QCD at $T > 0$ and $B > 0$

Kalman Szabo
Bergische Universitat, Wuppertal

Fairly well established (continuum, physical mass, staggered):

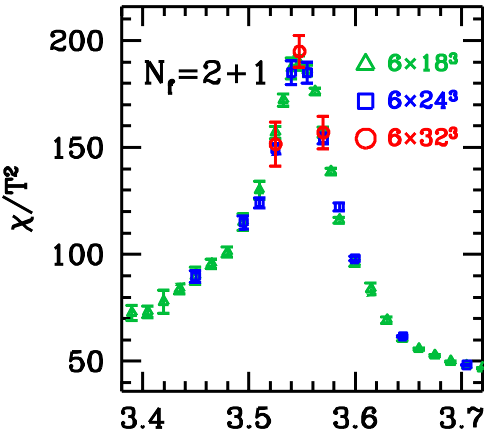
Crossover

T_c

EoS

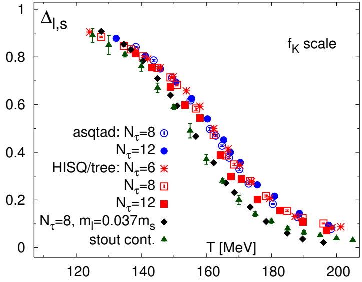
Crossover

[Wuppertal-Budapest,WB,'06]
volume dependence of chiral susceptibility:



T_c

$$T_c = \begin{cases} \sim 190 \text{ MeV} & \textbf{[BNL-Bielefeld-RIKEN-Columbia,'06]} \\ \sim 150 \text{ MeV} & \textbf{[WB,'06]} \end{cases} = ?$$

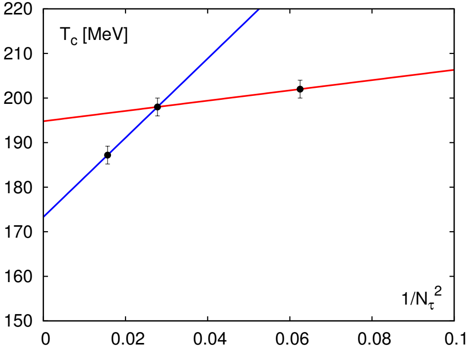


[WB'06'09'10] $T_c = 147(2)(3)$MeV

[hotQCD '12] $T_c = 154(9)$MeV

T_c

[hotQCD '12]

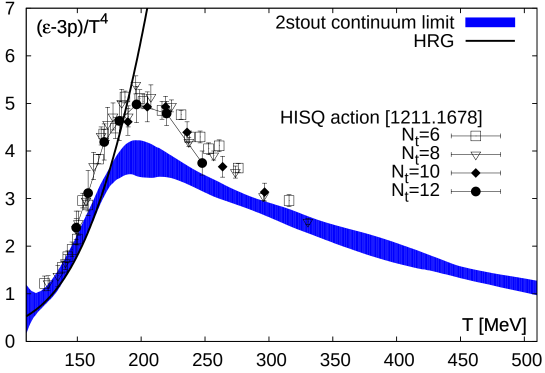


N_t	a[fm]
4	0.30
6	0.20
8	0.15

$N_t = 4$ results are unreliable

Equation of state

[WB,'13] 2stout 2+1, continuum from $N_t = 6, 8, 10, 12, 16$



[Krieg, Mon] height of the peak?

1. $N_t = 16$ at peak
2. different action (4stout)

[Bazavov,Tue] MILC-coll, HISQ 2+1+1 at $m_\pi \sim 300$ MeV, finite "a"

[Burger,Tue] tmT-coll, twisted mass 2+1+1 at $m_\pi = 400$ MeV, finite "a"

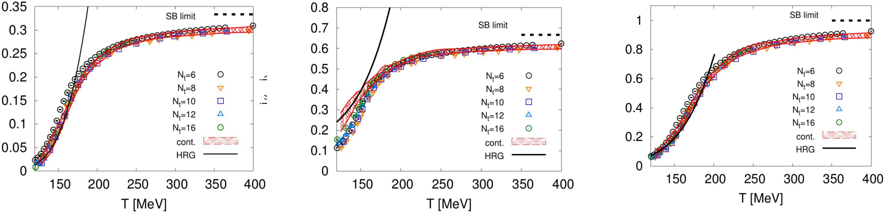
Fluctuations

Baryon, charge and strange

fluctuations $\equiv$ susceptibilities

$$\chi_{ijk}^{BQS} = \frac{1}{VT^3}\left[\frac{\partial^i}{\partial(\mu_B/T)^i}\frac{\partial^j}{\partial(\mu_Q/T)^j}\frac{\partial^k}{\partial(\mu_S/T)^k}\right]\log Z$$

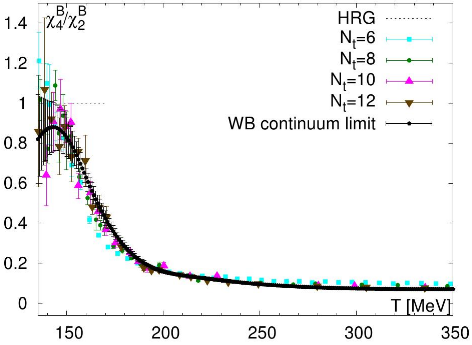
$\langle\delta B^2\rangle \sim \chi_2^B$ $\quad$ $\langle\delta Q^2\rangle \sim \chi_2^Q$ $\quad$ $\langle\delta S^2\rangle \sim \chi_2^S$



continuum extrapolation from $N_t = 6, 8, 10, 12$ and 16 **[WB '11]**

also by **[hotQCD,12]**, see **[Petreczky, Mon]** comparing it to HRG/HTL

Higher moments

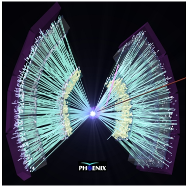


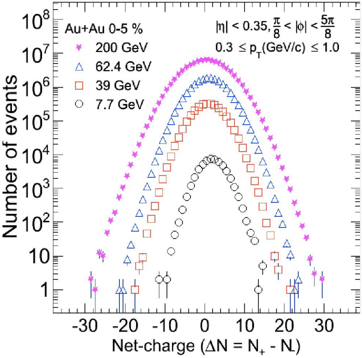
baryon-number kurtosis:

$$\chi_4^B \sim \langle \delta B^4 \rangle - 3\langle \delta B^2 \rangle^2$$

higher orders are increasingly difficult ($\rightarrow$ O(10^6) configurations), even **upto 8th-order** by **[Datta,Gavai,Gupta]**, not yet continuum

Event-by-event





What fluctuates in a heavy-ion collision, if you start with a fixed number of conserved charges (Z=82, A=207)?

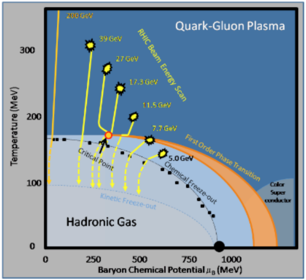
Consider particles coming only from a small part of the whole system, defined by imposing kinematical constraints.

Charges from subvolumes will fluctuate from one event to the other.

Experiment vs. lattice

$$\langle \text{fluct.} \rangle_{\text{exp}} \stackrel{?}{=} \langle \text{fluct.} \rangle_{\text{latt},T,\mu_B,\mu_Q,\mu_S}$$

1. use 4 fluctuations to determine 4 **freezout** T, μ_B, μ_Q, μ_S
2. do they originate from **thermal & chemical equilibrium**?



3. how is the freezout curve (latt+exp) related to QCD transition line (latt)? is the freezout curve close enough to the **QCD endpoint**?

Freezeout parameters

[BNL-Bielefeld '12]

How to get freezout T, μ_B, μ_Q, μ_S?

- in a lead-lead collision (Z=82, A=207):

$$\langle S \rangle = 0, \qquad \langle Q \rangle = \frac{82}{207} \langle B \rangle$$

  can be used to obtain $\mu_Q = \mu_Q(T, \mu_B)$ **and** $\mu_S = \mu_S(T, \mu_B)$

- choose two fluctuations (1. thermometer, 2. baryometer) and **find** T **and** μ_B, where

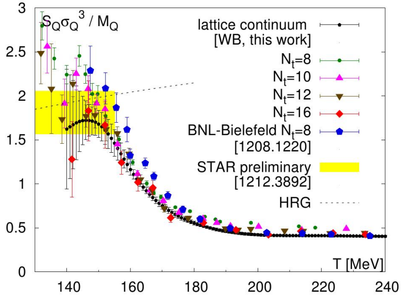
$$\langle \ldots \rangle_{\mathrm{exp}} = \langle \ldots \rangle_{\mathrm{latt}, T, \mu_B}$$

- to cancel Vol of the subsystem work with **fluctuation ratios**

Thermometer, baryometer

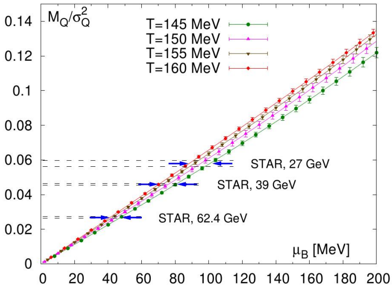
[WB,'13][Borsanyi, Mon]

$\langle \delta Q^3 \rangle / \langle Q \rangle$



$T_f \lesssim 157$MeV

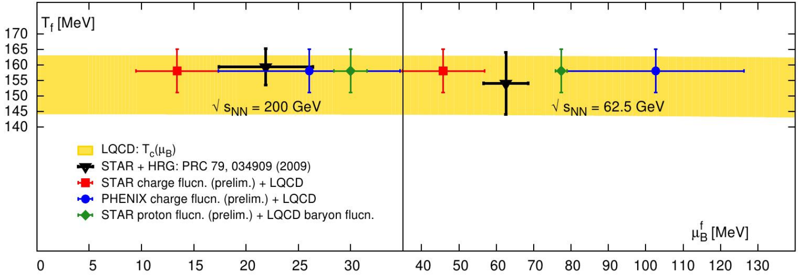
$\langle Q \rangle / \langle \delta Q^2 \rangle \sim \mu_B$



$\sqrt{s}[GeV]$	μ_B^f [MeV]
62.4	44(3)(1)(2)
39	75(5)(1)(2)
27	95(6)(1)(5)
	$()_{\delta T}()_{\rm lat}()_{\rm exp}$

Freezout vs. transition line

[BNL-Bielefeld,'13][Wagner,Mon]



ab-initio determinations of freezout T, μ:

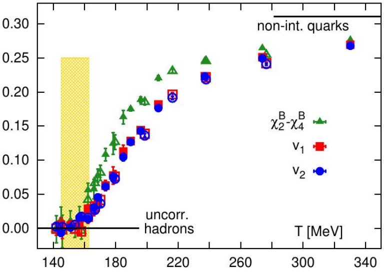
- **STAR** electric charge + lattice
- **PHENIX** electric charge + lattice
- **STAR** proton number + lattice baryon number

good agreement with QCD transition line

Flavor dependent transition?

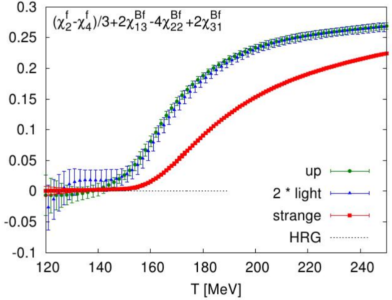
Do strange degrees of freedom have a higher deconfinement temperature, than that of the light quarks?

No. [BNL-Bielefeld,'13]



[Schmidt,Tue]

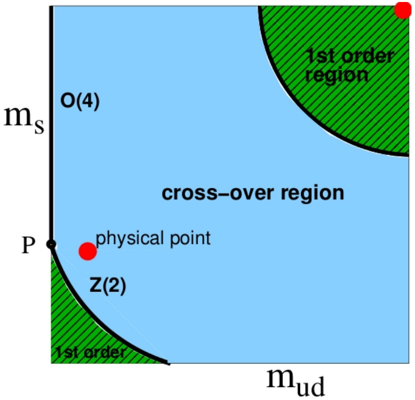
Yes. [WB,'13]



[Borsanyi,Mon]

$\sim$ 15 **MeV higher**

Columbia-plot

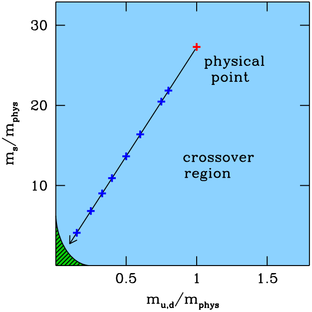


what is the order of the transition in the **chiral limit**?

Columbia, $n_f = 3$

Columbia-plot: $n_f = 3$, staggered

effective theory predicts first-order in the chiral limit, at the critical pion mass ($m_{\pi,c}$) it turns to crossover



$m_{\pi,c}$ decreases with a: **[de Forcrand, Philipsen '07][Schmidt '03][Endrodi '07]**

$N_t = 4$ unimproved	260 MeV
$N_t = 6$ unimproved	150 MeV
$N_t = 4$ p4-improved	70 MeV
$N_t = 6$ stout-improved	$\lesssim$ 50 **MeV**
continuum	**??**

no trace of the first order region

$+$

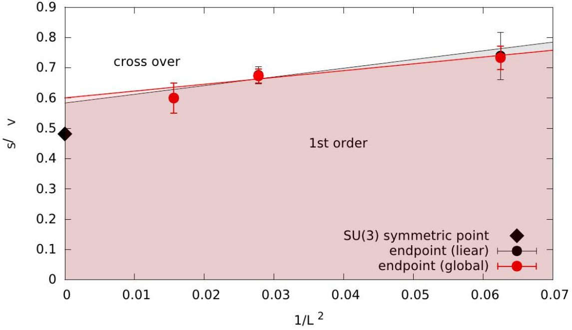
RMS pion mass is not decreasing, $\gtrsim$ 400 **MeV**

would need to reduce "m" and "a" simultaneously

Columbia-plot: $n_f = 3$, Wilson

[Nakamura, Thu] $N_t = 4, 6, 8$ improved Wilson

$m_{PS}^{E;sym} / m_{PS}^{phy;sym} \sim 1.2$ (preliminary)



critical $m_{\pi,c} \approx 500$ MeV with Wilson

Columbia, $n_f = 2$

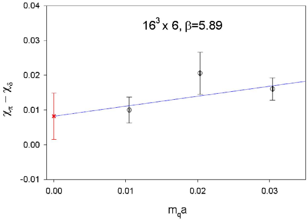
The effective theory prediction depends, wether $U(1)_A$ is

- restored at $T_c \rightarrow$ **first-order**
- not restored at $T_c \rightarrow$ **second-order**

Columbia-plot: $n_f = 2$ and fate of $U(1)_A$

- difference of pseudoscalar ($\pi = \overline{u}\gamma_5 d$) and scalar ($\delta = \overline{u}d$):

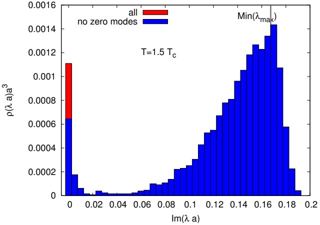
$$\langle \pi(x)\pi(0) \rangle - \langle \delta(x)\delta(0) \rangle$$



[Chiu,Thu] optimal domain-wall: "axial $U(1)$ symmetry is **restored** in the chirally symmetric phase"

[Coussu et al, '13] overlap: "axial $U(1)$ symmetry is effectively **restored** in the chirally symmetric phase"

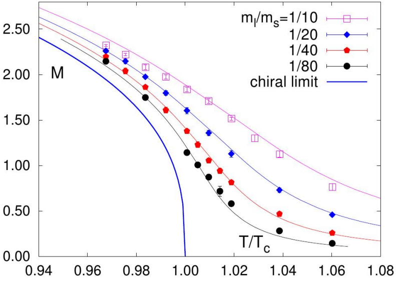
- absence of near-zero eigenvalues → $U(1)_A$ is restored

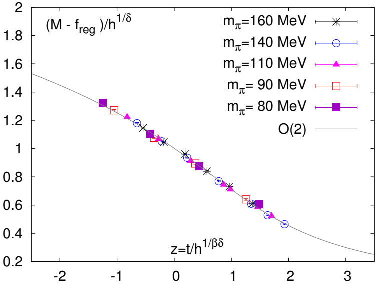


[Sharma,Thu] overlap on HISQ: "$U(1)_A$ is **not restored** effectively even at 1.5 times the pseudo critical temperature"

Columbia-plot: $n_f = 2$, staggered

[BNL-Bielefeld,'09] $N_t = 4$ p4-improved
[Ding, Mon] $N_t = 6$ HISQ-improved
downto $m_\pi \approx 80$ MeV ($m_\pi L \sim 3$)



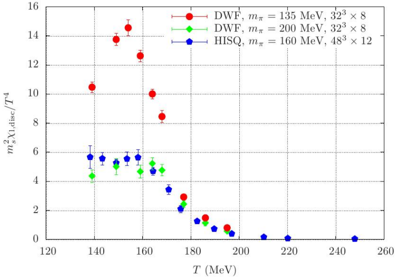


best evidence for second order O(4)

chiral limit, RMS pion mass remains finite

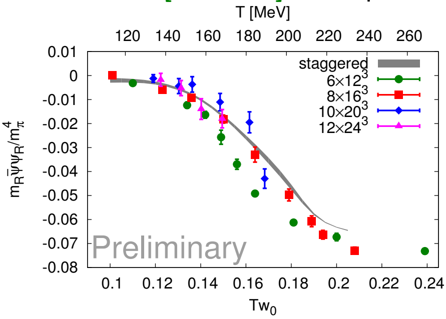
Columbia-plot: $n_f = 2$ chiral fermions

[Schroeder,Mon] domain-wall



$m_\pi = 135$ MeV, $N_t = 8$

[Toth,Thu] overlap

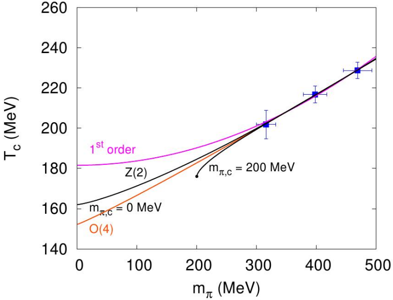


$m_\pi = 320$ MeV, $N_t = 6, 8, 10, 12$

agreement with previous staggered

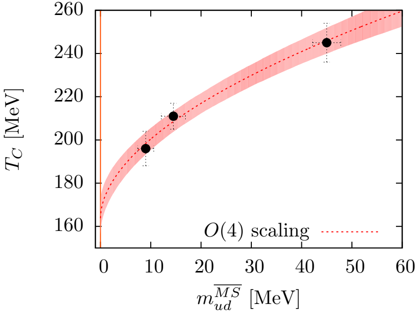
Columbia-plot: $n_f = 2$, Wilson

[Burger, Tue] twisted mass, tmT-coll



$m_\pi \gtrsim 300$ MeV, $N_t = 12$

[Brandt, Thu] improved Wilson, Frankfurt-Mainz group

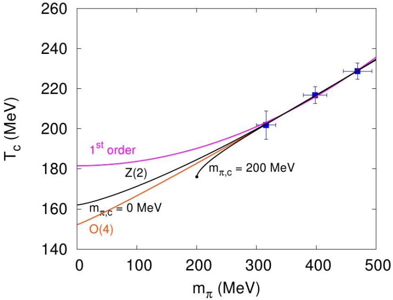


$m_\pi \gtrsim 200$ MeV, $N_t = 16$

not yet conclusive on the transition order

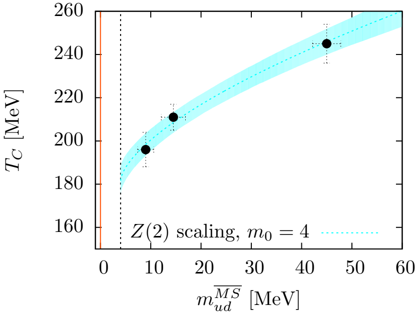
Columbia-plot: $n_f = 2$, Wilson

[Burger, Tue] twisted mass, tmT-coll



$m_\pi \gtrsim 300$ MeV, $N_t = 12$

[Brandt, Thu] improved Wilson, Frankfurt-Mainz group

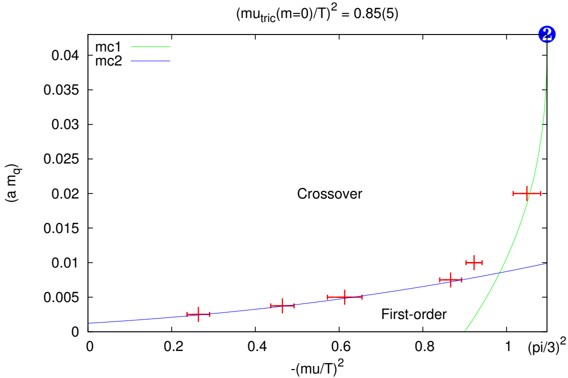


$m_\pi \gtrsim 200$ MeV, $N_t = 16$

not yet conclusive on the transition order

Columbia-plot from imaginary μ

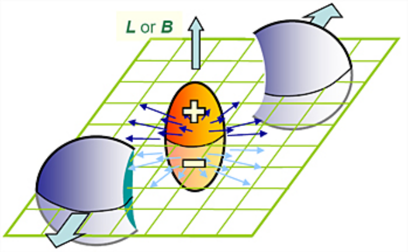
[de Forcrand, Philipsen ’10][Philipsen, Thu] **imaginary μ** might turn transition to 1st-order, $N_t = 4$ unimproved staggered



$$B > 0$$

$B > 0$ in ...

heavy-ion collisions $B \sim 10^{15}\,T$:



chiral magnetic effect: “electric charge separation along a magnetic field induced by a chirality imbalance” **[Kharzeev, McLerran, Warringa]**

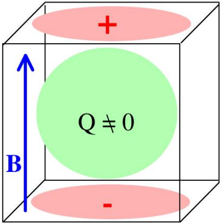
test non-Abelian nature of QCD

Chiral magnetic effect

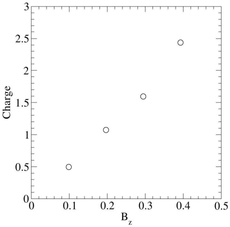
- on one instanton configuration the charge separation can be nicely reproduced:

$$\rho_{\mathrm{EM}}(x) = \sum_{\lambda} \frac{\overline{\psi}_\lambda \gamma_4 \psi_\lambda(x)}{i\lambda + m}$$

plot from **[Yamamoto]**



[Abramczyk et al '09]



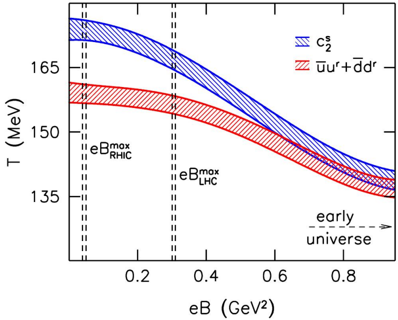
- on "real" configuration the effect is more complex: studied by **[Buividovich et al '09]** and **[Yamamoto '11]**

$T_c(B)$ and crossover

common wisdom supported by first lattice studies **[D'Elia et al '10]**:

- T_c increases with B

continuum lattice **[Bali et al '11][Endrodi,Wed][Bruckmann,Wed]**:



T_c decreases with B

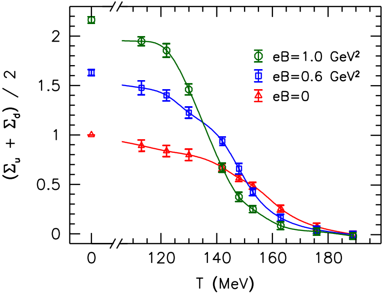
no change in width, no V-dependence upto → **crossover**

$B > 0$ chiral condensate

common wisdom:

- $\bar{\psi}\psi$ increases with B, **magnetic catalysis**

[Bali et al '12][Kovacs,Wed]



magnetic catalysis at $T = 0$

inverse magnetic catalysis at $T_c \to$ decreasing T_c

$$\langle\bar{\psi}\psi\rangle \sim \int \mathrm{Tr} D^{-1}(B) \cdot \det D(B)$$

- $\mathrm{Tr} D^{-1}(B)$ "valence" contribution increases
- $\det D(B)$ "sea" contribution decreases

Para- or diamagnetic?

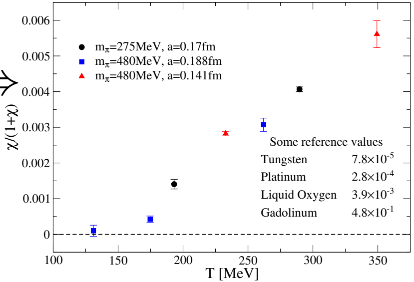
calculate magnetic susceptibility **[Bali et al '12]**:

$$\xi = \frac{T}{V} \frac{\partial^2}{\partial B^2}\bigg|_0 \log Z \sim \frac{\partial}{\partial B}\bigg|_0 \left(\langle \overline{\psi} \sigma_{xy} \psi \rangle + \langle \overline{\psi} L_{xy} \psi \rangle \right) = \begin{cases} > 0 & \text{para} \\ < 0 & \text{dia} \end{cases}$$

at zero temperature **[Bali et al '13]**: $\xi(T = 0) \equiv 0$

at finite temperature ξ from:

- finite difference **[Bonati,Wed]** $\Longrightarrow$
- Taylor-expansion at $B = 0$ **[DeTar,Wed]**
- integral method, anisotropy **[Endrodi,Wed]**



Quark-Gluon Plasma is paramagnetic

Crossover, T_c, EoS

Fluctuations

Columbia-plot

$B > 0$

Thanks to

Gergely Endrodi, Alexei Bazavov, Claudio Bonati, Bastian Brandt, Ting-Wai Chiu, Massimo d'Elia, Carleton DeTar, Heng-Tong Ding, Tamas Kovacs, Ludmila Levkova, Michael Mueller-Preussker, Yoshifumi Nakamura, Francesco Negro, Chris Schroeder, Sayantan Sharma, Mathias Wagner

$B > 0$ on the lattice

in fermion matrix $SU(3)$ links are multiplied with $U(1)$ phases:

$$U_\mu(x) \to \exp(ieA_\mu) U_\mu(x)$$
$$A_t = A_x = A_z = 0, \quad A_y = Bx$$

equivalent to an imaginary μ: $\mu_I(x) \to$ no sign problem

In a finite box (size L) with periodic boundary condition:

B is quantized:

$$eB = \frac{2\pi}{L^2} n$$

where $n = 0 \dots L^2 - 1$

B can be changed in discrete steps only

[AlHashimi,Wiese,'09]

OR

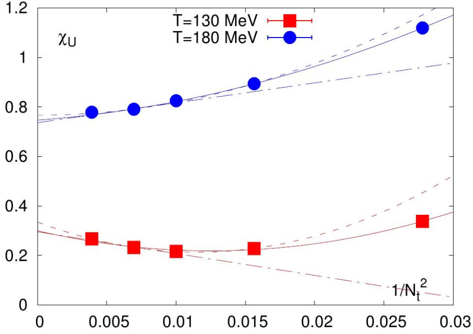
modify A_y:

$$A_y = \begin{cases} Bx & \text{for } x \le L/2 \\ B(L-x) & \text{for } x > L/2 \end{cases}$$

B has a jump, but can be changed continuously **[DeTar,Thu]**

Scaling regime

quark number susceptibility **[WB,'11]**



N_t	a[fm]	
6	0.20	✗
8	0.15	✗
10	0.12	✓
12	0.10	✓
16	0.075	✓

a^2-scaling for $N_t \gtrsim 10$ or $a \lesssim 0.12$ fm