Pseudoscalar decay constants f_K/f_π, f_D and f_{Ds} with $N_f=2+1+1$ ETMC configurations

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Outline

- Simulation Details
- Plateaux
- f_K^+ and f_K^+/f_π^+ Analysis
- f_{Ds} and f_D Analysis
- Summary and Conclusions

Simulation Details

Something on the action:

- Wilson Twisted Mass action at maximal twist with Nf=2+1+1 sea quarks
- Osterwalder-Seiler valence quark action
- Iwasaki gluon action

Simulation Details

Details of the ensembles of the gauge configurations used in this N_f =2+1+1 analysis

PRA027
"QCD simulations for flavor physics in the Standard Model and beyond" (35 millions of core-hours at the BG/P system in Julich from December 2010 to March 2011)

β	V	$a\mu_{sea}$	N_{cfg}	$a\mu_s$	$a\mu_c$
1.90	$32^3 \times 64$	0.0030	150	0.0145, 0.0185,	0.1800, 0.2200,
		0.0040	90	0.0225	0.2600, 0.3000,
		0.0050	150		0.3600, 0.4400
	$24^3 \times 48$	0.0040	150		
		0.0060	150		
		0.0080	150		
		0.0100	150		
1.95	$32^3 \times 64$	0.0025	150	0.0141, 0.0180,	0.1750, 0.2140,
		0.0035	150	0.0219	0.2530, 0.2920,
		0.0055	150		0.3510, 0.4290
		0.0075	75		
	$24^3 \times 48$	0.0085	150		
2.10	$48^3 \times 96$	0.0015	60	0.0118, 0.0151,	0.1470, 0.1795,
		0.0020	90	0.0184	0.2120, 0.2450,
		0.0030	90		0.2945, 0.3595

β	$L(fm)$	M_π(MeV)	$M_\pi L$
1.90	2.84	245.41	3.53
		282.13	4.06
		314.43	4.53
1.90	2.13	282.13	3.05
		343.68	3.71
		396.04	4.27
		442.99	4.78
1.95	2.61	238.67	3.16
		280.95	3.72
		350.12	4.64
		408.13	5.41
1.95	1.96	434.63	4.32
2.10	2.97	211.18	3.19
		242.80	3.66
		295.55	4.46

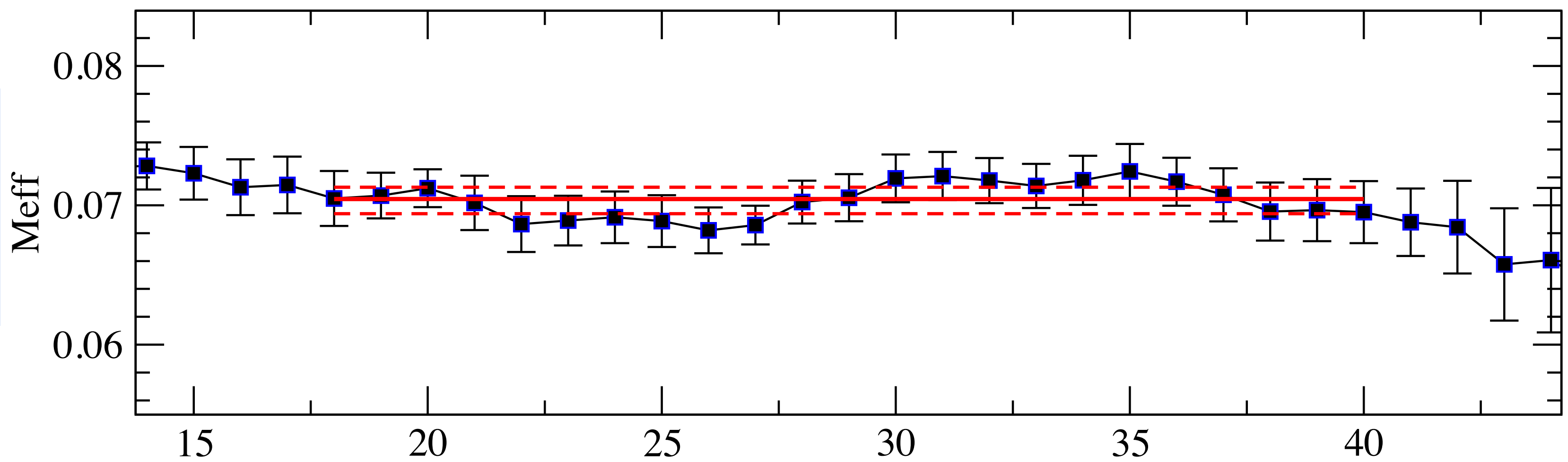
β	$Z_P^{\overline{MS}}(2\,\mathrm{GeV})(M_1)$	$Z_P^{\overline{MS}}(2\,\mathrm{GeV})(M_2)$	r_0/a
1.90	0.521(7)	0.564(6)	5.31(8)
1.95	0.506(4)	0.537(4)	5.77(6)
2.10	0.513(3)	0.540(2)	7.60(8)

Lattice Spacings	
a($\beta = 1.90$)	0.0885(36)fm
a($\beta = 1.95$)	0.0815(30)fm
a($\beta = 2.10$)	0.0619(18)fm

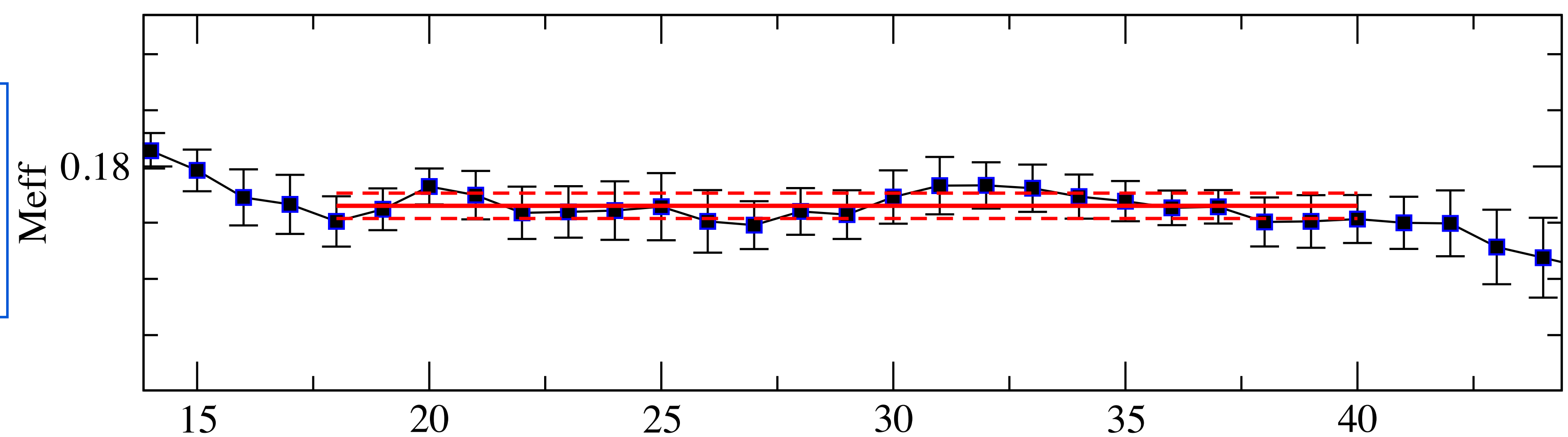
Values of the bare quark masses in the strange and heavy sector. The valence light quark mass is put equal to the sea quark mass

Plateaux

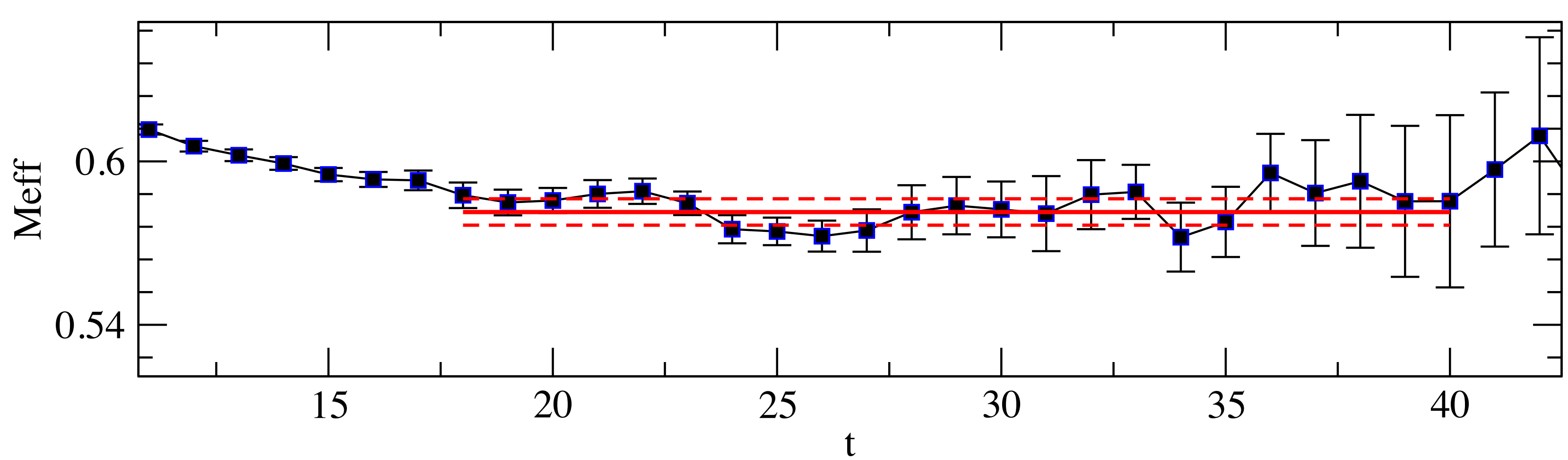
β=2.10
μ_{l1} =0.0015
μ_{l2} =0.0015
M_{eff} =0.0705(9)



β=2.10
μ_l =0.0015
μ_s =0.0184
M_{eff} =0.1772(9)



β=2.10
μ_l =0.0015
μ_h =0.1795
M_{eff} =0.581(5)



On π and K Analyses in general

Two different approaches for the discretization effects

- in units of r_0
  standard approach
- in units of $M_{<ss>}$
  handles discretization effects in a different way.
  <s> is a reference valence quark mass in the strange region

Two different approaches for the chiral extrapolation

- SU(2) ChPT
- Polynomial fit

Different formulae for FSE correction

- Colangelo, Dürr, Haefeli for M_K and f_K
  resummed formulae
  [Nucl.Phys.B721:136-174,2005]
- Colangelo, Wenger, Wu for M_π and f_π
  contribution from π_0 and π_+ splitting
  [Phys.Rev. D82 (2010) 034502]

K Analysis - Comparison with pion FSE

- GL: Gasser Leutwyler
  NLO ChPT
- CDH: Colangelo, Dürr, Haefeli
  resummed formulae
- CWW: Colangelo, Wenger, Wu
  contribution from π_0 and π_+ splitting due to discretization effects

we have two point with the same mass at different volume. Looking at them helps us understand how well corrections are working

$$M_{[24]} = M_{[\infty]} FSE_{M,[24]}$$

$$M_{[32]} = M_{[\infty]} FSE_{M,[32]}$$

Kaon FSE correction

	GL	CDH	Lattice data M_{32}/M_{24}
$FSE_{M_K,32}/FSE_{M_K,24}$	1	0.991	0.990(7)

	GL	CDH	Lattice data f_{32}/f_{24}
$FSE_{f_K,32}/FSE_{f_K,24}$	1.007	1.028	1.020(13)

Pion FSE correction

	GL	CDH	CWW	Lattice data M_{32}/M_{24}
$FSE_{M\pi,32}/FSE_{M\pi,24}$	0.994	0.985	0.981	0.972(13)

	GL	CDH	CWW	Lattice data f_{32}/f_{24}
$FSE_{f\pi,32}/FSE_{f\pi,24}$	1.023	1.040	1.054	1.050(19)

K Analysis - Comparison with pion FSE

- GL: Gasser Leutwyler
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  resummed formulae
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  contribution from π_0 and π_+ splitting due to discretization effects

we have two point with the same mass at different volume. Looking at them helps us understand how well corrections are working

$$M_{[24]} = M_{[\infty]} FSE_{M,[24]} = M_{[\infty]}(1 + R_{M,[24]})$$

$$M_{[32]} = M_{[\infty]} FSE_{M,[32]} = M_{[\infty]}(1 + R_{M,[32]})$$

Kaon FSE correction

	GL	CDH	Lattice data M_{32}/M_{24}
$FSE_{M_K,32}/FSE_{M_K,24}$	1	0.991	0.990(7)

	GL	CDH	Lattice data f_{32}/f_{24}
$FSE_{f_K,32}/FSE_{f_K,24}$	1.007	1.028	1.020(13)

	GL	CDH
$R_{M_K,24}$	0	0.0102
$R_{f_K,24}$	-0.009	-0.032

Pion FSE correction

	GL	CDH	CWW	Lattice data M_{32}/M_{24}
$FSE_{M\pi,32}/FSE_{M\pi,24}$	0.994	0.985	0.981	0.972(13)

	GL	CDH	CWW	Lattice data f_{32}/f_{24}
$FSE_{f\pi,32}/FSE_{f\pi,24}$	1.023	1.040	1.054	1.050(19)

	GL	CDH	CWW
$R_{M\pi,24}$	0.0070	0.0187	0.0243
$R_{f\pi,24}$	-0.0280	-0.0469	-0.0632

f_K: Chiral and continuum extrapolation

f_K

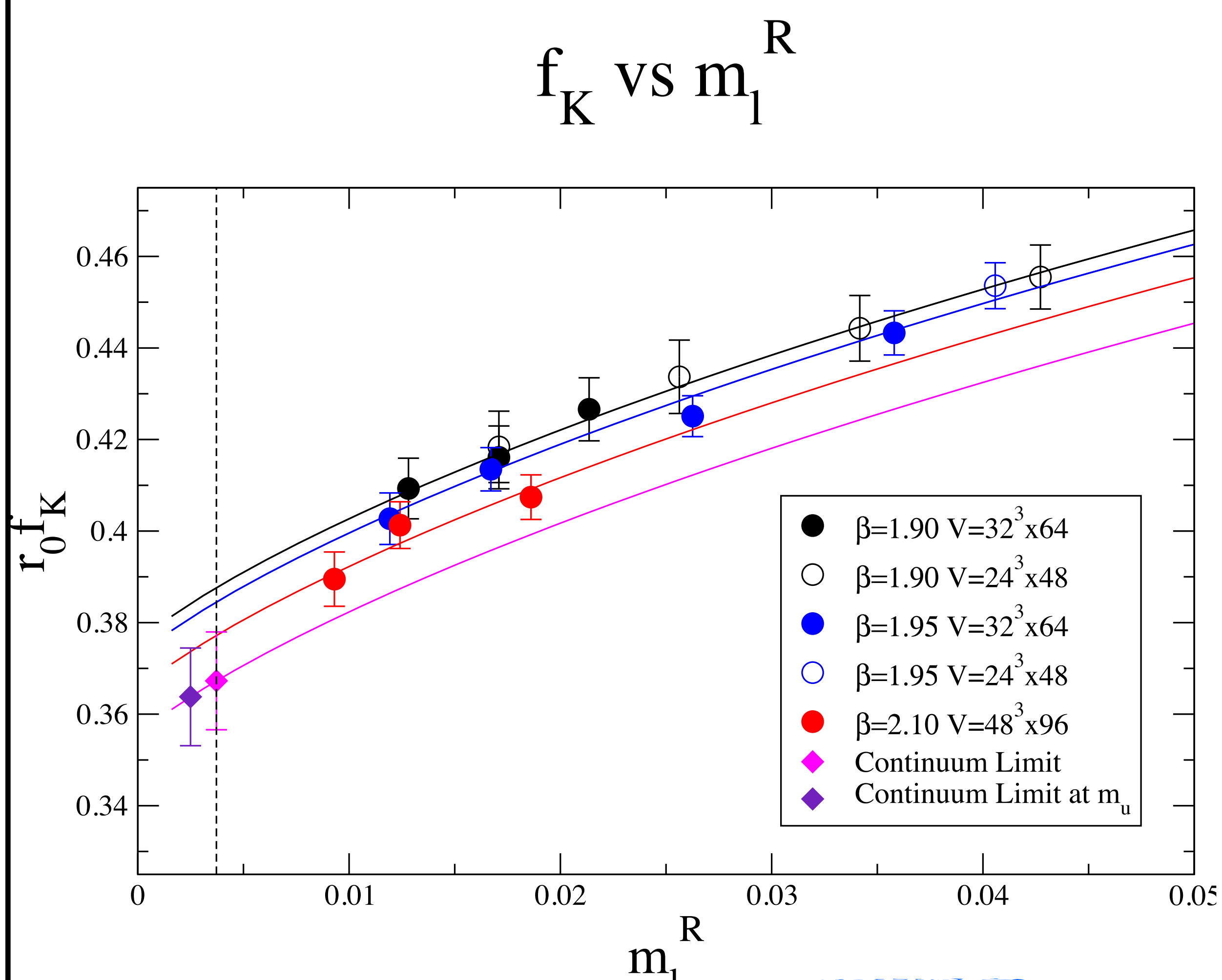
we performed a small interpolation in the data to arrive at $m_{s\,phys}$

different fit formulae have been used

Chiral fit: $(f_K r_0) = P_1\left(1 - \frac{3}{4}\xi_l \log \xi_l + P_2 \xi_l + P_3 \frac{a^2}{r_0^2}\right) \cdot [K_f]$

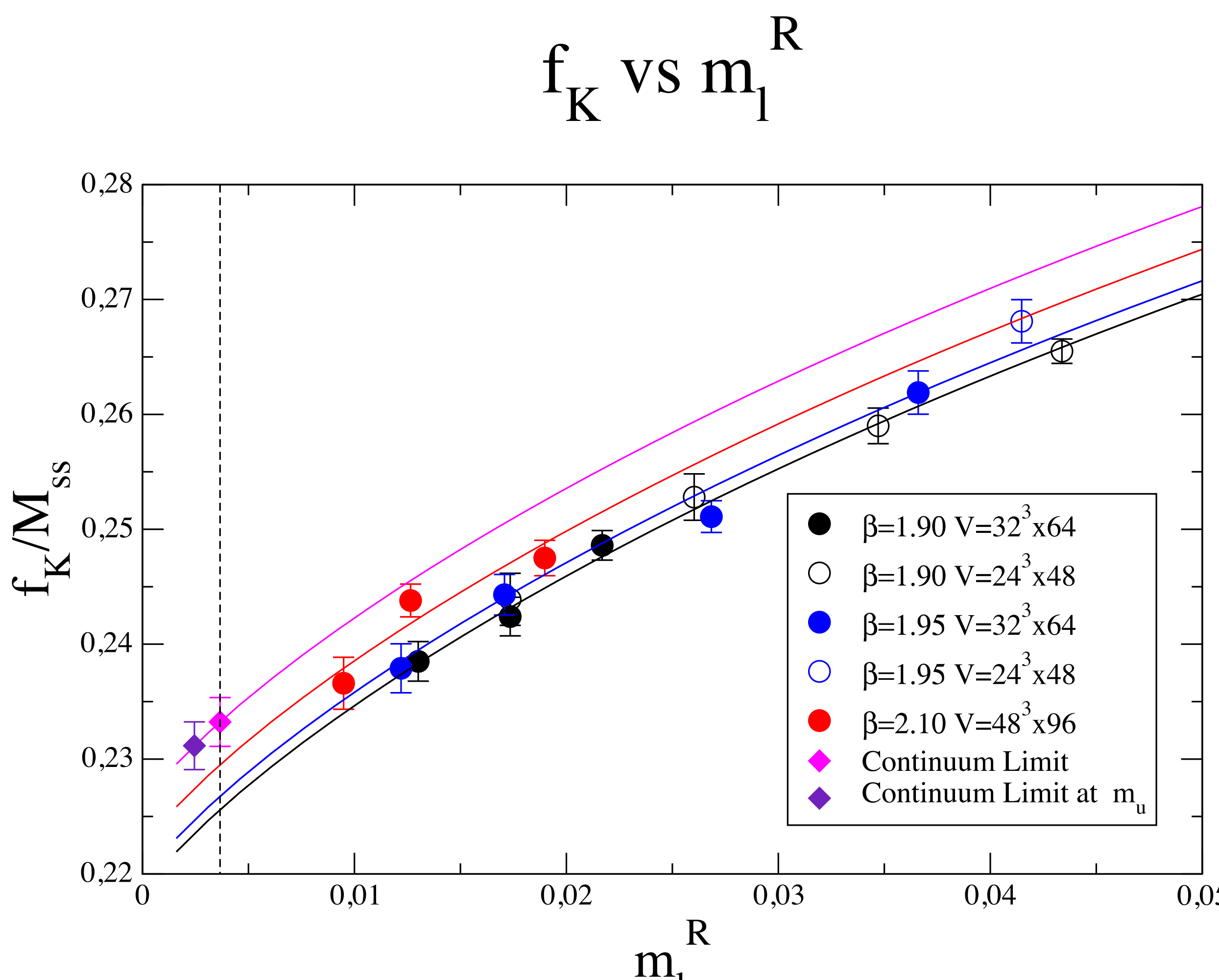
Polynomial fit: $(f_K r_0) = P_1\left(1 + P_2 m_l + P_3 a^2 + P_4 m_l^2\right) \cdot [K_f]$

SU(2) Chiral fit in units of r_0



(finer-continuum)/continuum ~ 3% r_0 units

SU(2) Chiral fit in units of $M_{<ss>}$



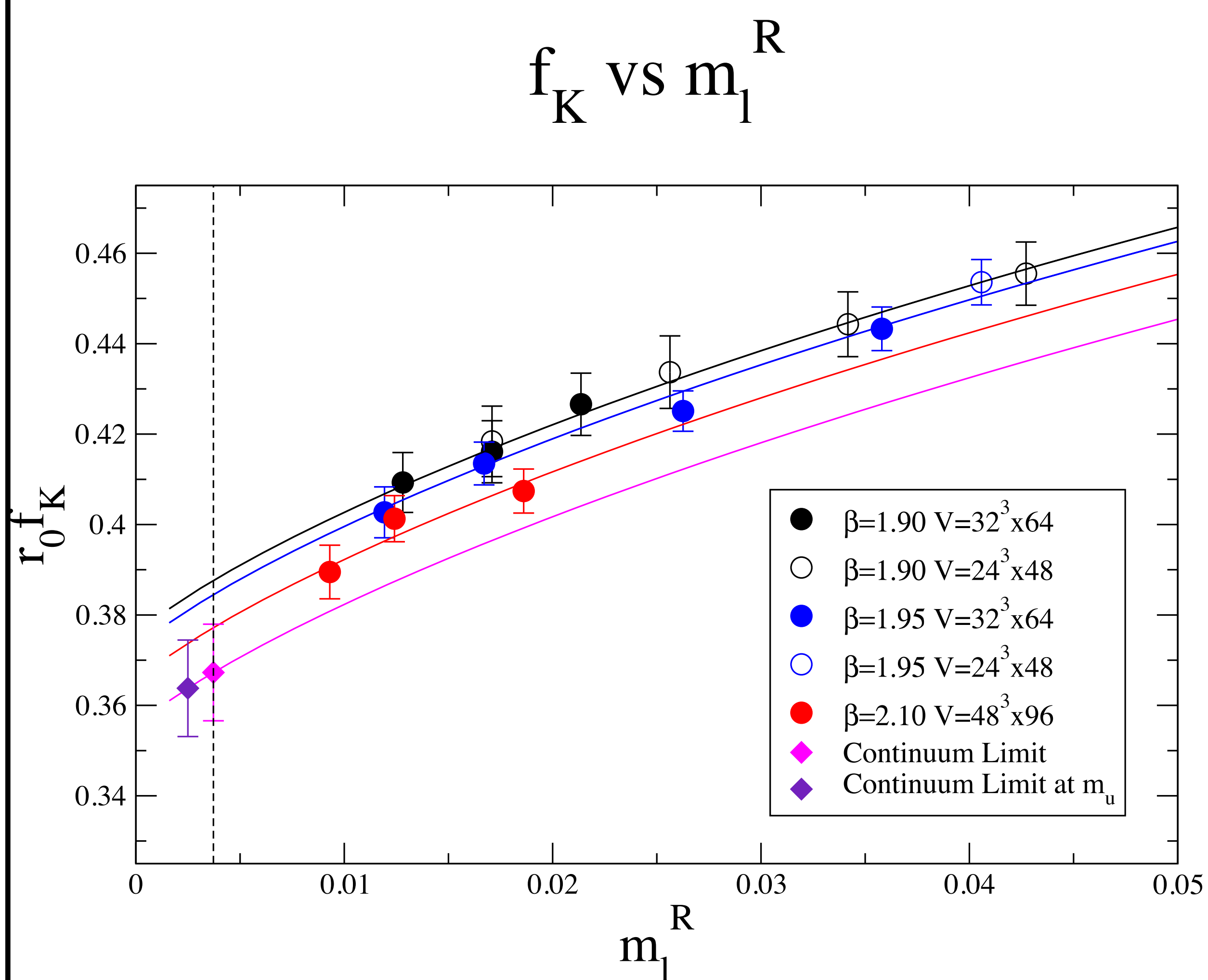
(finer-continuum)/continuum ~ -1.5% $M_{<ss>}$ units

f_K: Chiral and continuum extrapolation

f_K

we performed a small interpolation in the data to arrive at $m_{s\ phys}$

SU(2) Chiral fit in units of r_0



different fit formulae have been used

Chiral fit: $(f_K r_0) = P_1\left(1 - \frac{3}{4}\xi_l \log \xi_l + P_2 \xi_l + P_3 \frac{a^2}{r_0^2}\right) \cdot [K_f]$

Polynomial fit: $(f_K r_0) = P_1\left(1 + P_2 m_l + P_3 a^2 + P_4 m_l^2\right) \cdot [K_f]$

$$\boxed{\xi_l = \frac{2(B_0 r_0)(m_l r_0)}{16\pi^2 (f_0 r_0)^2}}$$

We corrected for FSE using the formulae calulated by Colangelo Dürr Haefeli [Nucl.Phys.B721:136-174,2005]

To calculate f_{K^+} we used m_u=2.47(11) MeV from $m_{u/d}$=3.70(17) MeV (see P. Lami's talk) and m_u/m_d =0.5 [10.1103/PhysRevD.87.114505]

our result:

$$f_{K^+} = 154.4(2.1)\text{MeV}$$

$$f_{K^+} = 154.4(1.8)_{stat+fit}(1.2)_{syst}\,\text{MeV} = 154.4(2.1)\text{MeV}$$

then using the experimental value of the pion decay constant we extracted the ratio f_K/f_π

f_K/f_π - results and sistematics

The results from our analysis:

$$f_{K^+}/f_{\pi^+} = 1.183(17)$$

in particular:

$$f_{K^+}/f_{\pi^+} = 1.183(14)_{stat+fit}(4)_{Chiral}(8)_{Disc.}(1)_{FSE}$$

stat+fit is referred to both the statistical uncertainties (including the total error on r_0/a Z_p and m_s) and the uncertainties due the fitting procedure

Chiral extrapolation systematic uncertainties have been evaluated comparing the results obtained from two different fit formulae i.e. SU(2) ChPT and polynomial expansion

Discretization systematic errors have been evaluated comparing the results obtained from the analysis done in units of r_0 and the analysis done in units of the unphysical pseudoscalar meson mass $M_{<ss>}$

Finally systematic effects on the **FSE** have been taken as the difference of f_K/f_π corrected with CDH (SU(3)) formulae and the value obtained applying no correction at all

On D&D_s Analyses in general

Two different approaches for the discretization effects

- in units of r_0
  standard approach
- in units of $M_{<cs>}$
  handles discretization effects in a different way.
  <s> and <c> are reference valence quark masses
  in the strange and charm region

Two different approaches for the chiral extrapolation depending on the physical quantity

f_{Ds}: Chiral and continuum extrapolation

f_{Ds}

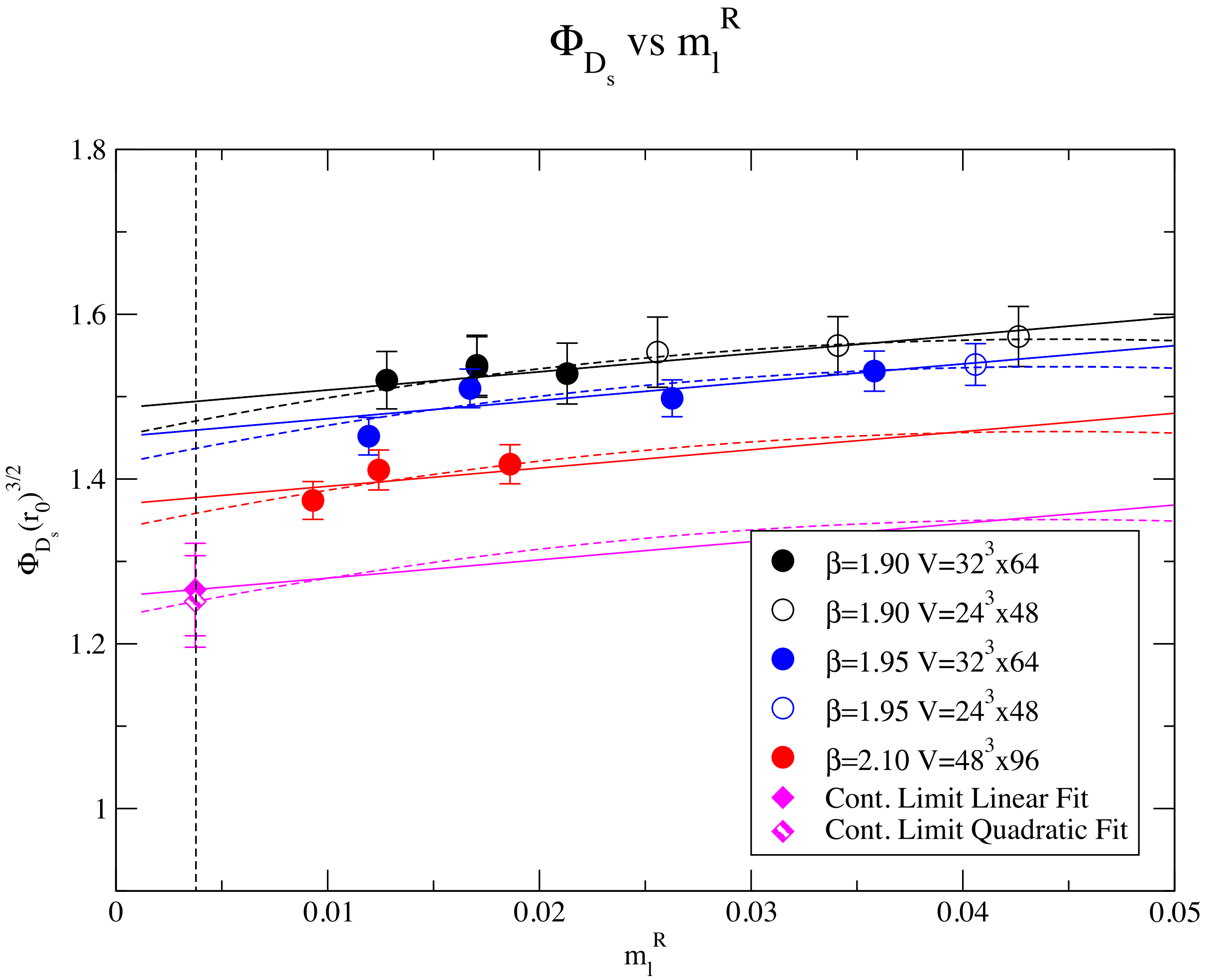
we performed a small interpolation in the data to arrive at $m_{c\ phys}$ and $m_{s\ phys}$

Polynomial fit: $\Phi_{D_s} = P_1(1 + P_2 m_l + P_3 a^2 + P_4 m_l^2)$

with $\Phi_{D_s} = f_{D_s}\sqrt{M_{D_s}}$

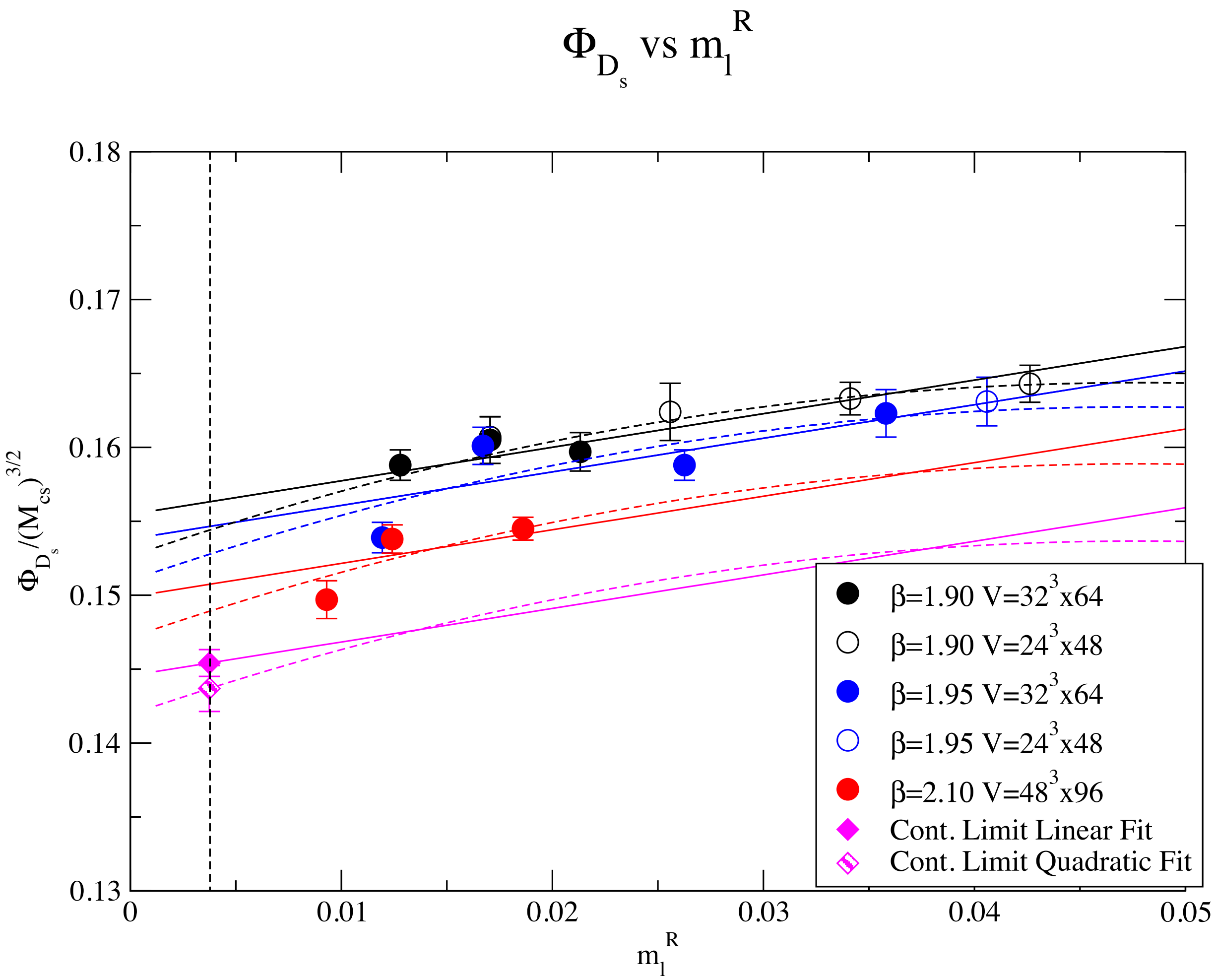
we tried both with linear and quadratic expressions in m_l

linear & quadratic fit in units of r_0



(finer-continuum)/continuum ~8% r_0 units

linear & quadratic fit in units of $M_{<cs>}$



(finer-continuum)/continuum ~3% $M_{<cs>}$ units

f_{Ds}: Chiral and continuum extrapolation

f_{Ds}

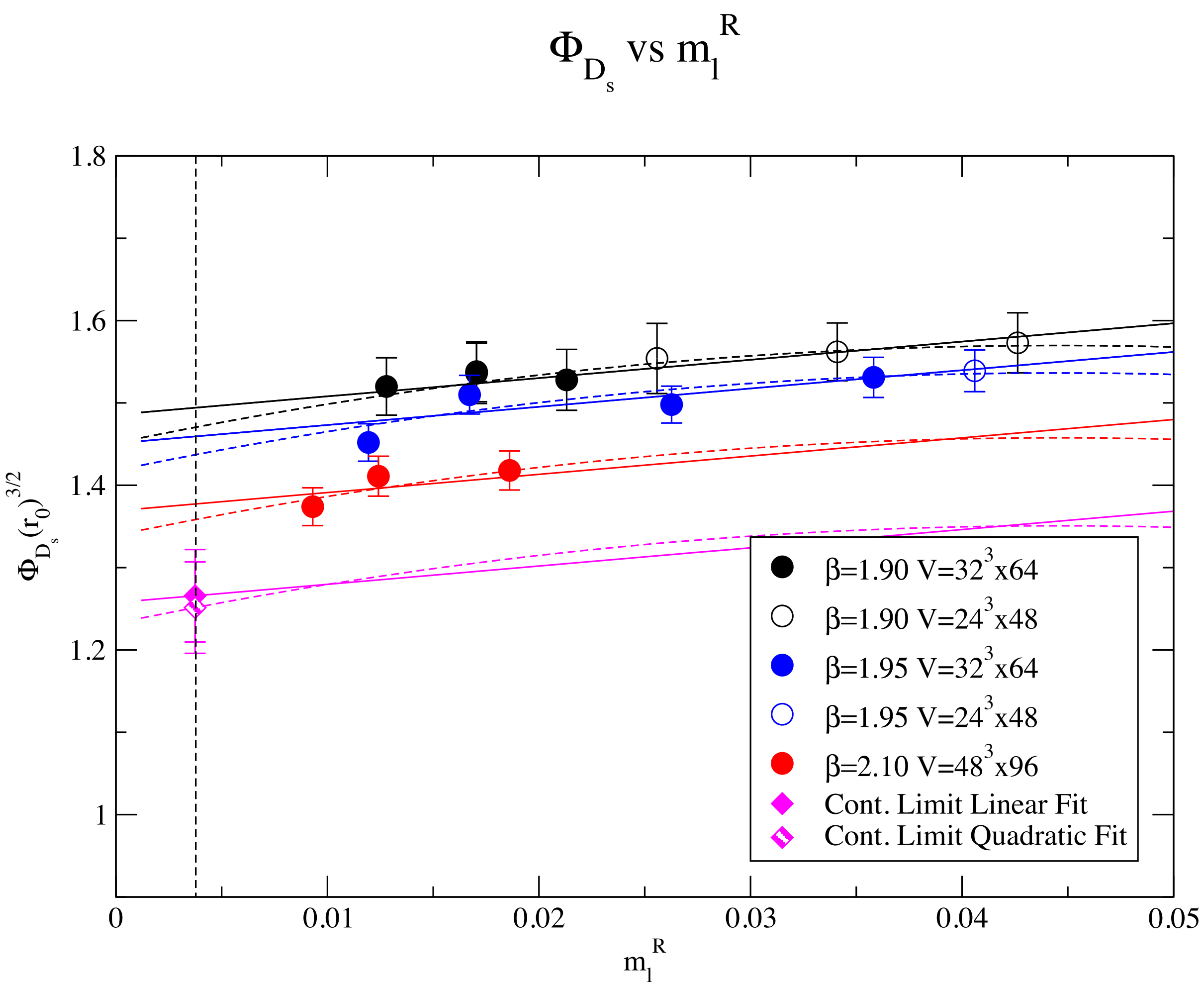
we performed a small interpolation in the data to arrive at $m_{c\ phys}$ and $m_{s\ phys}$

Polynomial fit: $\Phi_{D_s} = P_1(1 + P_2 m_l + P_3 a^2 + P_4 m_l^2)$

with $\Phi_{D_s} = f_{D_s}\sqrt{M_{D_s}}$

we tried both with linear and quadratic expressions in m_l

linear & quadratic fit in units of r_0



the continuum and dashed lines shown in the plot represent respectively the linear and quadratic fit curves

f_{D_s} - results and sistematics

The results from our analysis:

$$f_{D_s} = 242.1(8.3)\mathrm{MeV}$$

in particular:

$$f_{D_s} = 242.1(7.1)_{stat+fit+scale}(1.4)_{Chiral}(2.9)_{Disc.}(2.4)_{m_c}(1.4)_{m_s}\ \mathrm{MeV}$$

stat+fit+scale includes both the statistical uncertainties, the uncertainties due to the fitting procedures and all the uncertainties coming from the scale

Chiral extrapolation systematic uncertainties have been evaluated comparing the results obtained from two different fit formulae i.e. linear and quadratic expansion in m_l

Discretization systematic errors have been evaluated comparing the results obtained from the analysis done in units of r_0 and the analysis done in units of the unphysical pseudoscalar meson mass $M_{<cs>}$

Finally we included the uncertainties due to the total errors in the determination of the **strange and charm quark masses**

f_{D_s}/f_D: Chiral and continuum extrapolation

f_{D_s}/f_D

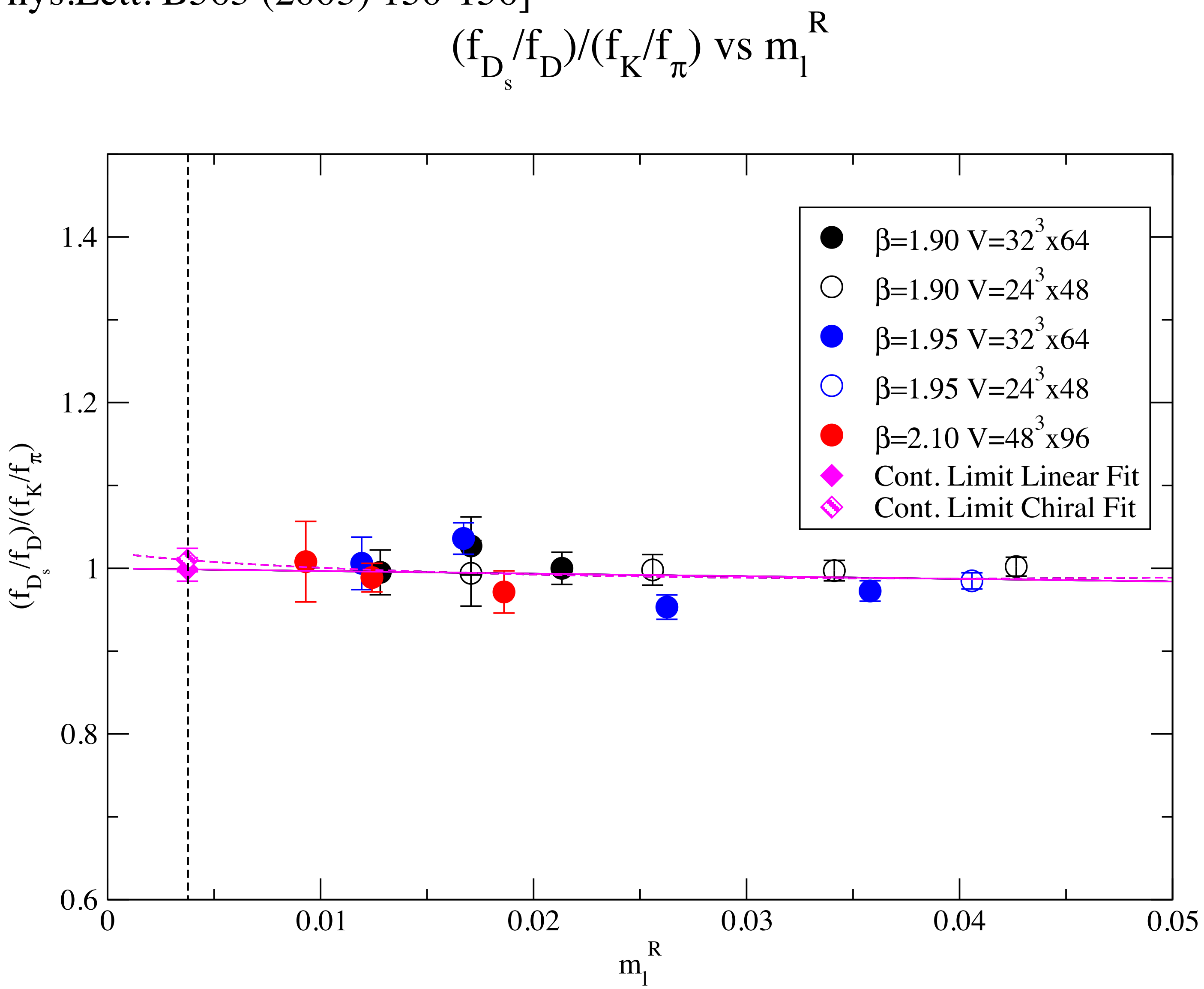
Chiral fit: $$\frac{f_{D_s}/f_D}{f_K/f_\pi} = P_1\left(1 + P_2 m_l + \left(\frac{9}{4}\hat{g}^2 - \frac{1}{2}\right)\xi_l \log \xi_l\right)$$

We found that analyzing the double ratio $(f_{D_s}/f_D)/(f_K/f_\pi)$ increases the precision on the determination of f_{D_s}/f_D because of the mild dependence of this quantity on m_l as suggested in the paper from Becirevic et. al. [Phys.Lett. B563 (2003) 150-156]

we also tried to remove the chiral log and fit the quantity with a linear expression in m_l

we took g=0.61(7) from

[K. Nakamura et al. [Particle Data Group], J. Phys. G 37 (2010) 075021]



First of all we performed a small interpolation in the data to arrive at $m_{c\ phys}$ and $m_{s\ phys}$

the continuum and dashed line shown in the plot represent respectively the Chiral and linear fit curves

using the value we obtained for f_K/f_π we can estimate the ratio f_{D_s}/f_D

f_{D_s}/f_D - results and sistematics

The results from our analysis:

$$f_{D_s} / f_D = 1.199(25)$$

in particular:

$$f_{D_s} / f_D = 1.199(17)_{stat+fit+scale}(7)_{Chiral}(16)_{f_K/f_\pi}$$

stat+fit+scale includes both the statistical uncertainties, the uncertainties due to the chiral and continuum extrapolation and all the uncertainties coming from the scale

Chiral extrapolation systematic uncertainties have been evaluated comparing the results obtained from two different fit formulae i.e. linear fit and Chiral fit

the total error on $\mathbf{f_K/f_\pi}$ as been included as well. Notice that in this case we use the quantity f_K/f_π extrapolated to m_{ud} which has a slightly smaller error.

Discretization effects are negligible compared for example to the statistical errors thus we removed the *a* dependence from the fit. We tried also to perform the same fit excluding all the data corresponding to the coarser lattice spacing obtaining the same result.

the double ratio proved to be also very stable with respect to changes of the **strange and charm quark masses** which if variated within their total error produce no changes.

Combining the results: f_D

Combining our determination of f_{Ds}/f_D and f_{Ds} we can estimate f_D as well.

$$f_D = 201.9(8.0)\,\mathrm{MeV}$$

Conclusions

We presented N_f=2+1+1 results for decay constants and in particular:

- f_K^+ and f_K^+/f_π^+
- f_{Ds}, f_D and the ratio f_{Ds}/f_D

Summary of the results in comparison with FLAG averages:

Quantity	Our Result	FLAG$_{N_f=2}$	FLAG$_{N_f=2+1}$	FLAG$_{N_f=2+1+1}$
f_{K^+} (MeV)	154.4(2.1)	-	-	-
f_K (MeV)	155.6(2.1)	158.1(2.5)	156.3(0.8)	-
f_{K^+}/f_{π^+}	1.183(17)	1.205(6)(17)	1.192(5)	1.195(3)(4)
f_D (MeV)	201.9(8.0)	212(8)	209.2(3.3)	-
f_{D_s} (MeV)	242.1(8.3)	248(6)	248.6(2.7)	-
f_{D_s}/f_D	1.199(25)	1.17(5)	1.187(12)	-

Thanks for the attention!

Backup

Other results from the Pion Analysis

A collection of results from the simultaneous fit of M^2_π and f_π

$$r_0 = 0.474(14)\,fm = 0.474(13)_{stat+fit}(3)_{Chiral}(6)_{FSE}\,fm$$

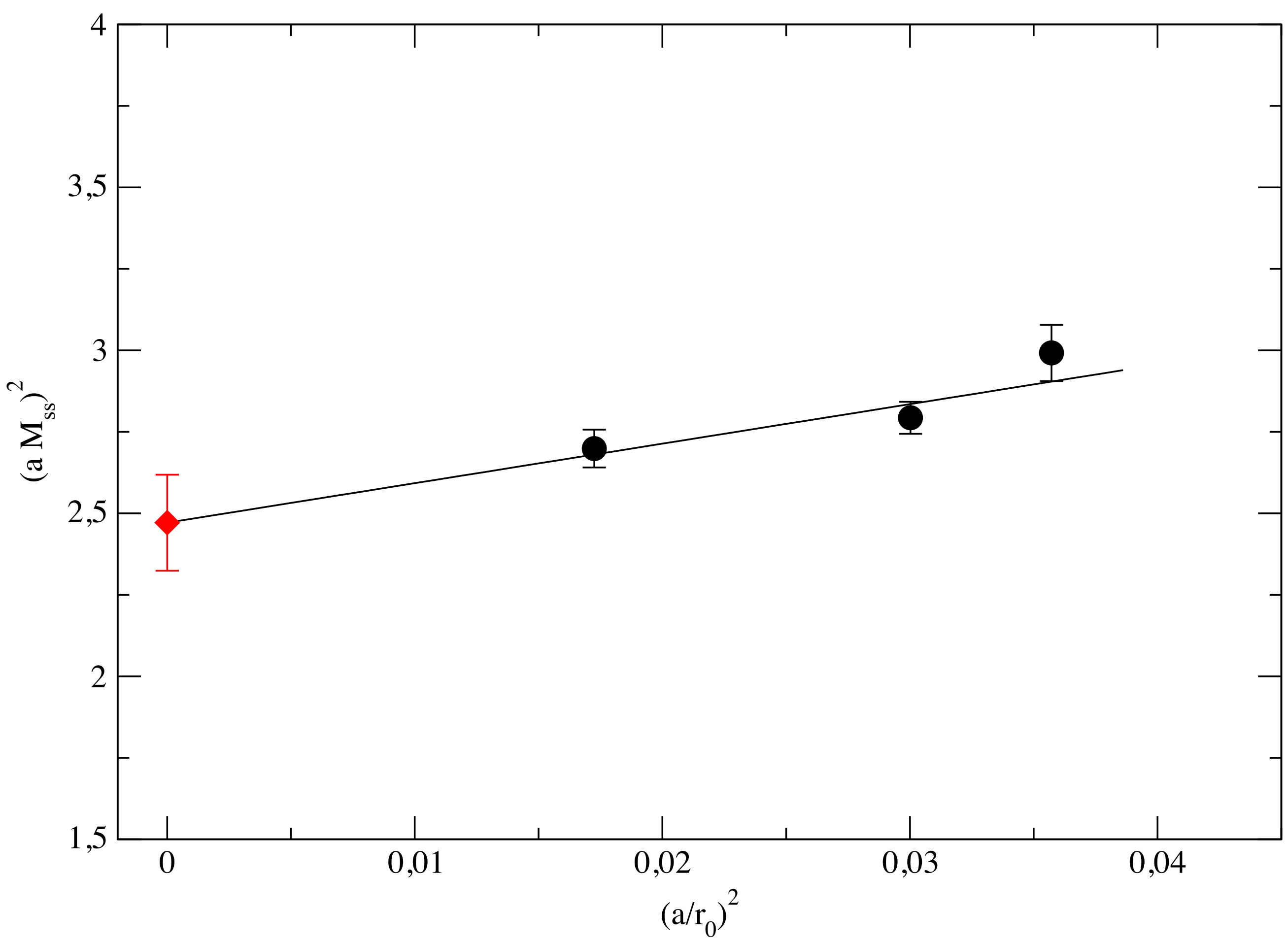
$$B_0 = 2571(97)\,MeV = 2571(80)_{stat+fit}(22)_{Disc.}(33)_{FSE}(38)_{Z_p}\,MeV$$

$$f_0 = 121.2(4)\,MeV = 121.2(2)_{stat+fit}(1)_{Disc.}(3)_{FSE}\,MeV$$

$$\bar{l}_3 = 3.11(34) = 3.11(23)_{stat+fit}(17)_{Disc.}(18)_{FSE}$$

$$\bar{l}_4 = 4.69(17) = 4.69(9)_{stat+fit}(2)_{Disc.}(14)_{FSE}$$

$(aM_{ss})^2$ vs a^2



Fit of $(a\ M_{ss})^2$ as a function of a^2.
It is important to remove discretization effects from $(a\ M_{ss})$ to get the correct lattice spacings.

More on FSE

A comparison of the effects of the different FSE correction on the physical quantities from the pion and kaon fits.

Quantity	No Correction	GL	CDH	CWW
$m_l(\mathrm{MeV})$	3.68(14)	3.76(14)	3.73(13)	3.72(13)
$r_0(fm)$	0.464(12)	0.466(12)	0.468(12)	0.470(12)
$\bar{l}_3$	3.42(20)	3.35(20)	3.34(21)	3.24(25)
$\bar{l}_4$	4.83(9)	4.77(9)	4.76(9)	4.69(10)
$B_0(\mathrm{MeV})$	2548(99)	2497(97)	2500(93)	2515(90)
$f_0(\mathrm{MeV})$	120.8(1)	120.9(1)	120.9(1)	121.1(2)

Quantity	No Correction	GL	CDH
$m_s(\mathrm{MeV})$	101.1(4.4)	101.1(4.4)	101.6(4.4)
$f_K(\mathrm{MeV})$	151.8(2.6)	152.2(2.6)	152.3(2.6)
f_K/f_π	1.164(20)	1.167(20)	1.168(20)

K Analysis - plot

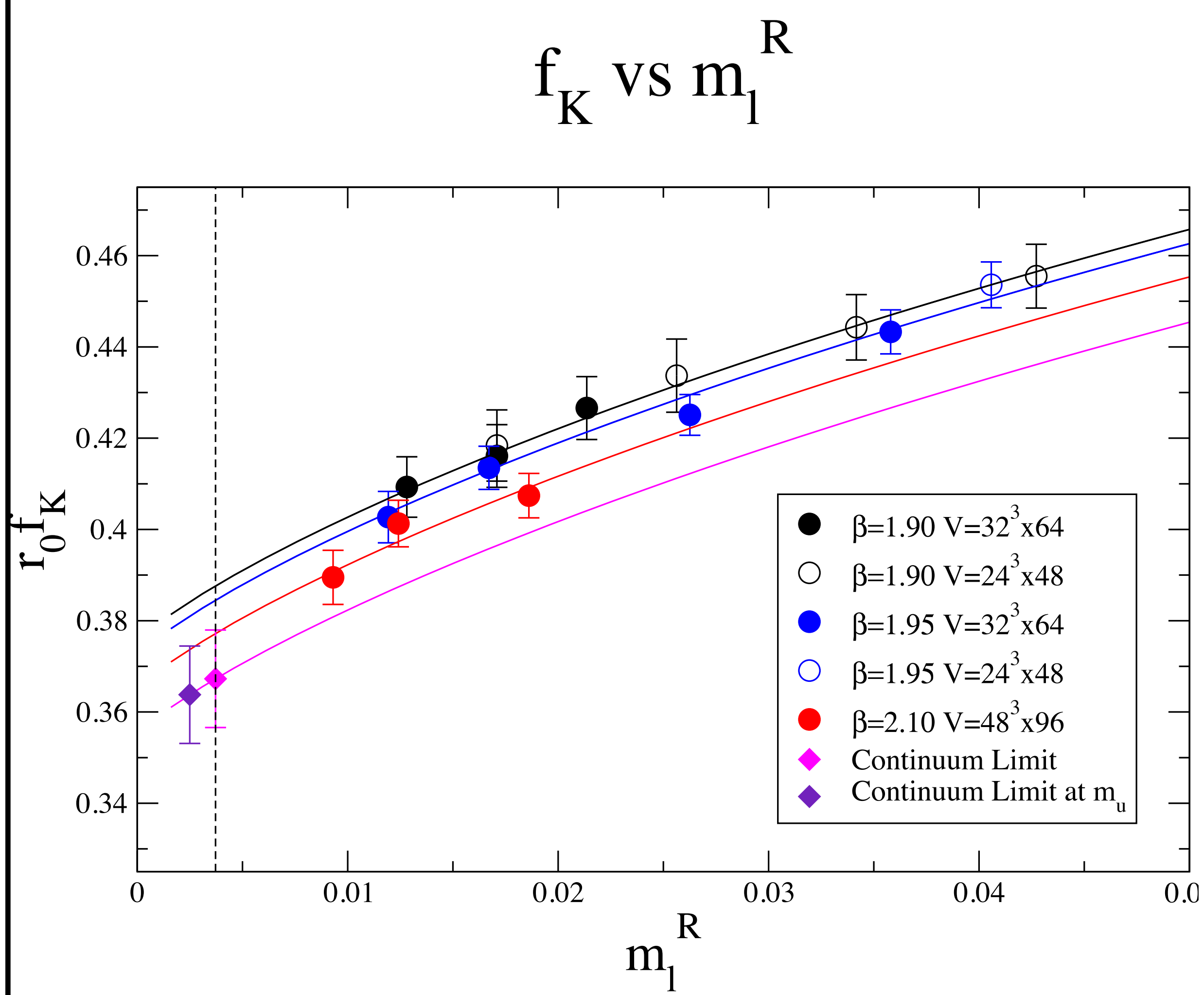
f_K

different fit formulae have been used

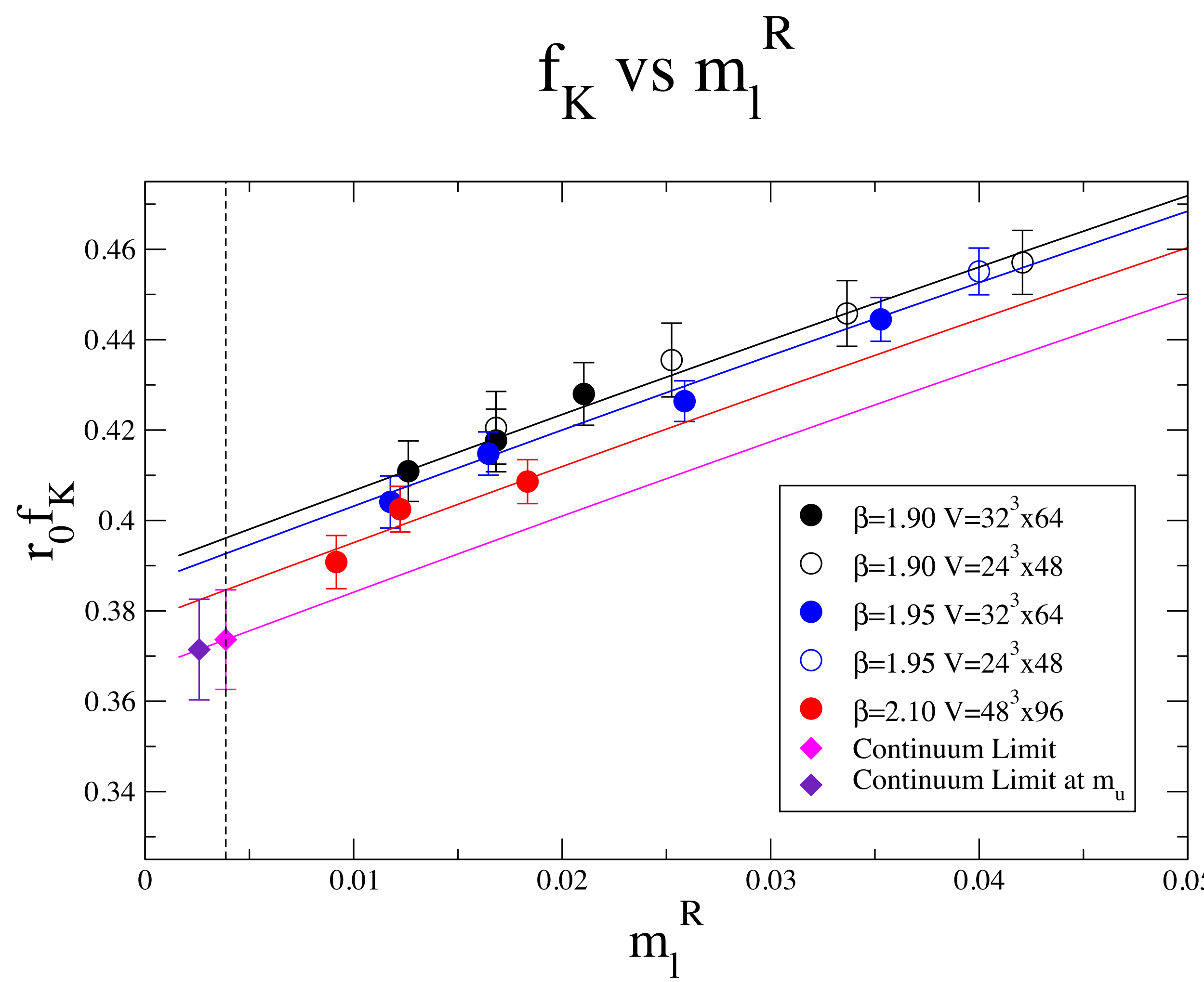
Chiral fit: $$(f_K r_0) = P_1\left(1 - \frac{3}{4}\xi_l \log \xi_l + P_2 \xi_l + P_3 \frac{a^2}{r_0^2}\right)\cdot[K_f]$$

Polynomial fit: $$(f_K r_0) = P_1\left(1 + P_2 m_l + P_3 a^2 + P_4 m_l^2\right)\cdot[K_f]$$

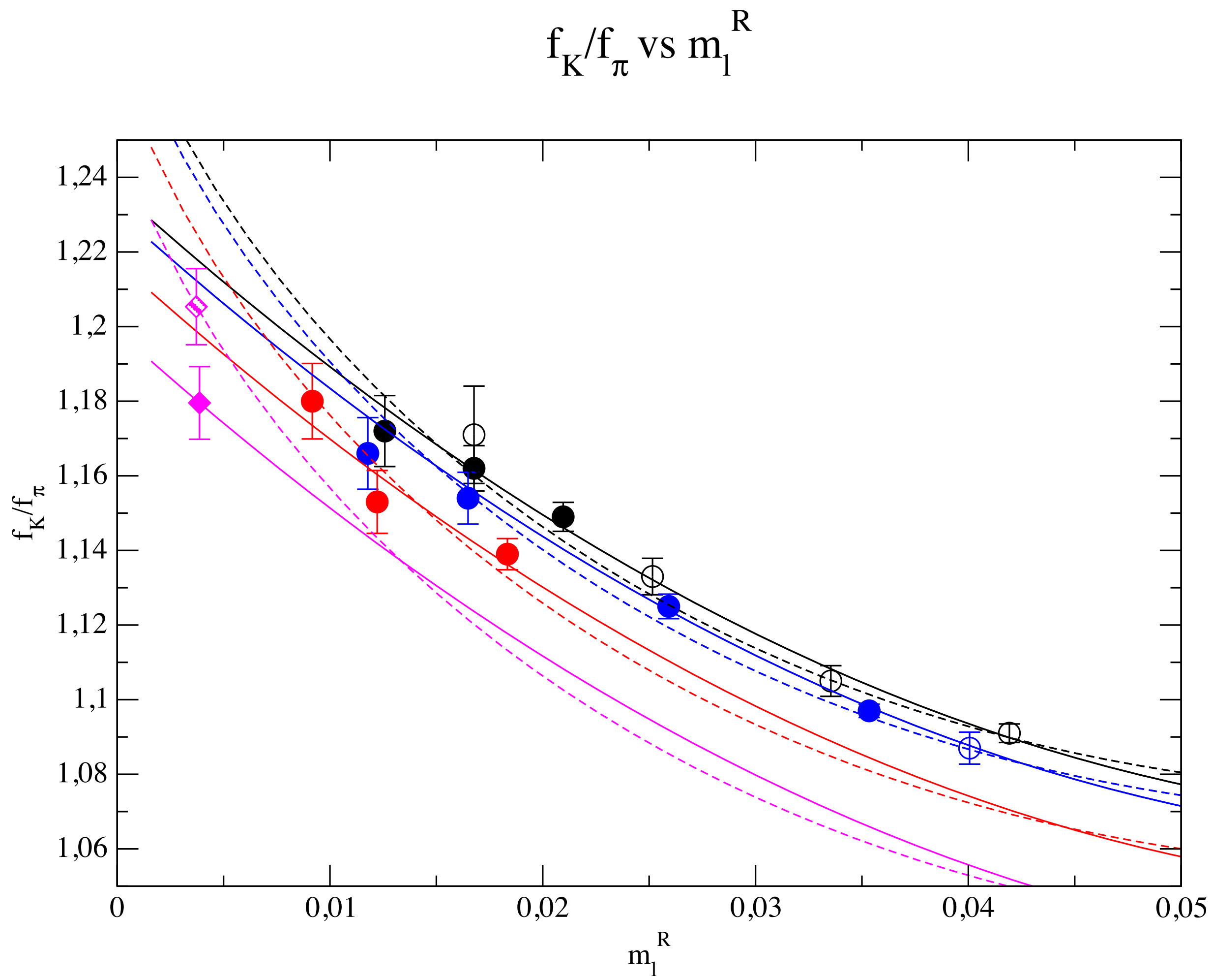
SU(2) Chiral fit in units of r_0



Polynomial fit in units of r_0



The ratio f_K/f_π



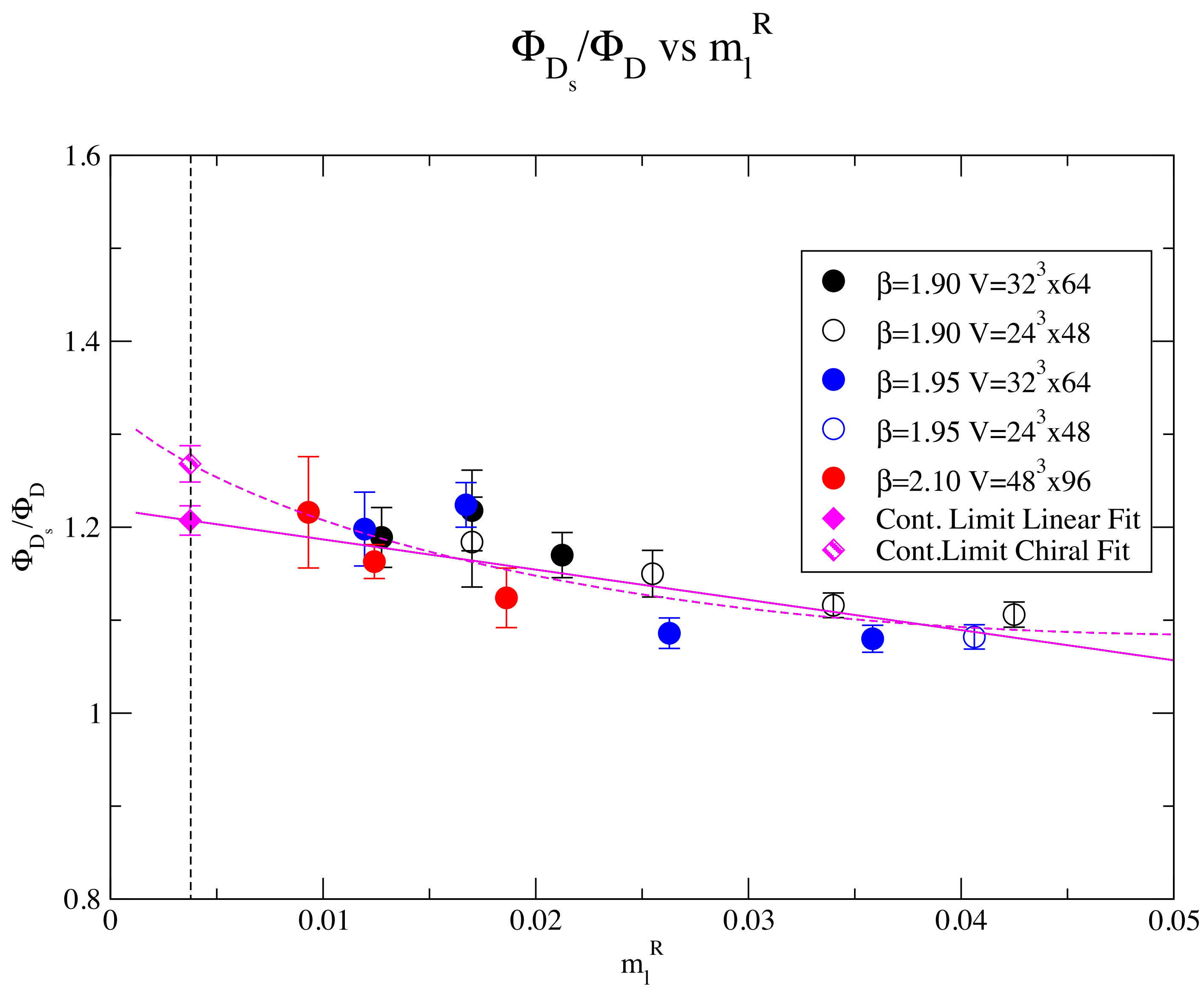
Preliminary

$f_K/f_\pi(m_{ud})$=1.192(16) in agreement with the determination from f_K 1.193(16)

Results from the K analyses

	r_0 Analysis		M_{ss} Analysis	
Quantity	Chiral Fit	Polynomial Fit	Chiral Fit	Polynomial Fit
m_s(MeV)	101.6(4.4)	102.5(3.9)	99.4(2.9)	100.8(3.2)
f_K(MeV)	153.8(2.5)	154.2(1.9)	156.6(1.3)	155.1(1.6)
f_K at m_{up}(MeV)	152.3(2.6)	153.3(2.0)	155.2(1.4)	154.2(1.7)
f_K/f_π	1.179(20)	1.182(15)	1.201(09)	1.189(12)
f_K/f_π at m_{up}	1.168(20)	1.175(16)	1.190(11)	1.182(13)

The ratio Φ_{Ds}/Φ_D



$$f_{D_s} / f_D = 1.200(80)$$

The systematic error due to the chiral extrapolation is in this case much bigger.

The result is however fully compatible with the one found fitting the double ratio

Masses results summary

	Our results	$\mathrm{FLAG}_{N_f=2}$	$\mathrm{FLAG}_{N_f=2+1}$
$m_{u/d}(\mathrm{MeV})$	$3.70(17)$	$3.6(2)$	$3.42(9)$
$m_s(\mathrm{MeV})$	$99.2(4.0)$	$101(3)$	$93.8(2.4)$
$m_s/m_{u/d}$	$27.0(1.3)$	$28.1(1.2)$	$27.5(4)$
m_u/m_d	$0.49(5)$	$0.50(4)$	$0.46(2)$
$m_c(\mathrm{GeV})$	$1.350(46)$		
m_c/m_s	$11.86(59)$		