On the decoupling of mirror fermions

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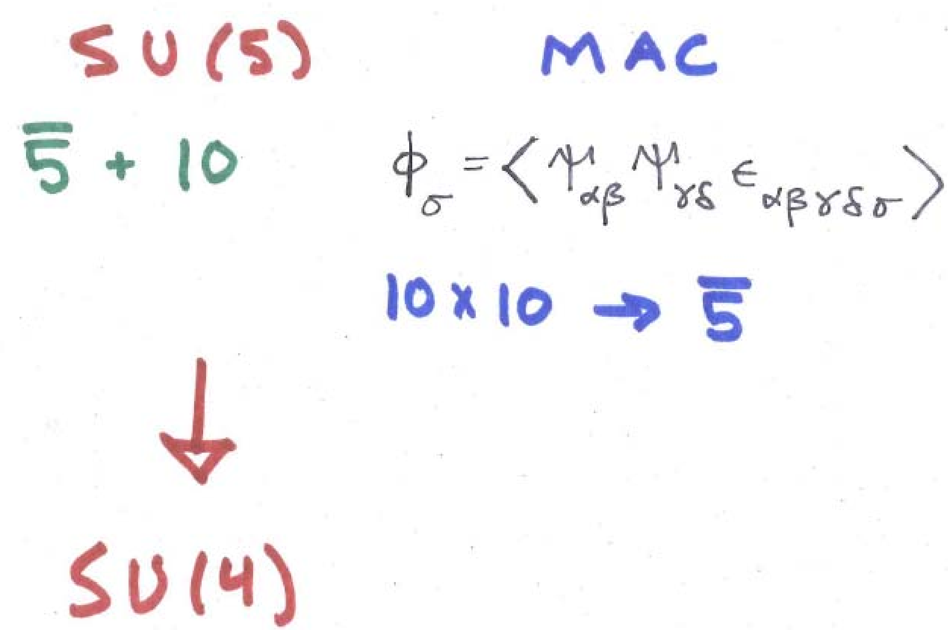
Motivation

- Strong interaction mechanisms to break gauge symmetry (w/o scalars) and generate hierarchies of scales involve tumbling dynamics in chiral gauge theories.

Georgi 1979

Raby, Dimopoulos & Susskind 1980



- This has been built up into realistic models of extended technicolor (ETC).

Appelquist & Terning 1993

$$SU(5)_{ETC} \times SU(2)_{HC} \times SU(4)_{PS} \times SU(2)_L \times U(1)_R$$

$$(\mathbf{5},\mathbf{1},\mathbf{4},\mathbf{2})_0 \quad (\overline{\mathbf{5}},\mathbf{1},\overline{\mathbf{4}},\mathbf{1})_{-1} \quad (\overline{\mathbf{5}},\mathbf{1},\overline{\mathbf{4}},\mathbf{1})_1$$

$$(\mathbf{1},\mathbf{1},\mathbf{6},\mathbf{1})_0 \quad (\mathbf{1},\mathbf{2},\mathbf{6},\mathbf{1})_0$$

$$(\mathbf{10},\mathbf{1},\mathbf{1},\mathbf{1})_0 \quad (\mathbf{5},\mathbf{1},\mathbf{1},\mathbf{1})_0$$
$$(\overline{\mathbf{10}},\mathbf{2},\mathbf{1},\mathbf{1})_0 \quad .$$

$$8(5+\bar{5}) + (10+\overline{10}) + (5+\overline{10})$$

$$SU(5)_{ETC} \to SU(4)_{ETC} \to SU(3)_{ETC} \to SU(2)_{TC}$$

- Most models of dynamical supersymmetry breaking require chiral gauge theories that are strongly coupled.

Classic example: The SU(3) x SU(2) model of Affleck, Dine & Seiberg (1984-85)

	$SU(3) \times SU(2)$
P	$(3, 2)$
L	$(1, 2)$
$\bar{U}$	$(\bar{3}, 1)$
$\bar{D}$	$(\bar{3}, 1)$

$$Z = P^2 \bar{U} \bar{D}, \quad X_1 = P L \bar{D}, \quad Y = P^3 L$$

$$W = \frac{\Lambda_3^7}{Z} + \mathcal{A}(Y - \Lambda_2^4) + \lambda X_1$$

- Chiral dynamics in a hidden sector could give rise to novel forms of dark matter.
- We would like a first principles approach to study strongly coupled chiral gauge dynamics.

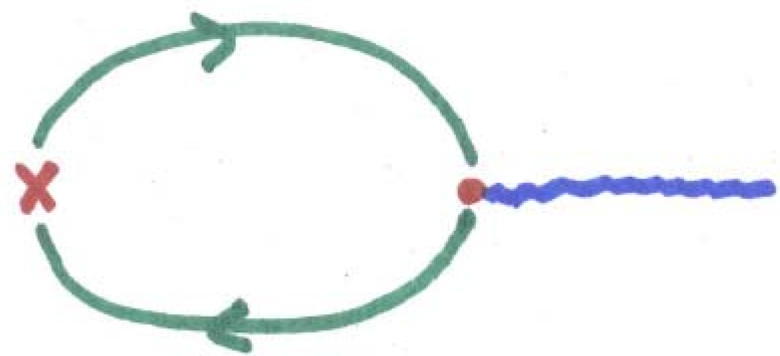
3-4-5 model

- We work with an Abelian gauge theory in two dimensions since this gives the simplest, most numerically feasible example of chiral gauge dynamics.

Light Field	Mirror Field	Q
A_+	A_-	3
B_+	B_-	4
C_-	C_+	5
X_-	X_+	0
—	ϕ	-1

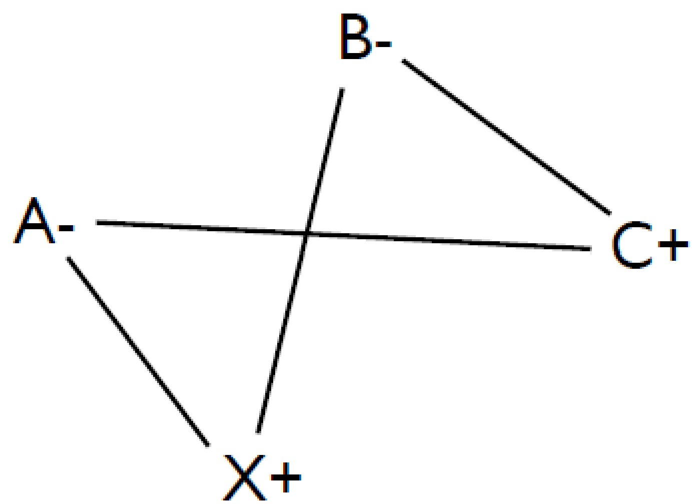
Table 1: Summary of the field content in the 3-4-5 model.

Two-dimension anomaly cancellation



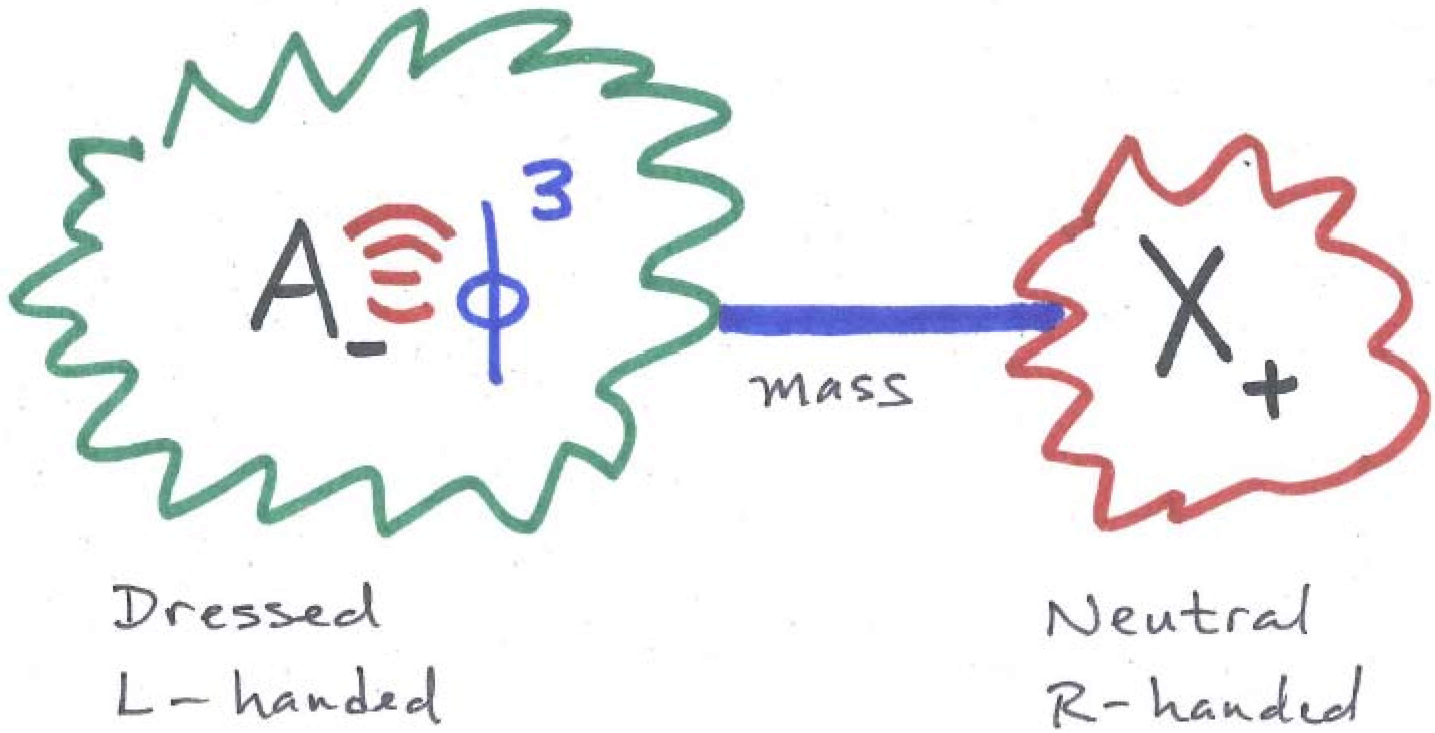
$$A = \sum q_R^2 - \sum q_L^2$$

Mirror sector "Higgs" interactions



Strong Yukawa couplings, symmetric phase

Mass in the mirror sector through dressed fermions (strong Yukawa)



$$
\begin{aligned}
S &= S_{\text{light}} + S_{\text{mirror}} \\
S_{\text{light}} &= -(\bar{A}_+ \cdot D_3 \cdot A_+) - (\bar{B}_+ \cdot D_4 \cdot B_+) - (\bar{C}_- \cdot D_5 \cdot C_-) - (\bar{X}_- \cdot D_0 \cdot X_-) \\
S_{\text{mirror}} &= S_\kappa - (\bar{A}_- \cdot D_3 \cdot A_-) - (\bar{B}_- \cdot D_4 \cdot B_-) - (\bar{C}_+ \cdot D_5 \cdot C_+) - (\bar{X}_+ \cdot D_0 \cdot X_+) \\
&\quad + S_{\text{Yuk.,Dirac}} + S_{\text{Yuk.,Maj}},
\end{aligned}
$$

$$
\begin{aligned}
S_{\text{Yuk.,Dirac}} &= y_{30}\bar{A}_- X_+ \phi^{-3} + y_{40}\bar{B}_- X_+ \phi^{-4} + y_{35}\bar{A}_- C_+ \phi^2 + y_{45}\bar{B}_- C_+ \phi \\
&\quad + y_{30}\bar{X}_+ A_- \phi^3 + y_{40}\bar{X}_+ B_- \phi^4 + y_{35}\bar{C}_+ A_- \phi^{-2} + y_{45}\bar{C}_+ B_- \phi^{-1} \\
S_{\text{Yuk.,Maj.}} &= h_{30} A_-^T \gamma_2 X_+ \phi^3 + h_{40} B_-^T \gamma_2 X_+ \phi^4 + h_{35} A_-^T \gamma_2 C_+ \phi^8 + h_{45} B_-^T \gamma_2 C_+ \phi^9 \\
&\quad - h_{30}\bar{X}_+ \gamma_2 \bar{A}_-^T \phi^{-3} - h_{40}\bar{X}_+ \gamma_2 \bar{B}_-^T \phi^{-4} - h_{35}\bar{C}_+ \gamma_2 \bar{A}_-^T \phi^{-8} \\
&\quad - h_{45}\bar{C}_+ \gamma_2 \bar{B}_-^T \phi^{-9}.
\end{aligned}
$$

$$
\hat{\gamma}_{5q} = \frac{1}{\sqrt{X_q X_q^\dagger}} X_q \gamma_5 \,, \qquad S_\kappa = \frac{\kappa}{2} \sum_x \sum_\mu [2 - (\phi_x^* U_\mu^*(x) \phi_{x+\hat{\mu}} + h.c.)]
$$

$$
D_q = 1 - \hat{\gamma}_{5q}\gamma_5, \qquad U_\mu(x) = e^{iA_\mu(x)} \qquad \phi_x = e^{i\eta_x}
$$

$$
X_{q,xy} = (M - 2r)\delta_{xy} + \frac{1}{2}\sum_\mu \left[(r - \gamma_\mu)\delta_{y,x+\hat{\mu}} U_\mu^q(x) + (r + \gamma_\mu)\delta_{y,x-\hat{\mu}} U_\mu^{q\dagger}(y) \right]
$$

- The polarization tensor is a unique probe of the charged states in the spectrum.

$$\Pi_{\mu\nu}(x,y) \equiv \frac{\delta^2 \ln Z[A]}{\delta A(x)\delta A(y)}\bigg|_{A=0}$$

- Here Z[A] is the partition function with A_μ treated as a background field.
- We can do this because it is the strong Yukawa dynamics (interactions between scalars and fermions) that is supposed to be the operative feature in the mirror sector.

- If there are massless particles in the spectrum, then

$$\tilde{\Pi}_{\mu\nu}(k) = 2C\,\frac{\delta_{\mu\nu}k^2 - k_\mu k_\nu}{k^2}$$

- This gives a directional discontinuity as k → 0:

$$\tilde{\Pi}_{11}(\phi)\Big|_{k\to 0} = C(1 - \cos 2\phi)$$

- In particular, we use:

$$\tilde{\Pi}_{11}(0^o) = 0, \quad \tilde{\Pi}_{11}(45^o) = C, \quad \tilde{\Pi}_{11}(90^o) = 2C$$

- On the other hand, if all particles are massive, then as k → 0,

$$\tilde{\Pi}_{\mu\nu} \sim \frac{\delta_{\mu\nu}k^2 - k_\mu k_\nu}{m^2} \to 0$$

  independent of the direction.
- Of course these are all continuum relations, and they may be modified at finite lattice spacing *a*.
- (In fact that is what we find, even in the free case.)

- Virtually all of the computational cost of our calculation goes into the computation of the mirror sector polarization tensor.
- It is a very length expression, because of the appearance of the gauge field in the Lüscher projection operators that determine the chiral couplings to the Higgs.
- In what follows, $\delta_\mu = \frac{\delta}{\delta A_\mu(x)}$

$$\bar{A}_- = \sum_i \bar{\alpha}^i_- w^\dagger_{iA}, \quad \bar{B}_- = \sum_i \bar{\beta}^i_- w^\dagger_{iB}, \quad \bar{C}_+ = \sum_i \bar{\gamma}^i_+ u^\dagger_{iC}, \quad \bar{X}_+ = \sum_i \bar{\chi}^i_+ u^\dagger_{iX},$$

$$A_- = \sum_i \alpha^i_- t_i, \quad B_- = \sum_i \beta^i_- t_i, \quad C_+ = \sum_i \gamma^i_+ v_i, \quad X_+ = \sum_i \chi^i_+ v_i,$$

$$\gamma_5 v_i = v_i, \quad \gamma_5 t_i = -t_i \quad \hat{\gamma}_5 u_i = -u_i, \quad \hat{\gamma}_5 w_i = w_i$$

$$
\begin{aligned}
\Pi_{\mu\nu} = {} & \delta_\nu (j_\mu^{wA} + j_\mu^{wB} + j_\mu^{uC}) \\
& + \langle \bar{\alpha}_-^i \alpha_-^j \rangle (w_{iA}^\dagger \cdot (\delta_\nu \delta_\mu D_3 + \delta_\nu \hat{P}_{+A} \cdot \delta_\mu D_3) \cdot t_j) \\
& + \langle \bar{\beta}_-^i \beta_-^j \rangle (w_{iB}^\dagger \cdot (\delta_\nu \delta_\mu D_4 + \delta_\nu \hat{P}_{+B} \cdot \delta_\mu D_4) \cdot t_j) \\
& + \langle \bar{\gamma}_+^i \gamma_+^j \rangle (u_{iC}^\dagger \cdot (\delta_\nu \delta_\mu D_5 + \delta_\nu \hat{P}_{-C} \cdot \delta_\mu D_5) \cdot v_j) \\
& + \Big\langle [\bar{\alpha}_-^i \alpha_-^j (w_{iA}^\dagger \cdot \delta_\mu D_3 \cdot t_j) + \bar{\beta}_-^i \beta_-^j (w_{iB}^\dagger \cdot \delta_\mu D_4 \cdot t_j) \\
& + \bar{\gamma}_+^i \gamma_+^j (u_{iC}^\dagger \cdot \delta_\mu D_5 \cdot v_j)][\bar{\alpha}_-^k \alpha_-^l (w_{kA}^\dagger \cdot \delta_\nu D_3 \cdot t_l) \\
& + \bar{\beta}_-^k \beta_-^l (w_{kB}^\dagger \cdot \delta_\nu D_4 \cdot t_l) + \bar{\gamma}_+^k \gamma_+^l (u_{kC}^\dagger \cdot \delta_\nu D_5 \cdot v_l)] \Big\rangle^C \\
& + \frac{\kappa}{2} \langle (\phi^* \cdot \delta_\nu \delta_\mu U^* \cdot \phi) + \text{h.c.} \rangle \\
& + \frac{\kappa^2}{4} \langle [(\phi^* \cdot \delta_\mu U^* \cdot \phi) + \text{h.c.}][(\phi^* \cdot \delta_\nu U^* \cdot \phi) + \text{h.c.}] \rangle^C \\
& + \frac{\kappa}{2} \Big\langle [(\phi^* \cdot \delta_\mu U^* \cdot \phi) + \text{h.c.}][\bar{\alpha}_-^i \alpha_-^j (w_{iA}^\dagger \cdot \delta_\mu D_3 \cdot t_j) + \bar{\beta}_-^i \beta_-^j (w_{iB}^\dagger \cdot \delta_\mu D_4 \cdot t_j) \\
& + \bar{\gamma}_+^i \gamma_+^j (u_{iC}^\dagger \cdot \delta_\mu D_5 \cdot v_j)] + (\mu \leftrightarrow \nu) \Big\rangle^C \\
& - y_{30} \langle \bar{\alpha}_-^i \chi_+^j \rangle (w_{iA}^\dagger \cdot \delta_\nu (\hat{P}_{+A} \cdot \delta_\mu \hat{P}_{+A}) \cdot \phi^{-3} \cdot v_j) - y_{40} \langle \bar{\beta}_-^i \chi_+^j \rangle (w_{iB}^\dagger \cdot \delta_\nu (\hat{P}_{+B} \cdot \delta_\mu \hat{P}_{+B}) \cdot \phi^{-4} \cdot v_j) \\
& - y_{35} \langle \bar{\alpha}_-^i \gamma_+^j \rangle (w_{iA}^\dagger \cdot \delta_\nu (\hat{P}_{+A} \cdot \delta_\mu \hat{P}_{+A}) \cdot \phi^2 \cdot v_j) - y_{45} \langle \bar{\beta}_-^i \gamma_+^j \rangle (w_{iB}^\dagger \cdot \delta_\nu (\hat{P}_{+B} \cdot \delta_\mu \hat{P}_{+B}) \cdot \phi \cdot v_j) \\
& - y_{35} \langle \bar{\gamma}_+^i \alpha_-^j \rangle (u_{iC}^\dagger \cdot \delta_\nu (\hat{P}_{-C} \cdot \delta_\mu \hat{P}_{-C}) \cdot \phi^{-2} \cdot t_j) - y_{45} \langle \bar{\gamma}_+^i \beta_-^j \rangle (u_{iC}^\dagger \cdot \delta_\nu (\hat{P}_{-C} \cdot \delta_\mu \hat{P}_{-C}) \cdot \phi^{-1} \cdot t_j) \\
& + h_{30} \langle \bar{\chi}_+^i \bar{\alpha}_-^j \rangle (u_{iX}^\dagger \cdot \phi^{-3} \cdot \gamma_2 \cdot \delta_\nu (\delta_\mu \hat{P}_{+A}^T \cdot \hat{P}_{+A}^T) \cdot w_{jA}^*) \\
& + h_{40} \langle \bar{\chi}_+^i \bar{\beta}_-^j \rangle (u_{iX}^\dagger \cdot \phi^{-4} \cdot \gamma_2 \cdot \delta_\nu (\delta_\mu \hat{P}_{+B}^T \cdot \hat{P}_{+B}^T) \cdot w_{jB}^*) \\
& + h_{35} \langle \bar{\gamma}_+^i \bar{\alpha}_-^j \rangle [(u_{iC}^\dagger \cdot \delta_\nu (\hat{P}_{-C} \cdot \delta_\mu \hat{P}_{-C}) \cdot \phi^{-8} \cdot \gamma_2 \cdot w_{jA}^*) \\
& + (u_{iC}^\dagger \cdot \phi^{-8} \cdot \gamma_2 \cdot \delta_\nu (\delta_\mu \hat{P}_{+A}^T \cdot \hat{P}_{+A}^T) \cdot w_{jA}^*)] \\
& + h_{35} \langle \bar{\gamma}_+^i \bar{\alpha}_-^j \rangle [(u_{iC}^\dagger \cdot \delta_\mu \hat{P}_{-C} \cdot \phi^{-8} \cdot \gamma_2 \cdot \delta_\nu \hat{P}_{+A}^T \cdot w_{jA}^*) + (\mu \leftrightarrow \nu)] \\
& + h_{45} \langle \bar{\gamma}_+^i \bar{\beta}_-^j \rangle [(u_{iC}^\dagger \cdot \delta_\mu \hat{P}_{-C} \cdot \phi^{-9} \cdot \gamma_2 \cdot \delta_\nu \hat{P}_{+B}^T \cdot w_{jB}^*) + (\mu \leftrightarrow \nu)] \\
& + h_{45} \langle \bar{\gamma}_+^i \bar{\beta}_-^j \rangle [(u_{iC}^\dagger \cdot \delta_\nu (\hat{P}_{-C} \cdot \delta_\mu \hat{P}_{-C}) \cdot \phi^{-9} \cdot \gamma_2 \cdot w_{jB}^*) \\
& + (u_{iC}^\dagger \cdot \phi^{-9} \cdot \gamma_2 \cdot \delta_\nu (\delta_\mu \hat{P}_{+B}^T \cdot \hat{P}_{+B}^T) \cdot w_{jB}^*)]
\end{aligned}
$$

(continued)

$$
\begin{aligned}
&-\Big\langle \big[\bar{\alpha}_-^k \alpha_-^l (w_{kA}^\dagger \cdot \delta_\mu D_3 \cdot t_l) + \bar{\beta}_-^k \beta_-^l (w_{kB}^\dagger \cdot \delta_\mu D_4 \cdot t_l) \\
&+\bar{\gamma}_+^k \gamma_+^l (u_{kC}^\dagger \cdot \delta_\mu D_5 \cdot v_l)\big] \\
&\times \big[y_{30}\bar{\alpha}_-^i \chi_+^j (w_{iA}^\dagger \cdot \delta_\nu \hat{P}_{+A} \cdot \phi^{-3} \cdot v_j) + y_{40}\bar{\beta}_-^i \chi_+^j (w_{iB}^\dagger \cdot \delta_\nu \hat{P}_{+B} \cdot \phi^{-4} \cdot v_j) \\
&+y_{35}\bar{\alpha}_-^i \gamma_+^j (w_{iA}^\dagger \cdot \delta_\nu \hat{P}_{+A} \cdot \phi^2 \cdot v_j) + y_{45}\bar{\beta}_-^i \gamma_+^j (w_{iB}^\dagger \cdot \delta_\nu \hat{P}_{+B} \cdot \phi \cdot v_j) \\
&+y_{35}\bar{\gamma}_+^i \alpha_-^j (u_{iC}^\dagger \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-2} \cdot t_j) + y_{45}\bar{\gamma}_+^i \beta_-^j (u_{iC}^\dagger \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-1} \cdot t_j) \\
&-h_{30}\bar{\chi}_+^i \bar{\alpha}_-^j (u_{iX}^\dagger \cdot \phi^{-3} \cdot \gamma_2 \cdot \delta_\nu \hat{P}_{+A}^T \cdot w_{jA}^*) - h_{40}\bar{\chi}_+^i \bar{\beta}_-^j (u_{iX}^\dagger \cdot \phi^{-4} \cdot \gamma_2 \cdot \delta_\nu \hat{P}_{+B}^T \cdot w_{jB}^*) \\
&-h_{35}\bar{\gamma}_+^i \bar{\alpha}_-^j [(u_{iC}^\dagger \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-8} \cdot \gamma_2 \cdot w_{jA}^*) + (u_{iC}^\dagger \cdot \phi^{-8} \cdot \gamma_2 \cdot \delta_\nu \hat{P}_{+A}^T \cdot w_{jA}^*)] \\
&-h_{45}\bar{\gamma}_+^i \bar{\beta}_-^j [(u_{iC}^\dagger \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-9} \cdot \gamma_2 \cdot w_{jB}^*) \\
&+(u_{iC}^\dagger \cdot \phi^{-9} \cdot \gamma_2 \cdot \delta_\nu \hat{P}_{+B}^T \cdot w_{jB}^*)]\big] + (\mu \leftrightarrow \nu)\Big\rangle^C \\
&+\Big\langle \big\{y_{30}\bar{\alpha}_-^i \chi_+^j (w_{iA}^\dagger \cdot \delta_\mu \hat{P}_{+A} \cdot \phi^{-3} \cdot v_j) + y_{40}\bar{\beta}_-^i \chi_+^j (w_{iB}^\dagger \cdot \delta_\mu \hat{P}_{+B} \cdot \phi^{-4} \cdot v_j) \\
&+y_{35}\bar{\alpha}_-^i \gamma_+^j (w_{iA}^\dagger \cdot \delta_\mu \hat{P}_{+A} \cdot \phi^2 \cdot v_j) + y_{45}\bar{\beta}_-^i \gamma_+^j (w_{iB}^\dagger \cdot \delta_\mu \hat{P}_{+B} \cdot \phi \cdot v_j) \\
&+y_{35}\bar{\gamma}_+^i \alpha_-^j (u_{iC}^\dagger \cdot \delta_\mu \hat{P}_{-C} \cdot \phi^{-2} \cdot t_j) + y_{45}\bar{\gamma}_+^i \beta_-^j (u_{iC}^\dagger \cdot \delta_\mu \hat{P}_{-C} \cdot \phi^{-1} \cdot t_j) \\
&-h_{30}\bar{\chi}_+^i \bar{\alpha}_-^j (u_{iX}^\dagger \cdot \phi^{-3} \cdot \gamma_2 \cdot \delta_\mu \hat{P}_{+A}^T \cdot w_{jA}^*) - h_{40}\bar{\chi}_+^i \bar{\beta}_-^j (u_{iX}^\dagger \cdot \phi^{-4} \cdot \gamma_2 \cdot \delta_\mu \hat{P}_{+B}^T \cdot w_{jB}^*) \\
&-h_{35}\bar{\gamma}_+^i \bar{\alpha}_-^j [(u_{iC}^\dagger \cdot \delta_\mu \hat{P}_{-C} \cdot \phi^{-8} \cdot \gamma_2 \cdot w_{jA}^*) + (u_{iC}^\dagger \cdot \phi^{-8} \cdot \gamma_2 \cdot \delta_\mu \hat{P}_{+A}^T \cdot w_{jA}^*)] \\
&-h_{45}\bar{\gamma}_+^i \bar{\beta}_-^j [(u_{iC}^\dagger \cdot \delta_\mu \hat{P}_{-C} \cdot \phi^{-9} \cdot \gamma_2 \cdot w_{jB}^*) + (u_{iC}^\dagger \cdot \phi^{-9} \cdot \gamma_2 \cdot \delta_\mu \hat{P}_{+B}^T \cdot w_{jB}^*)]\big\} \\
&\times \big\{y_{30}\bar{\alpha}_-^k \chi_+^l (w_{kA}^\dagger \cdot \delta_\nu \hat{P}_{+A} \cdot \phi^{-3} \cdot v_l) + y_{40}\bar{\beta}_-^k \chi_+^l (w_{kB}^\dagger \cdot \delta_\nu \hat{P}_{+B} \cdot \phi^{-4} \cdot v_l) \\
&+y_{35}\bar{\alpha}_-^k \gamma_+^l (w_{kA}^\dagger \cdot \delta_\mu \hat{P}_{+A} \cdot \phi^2 \cdot v_l) + y_{45}\bar{\beta}_-^k \gamma_+^l (w_{kB}^\dagger \cdot \delta_\nu \hat{P}_{+B} \cdot \phi \cdot v_l) \\
&+y_{35}\bar{\gamma}_+^k \alpha_-^l (u_{kC}^\dagger \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-2} \cdot t_l) + y_{45}\bar{\gamma}_+^k \beta_-^l (u_{kC}^\dagger \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-1} \cdot t_l) \\
&-h_{30}\bar{\chi}_+^k \bar{\alpha}_-^l (u_{kX}^\dagger \cdot \phi^{-3} \cdot \gamma_2 \cdot \delta_\nu \hat{P}_{+A}^T \cdot w_{lA}^*) - h_{40}\bar{\chi}_+^k \bar{\beta}_-^l (u_{kX}^\dagger \cdot \phi^{-4} \cdot \gamma_2 \cdot \delta_\nu \hat{P}_{+B}^T \cdot w_{lB}^*) \\
&-h_{35}\bar{\gamma}_+^k \bar{\alpha}_-^l [(u_{kC}^\dagger \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-8} \cdot \gamma_2 \cdot w_{lA}^*) + (u_{kC}^\dagger \cdot \phi^{-8} \cdot \gamma_2 \cdot \delta_\nu \hat{P}_{+A}^T \cdot w_{lA}^*)] \\
&-h_{45}\bar{\gamma}_+^k \bar{\beta}_-^l [(u_{kC}^\dagger \cdot \delta_\nu \hat{P}_{-C} \cdot \phi^{-9} \cdot \gamma_2 \cdot w_{lB}^*) + (u_{kC}^\dagger \cdot \phi^{-9} \cdot \gamma_2 \cdot \delta_\nu \hat{P}_{+B}^T \cdot w_{lB}^*)]\big\}\Big\rangle^C
\end{aligned}
$$

(continued)

$$
\begin{aligned}
&-\frac{\kappa}{2}\Big\langle [(\phi^{*}\cdot\delta_{\mu}U^{*}\cdot\phi)+\mathrm{h.c.}] \\
&\times\{y_{30}\bar{\alpha}^{i}_{-}\chi^{j}_{+}(w^{\dagger}_{iA}\cdot\delta_{\nu}\hat{P}_{+A}\cdot\phi^{-3}\cdot v_{j})+y_{40}\bar{\beta}^{i}_{-}\chi^{j}_{+}(w^{\dagger}_{iB}\cdot\delta_{\nu}\hat{P}_{+B}\cdot\phi^{-4}\cdot v_{j}) \\
&+y_{35}\bar{\alpha}^{i}_{-}\gamma^{j}_{+}(w^{\dagger}_{iA}\cdot\delta_{\nu}\hat{P}_{+A}\cdot\phi^{2}\cdot v_{j})+y_{45}\bar{\beta}^{i}_{-}\gamma^{j}_{+}(w^{\dagger}_{iB}\cdot\delta_{\nu}\hat{P}_{+B}\cdot\phi\cdot v_{j}) \\
&+y_{35}\bar{\gamma}^{i}_{+}\alpha^{j}_{-}(u^{\dagger}_{iC}\cdot\delta_{\nu}\hat{P}_{-C}\cdot\phi^{-2}\cdot t_{j})+y_{45}\bar{\gamma}^{i}_{+}\beta^{j}_{-}(u^{\dagger}_{iC}\cdot\delta_{\nu}\hat{P}_{-C}\cdot\phi^{-1}\cdot t_{j}) \\
&-h_{30}\bar{\chi}^{i}_{+}\bar{\alpha}^{j}_{-}(u^{\dagger}_{iX}\cdot\phi^{-3}\cdot\gamma_{2}\cdot\delta_{\nu}\hat{P}^{T}_{+A}\cdot w^{*}_{jA})-h_{40}\bar{\chi}^{i}_{+}\bar{\beta}^{j}_{-}(u^{\dagger}_{iX}\cdot\phi^{-4}\cdot\gamma_{2}\cdot\delta_{\nu}\hat{P}^{T}_{+B}\cdot w^{*}_{jB}) \\
&-h_{35}\bar{\gamma}^{i}_{+}\bar{\alpha}^{j}_{-}[(u^{\dagger}_{iC}\cdot\delta_{\nu}\hat{P}_{-C}\cdot\phi^{-8}\cdot\gamma_{2}\cdot w^{*}_{jA})+(u^{\dagger}_{iC}\cdot\phi^{-8}\cdot\gamma_{2}\cdot\delta_{\nu}\hat{P}^{T}_{+A}\cdot w^{*}_{jA})] \\
&-h_{45}\bar{\gamma}^{i}_{+}\bar{\beta}^{j}_{-}[(u^{\dagger}_{iC}\cdot\delta_{\nu}\hat{P}_{-C}\cdot\phi^{-9}\cdot\gamma_{2}\cdot w^{*}_{jB}) \\
&+(u^{\dagger}_{iC}\cdot\phi^{-9}\cdot\gamma_{2}\cdot\delta_{\nu}\hat{P}^{T}_{+B}\cdot w^{*}_{jB})]\}+(\mu\leftrightarrow\nu)\Big\rangle^{C}. \qquad \text{(A.1)}
\end{aligned}
$$

- Chen Chen implemented this lengthy expression into computer code.
- After various optimizations, it takes up about 3000 lines of code.
- Checks using transversality and symmetry w.r.t. interchange of indices.
- Around 500,000 core-hours to compute on NxN lattices, N=6,8,10
- Scaling of code is N^{10}(10 nested loops), so constrained to small lattices.

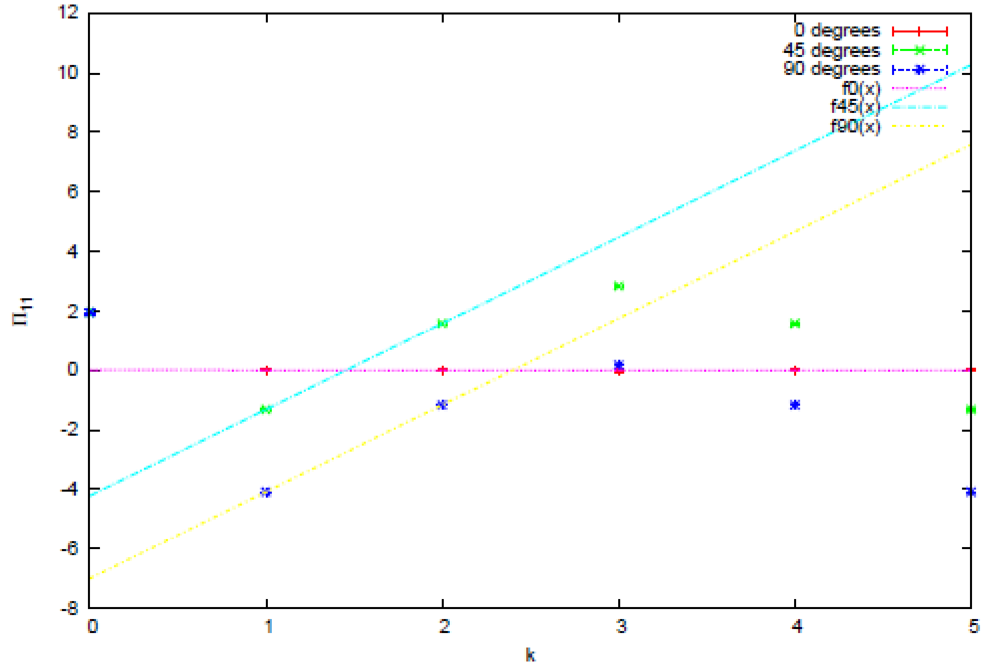


Figure 3. $\tilde{\Pi}^{\mathrm{mirror},\prime}_{11}(k)$ on a 6×6 lattice. The lines show the extrapolation $k \to 0$ for different angles of approach. A clear discontinuity in the directional limit can be seen.

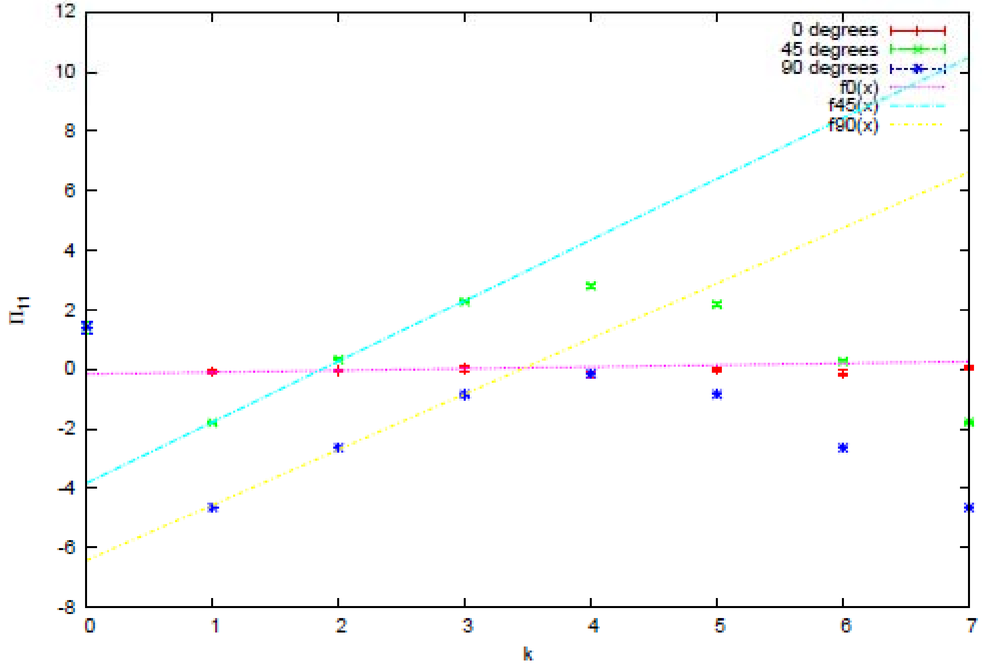


Figure 4. $\tilde{\Pi}_{11}^{\mathrm{mirror},\prime}$ on an 8×8 lattice with the couplings given in Table 2.

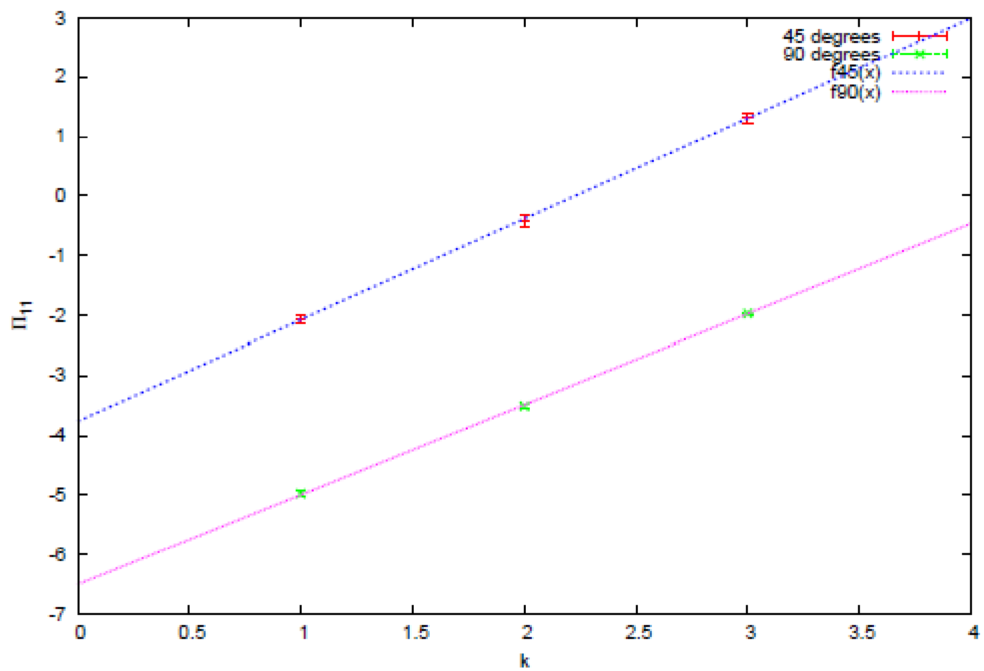


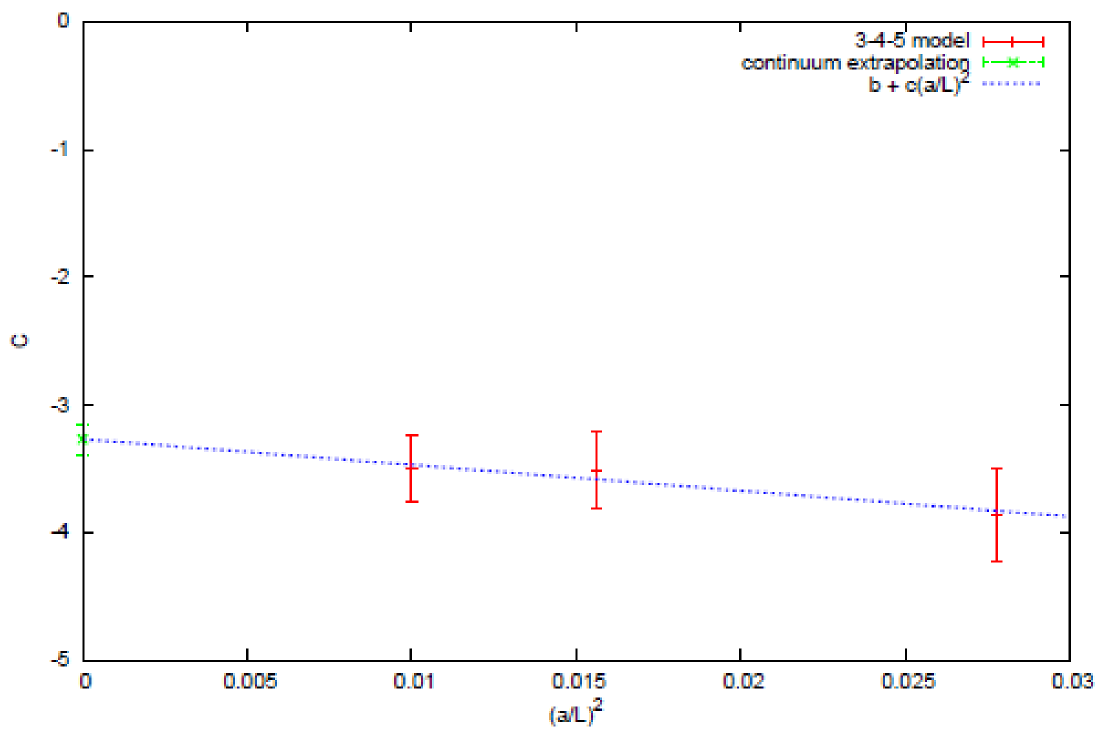
Figure 5. $\tilde{\Pi}_{11}^{\text{mirror},\prime}$ on a 10×10 lattice with the couplings given in Table 2. Only the smallest values of k were studied and the 0^o approach to the origin was omitted because of the expense of the calculation.

- From these we can extract the discontinuity C on each lattice.
- It is approximately constant with increasing N.
- Overlap fermions have $O(a^2)$ discretization error.
- So the discontinuity in the continuum limit is described by:

$$C = b + c(a/L)^2 + \mathcal{O}(a/L)^4 = b + cN^{-2} + \mathcal{O}(N^{-4}), \quad L = Na$$

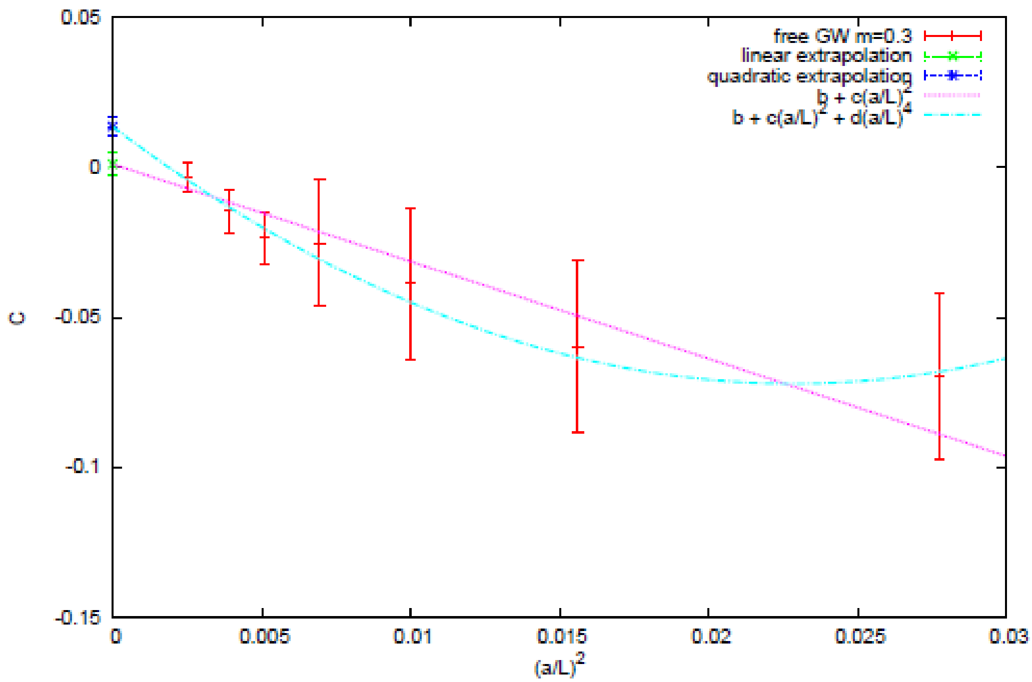
- Holding the physical volume L^2 fixed we can take the continuum limit by increasing N.

Continuum limit of 3-4-5 mirror sector directional limit "discontinuity"



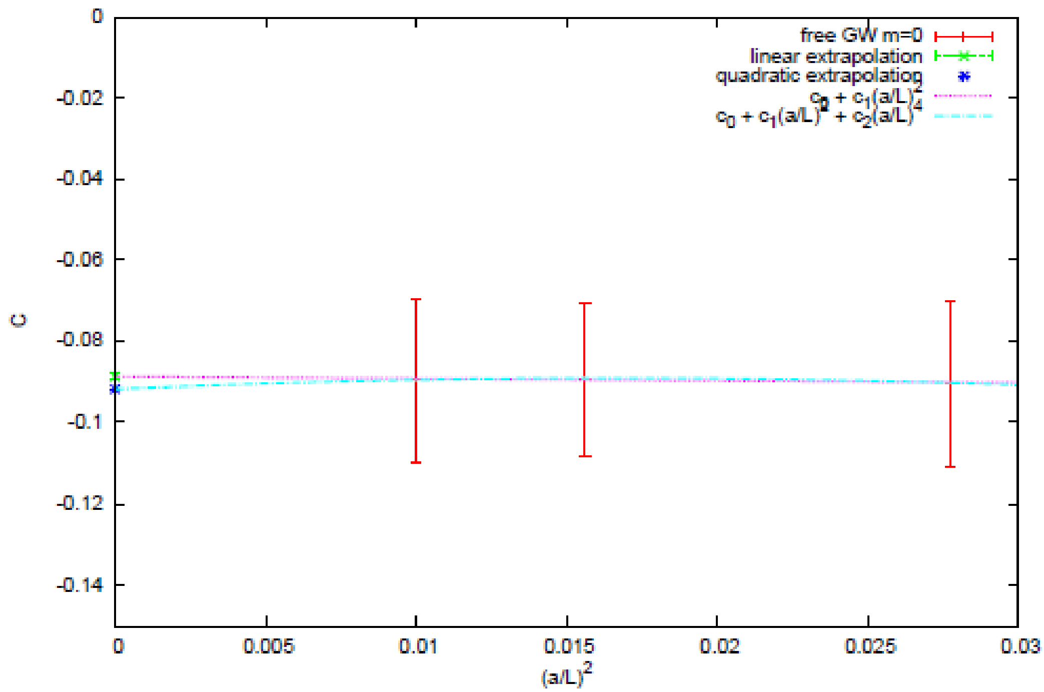
It is nonzero in the continuum limit, consistent with a massless theory.

Massive free overlap



Also see discontinuity nonzero at finite a, extrapolating to zero (within errors).

Massless free overlap



Clearly extrapolates to nonzero directional discontinuity in the massless case.

Conclusions

- Looking at the mirror sector polarization tensor, we see that the continuum limit is consistent with a directional discontinuity.
- Most similar to the free massless overlap theory.
- Very different from the free massive overlap theory.
- It appears that there are massless modes in the mirror sector.
- It seems that decoupling of the mirror sector is not successful.

Future work

- The Lüscher consistency condition approach.
- Applied to the 3-4-5 model.
- Numerical results.
- Compare to bosonization results.