Hadron Spectroscopy Review

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Outline

- Introduction
- Mesons
  - 'Single-meson' spectroscopy
  - Resonances etc
- Baryons
- Summary

Won't say much on precision determinations of low-lying states

Concentrate on higher-lying / excited states / resonances
– not all extrapolated in a, m_π, ...

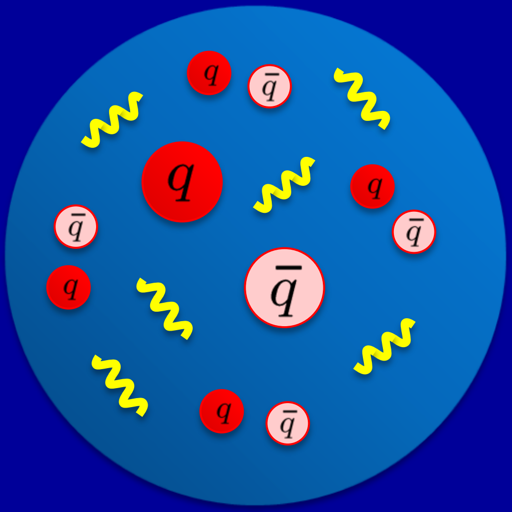
|baryon number| = 0 and 1

Thank you for sending material and apologies if I don't cover your work

More on scattering and resonances in Michael Döring's talk (next)

Hadron Spectroscopy – Mesons

Relevant degrees of freedom?
Role of gluons?



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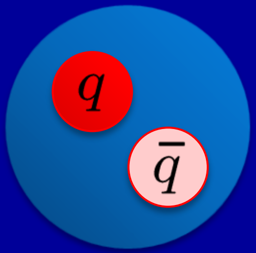

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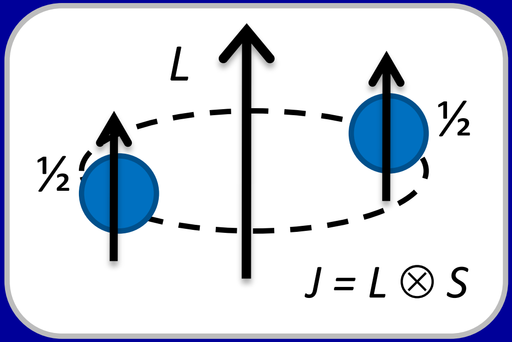

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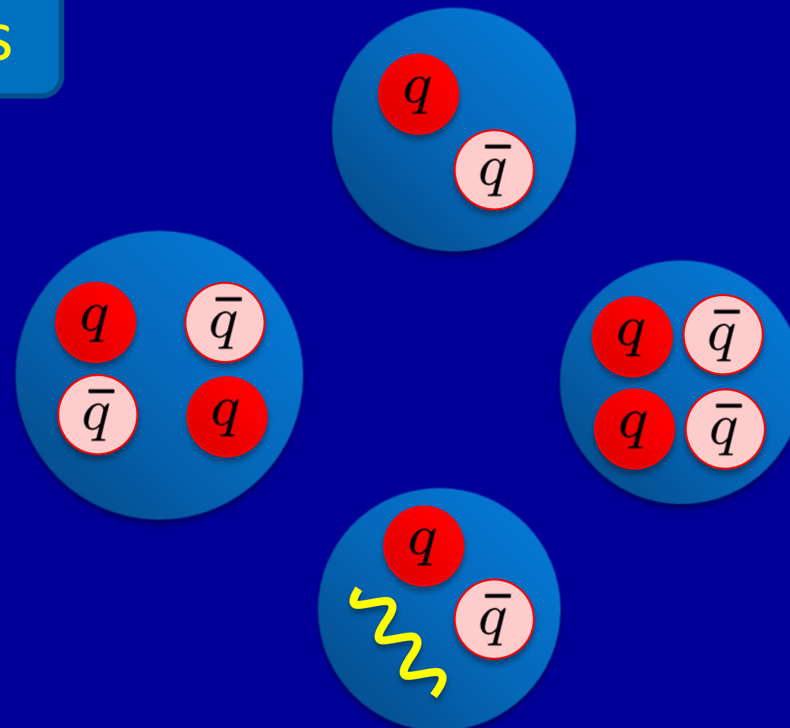
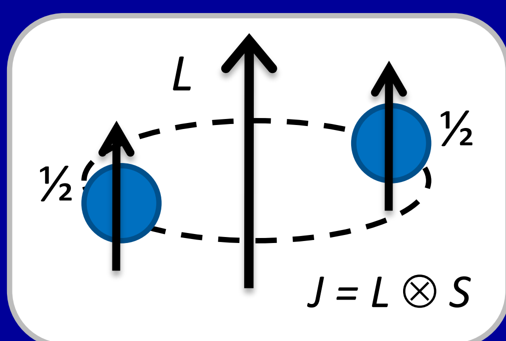


Relevant degrees of freedom?
Role of gluons?

Exotic J^{PC} **(0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, …)**
or flavour quantum numbers
– can't just be a $q\bar{q}$ pair



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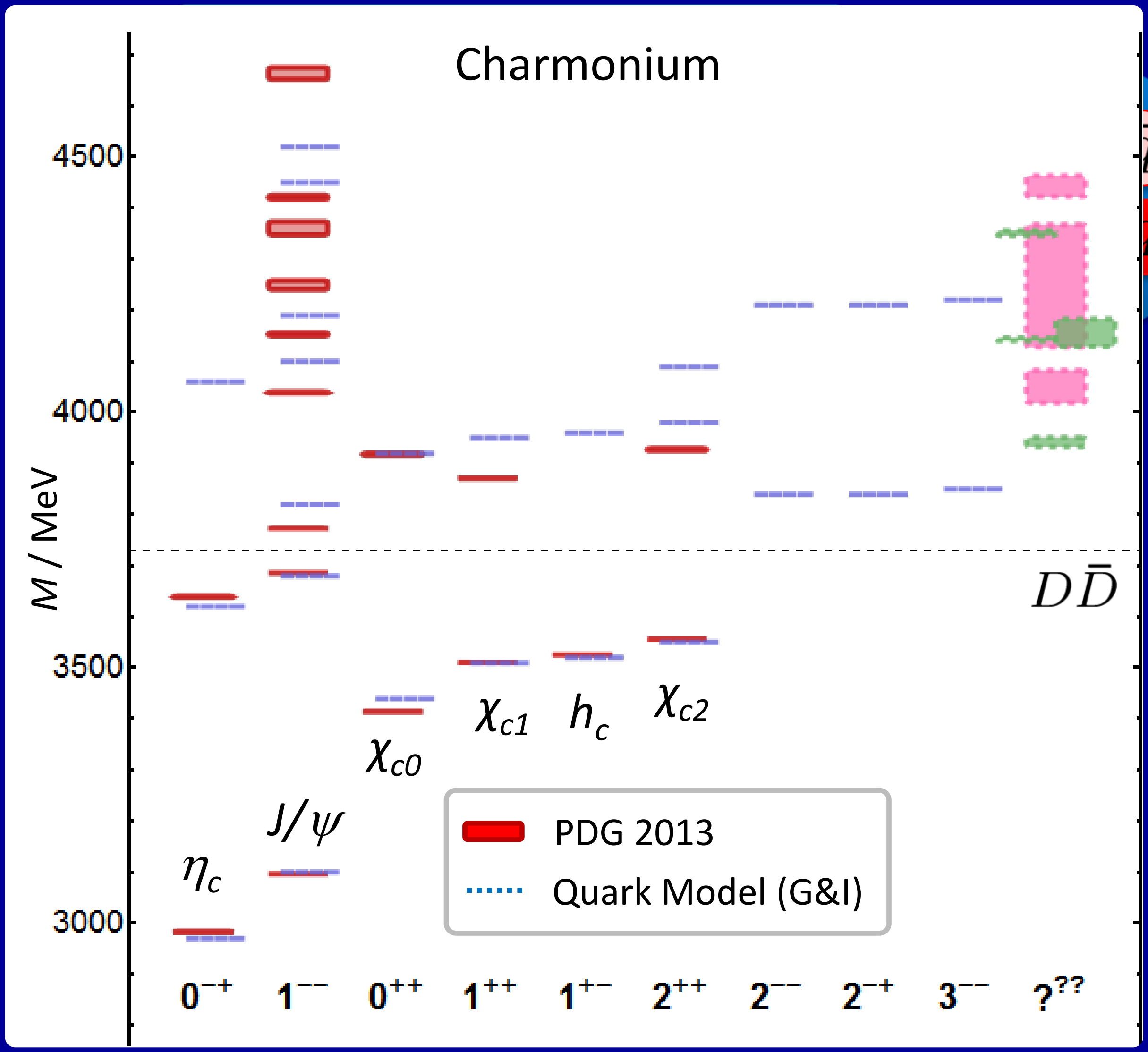

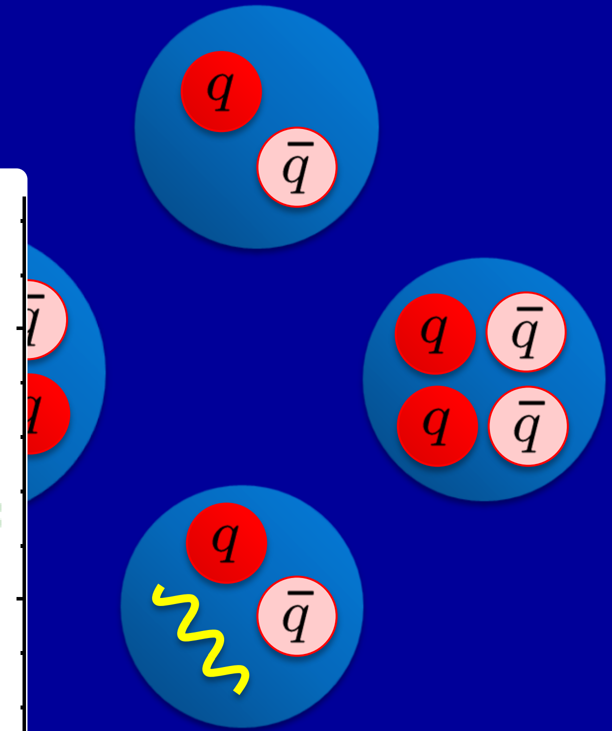

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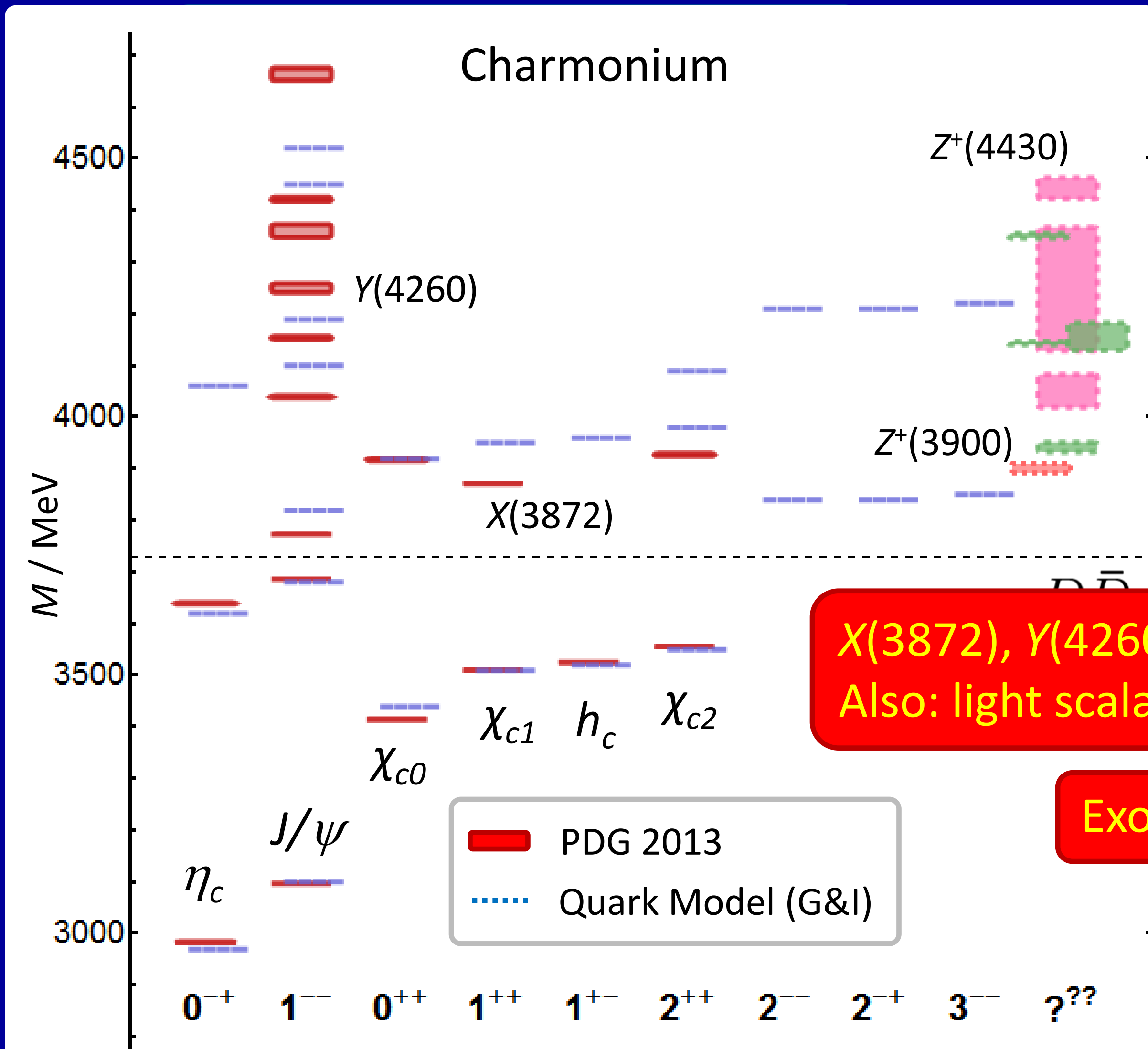

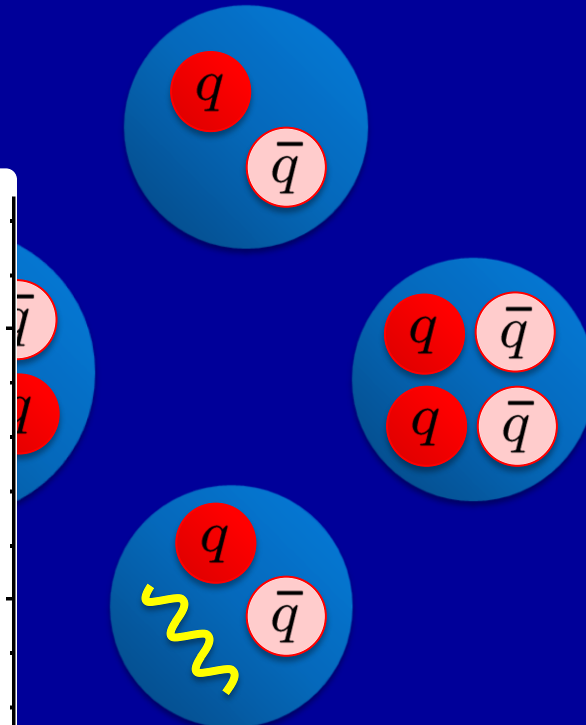

$X(3872)$, $Y(4260)$, $Z^+(4430)$, $Z^+(3900)$, ... ?
Also: light scalars, $D_s(2317)$, ...

Exotic 1^{-+} ?

Hadron Spectroscopy

Experiments

LHCb

ATLAS CMS

CLAS12

+ others at 12 GeV JLab

BESIII KLOE2

+ others at GSI

ELSA MAMI J-PARC Spring-8

Lattice Spectroscopy

Interpolating operators

$$C_{ij}(t) = < 0\, \mathcal{O}_i(t)\, \mathcal{O}_j^\dagger(0) | 0 >$$

$\bar{\psi} \Gamma \psi$ $\quad \epsilon^{abc} \psi_a \psi_b \psi_c$ $\quad + \overleftrightarrow{D}_i$

$$C_{ij}(t) = \sum_n \frac{e^{-E_n\ t}}{2\ E_n} Z_i^{(n)} Z_j^{(n)*}$$

$$Z_i^{(n)} \equiv < 0 | \mathcal{O}_i | n >$$

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Generalised Eigenvalue Problem / Variational Method

$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

Matrix of correlators

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$$\lambda^{(n)}(t) \to e^{-E_n(t-t_0)}\left[1 + O\left(e^{-\Delta E(t-t_0)}\right)\right]$$

Eigenvectors → $Z^{(n)}$

Probe structure, spin i.d., ...

$$\Omega^{(n)} \sim \sum_i v_i^{(n)} O_i$$

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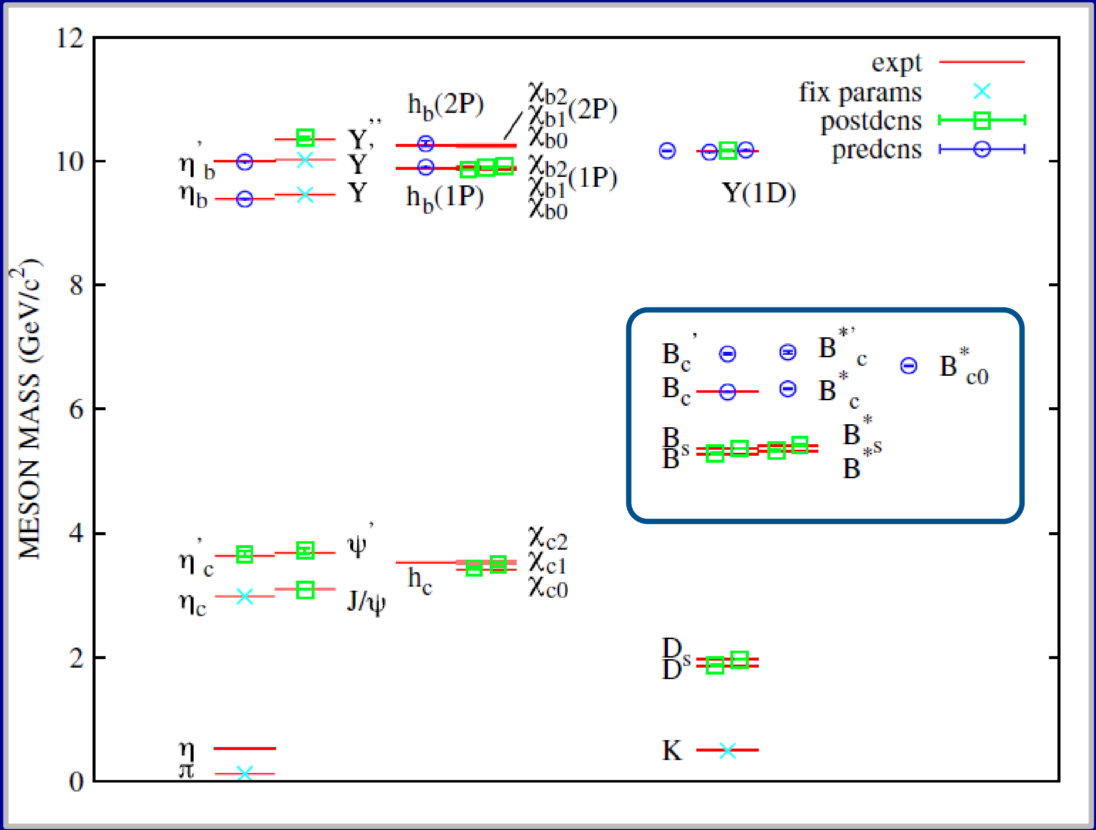
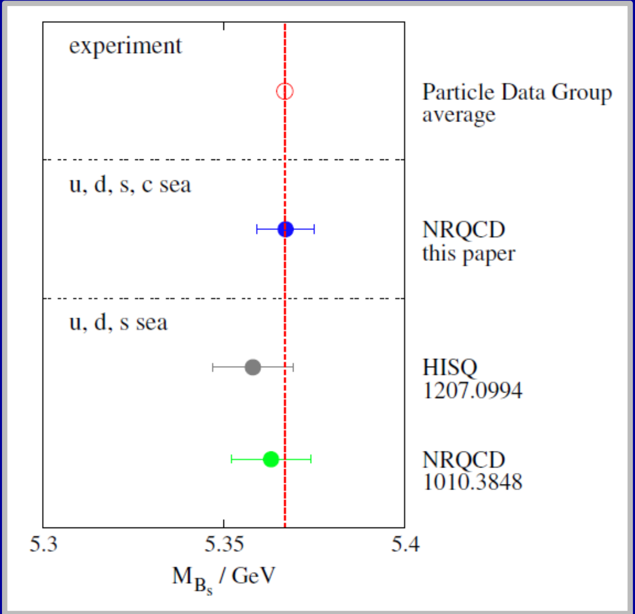
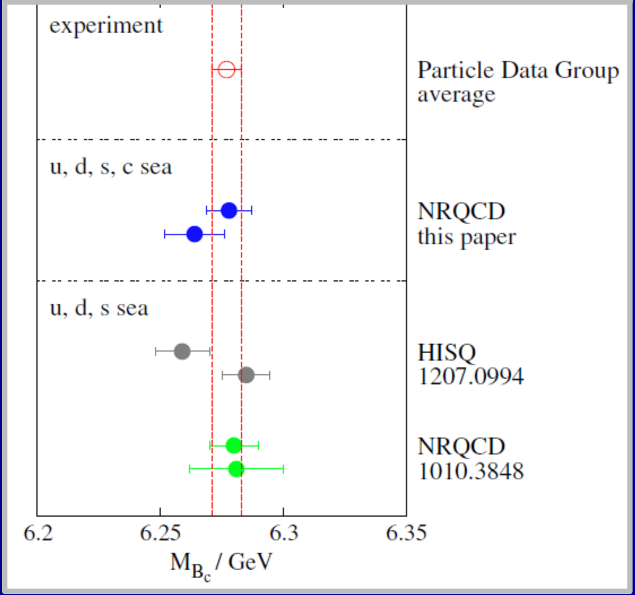
With single-hadron ops, generally don't see multi-hadron energies clearly (more later)

Quarkonia and heavy-light mesons

Dowdall et al (HPQCD) [PR D86, 094510 (2012)]





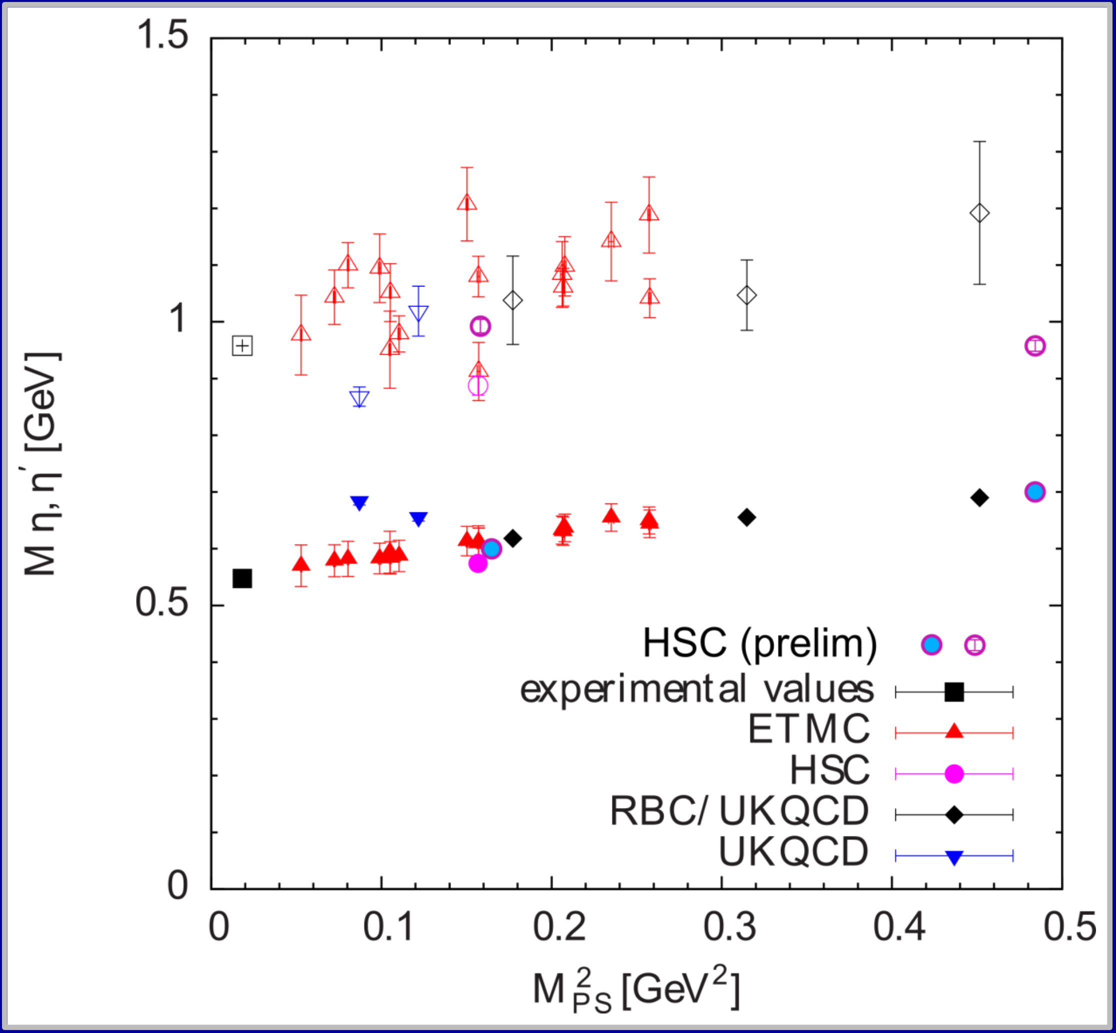


Dynamical (HISQ) N_f = 2+1+1 (u,d,s,c) with non-rel b quark [O(α_s) corrections]

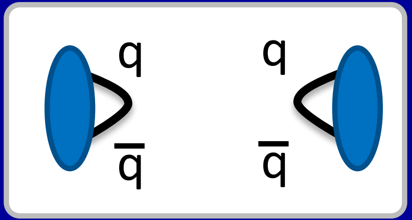
c.f. N_f = 2+1 (HISQ) with HISQ or non-rel b quark

Light isoscalar (I=0) mesons

ETMC preliminary → talks by
C. Urbach and K. Ottnad [Thrs, 8G]



Disconnected diagrams



Twisted mass [N_f = 2+1+1], extrapolate in a and M_π:

η: 552(10) MeV,

η′: 1005(54) MeV

+ Had Spec Collab prelim (Aniso. Clover, N_f = 2+1) – larger vol ($M_\pi L \sim 6$) and more M_π
c.f. [PR D83, 111502 (2011)] ($M_\pi L \sim 4$) *(many other isoscalar states in addition to η, η′)*

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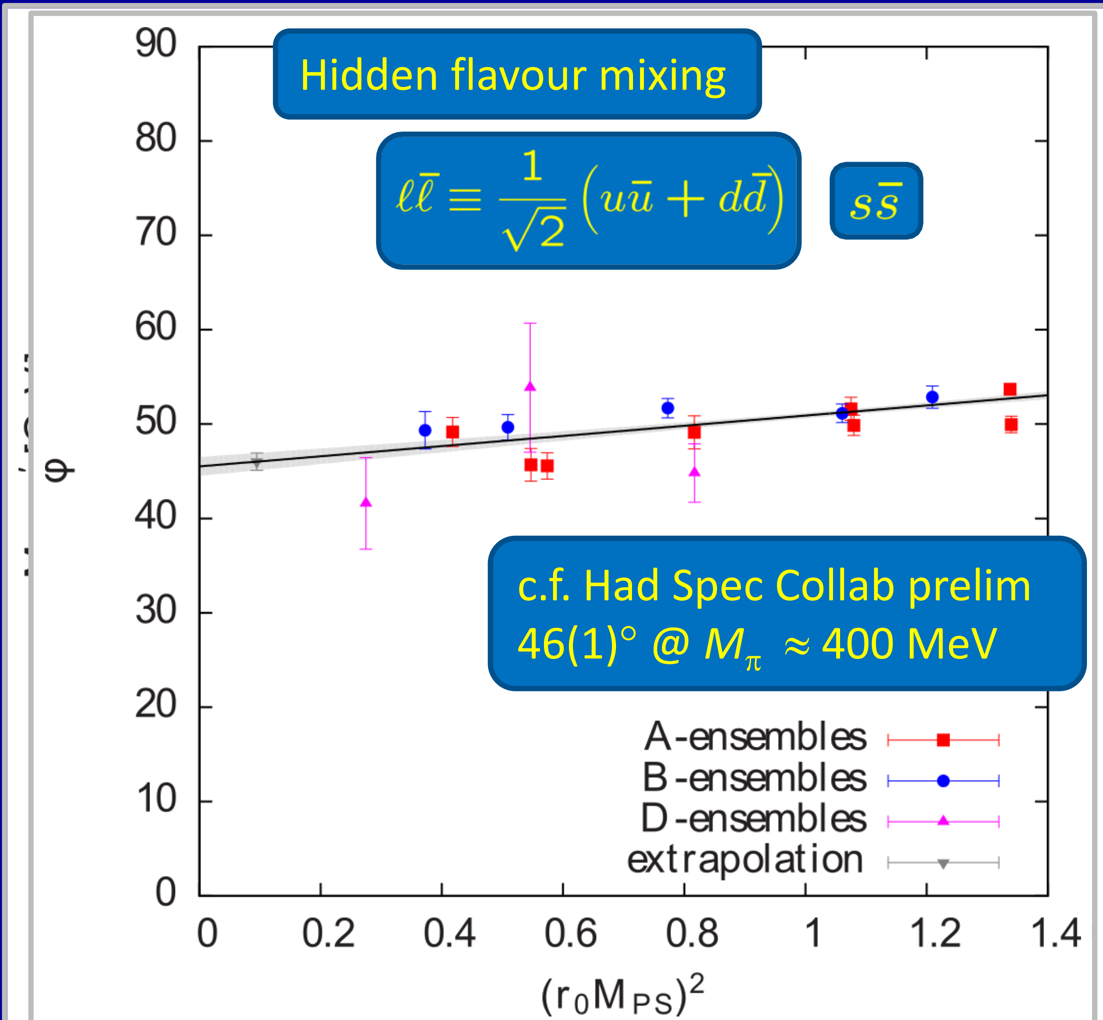

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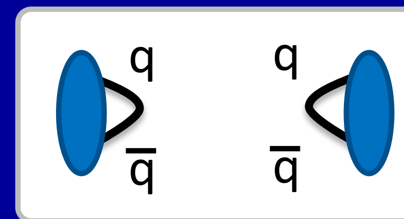

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Excited spectroscopy

Had Spec Collab single-hadron 'subduced ops'

$$O(t) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \; \bar{\psi}(x) \left[\Gamma \times \overleftrightarrow{D} \times \overleftrightarrow{D} \dots \right] \psi(x)$$

[similarly for baryons and 'subduced helicity ops' for **p** ≠ **0**]

Definite $J^{P(C)}$ in infinite vol. continuum (**p** = **0**)

'Subduce' ops → irreps. of reduced sym group

Spin identification using Z's

[PR D80 054506, PRL 103 262001, PR D82 034508, D84 074508, D85 014507]

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'Distillation'

$$\Box_{xy}(t) = \sum_{k=1}^{N} v_x^{(k)}(t) v_y^{(k)\dagger}(t)$$

e.g. $-\nabla^2 v^{(k)} = \lambda^k v^{(k)}$

$$\psi(x) \rightarrow \tilde{\psi}(x) = \Box\psi(x)$$

factorisation, smearing, ...

$$\langle 0 | \bar{\psi}' \Box(t') \Gamma_{t'}^{A} \Box(t') \psi'(t') \quad \cdot \quad \bar{\psi} \Box(t) \Gamma_{t}^{B} \Box(t) \psi(t) | 0 \rangle$$

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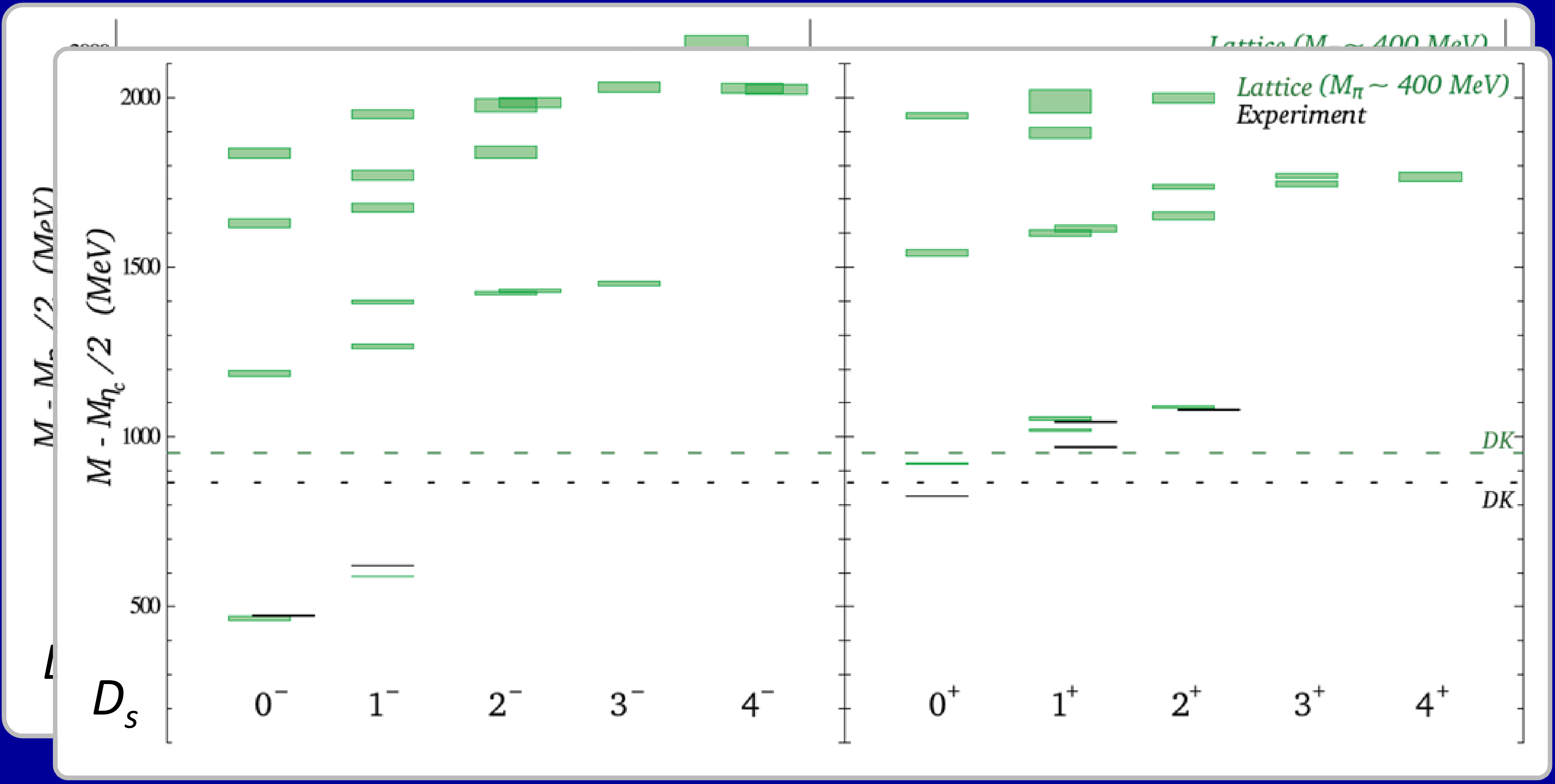
Up to 3 derivs: many ops in each channel, different spin and angular structures

include ~ $[D_i, D_j]$

[PR D80 054506, PRL 103 262001, PR D82 034508, D84 074508, D85 014507]

Charmed (D/D_s) mesons

Graham Moir et al [JHEP 05 (2013) 021]
→ talk by G. Moir [Weds, 5G]



Clover [N_f = 2+1], **anisotropic** ($a_s/a_t \approx 3.5$), $a_s \approx 0.12$ fm, $24^3 \times 128$ (L ≈ 2.9 fm, $M_\pi L \sim 6$) (also 16^3), $M_\pi \approx 400$ MeV, relativistic charm quark

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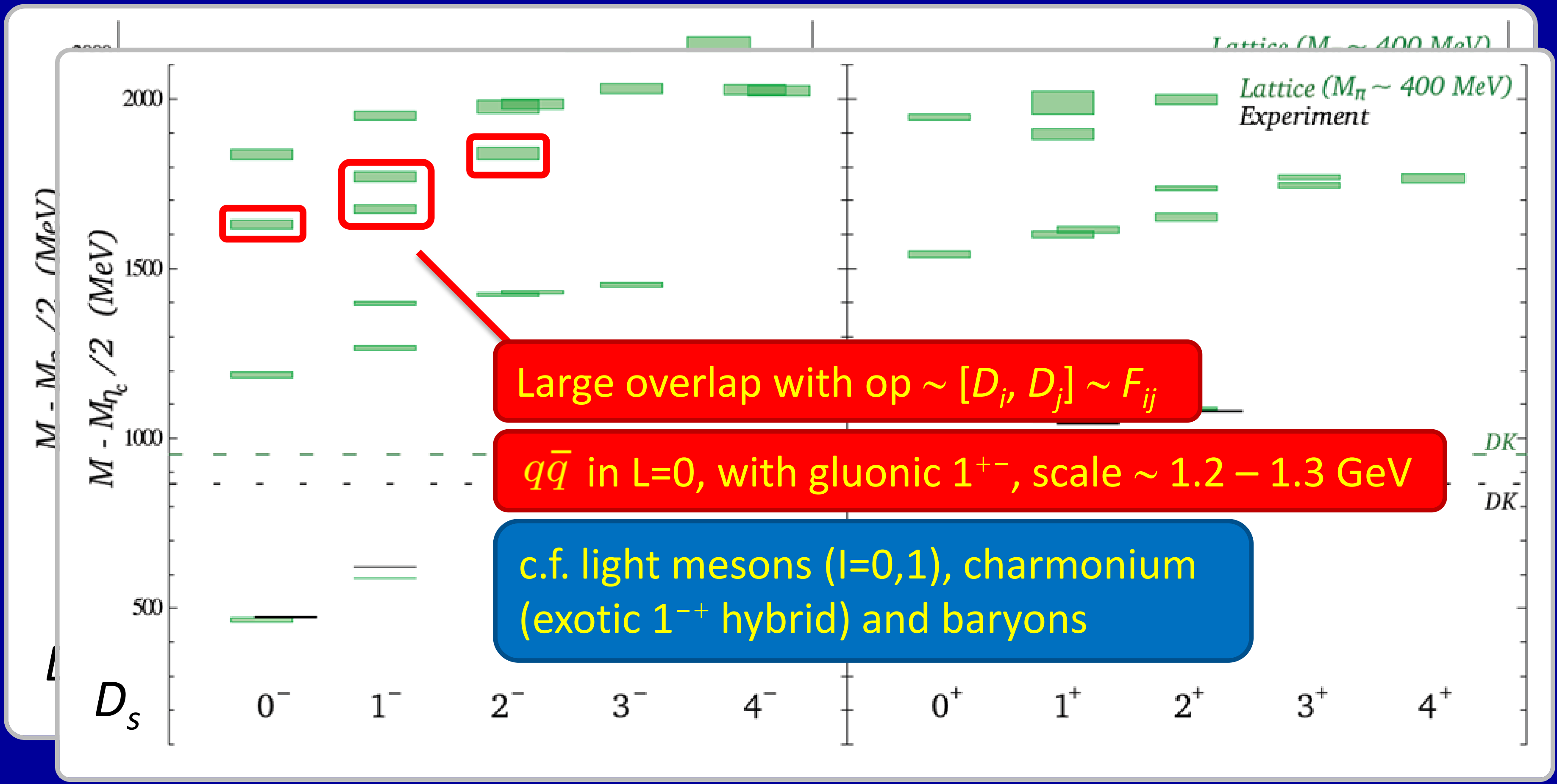

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Mesons – scattering

Elastic scattering – generalisation and many more details in Michael Döring's talk

Lüscher (elastic): energy levels in **finite vol.** → **infinite vol.** scattering phase shift at E_{cm}

Finite box → discrete spectrum

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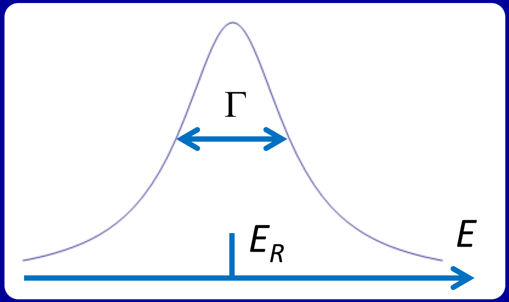
Map out phase shift → resonance parameters etc



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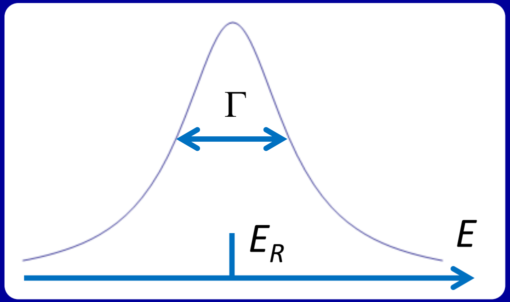

$$\sigma_l(E) \propto \sin^2 \delta_l(E) = \frac{(\Gamma/2)^2}{(E-E_R)^2 + (\Gamma/2)^2}$$

Need many (multi-hadron) energy levels

Single and multi-hadron ops

Non-zero $\boldsymbol{P}_{cm}$, different box sizes and shapes, twisted b.c.s, ...

Note: reduced symmetry → mixing between partial waves

The ρ resonance

P-wave $\pi\pi$ scattering
($J^{PC} = 1^{--}$, I = 1)

$\mathrm{BR}(\rho \to \pi\pi) \sim 100\%$

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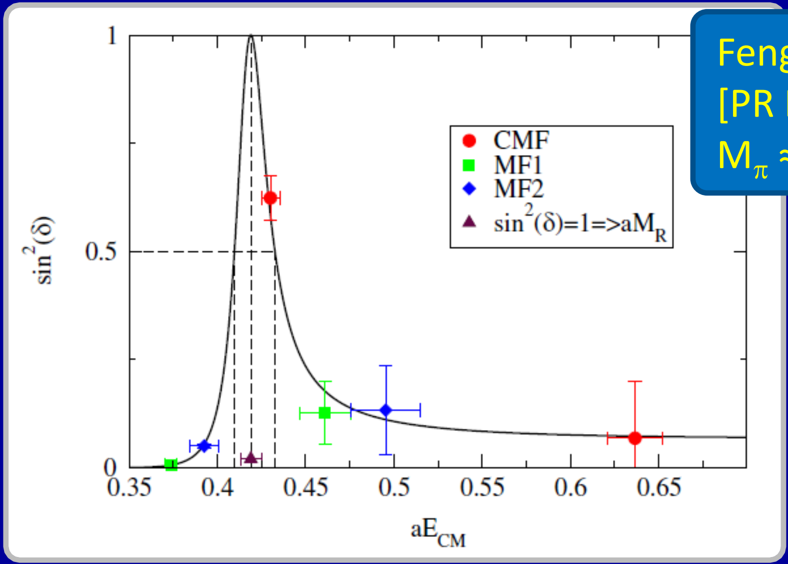
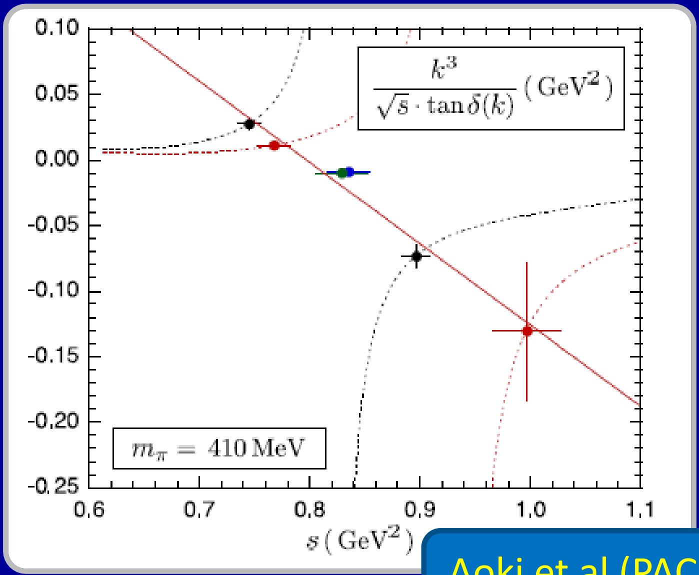
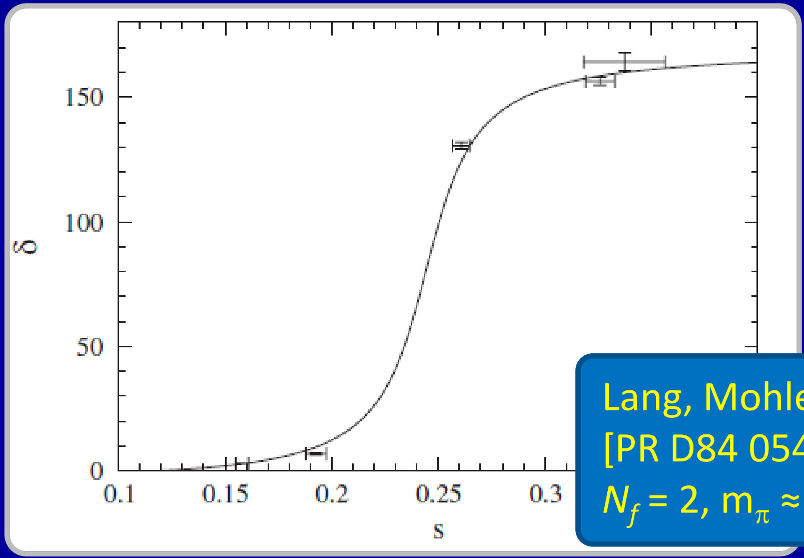


Feng, Jansen, Renner (ETMC), [PR D83 094505 (2011)] $N_f = 2$, $M_\pi \approx$ 480, 420, 330, 290 MeV



Aoki et al (PACS-CS), [PR D84 094505 (2011)] N_f = 2+1, $m_\pi \approx$ 410, 300 MeV

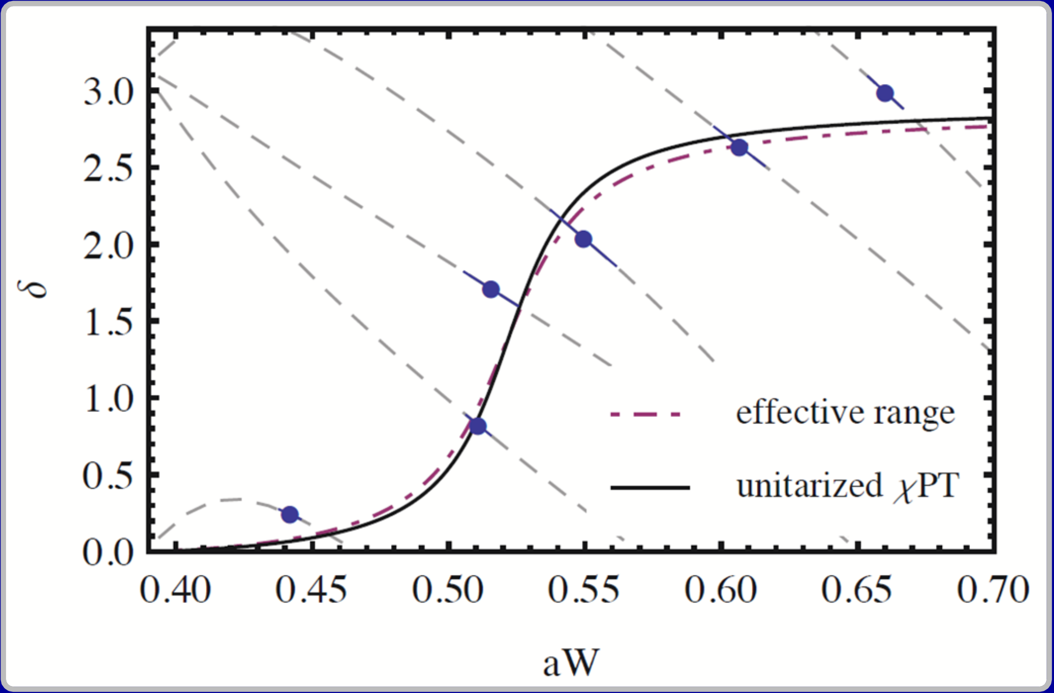


Lang, Mohler, Prelovsek, Vidmar, [PR D84 054503 (2011)] N_f = 2, $m_\pi \approx$ 266 MeV

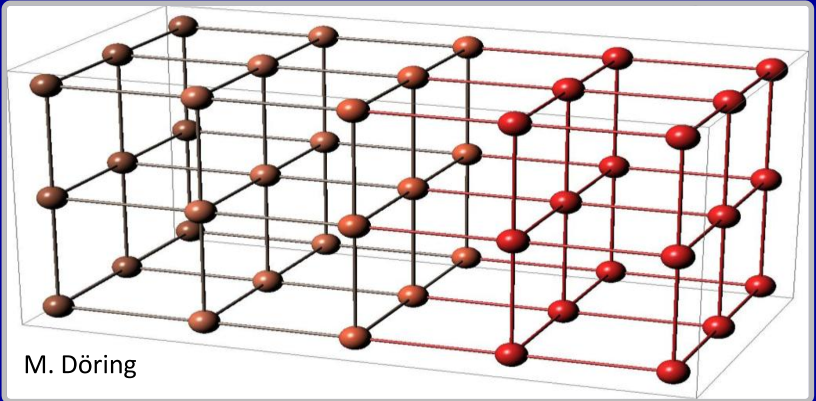
The ρ resonance

P-wave $\pi\pi$ scattering
($J^{PC} = 1^{--}$, I = 1)

$\mathrm{BR}(\rho \to \pi\pi) \sim 100\%$



Pelissier, Alexandru,
[PR D87 014503 (2013)]
$N_f = 2$, $M_\pi \approx 300$



M. Döring

The ρ resonance

Dudek, Edwards, CT (Had Spec Collab) [PR D87, 034505]

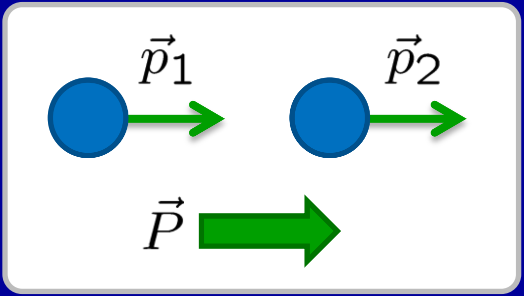
Use many single and multi-hadron ops

'Distillation'

$$\mathcal{O}(\vec{P}) = \sum_{\vec{p}_1,\ \vec{p}_2} \mathcal{C}_{\Lambda}(\vec{P},\vec{p}_1,\vec{p}_2)\, \mathcal{O}_{\pi}(\vec{p}_1)\, \mathcal{O}_{\pi}(\vec{p}_2)$$

Variationally optimised π ops

$\vec{P} = \vec{p}_1 + \vec{p}_2$ $\quad$ $\vec{P} = [0,0,0],\ [0,0,1],\ [0,1,1],\ [1,1,1]$



Aniso. Clover [N_f = 2+1], $M_\pi \approx 400$ MeV

Three volumes 16^3, 20^3, 24^3 ($M_\pi L \sim 4 - 6$)

The ρ resonance

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Use many single and multi-hadron ops

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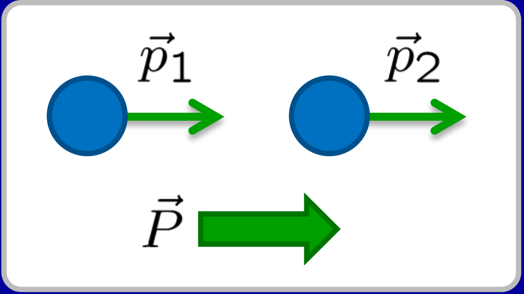
Variationally optimised π ops

, 1], [0, 1, 1], [1, 1, 1]

$\vec{P}$	Irrep	Single-meson	$\pi\pi$ $20^3, 24^3$ (16^3)
[0, 0, 0]	T_1^-	26	2 (3)
[0, 0, 1]	A_1	18	4 (4)
	E_2	29	2 (2)
	B_1	9	1 (0)
	B_2	9	1 (0)
[0, 1, 1]	A_1	27	3 (3)
	B_1	29	3 (3)
	B_2	29	2 (2)
[1, 1, 1]	A_1	21	3 (3)
	E_2	35	2 (2)
[0, 0, 2]	A_1	18	1 (1)

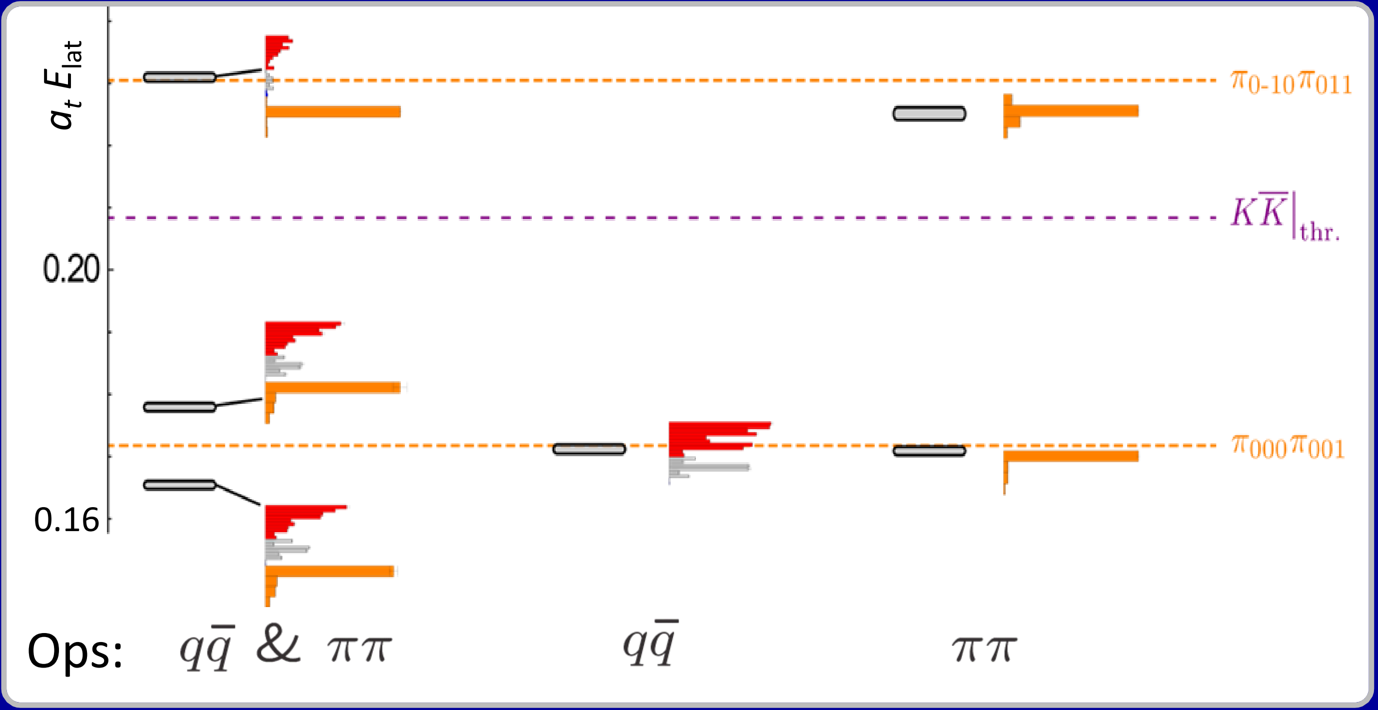


Consider various irreps – constrain higher partial waves

Operators and energy levels

24^3, P = [0,0,1] A_1

[PR D87, 034505]



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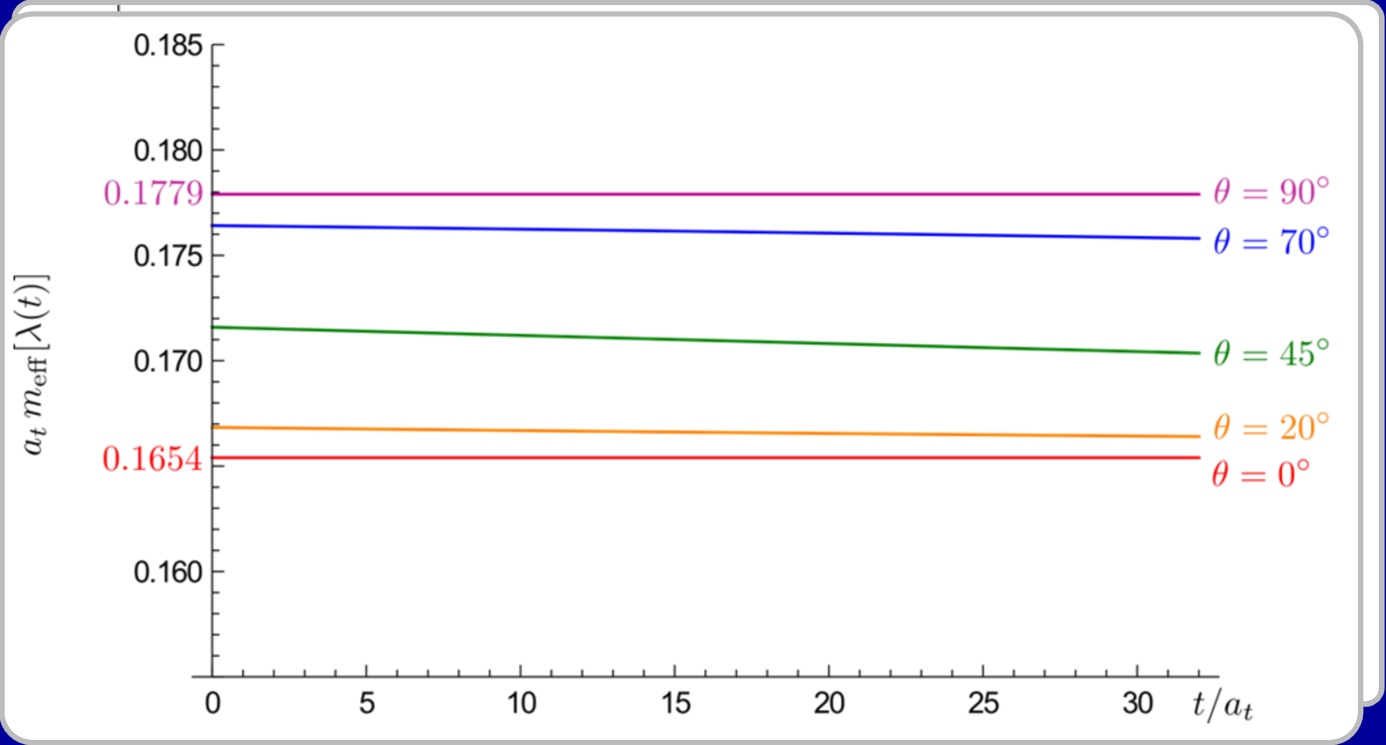

$$|E_1\rangle = +\cos\theta\,|\rho\rangle + \sin\theta\,|\pi\pi\rangle$$
$$|E_2\rangle = -\sin\theta\,|\rho\rangle + \cos\theta\,|\pi\pi\rangle$$

$$\left\langle \rho \left| e^{-Ht} \right| \rho \right\rangle = \cos^2\theta\; e^{-E_1 t} + \sin^2\theta\; e^{-E_2 t}$$

$a_t\, E_1 = 0.1654$, $a_t\, E_2 = 0.1779$

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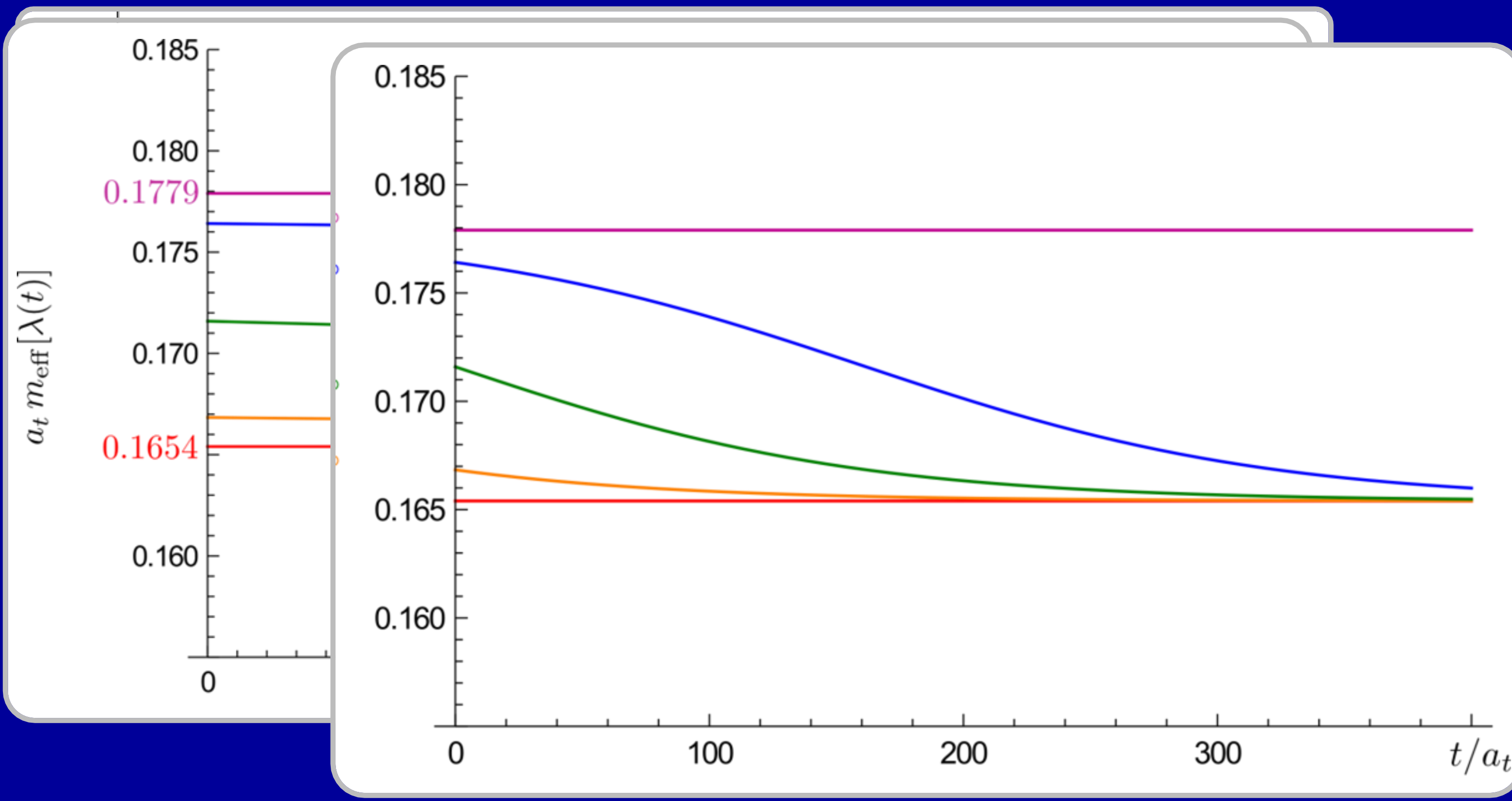

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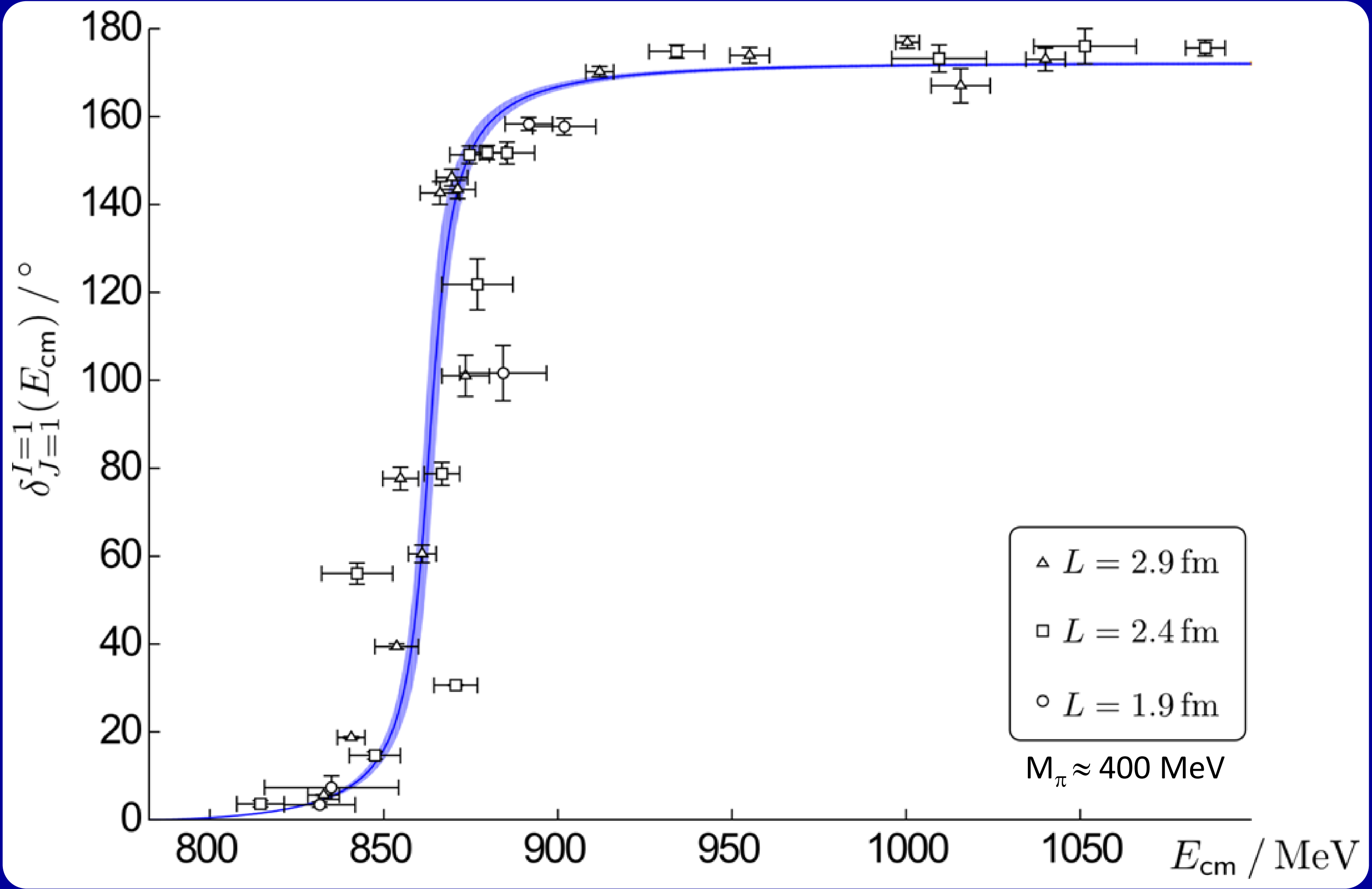
The ρ resonance

Mapped out in detail

[PR D87, 034505]

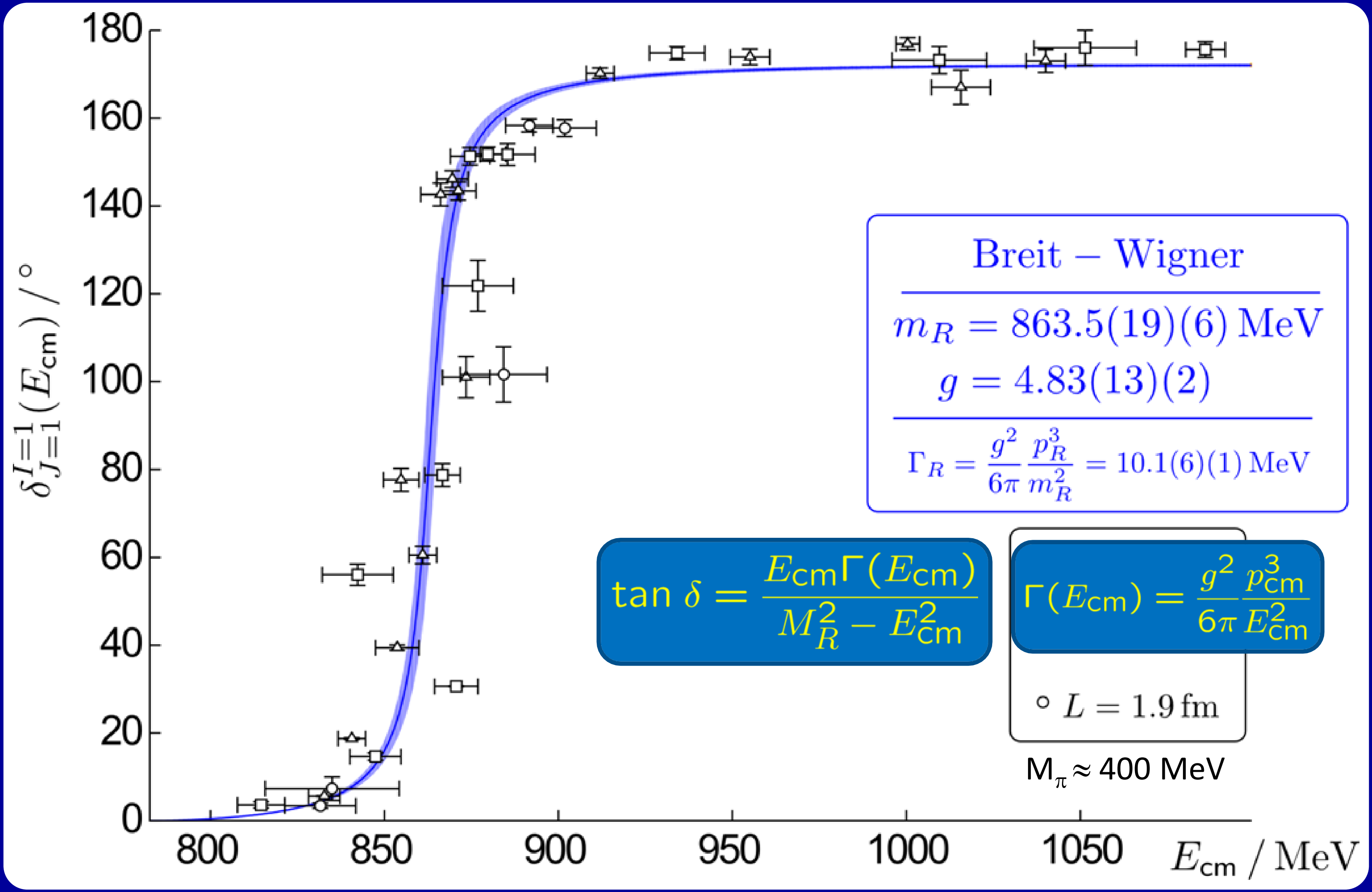


$M_\pi \approx 400$ MeV

The ρ resonance

Mapped out in detail

[PR D87, 034505]



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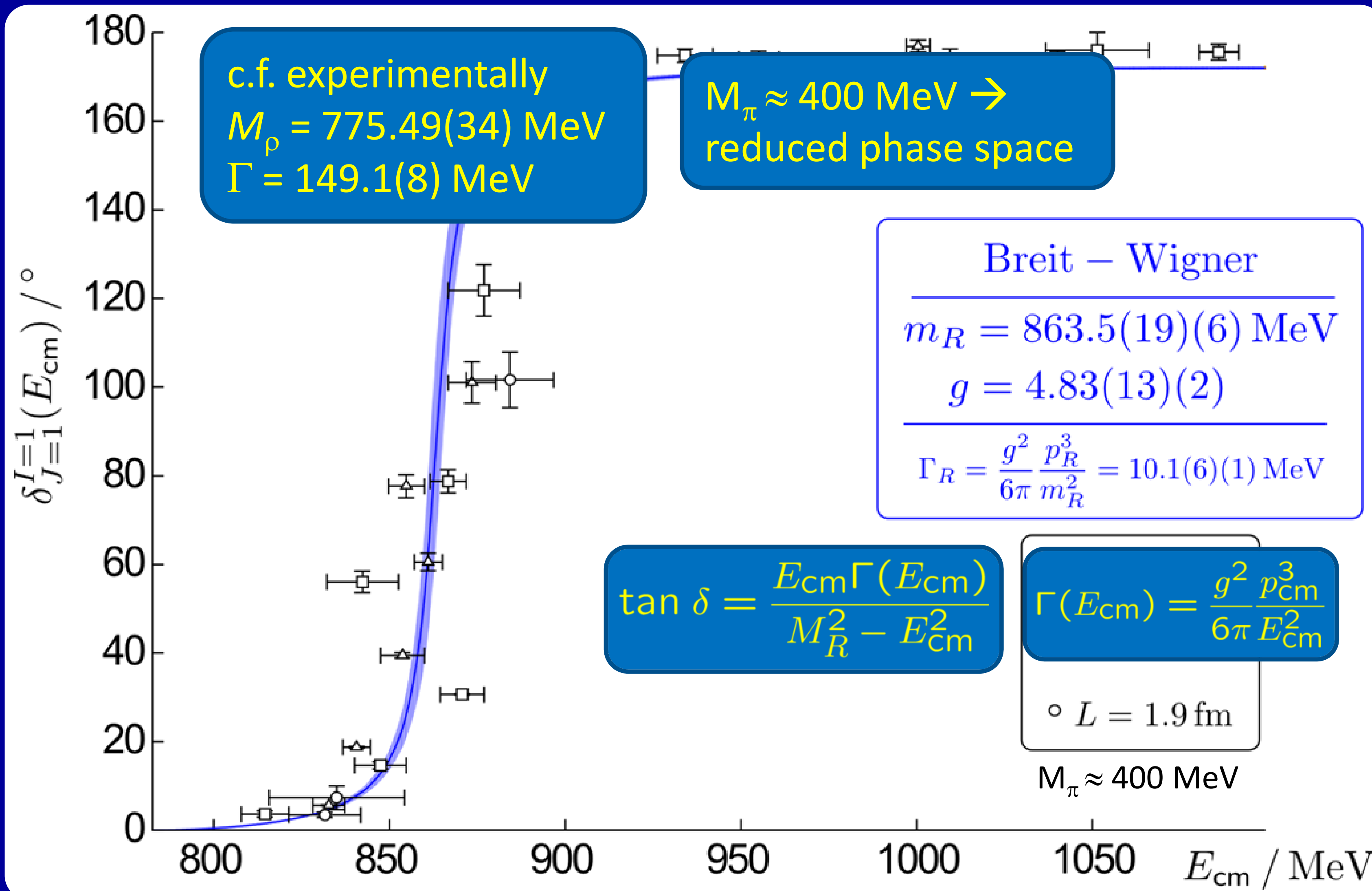
Mapped out in detail

[PR D87, 034505]



The ρ resonance

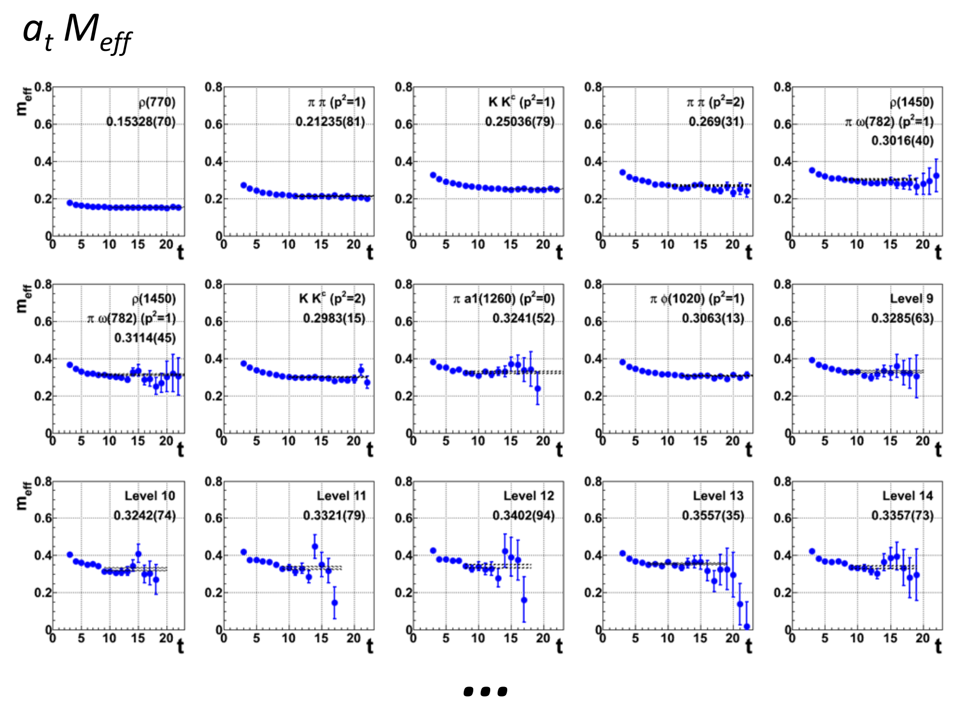
Morningstar et al preliminary
→ talk [Mon, 2G]

Different op constructions, stochastic distillation

56 ops $q\bar{q},\ \pi\pi,\ \eta\pi,\ \phi\pi,\ K\bar{K}$

Had Spec Collab lattice
(24^3 x 128, $M_\pi \approx 400$ MeV)

(also K* channel)



Time-like π form factor X. Feng [poster]
→ Hadron Structure Review

Mesons – scattering

Some other light and strange channels → Michael Döring's talk

Charmed mesons

Liu et al [PR D87, 014508] → talk by L. Liu [Weds, 5G]

Mohler, Prelovsek, Woloshyn [PR D87, 034501] and preliminary results → talk by D. Mohler [Weds, 5G]

Had Spec Collab preliminary → talk by G. Moir [Weds, 5G]

Charmonium

Ozaki, Sasaki [PR D87, 014506]

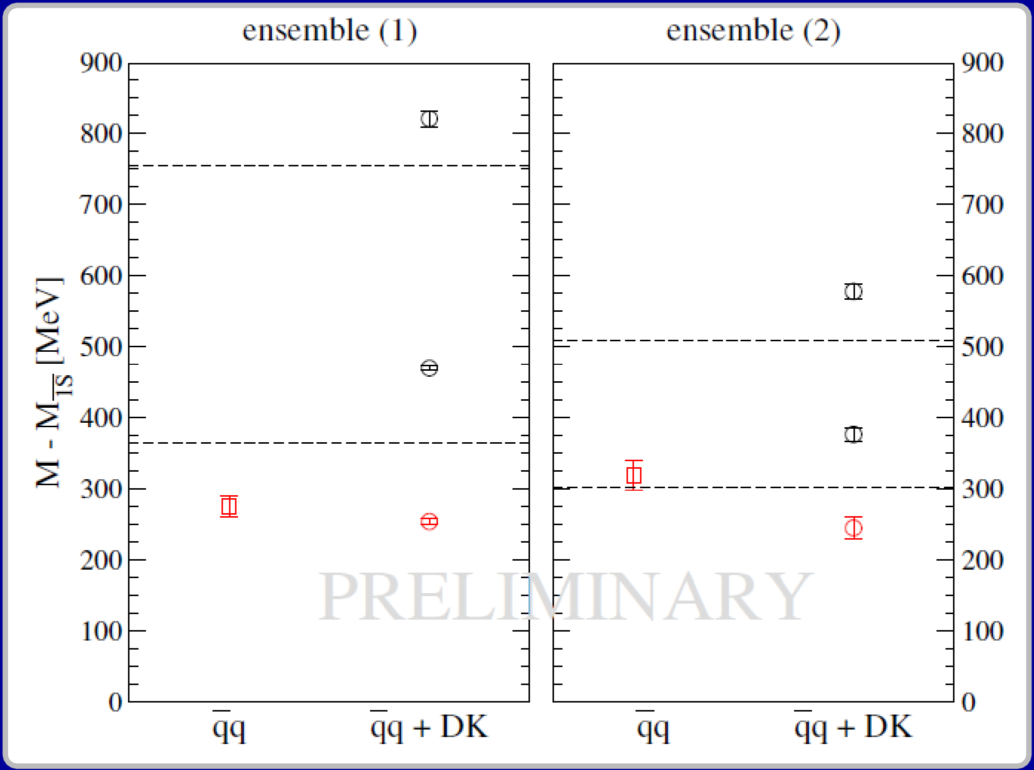
Prelovsek and Leskovec [arXiv:1307.5172]
→ talk by S. Prelovsek [Thrs, 8G]

D and D_s mesons

Mohler et al (preliminary) → talk by D. Mohler [Weds, 5G]

$D_s(2317)$ with $J^P = 0^+$

D_s and $D\ K$ ops



ID	$N_L^3 \times N_T$	N_f	a[fm]	L[fm]	#configs	m_π[MeV]	m_K[MeV]
(1)	$16^3 \times 32$	2	0.1239(13)	1.98	280/279	266(3)(3)	552(2)(6)
(2)	$32^3 \times 64$	2+1	0.0907(13)	2.90	196	156(7)(2)	504(1)(7)

(1) nHYP from Hasenfratz et al [$M_\pi L \approx 2.7$], (2) Clover from PACS-CS [$M_\pi L \approx 2.3$]
Use distillation; for (2) use stoch. distillation [PRD 83, 114505]. Fermilab approach for c

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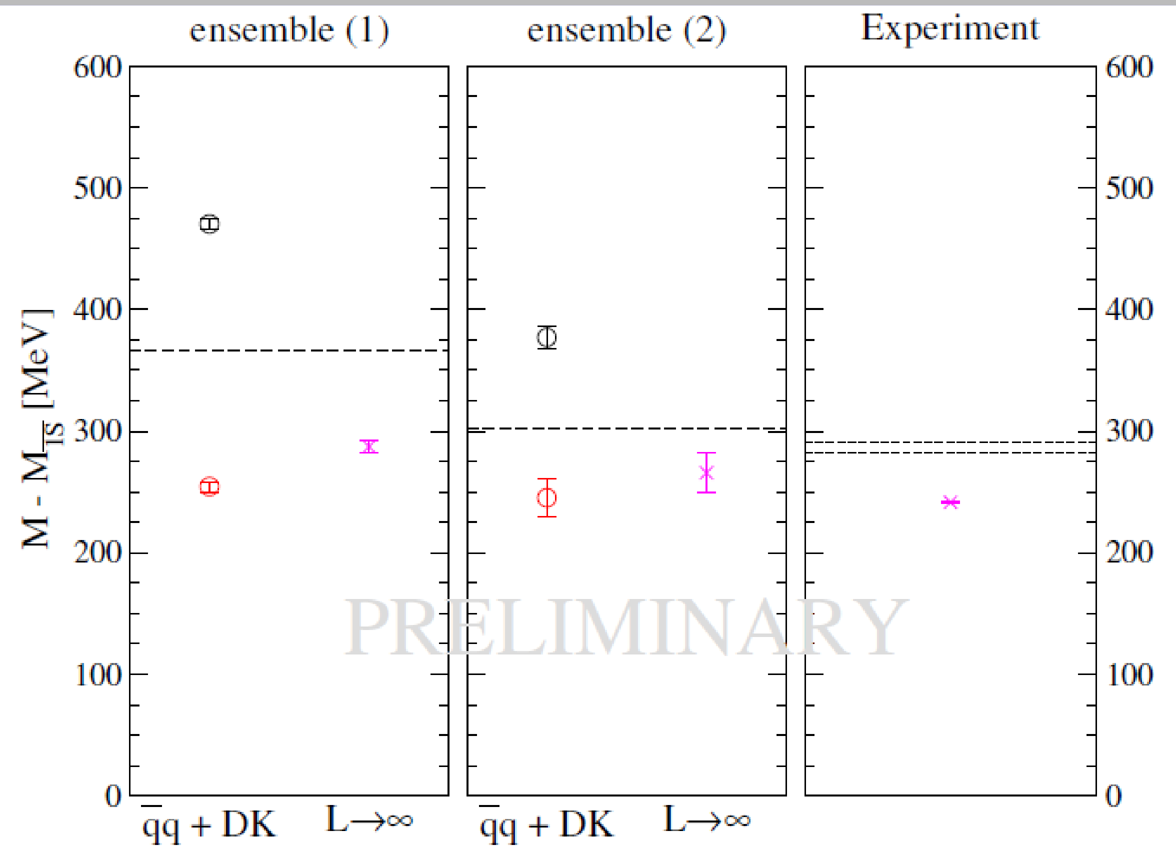
D_s and *D K* ops

$$p\cot\delta(p) \approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

$a_0 = -0.756 \pm 0.025$fm
$a_0 = -1.33 \pm 0.20$fm

$r_0 = 0.056 \pm 0.031$fm
$r_0 = 0.27 \pm 0.17$fm

[2 points per ensemble]



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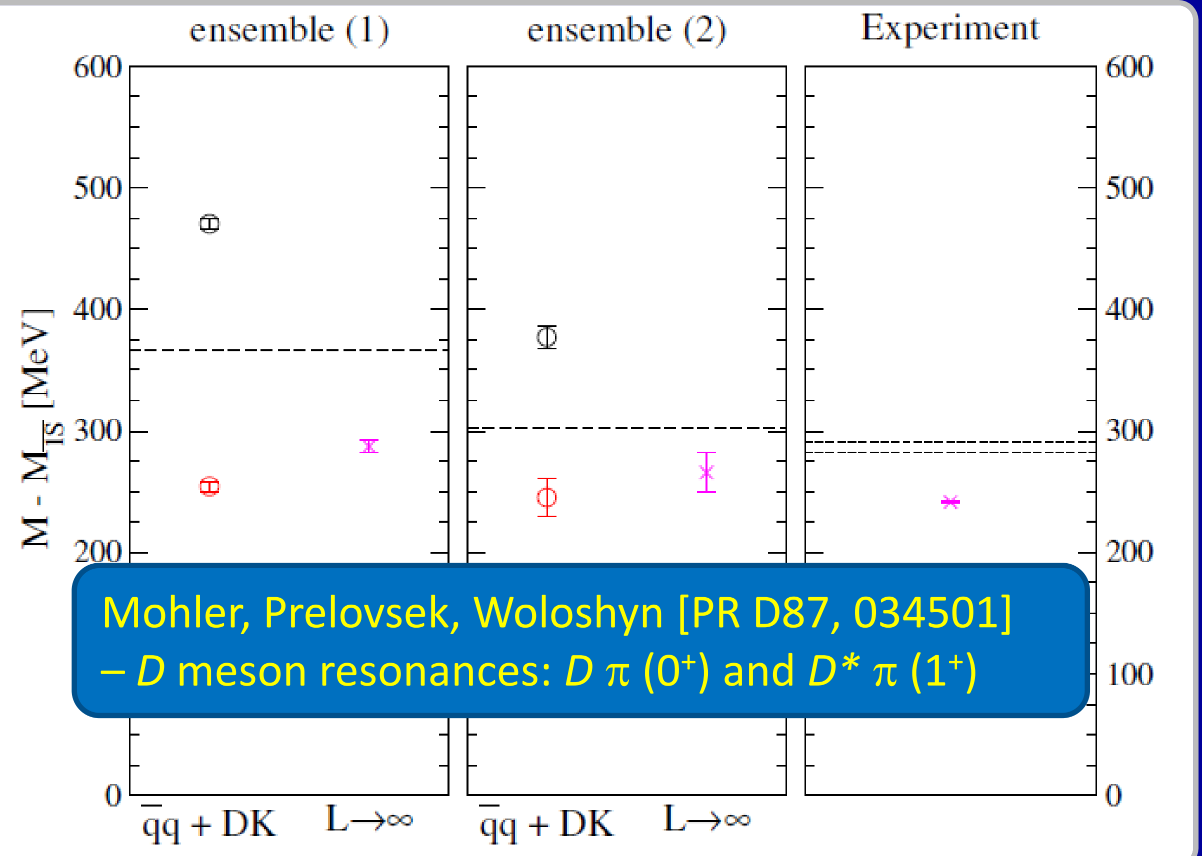

Mohler, Prelovsek, Woloshyn [PR D87, 034501]
– *D* meson resonances: *D* π (0$^+$) and *D** π (1$^+$)

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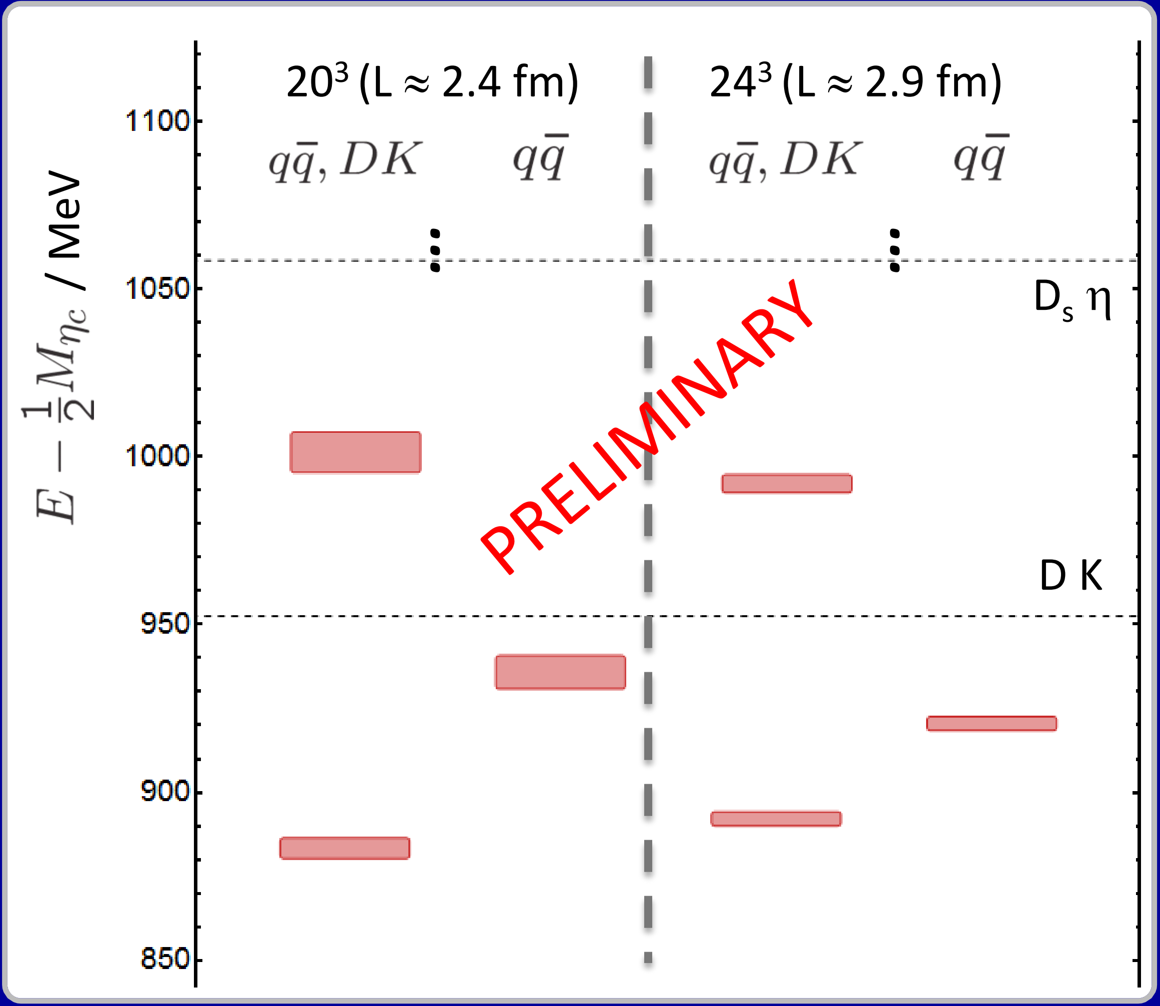
D and D_s mesons

Hadron Spectrum Collaboration preliminary

Preliminary D K (I=0) with $J^P = 0^+$ c.f. $D_s(2317)$

$M_\pi \approx 400$ MeV,
two vols,
$P = [0,0,0]$

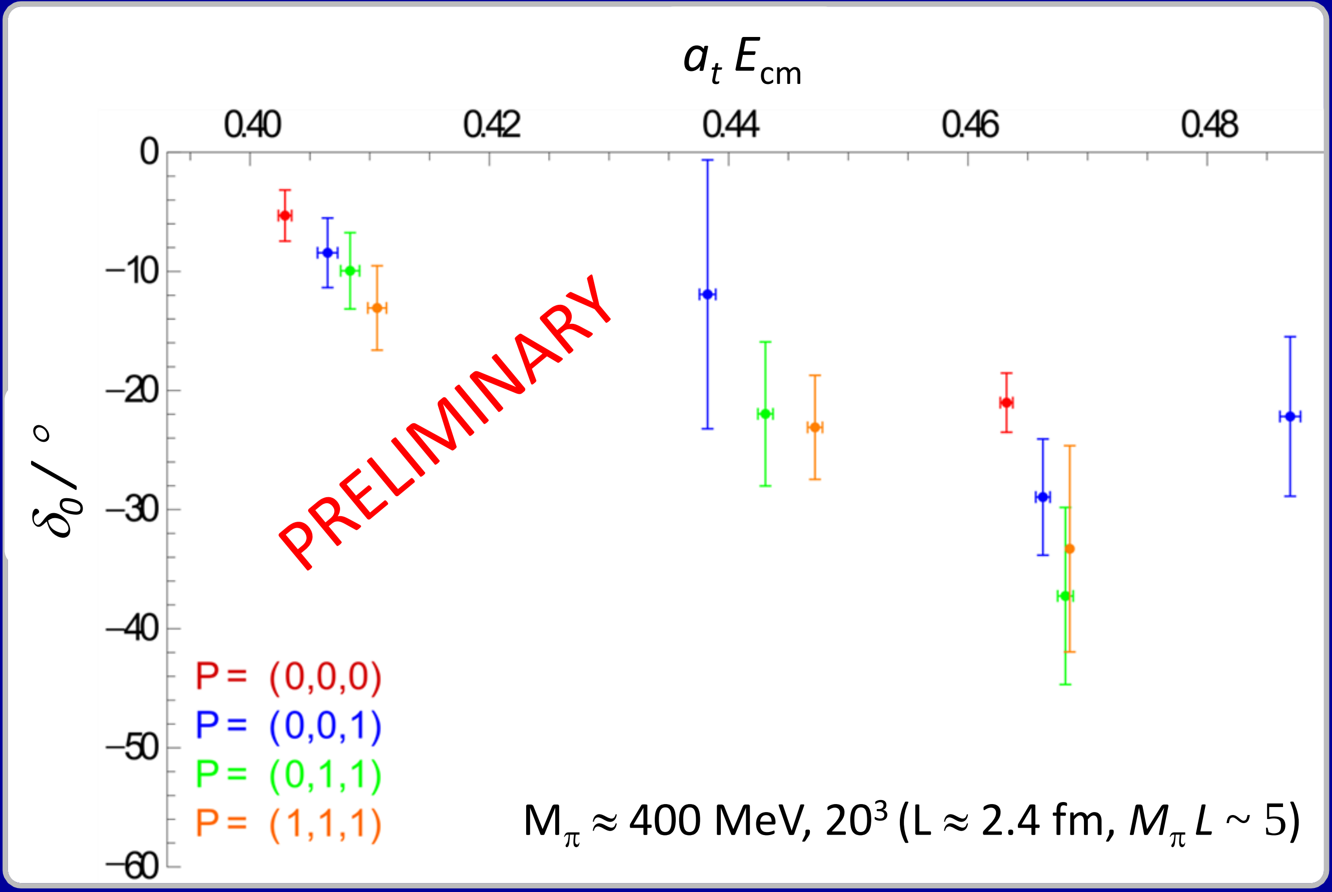
4 D K + 8 D_s ops



D and D_s mesons

Hadron Spectrum Collaboration preliminary

Preliminary D π scattering (I=3/2)
→ talk by G. Moir [Weds, 5G]



Charmonium

Prelovsek and Leskovec [arXiv:1307.5172 and prelim results] → talk by S. Prelovsek [Thrs, 8G]

X(3872) [$J^{PC} = 1^{++}$] near/below D D* threshold

Look in I=0
(one vol, one $\boldsymbol{P}_{cm}$)

$c\bar{c}, D\bar{D}^*, J/\psi\,\omega$ ops

Clover [$N_f = 2$] (Hasenfratz et al), M_π = 266(3)(3) MeV, a = 0.1239(13) fm
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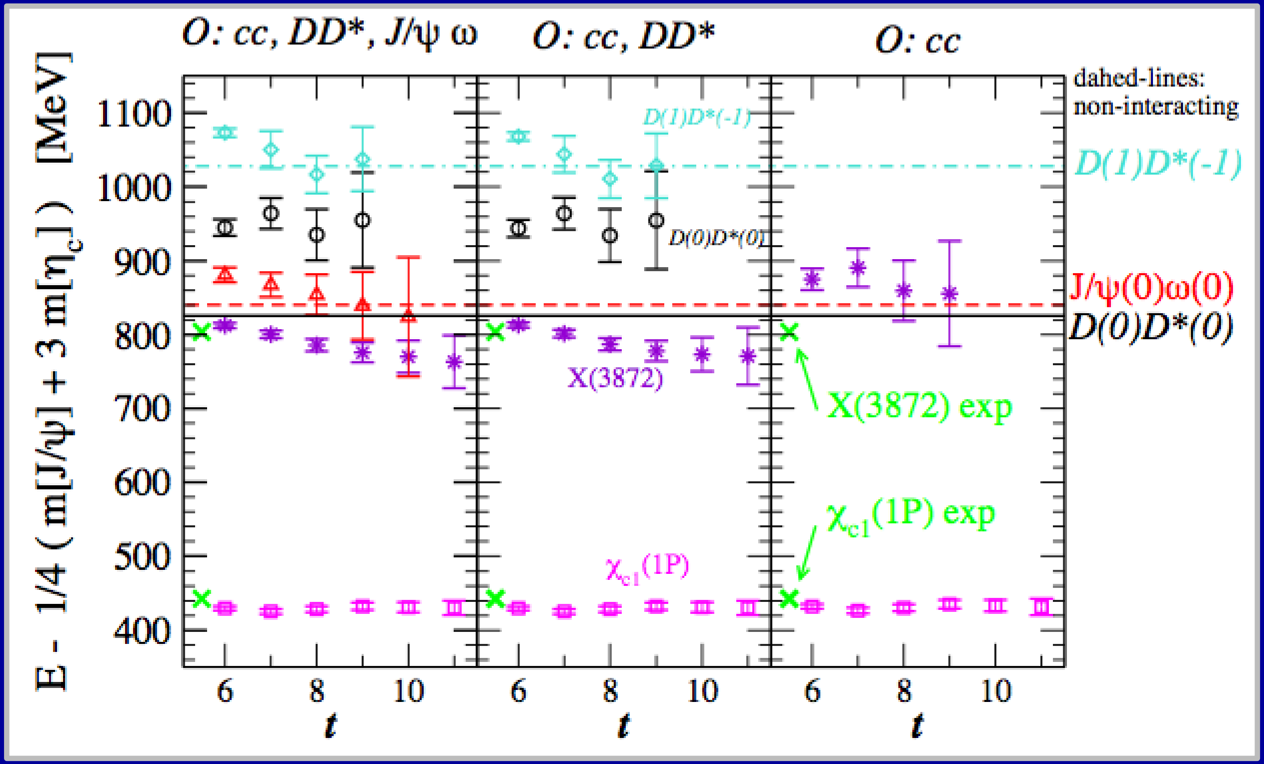

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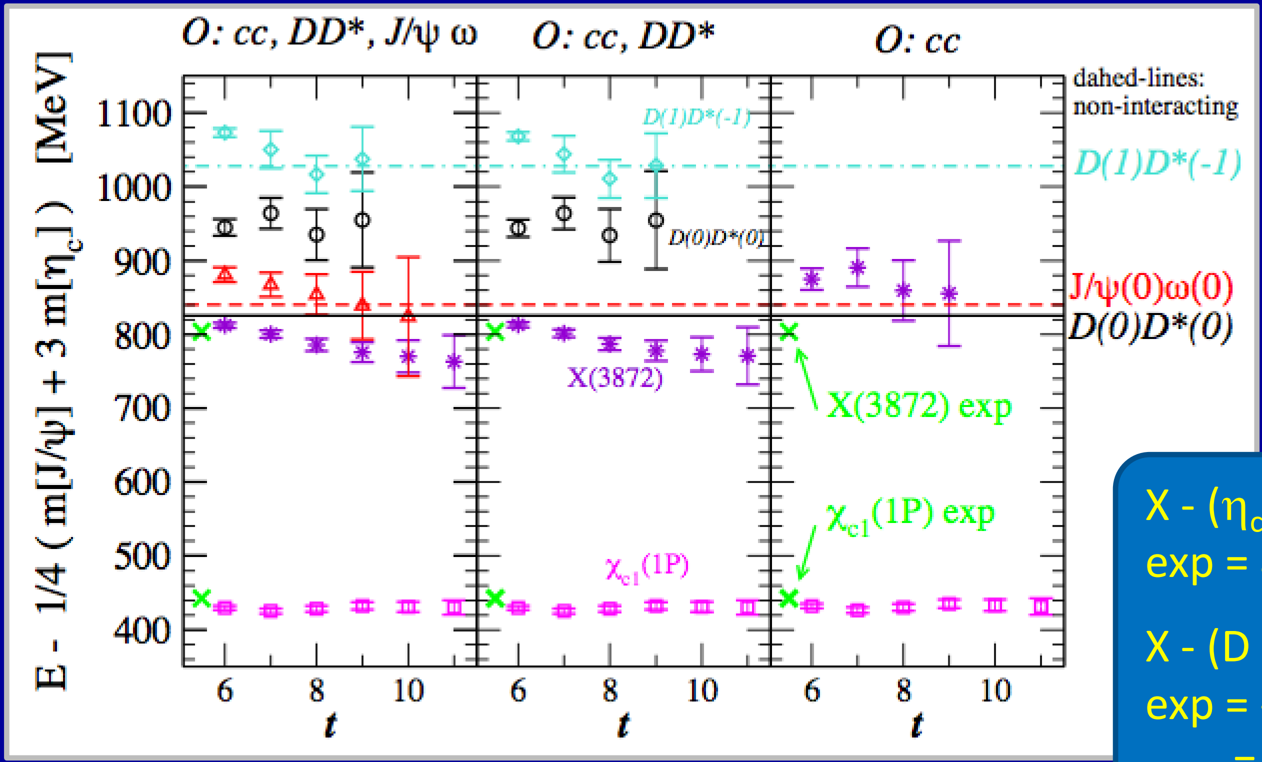

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$a_0 = -1.7 \pm 0.4\text{fm}$

$r_0 = 0.5 \pm 0.1\text{fm}$

2 points

X - (η_c + 3 J/ψ)/4 = 815(7) MeV
exp = 803.1(2) MeV

X - (D + D*) = -11(7) MeV
exp = -8.2(3) MeV [$D^+ D^{*-}$]
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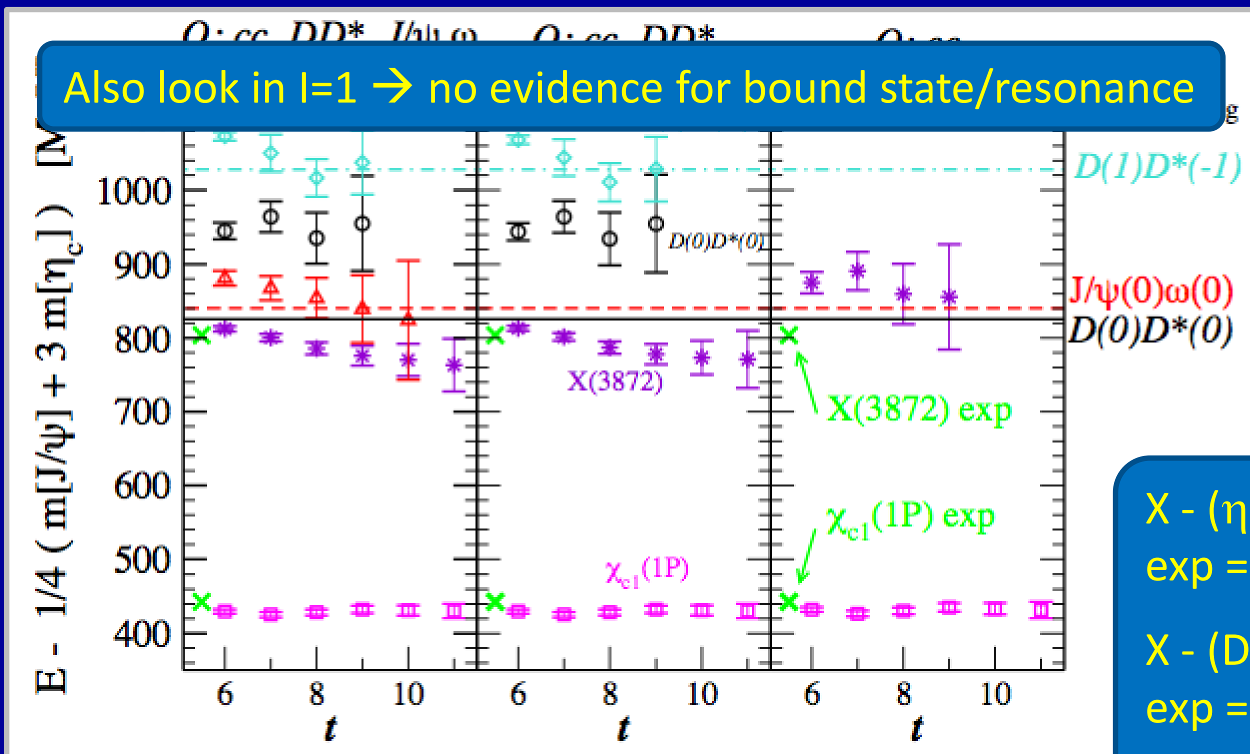
Look in I=0
(one vol, one $\boldsymbol{P}_{cm}$)

$c\bar{c}, D\bar{D}^*, J/\psi\,\omega$ ops

Also look in I=1 → no evidence for bound state/resonance



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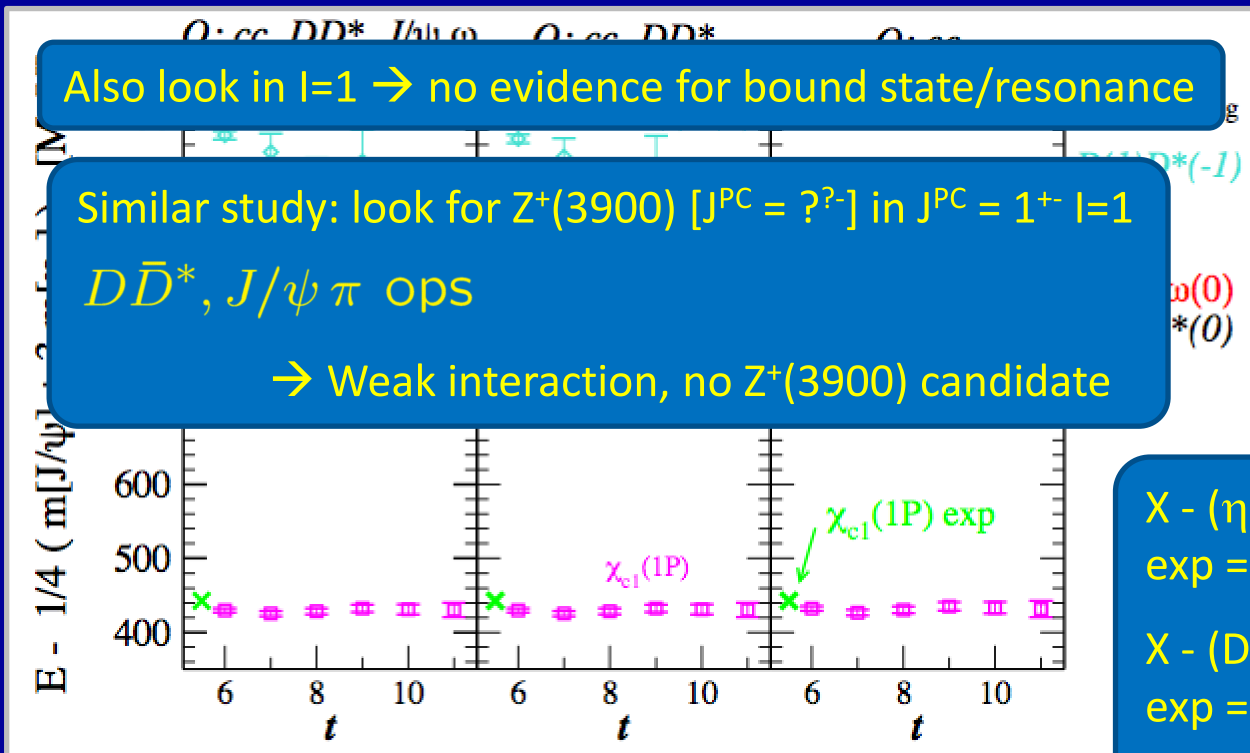

Also look in I=1 → no evidence for bound state/resonance

Similar study: look for $Z^+(3900)$ [$J^{PC} = ?^{?-}$] in $J^{PC} = 1^{+-}$ I=1

$D\bar{D}^*, J/\psi\,\pi$ ops

→ Weak interaction, no $Z^+(3900)$ candidate

$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

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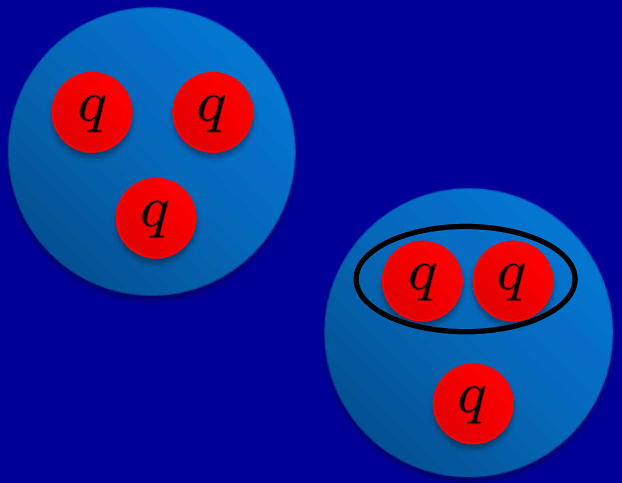
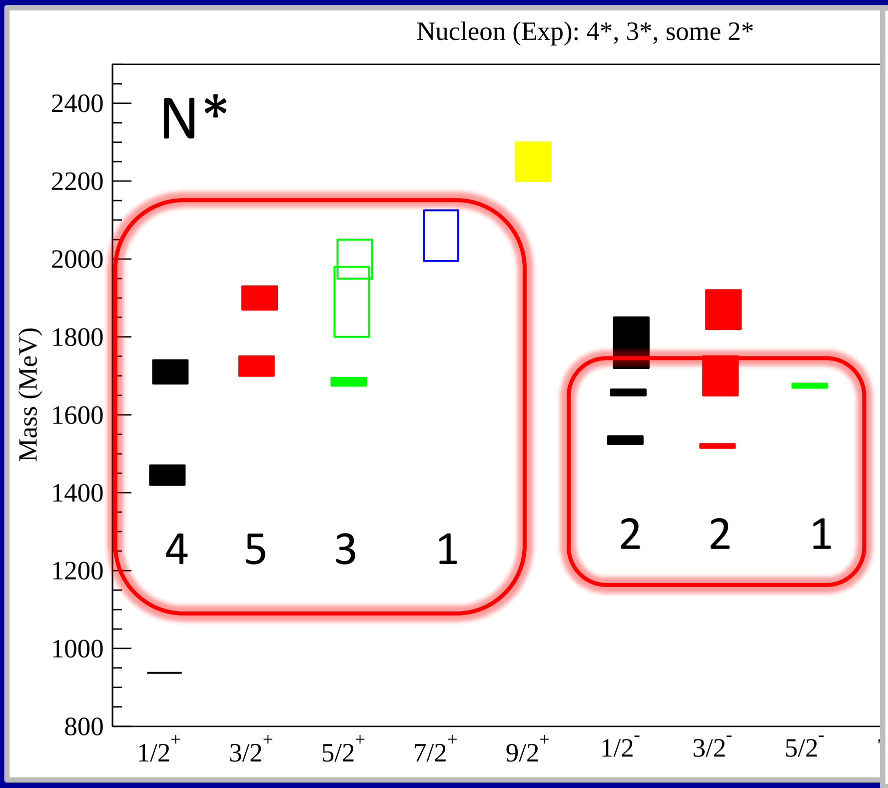
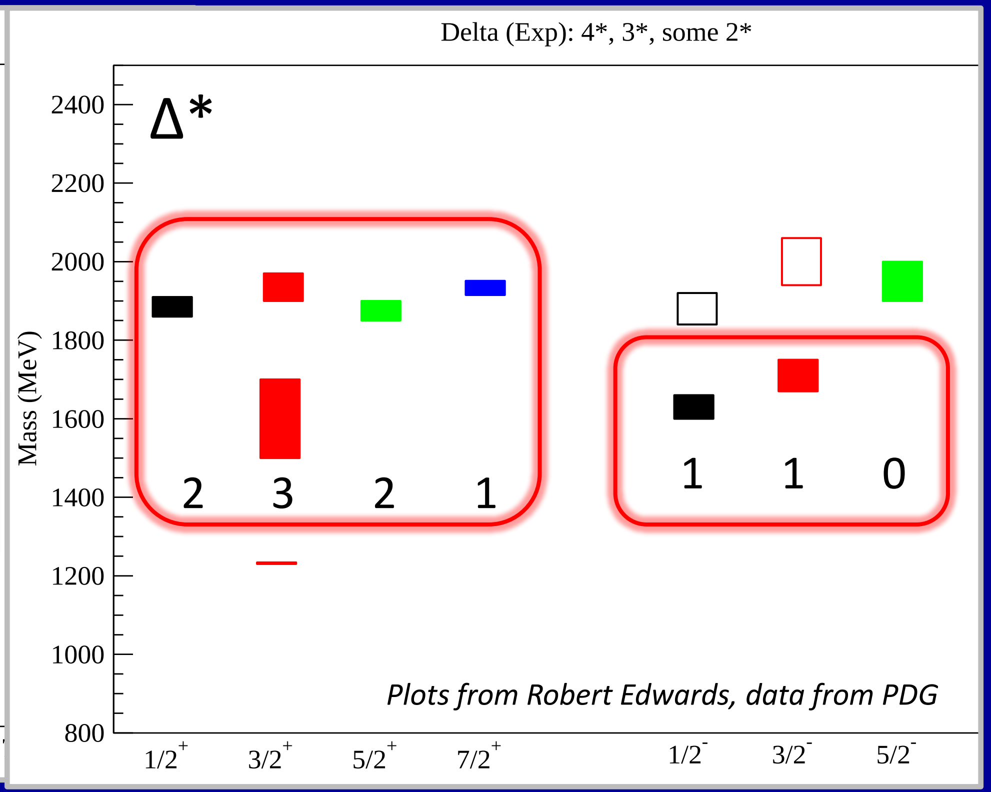
Hadron Spectroscopy – Baryons

- Missing states?
- 'Freezing' of degrees of freedom?
- Gluonic excitations?
- Flavour structure







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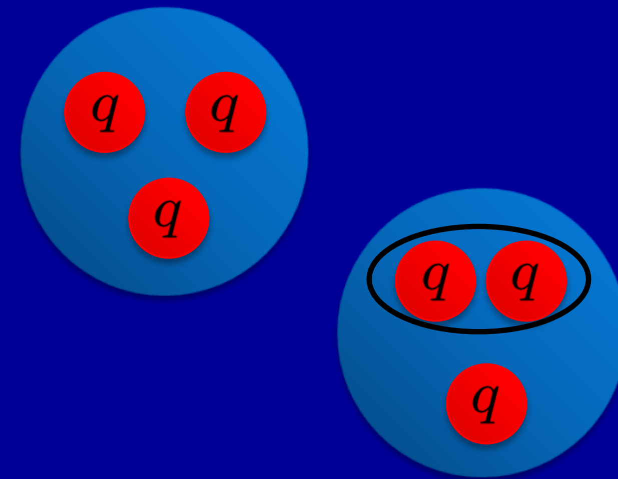

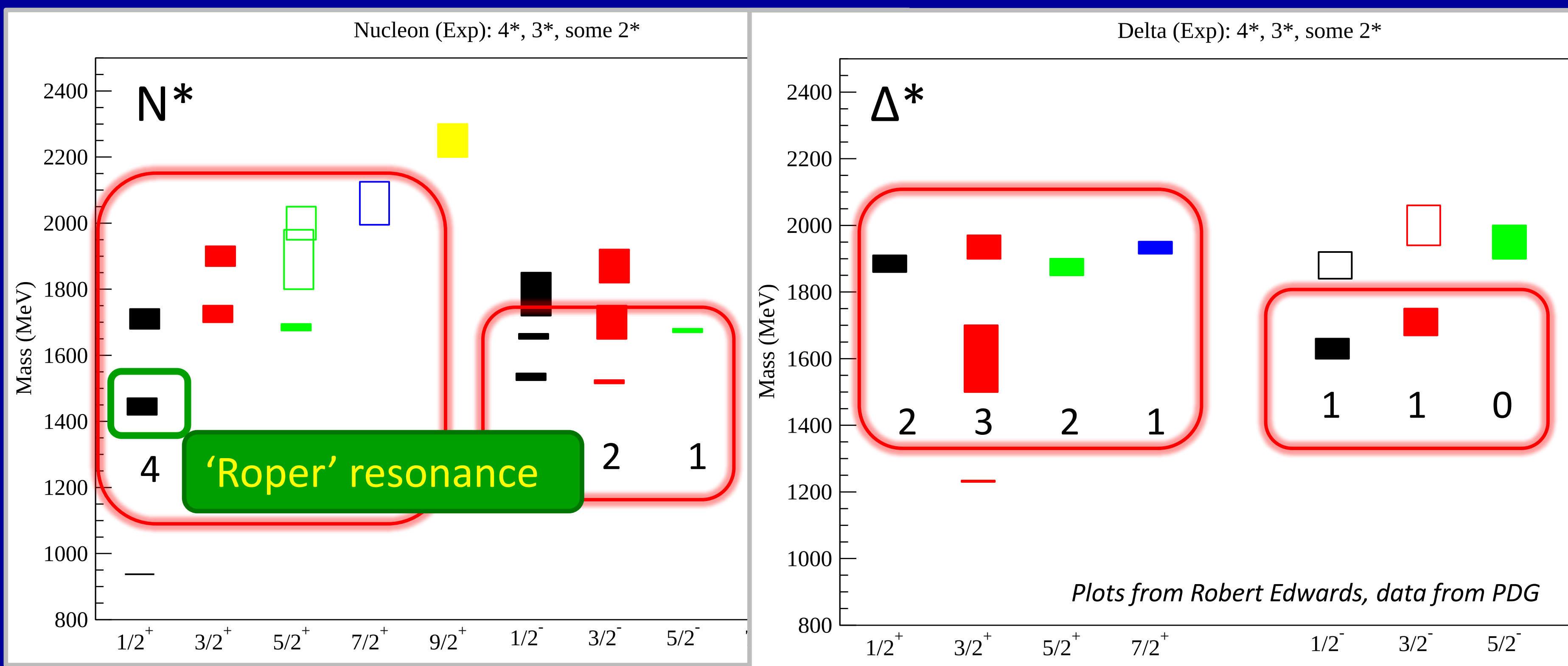

Flavour structure of excited baryons

Edwards et al (Hadron Spectrum Collaboration) [PR D87, 054506]

Light and strange baryons (all flavour combinations)

Lots of ops with different structures (same idea as mesons)

Aniso. Clover ($a_s/a_t \approx 3.5$), $a_s \approx 0.12$ fm, 16^3 x 128 (L ≈ 1.9 fm, $M_\pi L \geq 4$)

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Lots of ops with different structures (same idea as mesons)

Definite J^P in infinite vol. continuum [**p** = **0**]

$$\langle 1, m_1; 1, m_2 | L, m_l \rangle \langle L, m_l; S, m_s | J, m_J \rangle \overrightarrow{D}_{m_1} \overrightarrow{D}_{m_2} [\psi\psi\psi]_{S,m_s}$$

Up to 2 derivs: $1 \otimes 1 \otimes S \rightarrow \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$

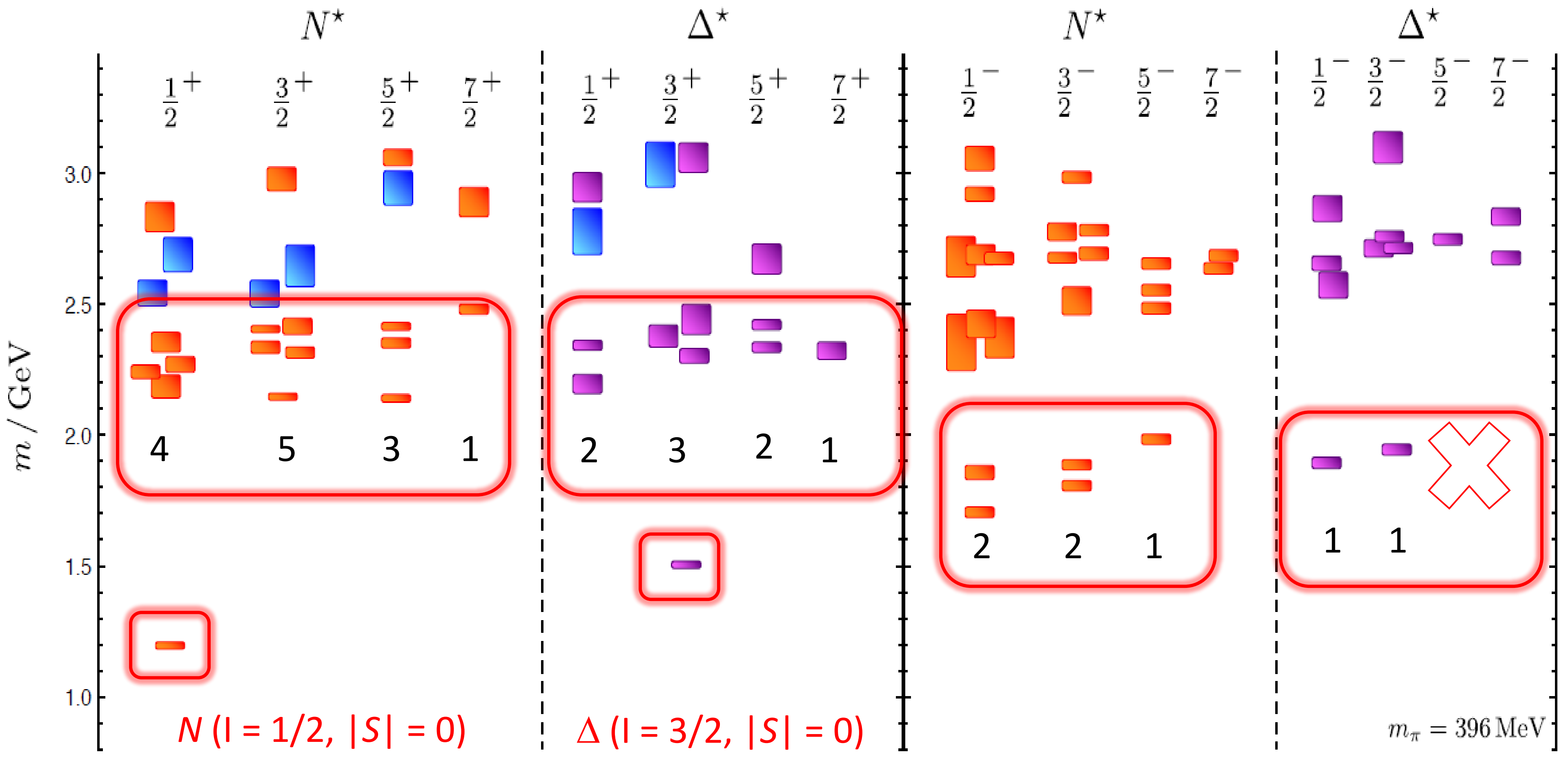
'Subduce' ops → irreps. of octahedral sym group

Spin identification

Aniso. Clover ($a_s/a_t \approx 3.5$), $a_s \approx 0.12$ fm, $16^3 \times 128$ (L ≈ 1.9 fm, $M_\pi L \geq 4$)

N and Δ baryons

[PR D84 074508; D85 054016]



Counting expected in non. rel. quark model, SU(6) x O(3)

N_f = 2+1, $M_\pi \approx$ 400 MeV

N and Δ baryons

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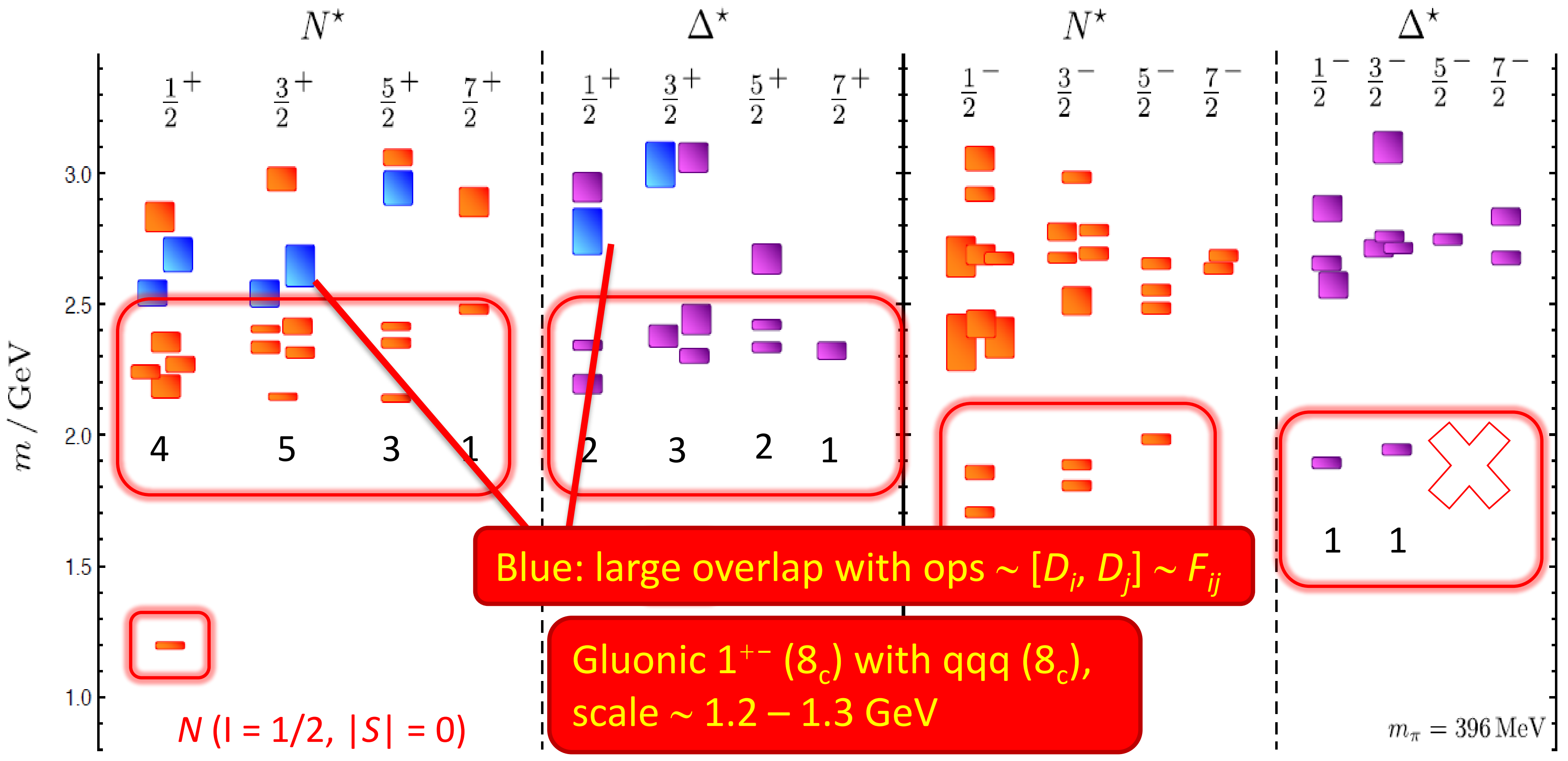

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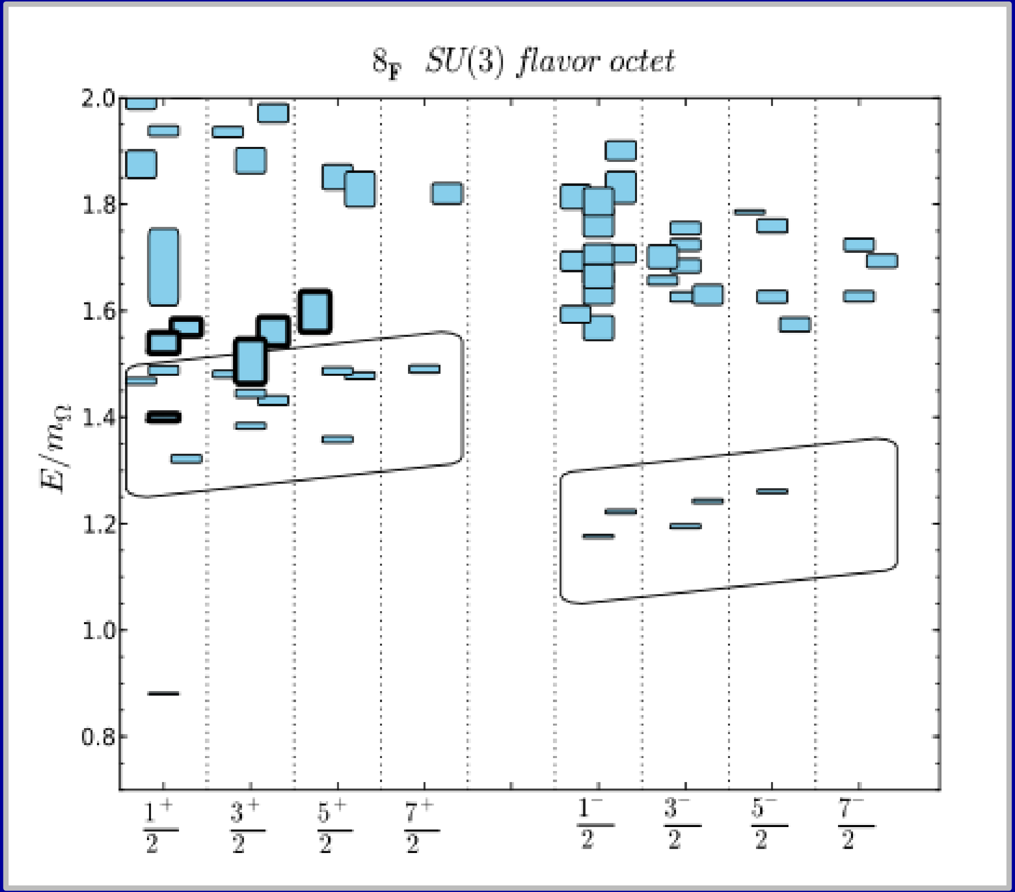
Flavour structure of excited baryons

[PR D87, 054506]

$N_f = 3$, $M_\pi \approx 700$ MeV

SU(3) flavour symmetry

$\rightarrow 1_F \oplus 8_F \oplus 10_F$



Multiplicities as expected in non. rel. quark model
SU(6) x O(3)
(flavour x spin x space)

No ‘freezing’ of d.o.f.

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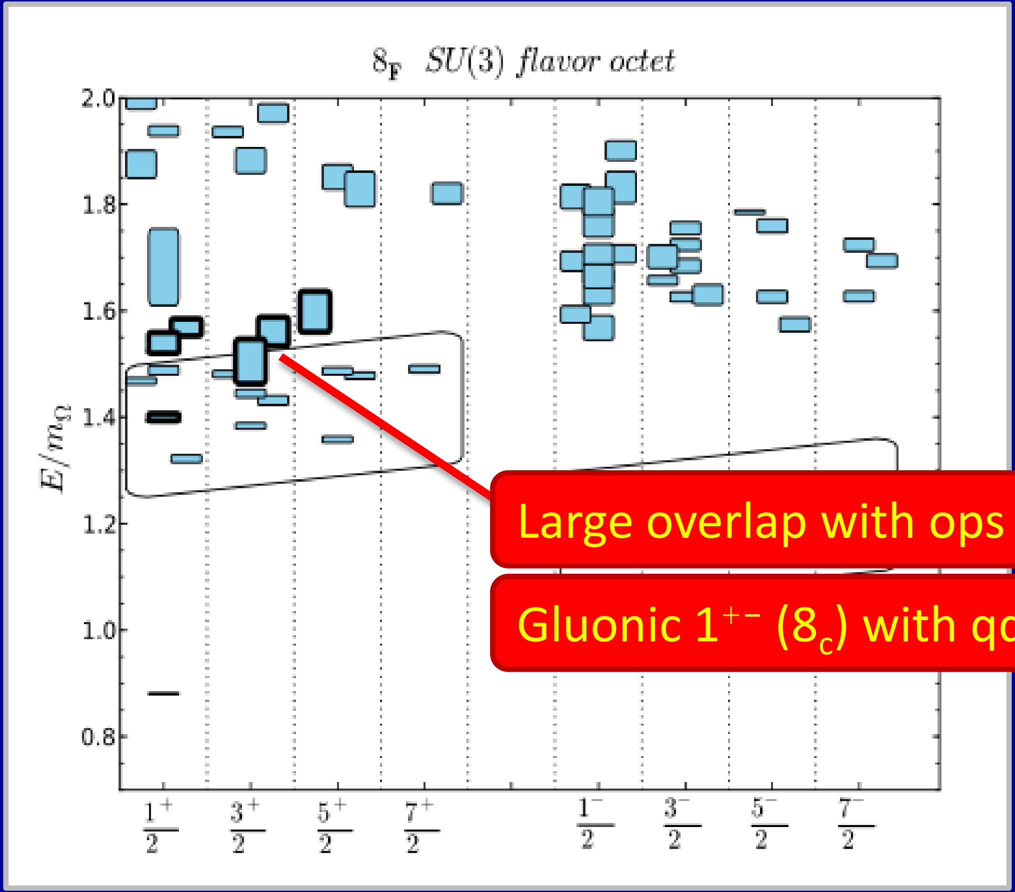

Multiplicities as expected in non. rel. quark model
SU(6) x O(3)
(flavour x spin x space)

No 'freezing' of d.o.f.

Large overlap with ops ~ $[D_i, D_j] \sim F_{ij}$

Gluonic 1^{+-} (8_c) with qqq (8_c), scale ~ 1.2 – 1.3 GeV

Flavour structure of excited baryons

[PR D87, 054506]

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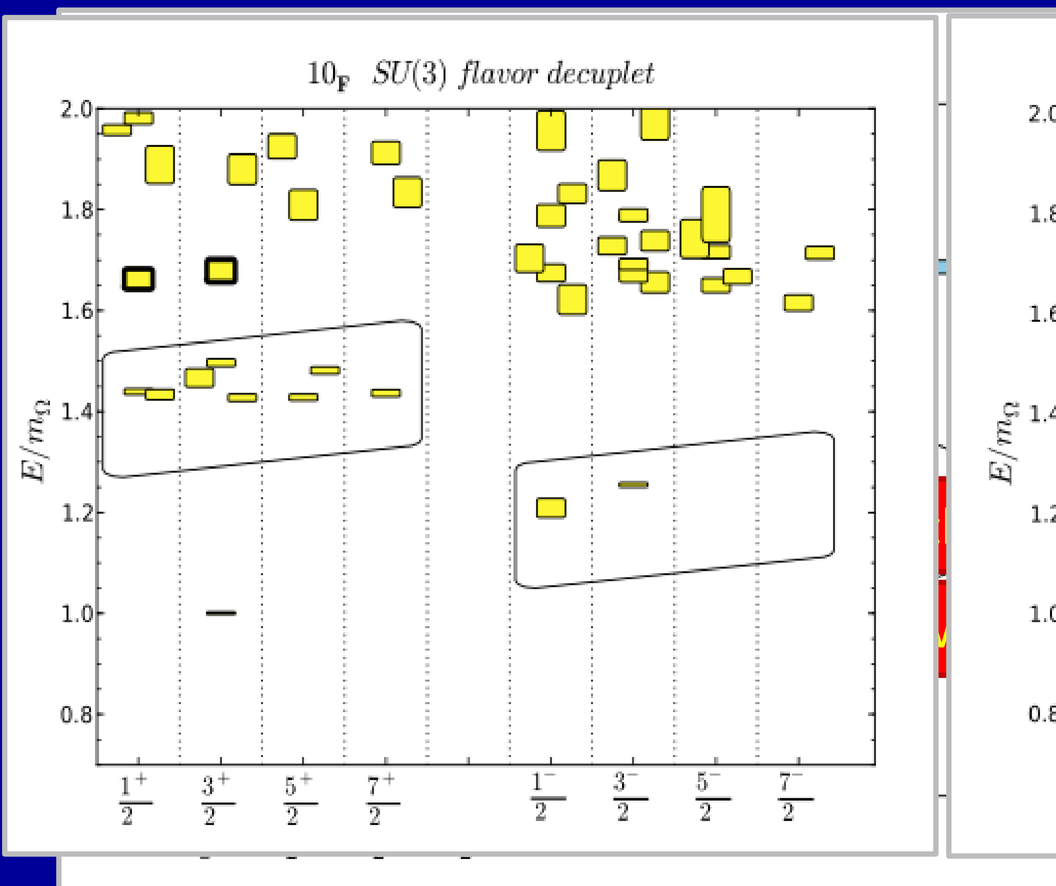
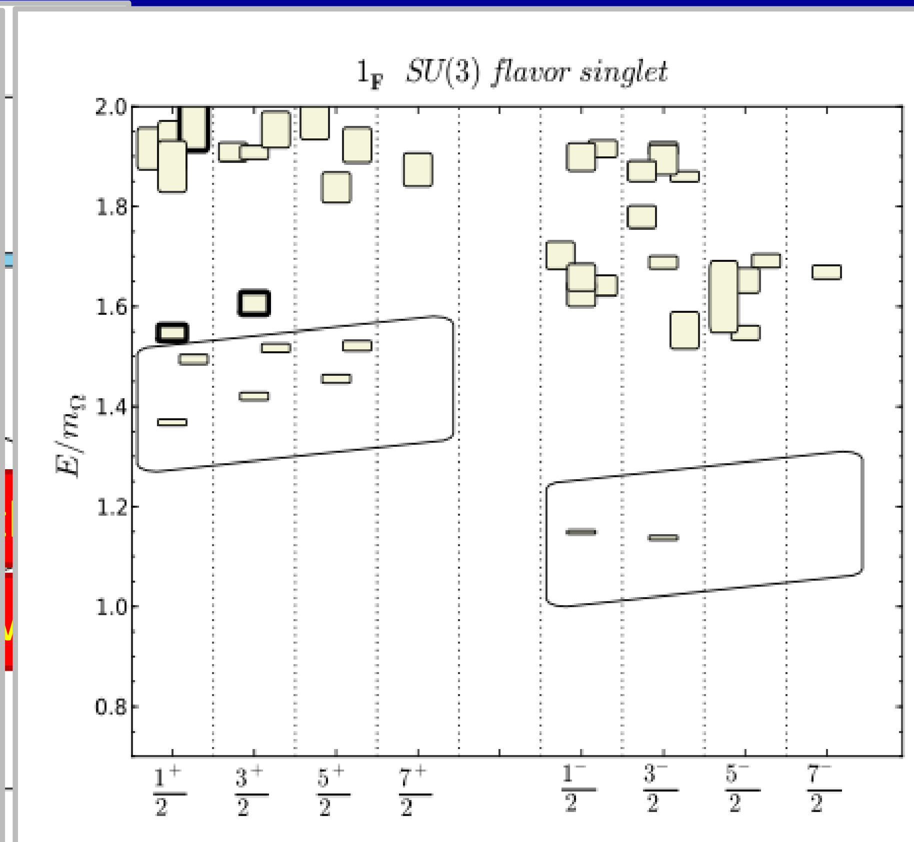
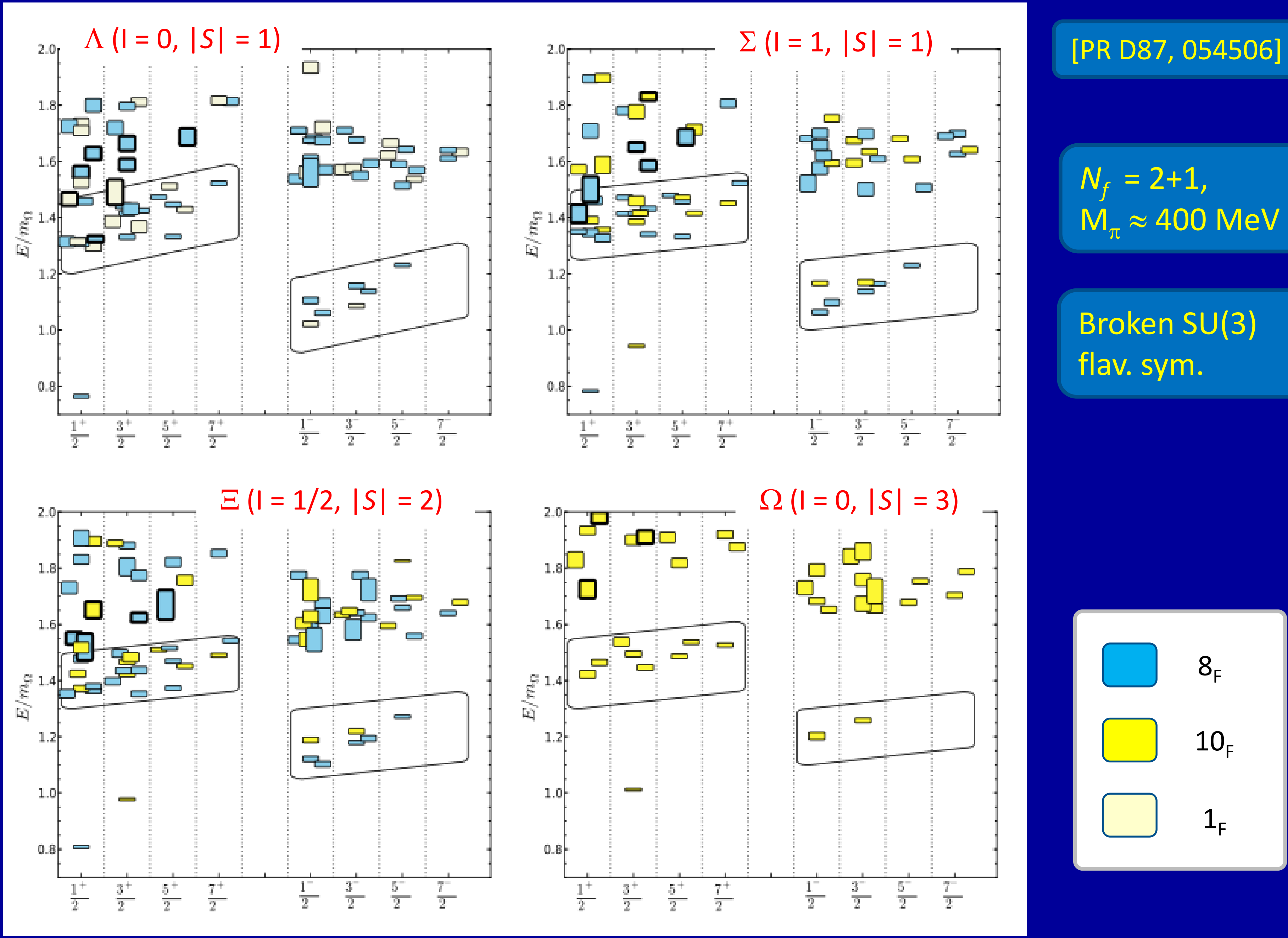




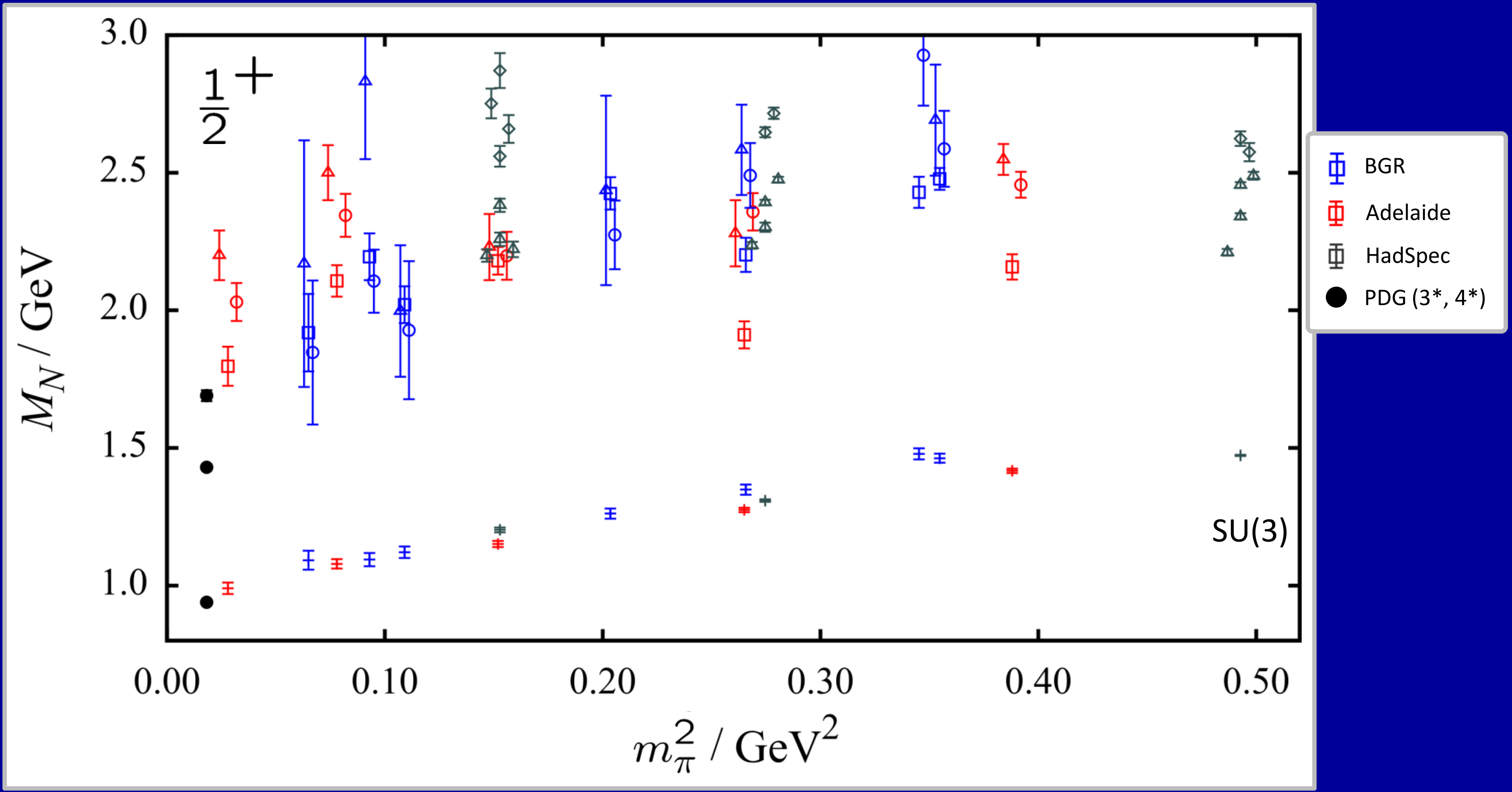


[PR D87, 054506]

N_f = 2+1,
$M_\pi \approx$ 400 MeV

Broken SU(3) flav. sym.

Excited nucleons: positive parity



BGR [PR D87, 074504]: chirally improved, $N_f = 2$ (also $3/2^{\pm}$ and low-lying strange baryons)
Adelaide [PR D87, 094506]: PACS-CS, $N_f = 2+1$ → talk by W. Kamleh [Thrs, 7G]
HadSpec [PR D87, 054506]: Aniso Clover, $N_f = 2+1$, lots of operators

Excited nucleons: positive parity

$$\chi_1(x) = \epsilon^{abc}(u^{Ta}(x)C\gamma_5 d^b(x))u^c(x),$$

$$\chi_2(x) = \epsilon^{abc}(u^{Ta}(x)Cd^b(x))\gamma_5 u^c(x),$$

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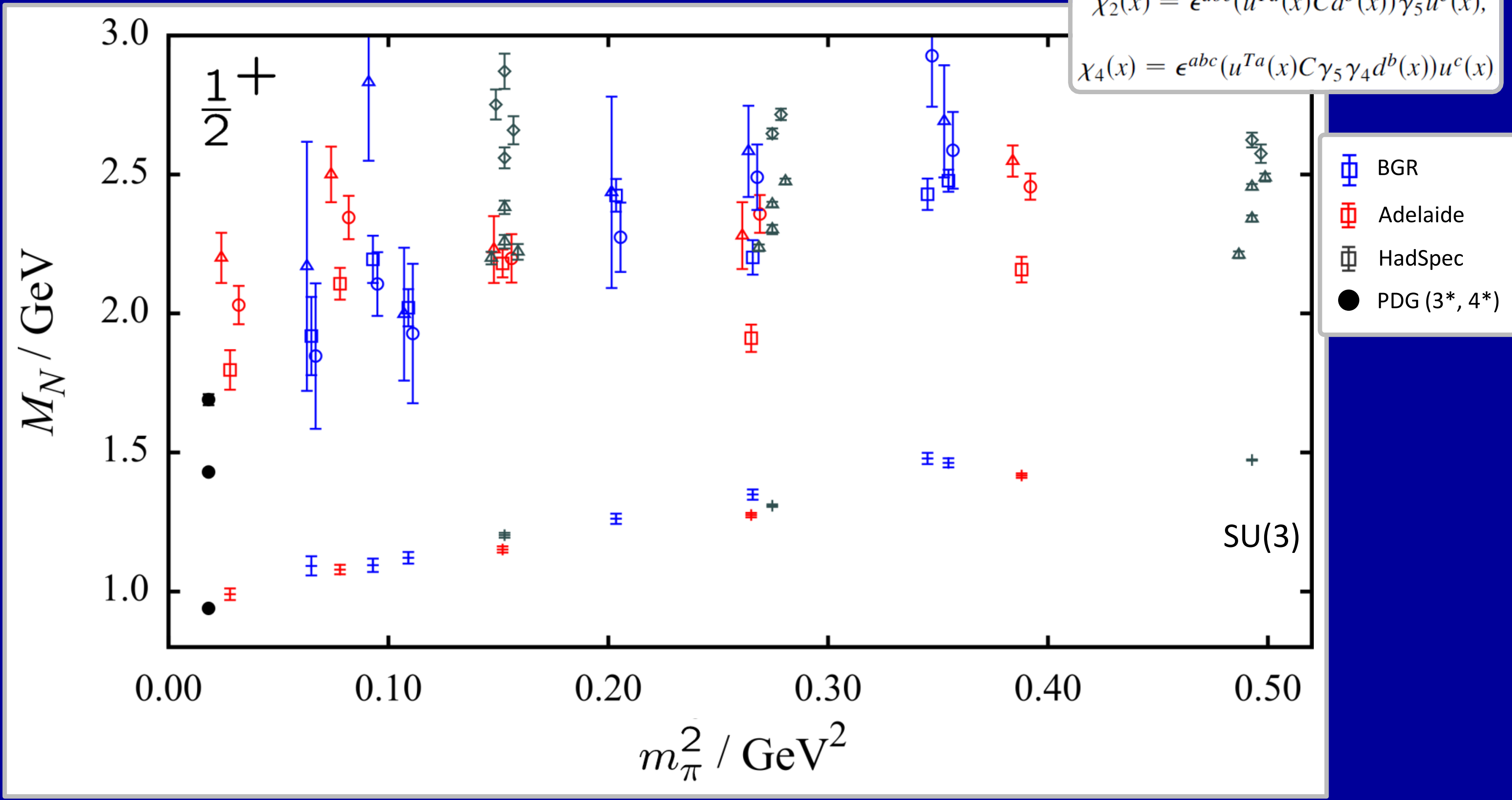

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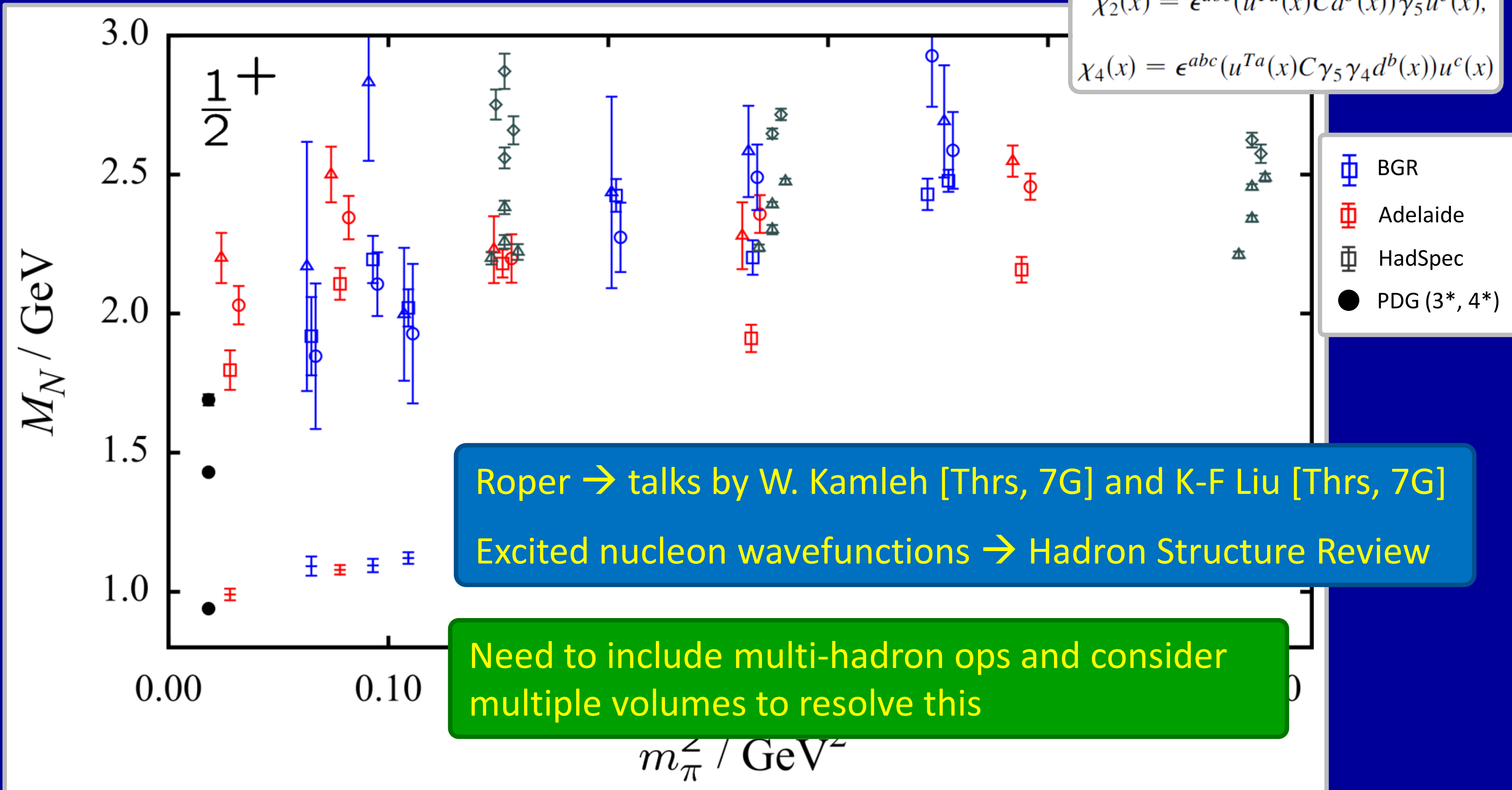

Roper → talks by W. Kamleh [Thrs, 7G] and K-F Liu [Thrs, 7G]

Excited nucleon wavefunctions → Hadron Structure Review

Need to include multi-hadron ops and consider multiple volumes to resolve this

BGR [PR D87, 074504]: chirally improved, $N_f = 2$ (also $3/2^\pm$ and low-lying strange baryons)
Adelaide [PR D87, 094506]: PACS-CS, $N_f = 2+1$ → talk by W. Kamleh [Thrs, 7G]
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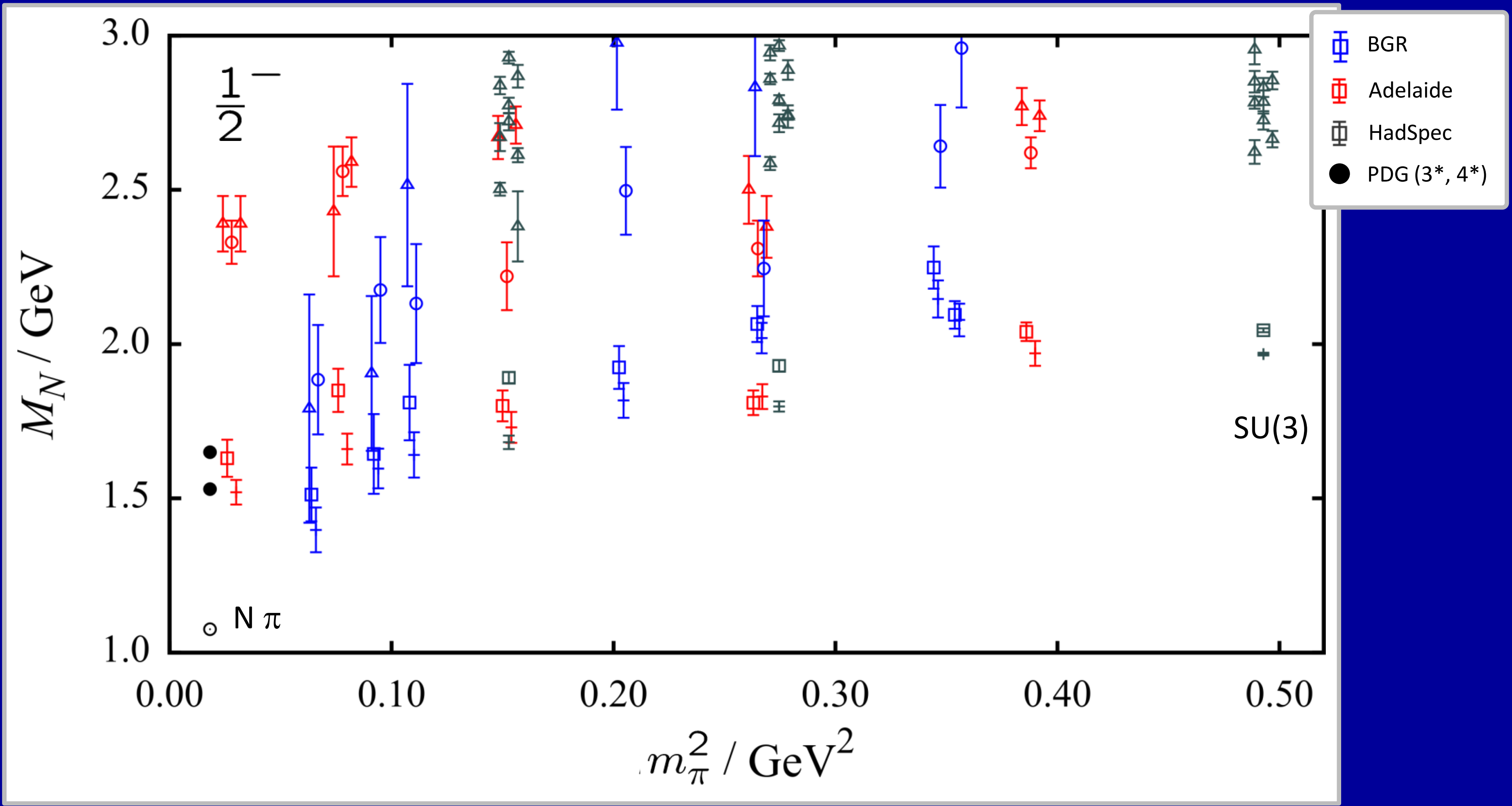
Excited nucleons: negative parity



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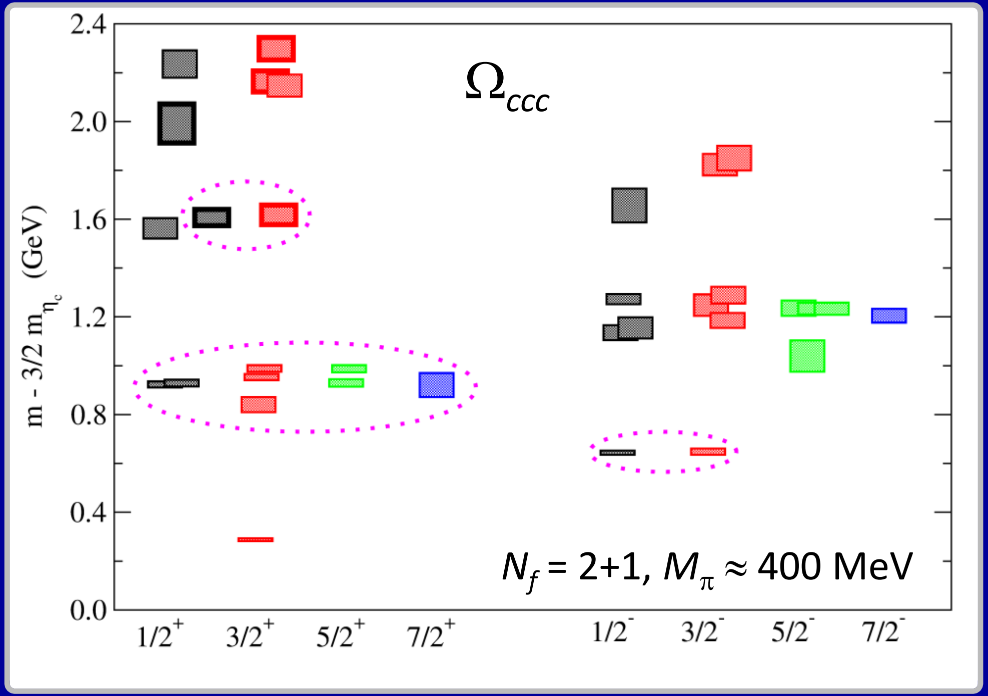
Charm baryons

Padmanath et al (Had Spec Collab) [arXiv:1307.7022]
Excited ccc baryons
→ talk by Padmanath [Thrs, 7G]
(also cc baryons)



Recent lower-lying charm baryons work:

- Briceño, Lin, Bolton [PR D86, 094504], N_f = 2+1+1
- Namekawa et al (PACS-CS) [PR D87, 094512], N_f = 2+1 (physical m_π)
- Talk by Z. Brown [Thrs, 7G]
- Talk by R. Horsley [Thrs, 7G]

Baryon resonances

See next talk...

Summary

- Significant process in studying **excited spectra**
  – **gluonic excitations**, degrees of freedom, **flavour** structure
- Lots of **experimental interest**
- Also beating down systematics → accurate low-lying masses
- **Resonances etc** (need appropriate multi-hadron ops)
  – Mesons: ρ studied in detail, still a lot of work to do for others
  – Baryons: a bit further behind
- Can **understand puzzles** in near future?
  (unusual charmonia, light scalars, Roper, ...)
- Challenges in scattering/resonances → next talk