Renormalization of HQET $\Delta B = 2$ operators: $O(a)$ improvement and $\frac{1}{m}$ matching with QCD

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Motivations: CP-violation in the SM

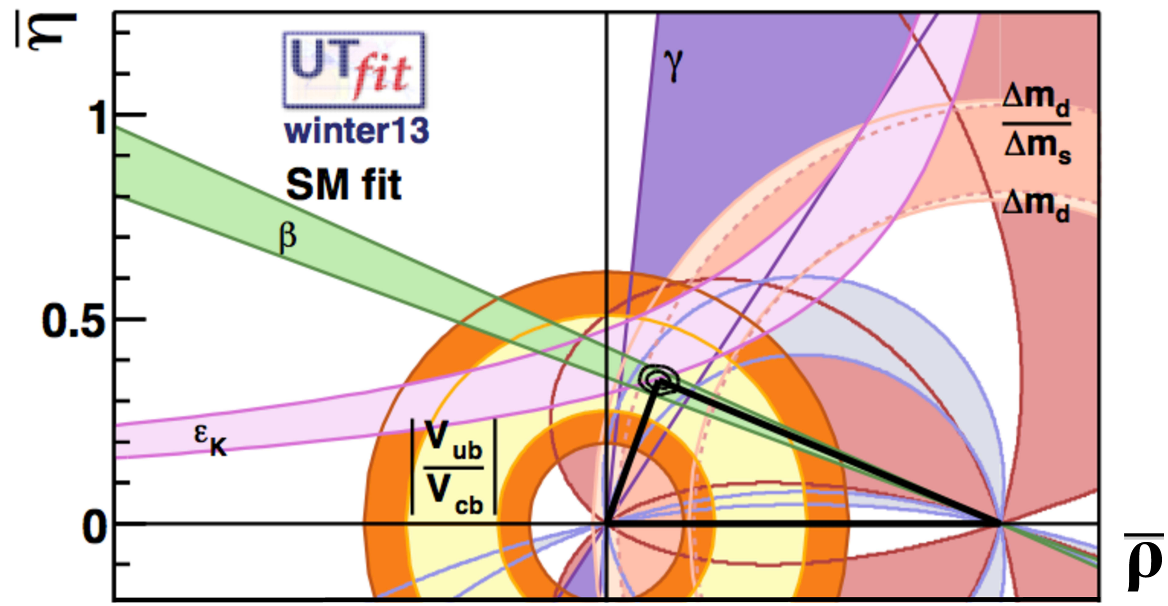
$$\Delta m_d = |\langle \bar{B}^0_d | \mathcal{H}_{\rm eff}^{\Delta B=2} | B^0_d \rangle| / m_{B_d} \propto f^2_{B_d} \, B_{B_d} \, A^2 \lambda^6 \big[(1-\bar\rho)^2 + \bar\eta^2 \big]$$

$$\frac{\Delta m_d}{\Delta m_s} \propto \frac{f^2_{B_d} \, B_{B_d}}{f^2_{B_s} \, B_{B_s}} \frac{|V_{td}|^2}{|V_{ts}|^2} \simeq$$

$$\xi^2 \, \frac{\lambda^2}{\left(1-\frac{\lambda^2}{2}\right)^2} \big[(1-\bar\rho)^2 + \bar\eta^2 \big]$$

$$a_{J/\psi K_s}(t) = \sin 2\beta \, \sin(\Delta m_d t)$$

$$2\beta = \arg \langle \bar{B}^0_d | \mathcal{H}_{\rm eff}^{\Delta B=2} | B^0_d \rangle$$



$$B_{B_q}(\mu) = \frac{\langle \bar{B}^0_q | \mathcal{O}_{LL}^{\Delta B=2}(\mu) | B^0_q \rangle}{\frac{8}{3} f^2_{B_q} m^2_{B_q}} \qquad \text{with} \qquad \mathcal{O}_{LL}^{\Delta B=2} = \bar{b} \gamma_\mu (1-\gamma_5) q \, \bar{b} \gamma_\mu (1-\gamma_5) q$$

Non-SM CP-violation could be detected as an inconsistency between the positions of the upper vertex as determined by different kinds of physics $\Rightarrow$ important to compute $f_{B_{d,s}}$, $B_{B_{d,s}}$ accurately!

Non-perturbative HQET

b-quark is too heavy ($m_b a > 1$) $\Rightarrow$ we use HQET for the b-quark and we want to compute at $\mathrm{O}(1/m)$ by matching with QCD in finite volume [Heitger & Sommer, 2004]

Heavy quark and anti-quark fields ψ_{h}, $\psi_{\bar{\mathrm{h}}}$ are now independent

$$P_+\psi_{\mathrm{h}} = \psi_{\mathrm{h}}\,, \quad \overline{\psi}_{\mathrm{h}}P_+ = \overline{\psi}_{\mathrm{h}}\,, \quad P_+ = \tfrac{1}{2}(1+\gamma_0), \quad H \equiv \psi_{\mathrm{h}} + \psi_{\bar{\mathrm{h}}}$$
$$P_-\psi_{\bar{\mathrm{h}}} = \psi_{\bar{\mathrm{h}}}\,, \quad \overline{\psi}_{\bar{\mathrm{h}}}P_- = \overline{\psi}_{\bar{\mathrm{h}}}\,, \quad P_- = \tfrac{1}{2}(1-\gamma_0), \quad \bar{H} \equiv \overline{\psi}_{\mathrm{h}} + \overline{\psi}_{\bar{\mathrm{h}}}$$

$$S_{\mathrm{HQET}} = a^4 \sum_x \{\bar{H}\gamma_0 D_0 H - \omega_{\mathrm{spin}} \underbrace{\bar{H}\sigma\cdot\mathbf{B}H}_{\mathcal{O}_{\mathrm{spin}}} - \omega_{\mathrm{kin}} \underbrace{\bar{H}\tfrac{1}{2}\nabla^2 H}_{\mathcal{O}_{\mathrm{kin}}} + \mathrm{O}(1/m^2)\}$$

also the composite fields have a $1/m$ expansion in the effective theory

$$\mathcal{O}^{\mathrm{QCD}} = Z_{\mathcal{O}}^{\mathrm{HQET}}\{\mathcal{O}^{\mathrm{stat}} + c_{\mathcal{O}}^{i}\mathcal{O}_i^{1/m} + \mathrm{O}(1/m^2)\}$$

with $\omega_{\mathrm{kin}} = \mathrm{O}(1/m)$, $\omega_{\mathrm{spin}} = \mathrm{O}(1/m)$, $c_{\mathcal{O}}^{i} = \mathrm{O}(1/m)$ and $Z_{\mathcal{O}}^{\mathrm{HQET}}$ to be determined.

in the path integral one expands the action in powers of $1/m$ and considers the higher orders (e.g. $\mathcal{O}_{\rm spin}$, $\mathcal{O}_{\rm kin}$) only as operator insertions

$$\begin{aligned}\langle\mathcal{O}\rangle^{\rm QCD} &= Z_{\mathcal{O}}^{\rm HQET}\{\langle\mathcal{O}^{\rm stat}\rangle_{\rm stat} + \omega_{\rm spin}\, a^4\sum_x\langle\mathcal{O}^{\rm stat}\mathcal{O}_{\rm spin}\rangle_{\rm stat} + \\ &+ \omega_{\rm kin}\, a^4\sum_x\langle\mathcal{O}^{\rm stat}\mathcal{O}_{\rm kin}\rangle_{\rm stat} + c_{\mathcal{O}}^i\langle\mathcal{O}_i^{1/m}\rangle_{\rm stat} + {\rm O}(1/m^2)\}\end{aligned}$$

at a fixed order in $1/m$ the theory is renormalizable (i.e. the continuum limit exists) and all the divergences can be reabsorbed by the parameters $m_{\rm bare}$, $Z_{\mathcal{O}}^{\rm HQET}$, $c_{\mathcal{O}}^i$, $\omega_{\rm spin}$, $\omega_{\rm kin}$ appearing at that order.

The presence of power divergences requires non-perturbative renormalization.

Non-perturbative matching between QCD and HQET

Determine the bare couplings of HQET at order n ($m_{\rm bare}$, $\omega_{\rm kin}$, $\omega_{\rm spin}$, $c_{\rm A}^{\rm HQET}$, $Z_{\rm A}^{\rm HQET}$, $\ldots$) by imposing:

$$\Phi_k^{\rm QCD}(M) = \Phi_k^{\rm HQET}(M) + O\left(\frac{1}{M^{n+1}}\right) \,, \quad k = 1, 2, \ldots, N_n^{\rm HQET}$$

In the l.h.s. the heavy quark must be treated relativistically: use observables Φ_k defined in finite volume and the fact that the parameters of the QCD and HQET lagrangians are independent of the volume.

Φ_k defined on $L = L_1 \approx 0.4\,\mathrm{fm} \ll 2\,\mathrm{fm} \Rightarrow$ simulate very fine a's where $m_{\rm b} a \ll 1$ and $1/(m_b L_1) \ll 1$ (to have a well behaved $1/m$-expansion in finite volume).

Physical observables (e.g. $B_{\rm B_s}$, $F_{\rm B_s}$) need a large volume, such that the B-meson fits comfortably: $L \approx 4L_1 \approx 1.6\,\mathrm{fm}$

Connection between L_1 and $4L_1$ achieved recursively in HQET using the Schrödinger Functional: no relativistic b-quark any more $\Rightarrow$ no problems with the size of a.

$\Delta B = 2$ operators in HQET

A complete basis of dimension 6 $\Delta B = 2$ operators (with two relativistic light quarks ψ of the same flavor) is:

$$
\begin{array}{rclrcl}
Q_1 & = & \mathcal{O}_{\mathrm{VV+AA}} & \mathcal{Q}_1 & = & \mathcal{O}_{\mathrm{VA+AV}} \\
Q_2 & = & \mathcal{O}_{\mathrm{SS+PP}} & \mathcal{Q}_2 & = & \mathcal{O}_{\mathrm{SP+PS}} \\
Q_3 & = & \mathcal{O}_{\mathrm{VV-AA}} & \mathcal{Q}_3 & = & \mathcal{O}_{\mathrm{VA-AV}} \\
Q_4 & = & \mathcal{O}_{\mathrm{SS-PP}} & \mathcal{Q}_4 & = & \mathcal{O}_{\mathrm{SP-PS}}
\end{array}
$$

$$
\text{with} \quad \mathcal{O}_{\Gamma_1\Gamma_2\pm\Gamma_2\Gamma_1} = \tfrac{1}{2}\left[(\bar{\psi}_{\mathrm{h}}\Gamma_1\psi)(\bar{\psi}_{\bar{\mathrm{h}}}\Gamma_2\psi) \pm (\bar{\psi}_{\mathrm{h}}\Gamma_2\psi)(\bar{\psi}_{\bar{\mathrm{h}}}\Gamma_1\psi)\right]
$$

Under the discrete axial transformation $\psi \to \gamma_5\psi$, $\bar{\psi} \to -\bar{\psi}\gamma_5$ we have:

$$
\begin{array}{ccccccccccccc}
Q_1 & \to & Q_1 & \quad & Q_2 & \to & Q_2 & \quad & Q_3 & \to & -Q_3 & \quad & Q_4 & \to & -Q_4 \\
\mathcal{Q}_1 & \to & \mathcal{Q}_1 & \quad & \mathcal{Q}_2 & \to & \mathcal{Q}_2 & \quad & \mathcal{Q}_3 & \to & -\mathcal{Q}_3 & \quad & \mathcal{Q}_4 & \to & -\mathcal{Q}_4
\end{array}
$$

The heavy quark action is to invariant anymore on $H(4)$ rotations but only on $H(3)$ spatial rotations and heavy quark spin symmetry (HQSS):

$$\psi_{\mathrm{h}}(x) \;\rightarrow\; e^{i\alpha_k \mathcal{S}_k}\,\psi_{\mathrm{h}}(x) \qquad \bar{\psi}_{\mathrm{h}}(x) \;\rightarrow\; \bar{\psi}_{\mathrm{h}}(x)\,e^{-i\alpha_k \mathcal{S}_k}$$

where $\mathcal{S}_k = \frac{1}{2}\epsilon_{ijk}\sigma_{ij} = \frac{i}{2}\epsilon_{ijk}\gamma_i\gamma_j$, and $\psi_{\bar{\mathrm{h}}}$ and $\bar{\psi}_{\bar{\mathrm{h}}}$ transform analogously with β_k independent of α_k.

With light fermions which respect chiral symmetry, by using HQSS one can prove that the operators in the new basis

$$\begin{aligned} Q'_1 &= Q_1 & \qquad \mathcal{Q}'_1 &= \mathcal{Q}_1 \\ Q'_2 &= Q_1 + 4Q_2 & \qquad \mathcal{Q}'_2 &= \mathcal{Q}_1 + 4\mathcal{Q}_2 \\ Q'_3 &= Q_3 + 2Q_4 & \qquad \mathcal{Q}'_3 &= \mathcal{Q}_3 + 2\mathcal{Q}_4 \\ Q'_4 &= Q_3 - 2Q_4 & \qquad \mathcal{Q}'_4 &= \mathcal{Q}_3 - 2\mathcal{Q}_4 \end{aligned}$$

renormalize multiplicatively [Becirevic & Reyes, 2003; Palombi,Papinutto,Pena,Wittig, 2005]

However, by using Wilson-like fermions chiral symmetry is explicitly broken and for the PE operators we have

$$\begin{pmatrix} Q'_1 \\ Q'_2 \\ Q'_3 \\ Q'_4 \end{pmatrix}_{\mathrm{R}} = \begin{pmatrix} Z_1 & 0 & 0 & 0 \\ 0 & Z_2 & 0 & 0 \\ 0 & 0 & Z_3 & 0 \\ 0 & 0 & 0 & Z_4 \end{pmatrix} \left[\mathbb{1} + \begin{pmatrix} 0 & 0 & \Delta_1 & 0 \\ 0 & 0 & 0 & \Delta_2 \\ \Delta_3 & 0 & 0 & 0 \\ 0 & \Delta_4 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} Q'_1 \\ Q'_2 \\ Q'_3 \\ Q'_4 \end{pmatrix}$$

In the PO sector, time reversal $\Rightarrow$ $\mathcal{Q}'_1$, $\mathcal{Q}'_2$, $\mathcal{Q}'_3$, $\mathcal{Q}'_4$ renormalize multiplicityively.

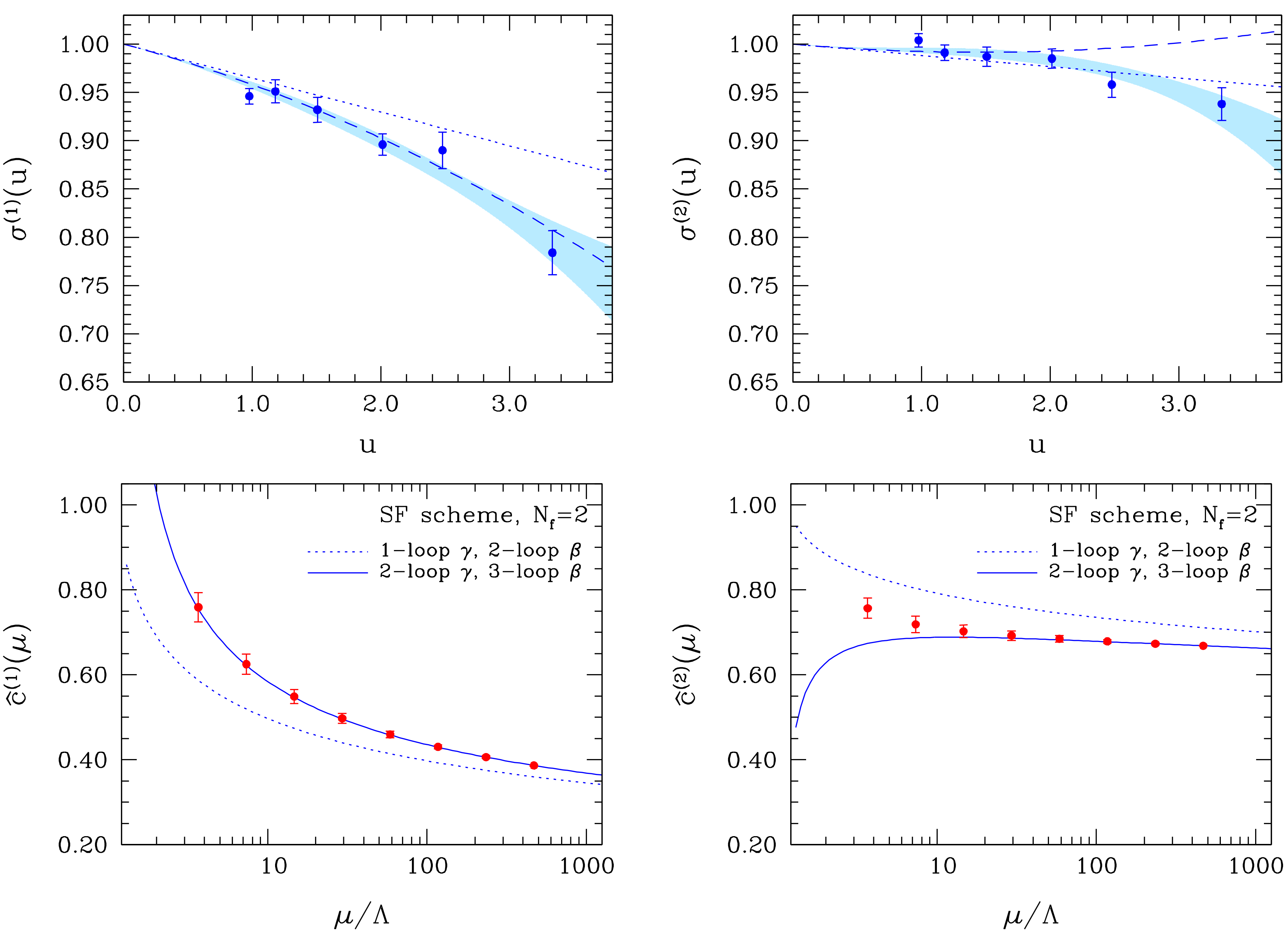
Since we are interested in computing the matrix element of $\mathcal{O}_{LL}^{\Delta B=2}$ (which under the axial transformation takes a plus sign) $\Rightarrow$ in matching QCD to HQET we will need (for the static part) only $(\mathcal{Q}'_1)^{\mathrm{R}}$ and $(\mathcal{Q}'_2)^{\mathrm{R}}$.

We thus use twisted mass QCD for the light quarks (at maximal twist). In this way $\langle\tilde{\mathcal{O}}_{\mathrm{R}}\rangle_{(m_r,0)} = \langle\mathcal{O}_{\mathrm{R}}\rangle_{(0,\mu_r)}$ where

$$\begin{array}{rclrcl} \tilde{\mathcal{O}}_{\mathrm{VV+AA}} & = & -i\mathcal{O}_{\mathrm{VA+AV}} & \tilde{\mathcal{O}}_{\mathrm{SS+PP}} & = & -i\mathcal{O}_{\mathrm{SP+PS}} \\ \tilde{\mathcal{O}}_{\mathrm{VV-AA}} & = & \mathcal{O}_{\mathrm{VV-AA}} & \tilde{\mathcal{O}}_{\mathrm{SS-PP}} & = & \mathcal{O}_{\mathrm{SS-PP}} \end{array}$$

and thus the first two operators $\tilde{\mathcal{Q}}'_1$ and $\tilde{\mathcal{Q}}'_2$ renormalize multiplicatively and have the correct chiral properties.

We have already computed non-perturbatively the renormalization constants and the running for $\mathcal{Q}'_1$ and $\mathcal{Q}'_2$ with $N_f = 0$ and $N_f = 2$ dynamical flavors: [Palombi,Papinutto,Pena,Wittig, 2007; Dimopoulos,Herdoiza,Palombi,Papinutto,Pena,Vladikas,Wittig 2008]


1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.0
1.0
2.0
3.0
u
$\sigma^{(1)}(u)$
$\sigma^{(2)}(u)$
1.00
0.80
0.60
0.40
0.20
10
100
1000
μ/Λ
$\hat{c}^{(1)}(\mu)$
$\hat{c}^{(2)}(\mu)$
SF scheme, $N_f=2$
1−loop γ, 2−loop β
2−loop γ, 3−loop β

O(a) improvement

We first consider the improvement terms in the massless limit. After using the equations of motion, Fierz identities and the properties of heavy quark fields (including local flavour symmetry) we obtain a basis of dimension 7 PO operators with the correct time reversal properties:

$$
\begin{aligned}
\delta\mathcal{Q}_1 &= \gamma_i \otimes \gamma_i\gamma_5(\boldsymbol{\gamma}\cdot\mathbf{D}) + \gamma_i\gamma_5(\boldsymbol{\gamma}\cdot\mathbf{D}) \otimes \gamma_i \\
\delta\mathcal{Q}_2 &= \gamma_i\gamma_5 \otimes \gamma_i(\boldsymbol{\gamma}\cdot\mathbf{D}) + \gamma_i(\boldsymbol{\gamma}\cdot\mathbf{D}) \otimes \gamma_i\gamma_5 \\
\delta\mathcal{Q}_3 &= \gamma_0 \otimes \gamma_0\gamma_5(\boldsymbol{\gamma}\cdot\mathbf{D}) + \gamma_0\gamma_5(\boldsymbol{\gamma}\cdot\mathbf{D}) \otimes \gamma_0 \\
\delta\mathcal{Q}_4 &= \gamma_0\gamma_5 \otimes \gamma_0(\boldsymbol{\gamma}\cdot\mathbf{D}) + \gamma_0(\boldsymbol{\gamma}\cdot\mathbf{D}) \otimes \gamma_0\gamma_5 \\
\delta\mathcal{Q}_5 &= \gamma_i \otimes (\boldsymbol{\gamma}\cdot\mathbf{D})\gamma_i\gamma_5 + (\boldsymbol{\gamma}\cdot\mathbf{D})\gamma_i\gamma_5 \otimes \gamma_i \\
\delta\mathcal{Q}_6 &= \gamma_i\gamma_5 \otimes (\boldsymbol{\gamma}\cdot\mathbf{D})\gamma_i + (\boldsymbol{\gamma}\cdot\mathbf{D})\gamma_i \otimes \gamma_i\gamma_5
\end{aligned}
$$

where $\mathbf{D}$ is the (spatial) covariant derivative acting on the light quark fields.

By using HQSS, we can constrain the form of the counterterms:

$$
\mathcal{Q}_1 \qquad \begin{cases} \delta\mathcal{Q}_{11} = \delta\mathcal{Q}_1 - \delta\mathcal{Q}_2 - \delta\mathcal{Q}_3 + \delta\mathcal{Q}_4 \\ \delta\mathcal{Q}_{12} = \delta\mathcal{Q}_1 + \delta\mathcal{Q}_2 + \delta\mathcal{Q}_3 + \delta\mathcal{Q}_4 \end{cases}
$$

$$\mathcal{Q}_2 \qquad \begin{cases} \delta\mathcal{Q}_{21} &= \delta\mathcal{Q}_1 - \delta\mathcal{Q}_2 + 3\delta\mathcal{Q}_3 - 3\delta\mathcal{Q}_4 \\ \delta\mathcal{Q}_{22} &= \delta\mathcal{Q}_3 - \delta\mathcal{Q}_4 + \delta\mathcal{Q}_5 + \delta\mathcal{Q}_6 \\ \delta\mathcal{Q}_{23} &= \delta\mathcal{Q}_1 + \delta\mathcal{Q}_2 - 3\delta\mathcal{Q}_3 - 3\delta\mathcal{Q}_4 \\ \delta\mathcal{Q}_{24} &= \delta\mathcal{Q}_3 + \delta\mathcal{Q}_4 - \delta\mathcal{Q}_5 + \delta\mathcal{Q}_6 \end{cases}$$

We wrote them in a form such that $\delta\mathcal{Q}_{11}$, $\delta\mathcal{Q}_{21}$ and $\delta\mathcal{Q}_{22}$ have the correct naïve chiral symmetry while $\delta\mathcal{Q}_{12}$, $\delta\mathcal{Q}_{23}$ and $\delta\mathcal{Q}_{24}$ have the wrong one.

With tmQCD at maximal twist, by using a discrete axial symmetry (or equivalently twisted parity) we find that $\delta\mathcal{Q}_{12}$, $\delta\mathcal{Q}_{23}$ and $\delta\mathcal{Q}_{24}$ do not contribute and we are thus left with $\delta\mathcal{Q}_{11}$, $\delta\mathcal{Q}_{21}$ and $\delta\mathcal{Q}_{22}$.

The same would be true by using fermions with exact chiral symmetry. In T. Ishikawa *et al.*, JHEP 1105 (2011) 040 the one-loop perturbative matching including O(a) improvement is computed for domain-wall fermions. In that work the counterterm $\delta\mathcal{Q}_{22}$ is missing: does it appears only at two loops?

At maximal twist we obtain one counterterm $\mathrm{O}(\mu a)$ for $\mathcal{Q}_1$ and one for $\mathcal{Q}_2$:

$$\delta\mathcal{Q}_{b1} = \mu Q_3 \qquad \delta\mathcal{Q}_{b2} = \mu Q_4$$

which have the same form of the chirally symmetric case if the twisted mass μ is replaced by the standard mass m.

$\mathrm{O}(1/\mathrm{m})$ terms, conlusions and outlook

The $1/m$ terms which we have to include in order to perform the matching are found by requiring the same (reduced) symmetries of $\mathcal{O}_{LL}^{\Delta B=2}$, namely $H(3)$ cubic invariance, parity, time reversal, flavor, but not HQSS and local flavor conservation anymore.

Neglecting $\mathrm{O}(\mu/m)$ terms, the corresponding terms in tmQCD at maximal twist are obtained from the operators found above for the $\mathrm{O}(a)$ improvement of Wilson-like fermions with $\mathbf{D}$ replaced by $(\mathbf{D} - \overleftarrow{\mathbf{D}})$ and $(\mathbf{D} + \overleftarrow{\mathbf{D}})$.

$\Rightarrow$ in the matching between QCD and HQET for $\mathcal{O}_{LL}^{\Delta B=2}$ twelve $1/m$ dim. 7 operators and two dim. 6 static operators ($\mathcal{Q}_1$ and $\mathcal{Q}_2$) contribute.

In the non-perturbative matching procedure, the coefficients $c^i_{\mathcal{O}_{LL}}$ of dim. 7 operators will provide also the $\mathrm{O}(a)$ improvement of the dim. 6 static operators and remaining discretization errors will be of $\mathrm{O}(a/m)$ and thus negligible in practice.

tmQCD not useful in excluding the half of these operators which have the wrong naïve chiraly because they enter the renormalization of the the other half. This depends only on the symmetries of Wilson-like fermions in the chiral limit.

With chirally symmetric fermions instead one is left with only six $1/m$ terms.

We are working to find a system of $3+2+12=17$ conditions to determine $m_{\rm bare}$, $\omega_{\rm kin}$, $\omega_{\rm spin}$, $Z_1^{\rm HQET}$, $Z_2^{\rm HQET}$, $c^i_{\mathcal{O}_{LL}}$ $(i=1,\ldots,12)$, where the first 3 conditions are practically independent from the other 14.

This should be possible by computing combinations of correlation functions with different "SF momenta" θ's (similarly to the matching of the currents [Blossier *et al.*, 2010-2011]) but also exploiting the larger freedom due to the variety of boundary operators in the case of four-fermion operators [Palombi,Papinutto,Pena,Wittig, 2005-2007].

Since we need tmQCD in the matching procedure, we have to work with chirally rotated boundary condition for the SF [S. Sint, 2005-2010].

A 1-loop computation of the HQET parameters could be useful to assess the size of the various terms and the stability of different possible matching conditions.

As first step in the non-perturbative computation we will perform the matching of QCD to the static theory.

$\mathrm{O}(a)$ improvement

A basis of independent dimension 7 operators (after using equations of motion, Fierz identities and heavy quark properties to reduce them) is:

$$
\begin{array}{ccc}
\gamma_j \otimes \gamma_5 D_j & \gamma_j\gamma_5 \otimes 1 D_j & \epsilon_{ijk}\ [\gamma_i\gamma_5 \otimes \gamma_j\gamma_5 D_k] \\
\gamma_5 \otimes \gamma_j D_j & 1 \otimes \gamma_j\gamma_5 D_j & \epsilon_{ijk}\ [\gamma_i \otimes \gamma_j D_k] \\
\gamma_j D_j \otimes \gamma_5 & \gamma_j\gamma_5 D_j \otimes 1 & \epsilon_{ijk}\ [\gamma_i\gamma_5 D_j \otimes \gamma_k\gamma_5] \\
\gamma_5 D_j \otimes \gamma_j & 1 D_j \otimes \gamma_j\gamma_5 & \epsilon_{ijk}\ [\gamma_i D_j \otimes \gamma_k]
\end{array}
$$

We can now pass from these operators to a new set of 12 operators as follows:

$$
\begin{aligned}
\gamma_i \otimes \gamma_i\gamma_5(\boldsymbol{\gamma}\cdot\mathbf{D}) &= -\gamma_j \otimes \gamma_5 D_j + \epsilon_{ijk}[\gamma_i \otimes \gamma_j D_k] \\
\gamma_i\gamma_5 \otimes \gamma_i(\boldsymbol{\gamma}\cdot\mathbf{D}) &= \gamma_j\gamma_5 \otimes 1 D_j - \epsilon_{ijk}[\gamma_j\gamma_5 \otimes \gamma_k\gamma_5 D_i] \\
\gamma_0 \otimes \gamma_0\gamma_5(\boldsymbol{\gamma}\cdot\mathbf{D}) &= 1 \otimes \gamma_j\gamma_5 D_j \\
\gamma_0\gamma_5 \otimes \gamma_0(\boldsymbol{\gamma}\cdot\mathbf{D}) &= -\gamma_5 \otimes \gamma_j D_j \\
\gamma_i \otimes (\boldsymbol{\gamma}\cdot\mathbf{D})\gamma_i\gamma_5 &= \gamma_j \otimes \gamma_5 D_j + \epsilon_{ijk}[\gamma_i \otimes \gamma_j D_k] \\
\gamma_i\gamma_5 \otimes (\boldsymbol{\gamma}\cdot\mathbf{D})\gamma_i &= \gamma_j\gamma_5 \otimes 1 D_j + \epsilon_{ijk}[\gamma_j\gamma_5 \otimes \gamma_k\gamma_5 D_i]
\end{aligned}
$$