overrightarrow{\not{D}} + m_d)\psi_d \\
& \mathcal{O}_6 &=& \; \overline{\psi}_s (\overrightarrow{\not{D}} + m_d)^2 \psi_d + \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)^2 \psi_d \\
& \mathcal{O}_7 &=& \; m_s \overline{\psi}_s (\overrightarrow{\not{D}} + m_d)\psi_d + m_d \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)\psi_d \\
& \mathcal{O}_8 &=& \; m_d \overline{\psi}_s (\overrightarrow{\not{D}} + m_d)\psi_d + m_s \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)\psi_d \\
& \mathcal{O}_9 &=& \; \overline{\psi}_s \overleftarrow{\not{\partial}} (\overrightarrow{\not{D}} + m_d)\psi_d - \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s) \overrightarrow{\not{\partial}} \psi_d \\
& \mathcal{O}_{10} &=& \; \overline{\psi}_s \overrightarrow{\not{\partial}} (\overrightarrow{\not{D}} + m_d)\psi_d - \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s) \overleftarrow{\not{\partial}} \psi_d \\
\text{dim. } 4 \quad & \mathcal{O}_{11} &=& \; i\, r_d\, \overline{\psi}_s \gamma_5 (\overrightarrow{\not{D}} + m_d)\psi_d + i\, r_s\, \overline{\psi}_s (-\overleftarrow{\not{D}} + m_s)\gamma_5 \psi_d \\
\text{dim. } 4 \quad & \mathcal{O}_{12} &=& \; i\,(r_d\, m_d + r_s\, m_s)\overline{\psi}_s \gamma_5 \psi_d \\
\text{dim. } 3 \quad & \mathcal{O}_{13} &=& \; \overline{\psi}_s\, \psi_d
\end{aligned}
$$

Renormalization Matrix

- $\mathcal{O}_i = \sum_{j=1}^{13} Z_{ij} \mathcal{O}_j^R \quad \Rightarrow \quad \mathcal{O} = Z \mathcal{O}^R$
- $Z_{ij} \equiv Z_{ij}^{X,Y}$ X: regularization,
  Y: renormalization scheme
- $Z = \mathbb{1} + \mathcal{O}(g^2)$
- $\mathcal{O}^R = Z^{-1}\, \mathcal{O}, \qquad Z^{-1} + Z = 2 \cdot \mathbb{1} + \mathcal{O}(g^4)$
- For $\mathcal{O}_1^R$: Only need first row: $Z_i \equiv Z_{1,i}$

  $Z_1 = 1 + g^2\, z_1 + \mathcal{O}(g^4), \quad Z_2\,, \ldots, Z_{13} = \mathcal{O}(g^2)$

Renormalization Matrix – Computation of Z_i

- Amputated quark–antiquark Green's function:

$$
\begin{aligned}
\langle \psi^R\, \mathcal{O}_1^R\, \overline{\psi}^R \rangle_{\rm amp} &= \langle \psi^R\, \overline{\psi}^R \rangle^{-1}\, \langle \psi^R\, \mathcal{O}_1^R\, \overline{\psi}^R \rangle\, \langle \psi^R\, \overline{\psi}^R \rangle^{-1} \\
&= \left(Z_\psi\, \langle \psi\, \overline{\psi} \rangle^{-1}\right)\, Z_\psi^{-1}\, \sum_{i=1}^{13} (Z^{-1})_{1i} \langle \psi\, \mathcal{O}_i\, \overline{\psi} \rangle\, \left(Z_\psi\, \langle \psi\, \overline{\psi} \rangle^{-1}\right) \\
&= Z_\psi\, \sum_{i=1}^{13} (Z^{-1})_{1i} \langle \psi\, \mathcal{O}_i\, \overline{\psi} \rangle_{\rm amp}
\end{aligned}
$$

◦ $\psi = Z_\psi^{1/2}\, \psi^R$: flavour independent (in mass-independent schemes)

- Amputated quark–antiquark–gluon Green's function:

$$
\langle \psi^R\, \mathcal{O}_1^R\, \overline{\psi}^R A_\nu^R \rangle_{\rm amp} = Z_\psi\, Z_A^{1/2} \sum_{i=1}^{13} (Z^{-1})_{1i} \langle \psi\, \mathcal{O}_i\, \overline{\psi}\, A_\nu \rangle_{\rm amp}\,, \quad A_\nu = \sqrt{Z_A}\, A_\nu^R
$$

- 4-pt Green's functions: Only for consistency checks
- 5-pt Green's functions, ... : Unnecessary (superficially convergent)

Renormalization Matrix – Computation of Z_i

- Amputated quark–antiquark Green's function:

$$\langle \psi^R \, \mathcal{O}_1^R \, \overline{\psi}^R \rangle_{\rm amp} = \langle \psi^R \, \overline{\psi}^R \rangle^{-1} \, \langle \psi^R \, \mathcal{O}_1^R \, \overline{\psi}^R \rangle \, \langle \psi^R \, \overline{\psi}^R \rangle^{-1}$$

$$= \left(Z_\psi \, \langle \psi \, \overline{\psi} \rangle^{-1} \right) \, Z_\psi^{-1} \sum_{i=1}^{13} (Z^{-1})_{1i} \langle \psi \, \mathcal{O}_i \, \overline{\psi} \rangle \, \left(Z_\psi \, \langle \psi \, \overline{\psi} \rangle^{-1} \right)$$

$$\langle \psi^R \, \mathcal{O}_1^R \, \overline{\psi}^R \rangle_{\rm amp} = Z_\psi \sum_{i=1}^{13} (Z^{-1})_{1i} \langle \psi \, \mathcal{O}_i \, \overline{\psi} \rangle_{\rm amp}$$

  - $\psi = Z_\psi^{1/2} \, \psi^R$: flavour independent (in mass-independent schemes)

- Amputated quark–antiquark–gluon Green's function:

$$\langle \psi^R \, \mathcal{O}_1^R \, \overline{\psi}^R A_\nu^R \rangle_{\rm amp} = Z_\psi \, Z_A^{1/2} \sum_{i=1}^{13} (Z^{-1})_{1i} \langle \psi \, \mathcal{O}_i \, \overline{\psi} \, A_\nu \rangle_{\rm amp} , \qquad A_\nu = \sqrt{Z_A} \, A_\nu^R$$

- 4-pt Green's functions: Only for consistency checks
- 5-pt Green's functions, ... : Unnecessary (superficially convergent)

Renormalization Matrix – Computation of Z_i

- Alternative definition of CMO:

$$\widetilde{\mathcal{O}}_{CM} \equiv m\,\mathcal{O}_{CM}$$

- Appears in the study of 4-fermi (dimension 6) operators

  $\Rightarrow \tilde{Z}_{ij} = Z_m\, Z_{ij}$

  ($m^R = Z_m^{-1}\, m$, Z_m : flavor independent)

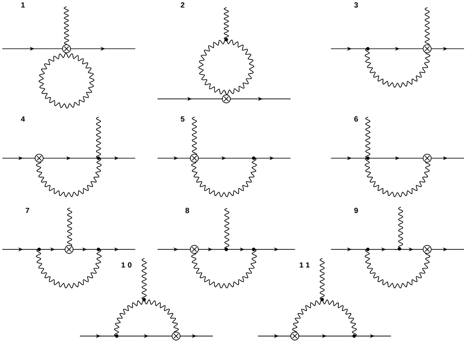
- Similarly, a factor of Z_g must be included in Z_{ij}, if using:

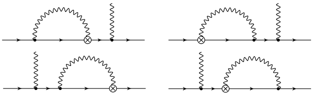
$$\overline{\psi}_s\, \sigma_{\mu\nu}\, G_{\mu\nu}\, \psi_d, \text{ rather than: } \quad g\, \overline{\psi}_s\, \sigma_{\mu\nu}\, G_{\mu\nu}\, \psi_d$$

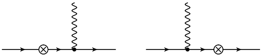
Feynman Diagrams

- $\langle \psi^R \, \mathcal{O}_1^R \, \overline{\psi}^R \rangle_{\mathrm{amp}} = Z_\psi \, \sum_{i=1}^{13} (Z^{-1})_{1i} \langle \psi \, \mathcal{O}_i \, \overline{\psi} \rangle_{\mathrm{amp}}$

- $\langle \psi^R \, \mathcal{O}_1^R \, \overline{\psi}^R A_\nu^R \rangle_{\mathrm{amp}} = Z_\psi \, Z_A^{1/2} \sum_{i=1}^{13} (Z^{-1})_{1i} \langle \psi \, \mathcal{O}_i \, \overline{\psi} \, A_\nu \rangle_{\mathrm{amp}}$







$\mathcal{O}_i, \; i = 1, \ldots, 13$

Dimensional Regularization and $\overline{\mathrm{MS}}$

- 2-pt: $\langle \psi^R \, \mathcal{O}_1^R \, \overline{\psi}^R \rangle_{\mathrm{amp}} = Z_\psi \, \sum_{i=1}^{10} (Z^{-1})_{1i} \langle \psi \, \mathcal{O}_i \, \overline{\psi} \rangle_{\mathrm{amp}}$
- General form of $\mathcal{O}(1/\epsilon)$ part:

$$\langle \psi \, \mathcal{O}_1 \, \overline{\psi} \rangle_{\mathrm{amp}}^{1-loop} \Big|_{1/\epsilon} = \rho_1 \, (q_s^2 + q_d^2) + \rho_2 \, (m_s^2 + m_d^2) + \rho_3 \, i \, (m_d \not{q}_d + m_s \not{q}_s) + \rho_4 \, i \, (m_s \not{q}_d + m_d \not{q}_s) + \rho_5 \, q_s \cdot q_d + \rho_6 \, \not{q}_s \, \not{q}_d + \rho_7 \, m_s m_d$$

- 3-pt: $\langle \psi^R \, \mathcal{O}_1^R \, \overline{\psi}^R A_\nu^R \rangle_{\mathrm{amp}} = Z_\psi \, Z_A^{1/2} \sum_{i=1}^{10} (Z^{-1})_{1i} \langle \psi \, \mathcal{O}_i \, \overline{\psi} \, A_\nu \rangle_{\mathrm{amp}}$
- General form of $\mathcal{O}(1/\epsilon)$ part (1PI):

$$\langle \psi \, \mathcal{O}_1 \, \overline{\psi} \, A_\nu \rangle_{\mathrm{amp}}^{1-loop} \Big|_{1/\epsilon} = R_1 \, g \, (q_s + q_d)_\nu + R_2 \, g \, (\gamma_\nu \not{q}_d + \not{q}_s \gamma_\nu) + R_3 \, i \, g \, (m_s + m_d) \gamma_\nu + R_4 \, (-2i \, g \, \sigma_{\rho\nu} q_{A\rho})$$

- 2^{nd} / 1^{st} degree polynomial in m_s, m_d, q_s, q_d, q_A
- Non-polynomial $\mathcal{O}(1/\epsilon)$ terms in 1PR-part of 3-pt relation: Cancel by virtue of the 2-pt relation
- Since $Z_{11} = 1 + (g^2/\epsilon) z_1$ and $Z_{1i} = (g^2/\epsilon) z_i$ ($i > 1$): 11 equations for 10 unknowns. Consistent.
- $\mathcal{O}(\epsilon^0)$ part of rhs: Renormalized Green's function

Dimensional Regularization and $\overline{\text{MS}}$

- Computing $\rho_i,\ R_i$ to 1 loop:

$$\rho_1 = \frac{g^2\, C_F}{16\,\pi^2}\,\frac{1}{\epsilon}(-3) \quad \rho_2 = \frac{g^2\, C_F}{16\,\pi^2}\,\frac{1}{\epsilon}(-6) \quad \rho_3 = \frac{g^2\, C_F}{16\,\pi^2}\,\frac{1}{\epsilon}(3)$$

$$\rho_4 = \rho_5 = \rho_6 = \rho_7 = 0$$

$$R_1 = \frac{g^2\, C_F}{16\,\pi^2}\,\frac{1}{\epsilon}(-6) \qquad R_3 = \frac{g^2}{16\,\pi^2}\,\frac{1}{\epsilon}\left(-\frac{3}{2\,N_c} + \frac{3\,N_c}{4}\right)$$

$$R_2 = \frac{g^2}{16\,\pi^2}\,\frac{1}{\epsilon}\left(\frac{3\,N_c}{4}\right) \quad R_4 = \frac{g^2}{16\,\pi^2}\,\frac{1}{\epsilon}\left(\frac{1}{N_c} - \frac{\alpha}{2\,N_c} + \frac{7\,N_c}{4} + \frac{3\,\alpha\,N_c}{4}\right)$$

- N_c : number of colors, $C_F = (N_c^2 - 1)/(2N_c)$
- α : gauge parameter

Z_i in Dimensional Regularization and $\overline{\mathrm{MS}}$

- Solving for $Z_i^{DR,\overline{\mathrm{MS}}}$

$$Z_1^{DR,\overline{\mathrm{MS}}} = 1 + \frac{g^2}{16\,\pi^2}\ \frac{1}{\epsilon}\left(-\frac{N_c}{2} + \frac{5}{2\,N_c}\right) \qquad Z_6^{DR,\overline{\mathrm{MS}}} = 0$$

$$Z_2^{DR,\overline{\mathrm{MS}}} = \frac{g^2}{16\,\pi^2}\ \frac{1}{\epsilon}\left(-3\,N_c + \frac{3}{N_c}\right) \qquad Z_7^{DR,\overline{\mathrm{MS}}} = \frac{g^2}{16\,\pi^2}\ \frac{1}{\epsilon}\left(-\frac{3\,N_c}{4} + \frac{3}{2\,N_c}\right)$$

$$Z_3^{DR,\overline{\mathrm{MS}}} = 0 \qquad Z_8^{DR,\overline{\mathrm{MS}}} = 0$$

$$Z_4^{DR,\overline{\mathrm{MS}}} = 0 \qquad Z_9^{DR,\overline{\mathrm{MS}}} = \frac{g^2}{16\,\pi^2}\ \frac{1}{\epsilon}\left(\frac{3\,N_c}{4} - \frac{3}{2\,N_c}\right)$$

$$Z_5^{DR,\overline{\mathrm{MS}}} = \frac{g^2}{16\,\pi^2}\ \frac{1}{\epsilon}\left(\frac{2\,N_c}{3} - \frac{3}{N_c}\right) \qquad Z_{10}^{DR,\overline{\mathrm{MS}}} = \frac{g^2}{16\,\pi^2}\ \frac{1}{\epsilon}\left(\frac{3\,N_c}{2} - \frac{3}{2\,N_c}\right)$$

- $Z_2 \neq 0$ (also $Z_3 \neq 0$, $Z_4 \neq 0$ beyond 1-loop)
  $\Rightarrow$ Mixing even on-shell
- $Z_9 \neq 0$, $Z_{10} \neq 0$: Non-gauge invariant operators DO mix

Z_i in Dimensional Regularization and $\overline{\text{MS}}$

- Solving for $Z_i^{DR,\overline{\text{MS}}}$

$$Z_1^{DR,\overline{\text{MS}}} = 1 + \frac{g^2}{16\,\pi^2}\;\frac{1}{\epsilon}\;\left(-\frac{N_c}{2} + \frac{5}{2\,N_c}\right) \qquad Z_6^{DR,\overline{\text{MS}}} = 0$$

$$Z_2^{DR,\overline{\text{MS}}} = \frac{g^2}{16\,\pi^2}\;\frac{1}{\epsilon}\;\left(-3\,N_c + \frac{3}{N_c}\right) \qquad Z_7^{DR,\overline{\text{MS}}} = \frac{g^2}{16\,\pi^2}\;\frac{1}{\epsilon}\;\left(-\frac{3\,N_c}{4} + \frac{3}{2\,N_c}\right)$$

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$$Z_5^{DR,\overline{\text{MS}}} = \frac{g^2}{16\,\pi^2}\;\frac{1}{\epsilon}\;\left(\frac{2\,N_c}{3} - \frac{3}{N_c}\right) \qquad Z_{10}^{DR,\overline{\text{MS}}} = \frac{g^2}{16\,\pi^2}\;\frac{1}{\epsilon}\;\left(\frac{3\,N_c}{2} - \frac{3}{2\,N_c}\right)$$

- By–product of $Z_1^{DR,\overline{\text{MS}}}$ (well-known):
  Anomalous dimension $\widetilde{\gamma}_{CM}$ for the operator $\widetilde{\mathcal{O}}_{CM}$

$$\widetilde{\gamma}_{CM} = \frac{g^2}{16\,\pi^2}\left(4\,N_c - \frac{8}{N_c}\right)$$

Lattice Regularization and $\overline{\rm MS}$

- Formally, same equations as before:

$$\langle\psi^R\,\mathcal{O}_1^R\,\overline{\psi}^R\rangle_{\rm amp} = Z_\psi\,\sum_{i=1}^{13}(Z^{-1})_{1i}\langle\psi\,\mathcal{O}_i\,\overline{\psi}\rangle_{\rm amp}$$

$$\langle\psi^R\,\mathcal{O}_1^R\,\overline{\psi}^R A_\nu^R\rangle_{\rm amp} = Z_\psi\,Z_A^{1/2}\sum_{i=1}^{13}(Z^{-1})_{1i}\langle\psi\,\mathcal{O}_i\,\overline{\psi}\,A_\nu\rangle_{\rm amp}$$

- lhs : Regularization independent; we evaluated it in DR
- Bare Green's functions now refer to Lattice regularization
- $Z_{ij} \equiv Z_{ij}^{L,\overline{\rm MS}}$, $Z_\psi \equiv Z_\psi^{L,\overline{\rm MS}}$, $Z_A \equiv Z_A^{L,\overline{\rm MS}}$
  (Prerequisite: Evaluate Z_ψ, Z_A beforehand... also Z_m , Z_g)
- Renormalizability $\Longrightarrow \langle\psi^R\,\mathcal{O}_1^R\,\overline{\psi}^R\rangle_{\rm amp} - \langle\psi\,\mathcal{O}_1\,\overline{\psi}\rangle_{\rm amp}$
  is polynomial in m's, q's :
  $2^{\rm nd}$ degree, but also $a^{-1}\cdot(1^{\rm st})$, $a^{-2}\cdot(0^{\rm th})$)
- Similarly for $\langle\psi^R\,\mathcal{O}_1^R\,\overline{\psi}^R A_\nu^R\rangle_{\rm amp} - \langle\psi\,\mathcal{O}_1\,\overline{\psi}\,A_\nu\rangle_{\rm amp}$:
  $1^{\rm st}$, but also $a^{-1}\cdot(0^{\rm th})$ degree polynomial
- $lhs = rhs \Longrightarrow$ the values of Z_{ij} must be determined in a way as to absorb this polynomial difference

Lattice Regularization and $\overline{\mathrm{MS}}$

- General form of $\langle\psi^R\,\mathcal{O}_1^R\,\overline{\psi}^R\rangle_{\rm amp} - \langle\psi\,\mathcal{O}_1\,\overline{\psi}\rangle_{\rm amp}$:

$$\begin{aligned} & \rho_1\,(q_s^2+q_d^2) + \rho_2\,(m_s^2+m_d^2) + \rho_3\,i\,(m_d\not{q}_d+m_s\not{q}_s) \\ + \quad & \rho_4\,i\,(m_s\not{q}_d+m_d\not{q}_s) + \rho_5\,q_s\cdot q_d + \rho_6\,\not{q}_s\,\not{q}_d + \rho_7\,m_s m_d \\ + \quad & \boldsymbol{\rho_8}\,(r_d\,\gamma_5\,\not{q}_d+r_s\,\not{q}_s\,\gamma_5) + \boldsymbol{\rho_9}\,i\,(r_d m_d+r_s m_s)\,\gamma_5 + \boldsymbol{\rho_{10}}\cdot 1 \end{aligned}$$

- General form of $\langle\psi^R\,\mathcal{O}_1^R\,\overline{\psi}^R A_\nu^R\rangle_{\rm amp} - \langle\psi\,\mathcal{O}_1\,\overline{\psi}\,A_\nu\rangle_{\rm amp}$:

$$\begin{aligned} & R_1\,g\,(q_s+q_d)_\nu + R_2\,g\,(\gamma_\nu\not{q}_d + \not{q}_s\gamma_\nu) \\ + \quad & R_3\,i\,g\,(m_s+m_d)\gamma_\nu + R_4\,(-2i\,g\,\sigma_{\rho\nu}q_{A\rho}) \\ + \quad & \mathbf{R_5}\,g\,(r_d - r_s)\,\gamma_5\,\gamma_\nu \end{aligned}$$

- ρ_i, R_i : independent of q, m. But depend on $a,\ \bar{\mu}$: $a^{-2},\ a^{-1},\ \log(a\,\bar{\mu})$ terms
- ρ_i, R_i also depend on: N_c , α, Symanzik coefficients.
- 15 equations for $Z_{1i},\ (i = 1,\ldots,13)$. Consistent.

Lattice Regularization and $\overline{\mathrm{MS}}$

- Evaluation of: $\langle \psi^R \, \mathcal{O}_1^R \, \overline{\psi}^R \rangle_{\mathrm{amp}} - \langle \psi \, \mathcal{O}_1 \, \overline{\psi} \rangle_{\mathrm{amp}}$

  and: $\langle \psi^R \, \mathcal{O}_1^R \, \overline{\psi}^R A_\nu^R \rangle_{\mathrm{amp}} - \langle \psi \, \mathcal{O}_1 \, \overline{\psi} \, A_\nu \rangle_{\mathrm{amp}}$
- Checks:
  - Coefficients of $\log(a)$:
    Must coincide with coefficients of $-1/(2\epsilon)$ in DR $\checkmark$
  - Polynomial dependence on m, q
    $\Rightarrow$ 2-pt Green's function $\checkmark$
    $\Rightarrow$ 3-pt Green's function: extremely complicated!
    (Even in massless case: Spence fn's of q_s, q_d, q_A)
    Still working on this $???$
- Easy way out: Evaluate 3-pt Green's function making any (nondegenerate) choice of q_s, q_d, q_A

Lattice Regularization and $\overline{\mathrm{MS}}$

- Evaluation of: $\langle\psi^R\,\mathcal{O}_1^R\,\overline{\psi}^R\rangle_{\rm amp} - \langle\psi\,\mathcal{O}_1\,\overline{\psi}\rangle_{\rm amp}$

  and: $\langle\psi^R\,\mathcal{O}_1^R\,\overline{\psi}^R\,A_\nu^R\rangle_{\rm amp} - \langle\psi\,\mathcal{O}_1\,\overline{\psi}\,A_\nu\rangle_{\rm amp}$
- Checks:
  - Coefficients of $\log(a)$:
    Must coincide with coefficients of $-1/(2\epsilon)$ in DR $\checkmark$
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    $\Rightarrow$ 2-pt Green's function $\checkmark$
    $\Rightarrow$ 3-pt Green's function: extremely complicated!
    (Even in massless case: Spence fn's of q_s, q_d, q_A)
    Still working on this ???
- Easy way out: Evaluate 3-pt Green's function making any (nondegenerate) choice of q_s, q_d, q_A , for example:

 Democratic choice: $q_s - q_d + q_A = 0,\ q_s^2 = q_d^2 = q_A^2 = \bar{\mu}^2$ $\checkmark$

Results for $Z_i^{L,\overline{MS}}$

- From 2-pt Green's function: $[\tilde{g}^2 \equiv (g^2 C_F)/(16\pi^2)]$

$\mathcal{O}_1 = g\,\overline{\psi}_s \sigma_{\mu\nu} G_{\mu\nu} \psi_d$	$Z_1 = ???$
$\mathcal{O}_2 = (m_d^2 + m_s^2)\overline{\psi}_s \psi_d$	$Z_2 = \tilde{g}^2(2.7677 + 6\,\log(a^2\bar{\mu}^2))$
$\mathcal{O}_3 = m_d\, m_s \overline{\psi}_s \psi_d$	$Z_3 = 0$
$\mathcal{O}_4 = \overline{\psi}_s \overleftarrow{D}_\mu \overrightarrow{D}_\mu \psi_d$	$Z_4 = 0$
$\mathcal{O}_5 = \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)(\overrightarrow{\not{D}} + m_d)\psi_d$	$Z_5 = ???$
$\mathcal{O}_6 = \overline{\psi}_s(\overrightarrow{\not{D}} + m_d)^2 \psi_d + \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)^2 \psi_d$	$Z_6 = ???$
$\mathcal{O}_7 = m_s \overline{\psi}_s(\overrightarrow{\not{D}} + m_d)\psi_d + m_d \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)\psi_d$	$Z_7 = -Z_5/2$
$\mathcal{O}_8 = m_d \overline{\psi}_s(\overrightarrow{\not{D}} + m_d)\psi_d + m_s \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)\psi_d$	$Z_8 = \tilde{g}^2(-3.9654) - Z_6$
$\mathcal{O}_9 = \overline{\psi}_s \overleftarrow{\not{\partial}}(\overrightarrow{\not{D}} + m_d)\psi_d - \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)\overrightarrow{\not{\partial}}\psi_d$	$Z_9 = Z_5/2$
$\mathcal{O}_{10} = \overline{\psi}_s \overrightarrow{\not{\partial}}(\overrightarrow{\not{D}} + m_d)\psi_d - \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)\overleftarrow{\not{\partial}}\psi_d$	$Z_{10} = \tilde{g}^2(5.5060 - 3\,\log(a^2\bar{\mu}^2)) - Z_6$
$\mathcal{O}_{11} = i\, r_d\, \overline{\psi}_s \gamma_5 (\overrightarrow{\not{D}} + m_d)\psi_d + i\, r_s\, \overline{\psi}_s(-\overleftarrow{\not{D}} + m_s)\gamma_5 \psi_d$	$Z_{11} = a^{-1}\tilde{g}^2(-4.0309)$
$\mathcal{O}_{12} = i\,(r_d\, m_d + r_s\, m_s)\overline{\psi}_s \gamma_5 \psi_d$	$Z_{12} = a^{-1}\tilde{g}^2(+4.0309)$
$\mathcal{O}_{13} = \overline{\psi}_s\, \psi_d$	$Z_{13} = a^{-2}\tilde{g}^2(47.793)$

- From 3-pt Green's function: 3 independent conditions $\Rightarrow$ fix the 3 remaining unknowns
- Z_3, Z_4 : nonzero beyond 1-loop

Results for $Z_i^{L,\overline{MS}}$

- Mixing with lower dimensional operators:

$$\mathcal{O}_{11}=i\,r_d\,\overline{\psi}_s\gamma_5(\overrightarrow{\not{D}}+m_d)\psi_d+i\,r_s\,\overline{\psi}_s(-\overleftarrow{\not{D}}+m_s)\gamma_5\psi_d \qquad Z_{11}=a^{-1}\tilde{g}^2(-4.0309)$$

$$\mathcal{O}_{12}=i\,(r_d\,m_d+r_s\,m_s)\overline{\psi}_s\gamma_5\psi_d \qquad Z_{12}=a^{-1}\tilde{g}^2(+4.0309)$$

$$\mathcal{O}_{13}=\overline{\psi}_s\,\psi_d \qquad Z_{13}=a^{-2}\tilde{g}^2(47.793)$$

- Perturbation theory: Only a ballpark estimate in such cases
- Non-perturbative estimates: Impose conditions such as

$$\lim_{m_s\,,\,m_d\to 0}\langle\pi(0)|\mathcal{O}_1^{\rm sub}|K(0)\rangle = \lim_{m_s\,,\,m_d\to 0}\langle\pi(0)|\mathcal{O}_1+\frac{c_1}{a^2}\mathcal{O}_{13}|K(0)\rangle = 0$$

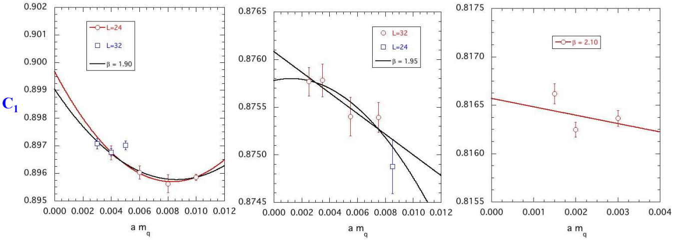
$$\langle 0|\mathcal{O}_1^{\rm sub}|K(0)\rangle_{m_s\,,\,m_d} = \langle 0|\mathcal{O}_1 + \frac{c_1}{a^2}\mathcal{O}_{13} + \frac{c_2}{a}\mathcal{O}_{12}|K(0)\rangle_{m_s\,,\,m_d} = 0$$

- $\mathcal{O}_{11}$ will vanish on shell

PRELIMINARY Results for c_1 , at various g

$$\hat{O}_{CM} = Z_{CM} O_{CM}^{sub} = Z_{CM} \left(O_{CM}^{bare} + \frac{c_1}{a^2} S + \frac{c_2}{a} (m_s + m_d) P + \right.$$
$$\left. + [c_3 (m_s^2 + m_d^2) + c_4 m_s m_d] S \right)$$

- The estimates fit quite well a g^2 dependence



- $\Rightarrow$ NP mixing of $\mathcal{O}_{13} = \overline{\psi}_s \psi_d$: $Z_{13} = a^{-2} \tilde{g}^2 (33.7)$
- Compare with perturbative result: $Z_{13} = a^{-2} \tilde{g}^2 (47.793)$
- As close as it gets for power divergent coefficients

Checks — Extensions

- Compare Renormalized Green's Functions in different Regularizations
- Check Consistency with 4-pt Green's functions
- Estimate Renormalization Coefficients Non-perturbatively
  - Power-divergent coefficients
  - Coefficients of dimension-5 operators
- Improved Perturbation theory (”boosted”, ”cactus”, ...)
- Compute $\mathcal{O}(a^2 g^2)$ corrections to Green's functions
  $\Rightarrow$ Improve Non-perturbative estimates

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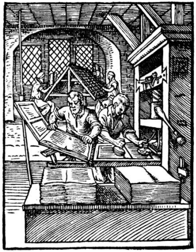

Vielen Dank