Non-perturbative fermion mass generation in Wilson lattice QCD

Roberto Frezzotti and Giancarlo Rossi

University & INFN of "Roma Tor Vergata"

Lattice 2013 – Mainz, August 2, 2013

Outline and main ideas of the talk

1. $O(\Lambda_S)$ contribution to m_{cr} seen in lattice QCD with plain Wilson quarks: a dynamically generated fermion mass? Not really: it comes entangled with the $1/a$–divergent mass term, their separation neither well defined nor "natural". Conjecture: a mechanism reproducing such an $O(\Lambda_S)$ mass term in chiral WTI's.
2. Toy model of two fermion species subjected to a non-Abelian gauge interaction and coupled to scalars by Wilson-like and Yukawa terms: given the model symmetries, we conjecture a non-perturbative (NP) mechanism analogous to that in item 1 yielding a well defined and natural $O(\Lambda_S)$ fermion mass term.
3. Outlook: applications of the conjectured NP mechanism for dynamical mass generation to fermion mass hierarchy and inclusion of electroweak-interactions:

   - each fermion species (except for neutrinos) receives a mass of the order of the Λ-parameter of the strongest gauge interaction in the model scaled by powers of the coupling of the interaction connecting it to the superstrong sector;

   - upon including SM-like weak gauge interactions weak gauge bosons naturally acquire via a "NP analog of the Higgs mechanism" a mass of the order of the superstrong Λ-parameter times g_W and an appropriate loop factor.

m_{cr} in Lattice QCD (LQCD) with Wilson quarks

$m_{cr} = m_{cr}(g_0^2)$: m_0–value(s) at which chiral symmetries are recovered up to $O(a)$

best determined from (flavour non-singlet) axial Ward-Takahashi identities (WTI):

$\nabla_\mu \hat{A}^i_\mu = 2m_0 \hat{P}^i + a\hat{O}^i_5$, $aO^i_5 = \delta^i_A(\mathrm{Wilson\ \ term})$ $i = 1, 2, \ldots N_f^2 - 1$

mixing $a\hat{O}^i_5 = Z^{-1}_{5,k}(\hat{O}^i_{5,k})_R - 2\bar{M}\hat{P}^i - (Z_A - 1)\partial_\mu A^i_\mu$ $\Rightarrow$ renormalized WTI

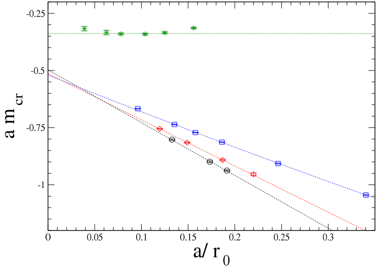
$\nabla_\mu(Z_A\hat{A}^i_\mu) = 2(m_0 - \bar{M})\hat{P}^i + \mathrm{O}(a)$, $Z_A = Z_A(g_0^2)$, $a\bar{M} = w(g_0^2, am_0)$

$w(g_0^2, am_{cr}) = am_{cr}$ determines $am_{cr} = f(g_0^2) = c_0 + c_1 a\Lambda_S + c_2(a\Lambda_S)^2 + \mathrm{O}(a^3)$

c_i polynomials in g_0^2, $a\Lambda_S \simeq \exp(-1/(2b_0 g_0^2))$ non-perturbative contribution(s)

one operator $(\hat{P}^i)$ associated to $\bar{M}$: any chiral symmetry recovery condition fixes

$m_{cr} = c_0/a + c_1\Lambda_S + \mathrm{O}(a)$ $\Rightarrow$ no symmetry-based criterion to isolate $c_1\Lambda_S$.



- Strong evidence of $c_1 \Lambda_S$ term in m_{cr} for plain ($c_{SW} = 0$) Wilson fermions:

  am_{cr} appears linear in a/r_0 in a wide a/r_0–"window", with $-c_1\Lambda_S \in [0.7, 1.0]$ GeV

"window": as $a \to 0$ terms polynomial in g_0^2, as $a \to \infty$ higher-a orders dominating

- Suggestive: "theory" with $m_0 = c_0/a \quad \Rightarrow \quad \nabla_\mu A^i_\mu = 2(-c_1\Lambda_S)\hat{P}^i + \mathrm{O}(a)$

  but $m_0 = c_0/a$ is most "unnatural" and not well defined within LQCD ...

Understanding & modelling the $c_1\Lambda_S$ term in m_{cr}

• Do non-Abelian gauge models exist where a symmetry criterion allows to fix the bare parameters so as to "naturally" obtain an effective fermion mass $\sim \Lambda_S$?

• Try to understand the NP mechanism responsible for the $c_1\Lambda_S$ term of m_{cr} in LQCD, then possibly "export" it to a theory where the answer may be positive.

• $am_{cr} = c_0(g_0^2) + c_1(g_0^2)a\Lambda_S + \ldots$: look for an O($a$) correction wrt leading term

• In Wilson's LQCD O(a) corrections arise due to chiral (Ch) symmetry breaking induced by the Wilson action term, can be described via Symanzik's LEL (SLEL) approach – O($a\Lambda_S$) provided dynamical ChSSB takes place in continuum QCD

• If $a\Lambda_S^2 \gg m_0 - m_{cr}, \mu_q \to 0$ the Wilson term dominates and acts as a seed of ChSSB: observed numerically, reflected in lattice ChPT (Aoki, Sharpe, Bär, ...)

$\Rightarrow$ the SLEL description of e.g. $O = S^1 S^1$ reads $\langle S^1(x)S^1(0)\rangle|^L_{m_0=m_{cr},\mu_q=0} =$

$$= \langle S^1(x)S^1(0)\rangle|^C_{m_q=0\ \mathrm{sgn}(r)} - a\int_y \langle L_5(y)S^1(x)S^1(0)\rangle|^C_{m_q=0\ \mathrm{sgn}(r)} + \mathrm{O}(a^2)$$

with $L_5|_{m_q=\mu_q=0} = b_{SW}(r)\bar{q}(i\sigma\cdot F)q + b_{D2}(r)\bar{q}(-\not{D}\not{D})q$, $\quad b_{SW,D2}$ odd in r.

- In fixed gauge, for $a\Lambda_S^2 \gg m_0 - m_{cr}, \mu_q \to 0$, expect $\mathrm{O}(a)$ corrections to

$\langle G^b_\mu(x) G^c_\nu(0)\rangle|^L_{m_0=m_{cr},\mu_q=0}$ and $\langle q_L(x)\bar{q}_L(0)\rangle|^L_{m_0=m_{cr},\mu_q=0}$ of the form

$$\Delta\langle G^b_\mu(x) G^c_\nu(0)\rangle|^L_{m_0=m_{cr},\mu_q=0} = -a\int_y \langle L_5(y) G^b_\mu(x) G^c_\nu(0)\rangle|^C_{m_q=0\ \mathrm{sgn}(r)} + \mathrm{O}(a^2)$$

$$\Delta\langle q_L(x)\bar{q}_L(0)\rangle|^L_{m_0=m_{cr},\mu_q=0} = -a\int_y \langle L_5(y) q_L(x)\bar{q}_L(0)\rangle|^C_{m_q=0\ \mathrm{sgn}(r)} + \mathrm{O}(a^2)$$

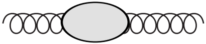
- $\mathrm{O}(a\Lambda_S)$ corrections in propagators may yield a mixing of $a\hat{O}^i_5$ with $\Lambda_S \hat{P}^i$...

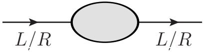
Conjecture: NP corrections to the mixing of $a\hat{O}^i_5$ can *effectively* be described by *lattice PT with* augmented Feynman rules for gluon and $q_{L/R}$ propagators

$$\Delta\Gamma^{bc}_{\mu\nu}(k)|^L_{m_0=m_{cr},\mu_q=0} = a\Lambda_S\alpha_S(\Lambda_S)\delta^{bc}\Pi_{\mu\nu}(k)k^{-2} f_{AA}(\Lambda_S^2/k^2)$$

$$\Delta S_{LL}(k)|^L_{m_0=m_{cr},\mu_q=0} = a\Lambda_S\alpha_S(\Lambda_S)\ \frac{ik_\mu(\gamma_\mu)_{LL}}{k^2+((ar/2)k^2)^2}\ f_{q\bar{q}}(\Lambda_S^2/k^2)$$

where $\quad f_{AA}(\Lambda_S^2/k^2) \overset{k^2\to a^{-2}}{\to} h_{AA} \neq 0 \quad$ and $\quad f_{q\bar{q}}(\Lambda_S^2/k^2) \overset{k^2\to a^{-2}}{\to} h_{q\bar{q}} \neq 0$





region $k^2 \sim a^{-2}$: relevant for the leading NP corrections to the mixing of $a\hat{O}^i_5$ ...

• Within our model (conjecture above): leading NP effects in the mixing of $a\hat{O}_5^i$ stem from $O(\alpha_S^2)$ "diagrams" of the kind of the following ones

⋆ the loop integral is dominated by loop momenta $k^2 \sim a^{-2}$, yielding a contribution

a^{-2}(loop UV divergency) $\times a$(insertion of $a\hat{O}_5^i$) $\times a\Lambda_S$(propagator correction) $\sim \mathrm{O}(\Lambda_S)$

⋆ the bare $a\hat{O}_5^i$ undegoes a mixing with $\mathrm{O}(\alpha_S^2\Lambda_S)\hat{P}^i$ that can not be canceled by any insertion of $m_{cr}\hat{P}^i$ (such insertions can give NP contributions to $O(\alpha_S^3\Lambda_S)$ at most).

• For $k^2 \sim a^{-2}$ the Δ(Feynman rules)$|^L_{m_0=m_{cr},\mu_q=0}$ above are just as if derived from $\Delta L(y) = a\Lambda_S\alpha_S(\Lambda_S)[h_{AA}(F^G \cdot F^G) + h_{q\bar{q}}(\bar{q}\not{D}q)](y)$

• This "model" of the NP mixing phenomenon for $a\hat{O}_5^i$ can be extended to theories where NP (SSB) effects are expected to give rise to new, peculiar operators in the mixing pattern of the operator of interest.

A (toy) gauge model with scalar fields and Wilson-like term

Model with two fermion species $Q = (Q_1, Q_2)^t$ coupled minimally to a $SU(N)$ gauge field G_μ and through Wilson-like and Yukawa terms to a complex scalar doublet ϕ.

$$\mathcal{L}_{\rm toy}(Q,G,\Phi) = \mathcal{L}_{kin}(Q,G,\Phi) + \mathcal{V}(\Phi) + \mathcal{L}_{Wil}(Q,G,\Phi) + \mathcal{L}_{Yuk}(Q,\Phi)\,,$$

$$\mathcal{L}_{kin}(Q,G,\Phi) = \frac{1}{4} F^{a\,G}_{\mu\nu} F^{a\,G}_{\mu\nu} + \bar{Q}_L \mathcal{D}\!\!\!/^{\,G} Q_L + \bar{Q}_R \mathcal{D}\!\!\!/^{\,G} Q_R + \frac{1}{2}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right]_{\rm tr}$$

$$\mathcal{V}(\Phi) = \frac{\mu_0^2}{2}\left[\Phi^\dagger\Phi\right]_{\rm tr} + \frac{\gamma_0}{4}\left(\left[\Phi^\dagger\Phi\right]_{\rm tr}\right)^2\,, \qquad \mathcal{L}_{Yuk}(Q,\Phi) = \eta_0\left(\bar{Q}_L \Phi Q_R + \bar{Q}_R \Phi^\dagger Q_L\right)$$

$$\mathcal{L}_{Wil}(Q,G,\Phi) = \frac{b^2}{2}\rho\left(\bar{Q}_L \overleftarrow{\mathcal{D}}^{\,G}_{\mu} \Phi \mathcal{D}^G_\mu Q_R + \bar{Q}_R \overleftarrow{\mathcal{D}}^{\,G}_{\mu} \Phi^\dagger \mathcal{D}^G_\mu Q_L\right)$$

with $\Phi = [\phi, i\tau^2\phi^*]$, $\phi = (\phi_1,\phi_2)^t$, $\mathcal{D}^{\,G}_\mu = \partial_\mu - ig_0 G_\mu$; $[M_{2\times 2}]_{\rm tr} = \frac{1}{2}{\rm tr}(M_{2\times 2})$.

Renormalizable: $1/b = \Lambda_{UV}$. No W-interactions (yet): ρ free parameter.

Symmetries: Poincaré group, C, P, T, gauge $SU(N)$ and $\chi_L \times \chi_R$, where

χ_L : $\tilde{\chi}_L \otimes (\Phi \to \Omega_L \Phi) \otimes (\Phi^\dagger \to \Phi^\dagger \Omega_L^\dagger)$, $\Omega_L \in {\rm SU}(2)_L$,

χ_R : $\tilde{\chi}_R \otimes (\Phi \to \Phi \Omega_R^\dagger) \otimes (\Phi^\dagger \to \Omega_R \Phi^\dagger)$, $\Omega_R \in {\rm SU}(2)_R$, ,

$\tilde{\chi}_{L,R}$: $Q_{L,R} \to \Omega_{L,R} Q_{L,R}$, $\bar{Q}_{L,R} \to \bar{Q}^\dagger_{L,R} \Omega^\dagger_{L,R}$

- Exact $\chi_L \times \chi_R$ symmatry: $\partial J_\mu^{L\,i}(x) = 0 \quad (i = 1,2,3)$, with conserved currents

$$J_\mu^{L\,i} = \bar{Q}_L \gamma_\mu \tfrac{\tau^i}{2} Q_L - \tfrac{1}{2}\big[\Phi^\dagger \tfrac{\tau^i}{2} \partial_\mu \Phi + \mathrm{h.c.}\big]_{\mathrm{tr}} - \tfrac{b^2}{2}\rho\,\big(\bar{Q}_L \tfrac{\tau^i}{2} \Phi \mathcal{D}_\mu^G Q_R + \mathrm{h.c.}\big)$$

$$J_\mu^{R\,i} = \bar{Q}_R \gamma_\mu \tfrac{\tau^i}{2} Q_R + \tfrac{1}{2}\big[\tfrac{\tau^i}{2} \Phi^\dagger \partial_\mu \Phi + \mathrm{h.c.}\big]_{\mathrm{tr}} + \tfrac{b^2}{2}\rho\,\big(\bar{Q}_L \overleftarrow{\mathcal{D}}_\mu^G \Phi \tfrac{\tau^i}{2} Q_R + \mathrm{h.c.}\big)$$

- In general $\tilde{\chi}_L$ and $\tilde{\chi}_R$ transformations are no symmetry: broken bare WTI's

$$\partial \tilde{J}_\mu^{L\,i}(x) = -\eta\big(\bar{Q}_L \tfrac{\tau^i}{2} \Phi Q_R - \mathrm{h.c.}\big)(x) - \tfrac{b^2}{2}\rho\,\big(\bar{Q}_L \overleftarrow{\mathcal{D}}_\mu^G \tfrac{\tau^i}{2} \Phi \mathcal{D}_\mu^G Q_R - \mathrm{h.c.}\big)(x)$$

$$\partial \tilde{J}_\mu^{R\,i}(x) = \ \eta\big(\bar{Q}_L \Phi \tfrac{\tau^i}{2} Q_R - \mathrm{h.c.}\big)(x) + \tfrac{b^2}{2}\rho\,\big(\bar{Q}_L \overleftarrow{\mathcal{D}}_\mu^G \Phi \tfrac{\tau^i}{2} \mathcal{D}_\mu^G Q_R - \mathrm{h.c.}\big)(x)$$

with non-conserved currents (differing from $J_\mu^{L/R\,i}$ by the absence of Φ-bilinear terms)

$$\tilde{J}_\mu^{L\,i} = \bar{Q}_L \gamma_\mu \tfrac{\tau^i}{2} Q_L - \tfrac{b^2}{2}\rho\,\big(\bar{Q}_L \tfrac{\tau^i}{2} \Phi \mathcal{D}_\mu^G Q_R + \mathrm{h.c.}\big)$$

$$\tilde{J}_\mu^{R\,i} = \bar{Q}_R \gamma_\mu \tfrac{\tau^i}{2} Q_R + \tfrac{b^2}{2}\rho\,\big(\bar{Q}_L \overleftarrow{\mathcal{D}}_\mu^G \Phi \tfrac{\tau^i}{2} Q_R + \mathrm{h.c.}\big)$$

- For given μ_0^2, γ_0, ρ_0: a family of models parameterized by η_0; just one model (with $\eta_0 = \eta_{cr}$) selected by requiring "maximal $\tilde{\chi}$–symmetry enhancement".

$\tilde\chi$ symmetry enhancement (restoring) for $\mu_R^2 > 0$

• Recovery of $\tilde\chi$ symmetry has to do with the mixings of $d = 6$ oper.s in bare WTI

$$O_6^{L\,i} = \frac{\rho}{2}\left(\bar Q_L \overleftarrow{\mathcal{D}}{}^G_\mu \frac{\tau^i}{2}\Phi \mathcal{D}^G_\mu Q_R - \mathrm{h.c.}\right), \qquad O_6^{R\,i} = -\frac{\rho}{2}\left(\bar Q_L \overleftarrow{\mathcal{D}}{}^G_\mu \Phi \frac{\tau^i}{2}\mathcal{D}^G_\mu Q_R - \mathrm{h.c.}\right)$$

which to all orders in PT reads

$$O^{L\,i}_{6\,\mathrm{bare}} = O^{L\,i}_{6\,\mathrm{sub}} + \frac{Z_{\tilde J}-1}{b^2}\partial_\mu \tilde J^{Li}_\mu - \frac{c_0}{b^2}\left[\bar Q_L \frac{\tau^i}{2}\Phi Q_R - \mathrm{h.c.}\right]$$

$$O^{R\,i}_{6\,\mathrm{bare}} = O^{R\,i}_{6\,\mathrm{sub}} + \frac{Z_{\tilde J}-1}{b^2}\partial_\mu \tilde J^{Ri}_\mu - \frac{c_0}{b^2}\left[\bar Q_R \frac{\tau^i}{2}\Phi^\dagger Q_L - \mathrm{h.c.}\right]$$

• If $\mu_R^2 > 0$ the mixing patterns above are expected not to be altered by NP effects.

As $\langle[\Phi]_{\mathrm{tr}}\rangle = 0$, $\Phi = \sigma 1 + i\vec\pi\vec\tau$, no contribution from $\mathcal{L}_{Wil}$ to Q-propagator $\Rightarrow$

no seed of dynamical $\tilde\chi$ *SSB* – at variance with the situation in Wilson LQCD

• At $\eta_0 = \eta_{cr} = c_0$: $\partial_\mu Z_{\tilde J}\tilde J^{L\,i}_\mu = \mathrm{O}(b^2)$, $\partial_\mu Z_{\tilde J}\tilde J^{R\,i}_\mu = \mathrm{O}(b^2)$

$\tilde\chi_L \times \tilde\chi_R$ restored $\Leftrightarrow$ Φ decoupled – up to (negligible) O(b^2)

η_{cr} can be determined e.g. by $\partial_\mu \langle \tilde J^{L\,i}_\mu(x)\Big(\bar Q_L \frac{\tau^i}{2}\Phi Q_R - \mathrm{h.c.}\Big)\rangle|_{\eta_0=\eta_{cr}} = 0$

$\eta_{cr} = f_{cr}(\rho_0; g_0^2, \gamma_0, b^2\mu_R^2 Z_{\Phi^\dagger\Phi})$ is odd in ρ_0 and essentially μ_R–independent

$\tilde{\chi}$ symmetry enhancement (no restoring?) for $\mu_R^2 < 0$

- If $\mu_R^2 < 0$, χ SSB takes place. Without loss of generality choose the vacuum where $\langle [\Phi]_{\rm tr} \rangle = v$, $\Phi = (v+\sigma)1 + i\vec{\pi}\vec{\tau}$, $\pi^{1,2,3}$ massless Goldstone bosons and $m_\sigma^2 \sim v^2 \sim |\mu_R^2|/\gamma_R$. Moreover we work with bare parameters s.t. $v \gg \Lambda_S$

- $\mathcal{L}_{Wil}$ contributes to the Q-propagator: $i\frac{\not{k}}{k^2} + \frac{\rho}{2} b^2 v$ in mom. space $\Rightarrow$ $\mathcal{L}_{Wil}$ provides the seed for dynamical $\tilde{\chi}$ SSB – a NP phenomenon, as in Wilson LQCD

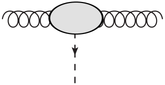
- Expect $\mathrm{O}(b^2\Lambda_S)$ relative corrections in correlators (non-vanishing even) at $\eta_0 = \eta_{cr}$:

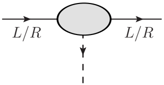
$$\langle O(x,y,..)\rangle\Big|_{\eta_0=\eta_{cr}}^{1/b} = \Big[\langle O(x,y,..)\rangle - b^2 \int_w \langle L_6(w) O(x,y,..)\rangle + \mathrm{O}(b^4)\Big]^{C}_{\delta\eta\to 0\,\mathrm{sgn}(\rho v)}$$

$$L_6 = L_6^{\tilde{\chi}-\mathrm{inv}} + \Big[b_{SW}(\bar{Q}_L \Phi i\sigma\cdot F Q_R + \mathrm{h.c.}) + b_{D2}(\bar{Q}_L \overleftarrow{\not{\mathcal{D}}}^G \Phi \not{\mathcal{D}}^G Q_R + \mathrm{h.c.})\Big]$$

owing to the $\tilde{\chi}$–violating piece of L_6 & dynamical $\tilde{\chi}$ SSB (order parameter $\langle [\bar{Q}Q]_{\rm sub}\rangle \sim \Lambda_S^3$)

e.g. for $O(x,y,..) \to G_\mu^a(x)G_\nu(y)$, $G_\mu^a(x)G_\nu(y)\sigma(z)$, $Q_L(x)\bar{Q}_L(y)$, $Q_L(x)\bar{Q}_L(y)\sigma(z)$





- Conjecture: NP corrections to the mixings of $b^2 O_6^{L\,i}$, $b^2 O_6^{R\,i}$ can *effectively* be described via PT with Feynman rules augmented as dictated by

$$\Delta L(y) = b^2 \alpha_S(\Lambda_S) \Big\{ h_{AA} \Lambda_S [U^\dagger \Phi + \Phi^\dagger U]_{\rm tr} (F^G \cdot F^G) + h_{q\bar{q}} \Lambda_S [U^\dagger \Phi + \Phi^\dagger U]_{\rm tr} (\bar{q} \not{\mathcal{D}} q) \Big\}(y)$$

with $U(\sigma, \vec{\pi})$: $U \to \Omega_L U \Omega_R^\dagger$ under $\chi_L \times \chi_R$ so as to preserve χ invariance

$U(\sigma, \vec{0}) = 1$ ($\vec{\pi} = \vec{0}$, no phases) $\Rightarrow$ unique $U(\sigma, \vec{\pi}) = \exp(i \mathrm{Arg}(\Phi))_{\rm non-Abelian}$

- Extra "diagrams" contribute to the mixings of $b^2 O_6^{L/R\,i}$ extra $\mathrm{O}(b^0 \Lambda_S)$ terms

$$\Rightarrow \quad \partial_\mu Z_{\tilde{J}} \tilde{J}_\mu^{L\,i}|_{\eta_0 = \eta_{cr}} = -c_1 \Lambda_S (\bar{Q}_L \tfrac{\tau^i}{2} U Q_R - \mathrm{h.c.})|_{\eta_0 = \eta_{cr}} + \mathrm{O}(b^2), \quad c_1 = O(\alpha_S^2)$$

⋆ r.h.s. is RG-invariant (U unique): a natural, NP.ly generated mass $\bar{m}_Q(1/b) = -c_1 \Lambda_S$

⋆ Φ NP.ly coupled at $\eta_0 = \eta_{cr}$ if $\mu_R^2 < 0$: $\tilde{\chi}$ symmetry broken by $\mathrm{O}(b^0 \Lambda_S)$ terms & c_1 depending on the $1/b$-scale details, order of magnitude of $m_Q(1/b)$ "universal"

⋆ no term $\sim \Lambda_S (\bar{Q}_L U Q_R + \mathrm{h.c.})$ can be added to $\mathcal{L}_{\rm toy}$: only polynomial of local fields compatible with (power counting) renormalizability

Outlook 1: fermion mass hierarchy

To describe t, b quark (and W, Z) masses need a superstrong force with $\Lambda_T \gg \Lambda_{QCD}$

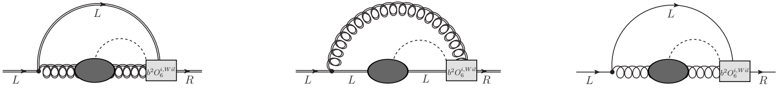
Simplest case: m_q/m_Q in model with strong and superstrong gauge fields (A and G):

$$\mathcal{L}_{\rm toy}^{(qQAG)} = (F^A \cdot F^A) + (F^G \cdot F^G) + (\bar{q}\tilde{D}\!\!\!/^A q) + (\bar{Q}\tilde{D}\!\!\!/^{G,A} Q) + [\partial_\mu \Phi^\dagger \partial_\mu \Phi] + V(\Phi) +$$
$$+ \mathcal{L}_{Wil}[q, A, \Phi] + \mathcal{L}_{Wil}[Q, A, G, \Phi] + \mathcal{L}_{Yuk}[q, \Phi] + \mathcal{L}_{Yuk}[Q, \Phi]\,, \qquad \mu_0^2 \,:\, \mu_R^2 < 0$$

$$\bar{m}_Q \sim -c_{1Q}^{(2)} \alpha_T(b^{-1}) \alpha_T(\Lambda_T) \Lambda_T + O(\Lambda_S) \quad \text{at} \quad \eta_Q = \eta_{Q,cr}$$
$$\bar{m}_q \sim -c_{1\,q}^{(2)} \alpha_S(b^{-1}) \alpha_S(\Lambda_T) \Lambda_T + O(\Lambda_S) \quad \text{at} \quad \eta_q = \eta_{q,cr}$$

Assuming $\alpha_T(\mu) \simeq \alpha_S(\mu)$ for $v \le \mu \le b^{-1}$ (coupling unification) and similar coefficients for the $d = 6$ Wilson-like terms $\mathcal{L}_{Wil}[q, A, \Phi]$ and $\mathcal{L}_{Wil}[Q, A, G, \Phi]$

$\bar{m}_q / \bar{m}_Q \sim \alpha_S(\Lambda_T)/\alpha_T(\Lambda_T) \sim 0.1$ essentially independent on UV-scale details



Taking $|c_1^{(2)}|$ as in Wilson LQCD & $\rho_{cr} \sim 0.2$ (Outlook 2) $\Rightarrow \bar{m}_Q \sim \Lambda_T \sim$ a few TeV

Outlook 2: weak interactions

Three massless Goldstones $\pi^{1,2,3}$ in the toy model(s) above: this calls for introducing weak interactions, global χ_L symmetry being ready to be gauged

Once this is done ($g_w > 0$) e.g. in the toy model specified by $\mathcal{L}_{\rm toy}^{(qQAG)}$: $W^{1,2,3}$ weak gauge bosons appear in $(F^W \cdot F^W)$ term and in cov. derivatives acting on χ_L-doublets

No $U_Y(1)$ gauge interaction yet, $\mu_R^2 < 0$: custodial $SU(2)_V$ preserved by χSSB.

A new term $\sim ig_w \Big[\Phi^\dagger[\frac{\tau^i}{2}, W_\mu]D_\mu^W\Phi + \mathrm{h.c.}\Big]_{\rm tr}$ in the r.h.s. of broken bare $\tilde\chi_L$–WTI $\leftrightarrow$ a new mixing of $b^2 O_6^{L\,i}$: maximal $\tilde\chi$ enhancement implies $(\rho, \eta_0) = (\rho_{cr}, \eta_{cr})$, with

$$\partial_\mu Z_{\tilde J} \tilde J_\mu^{L\,i} = -ig_w c_{1W}\Lambda_T \Big[U^\dagger[\frac{\tau^i}{2}, W_\mu]D_\mu^W\Phi + \mathrm{h.c.}\Big]_{\rm tr} + \mathrm{O}(c_{1Q,1q}\Lambda_T) + \mathrm{O}(b^2)$$

dynamically generated $\bar m_W \sim g_w \Lambda_T \mathrm{O}(\frac{\alpha_T(\Lambda_T)}{4\pi^2}) \sim 100$ GeV, while $\bar m_{Q,q}$ as above

Due to $WQ\bar Q$ coupling, WW bound states with binding energies O(50) GeV: LHC "Higss boson" ? If yes, get low ($\ll \Lambda_T$) energy effective theory very similar to SM ...

W_μ^i couples to conserved currents $J_\mu^{L\,i}$, with a non-SM-like piece: $J_\mu^{L\,i}$-matrix elem.s in principle differ by $\mathrm{O}(\alpha(b^{-1})\rho_{cr}^2)$ from their SM counterparts, tiny effect if $\rho_{cr} \sim 0.2$ ...