Determination of Light and Strange Quark Condensates

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Introduction

- I will present results for the first lattice QCD calculation of the quark condensates at the strange and light quark masses.

The talk will largely be based on the paper below

- Direct determination of the strange and light quark condensates from full lattice QCD. arXiv:1211.6577 Phys.Rev. D87 (2013) 03450
- A. Bazavov, C. T. H. Davies, R. J. Dowdall, K. Hornbostel,, G. P. Lepage, C. McNeile H. D. Trottier

Introduction

$$S_f = \overline{\psi} M_f \psi$$

and

$$\langle \overline{\psi}\psi \rangle = \langle 0 | \overline{\psi}_f \psi_f | 0 \rangle = -\frac{1}{V} \langle \mathrm{Tr} M_f(U)^{-1} \rangle_U,$$

The well-known Gell-Mann, Oakes, Renner (GMOR) relation:

$$\frac{f_\pi^2 M_\pi^2}{4} = -\frac{m_u + m_d}{2} \frac{\langle 0 | \overline{u}u + \overline{d}d | 0 \rangle}{2}$$

connects the u/d quark masses times condensate to the square of the mass times decay constant.

- Goal extract $\langle \overline{\psi}\psi \rangle$ for $m_q > 0$.
- Expect that as $m \to \infty$ then $\langle \overline{\psi}\psi \rangle \to 0$.
- Is the strange quark heavy enough?

History of condensate calculations

A rough count of chiral (zero quark mass) condensate calculations from lattice QCD using a variety of methods.

n_f=0	9 (Starting 1981)
n_f=2	10 (Starting 2007)
n_f=2+1	12 (Starting 2005)
n_f=2+1+1	1 (Starting 2012)

- Only one previous attempt to compute mass dependence of the condensates from lattice QCD in Altmeyer et al., Nucl.Phys. B389 (1993) 445.

$\langle \bar{s}s \rangle$ and $\langle \bar{l}l \rangle$ applications

- Some SUM rule calculations use $\frac{\langle \bar{s}s \rangle}{\langle \bar{l}l \rangle}$ as input parameters. (see Jamin hep-ph/0201174 for discussion [1]).
- Extract $\langle \bar{s}s \rangle$ from kaons, baryon with valence strange quarks, or B_s meson.
- Sum rules have found $\frac{\langle \bar{s}s \rangle}{\langle \bar{l}l \rangle}$ between 0.6 and 1.2.

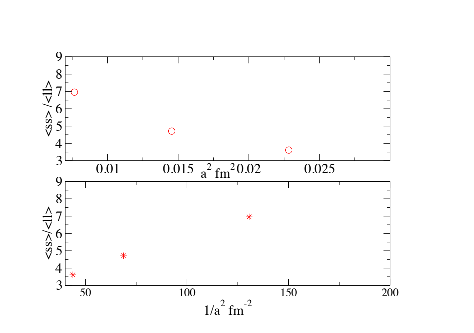
Some values used in lattice QCD calculations.

Group	application	$\frac{\langle \bar{s}s \rangle}{\langle \bar{l}l \rangle}$
JLQCD (1002.0371)	α_s	1
HPQCD (1011.1208)	strange-heavy moments	0.7
Borsanyi et al. (1005.3508)	hadron resonance gas	0.8

[1] The numbers have been updated in 0811.1590 Maltman

Back of the envelope calculation

- First attempt just use condensates at mass of strange quark and physical light quarks.
- 2+1+1 HISQ ensembles generated by MILC (details later).



Evidence of $1/a^2$ divergence.

The main problem

$$
\begin{aligned}
-\langle\overline{\psi}\psi\rangle &= \int_0^{\Lambda} \frac{d^4k}{(2\pi)^4}\frac{12m}{k^2+m^2} \\
&= \frac{3}{4\pi^2}\left(m\Lambda^2 + m^3\log\frac{m^2}{\Lambda^2+m^2}\right)
\end{aligned}
$$

- The $m\Lambda^2$ divergence depends on the regulator used.
- The $m^3\log(m/\Lambda)$ term is universal since it arises from infrared part of the integral.
- Additional potential divergence with Wilson like fermions. (Hamber and David, Nucl.Phys. B248 (1984) 381).

Lattice perturbation theory

Basic idea: subtract $\frac{1}{a^2}$ using perturbation theory

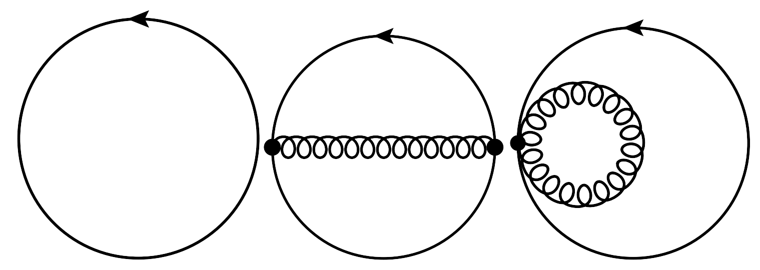
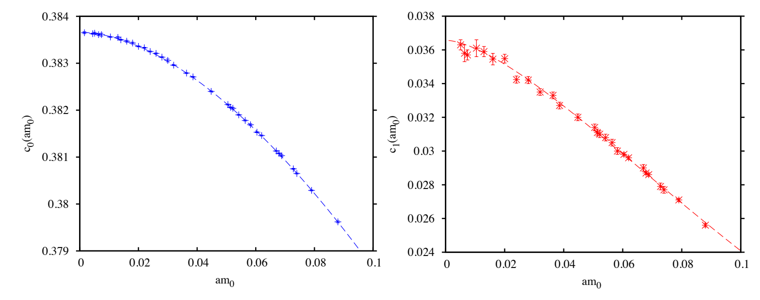


Figure: Perturbative contribution to the quark condensate through $\mathcal{O}(\alpha_s)$.

$$-a^3\langle\overline{\psi}\psi\rangle_{\mathrm{PT,HISQ}} = am_0\left[c_0(am_0) + c_1(am_0)\alpha_s + O(\alpha_s^2)\right]$$

Condensate in perturbation theory



$$-a^3\langle\overline{\psi}\psi\rangle_{\mathrm{PT,HISQ}} = am_0 \times \left[c_0(am_0) + c_1(am_0)\alpha_s + O(\alpha_s^2)\right]$$

$$c_0(am_0) = c_{00} + (am_0)^2\left[c_{01}\log(am_0) + c_{02}\right]$$

$$c_1(am_0) = c_{10} + (am_0)^2\left[c_{11}\log^2(am_0) \quad + c_{12}\log(am_0) + c_{13}\right]$$

Condensates and perturbation theory

- The condensate of strange or light quark extracted from the lattice needs to be compared against the condensates used in sum rule calculations.
- The extraction used a ratio of lattice to continuum perturbative factors (see appendix of arXiv:1211.6577 for some formalism).
- This also means that any issues with “renormalons” should cancel.

For convenience modern sum rule calculations don’t normal order the perturbation theory, as a consequence $m_q \langle \overline{\psi_q} \psi_q(\mu) \rangle$ depends slightly on the renormalisation scale.

Perturbative subtraction

$$-\langle\overline{\psi}\psi\rangle^{(\mu)}_{\mathrm{PT},\overline{\mathrm{MS}}} = \overline{m}^3(\mu)\times\Big[d_{01}l_m + d_{02} + \alpha_s\left(d_{11}l_m^2 + d_{12}l_m + d_{13}\right) + \ldots\Big]$$

where $l_m = \log(\overline{m}(\mu)/\mu)$.

$$-a^3\langle\overline{\psi}\psi\rangle_{\mathrm{PT,HISQ}} = am_0 \times \Big[c_0(am_0) + c_1(am_0)\alpha_s + O(\alpha_s^2)\Big]\,,$$

$$\begin{aligned}\Delta_{\mathrm{PT}} &= -a^4\left(\langle m_0\overline{\psi}\psi\rangle_{\mathrm{PT,HISQ}} - \langle\overline{m}(\mu)\overline{\psi}\psi\rangle_{\mathrm{PT},\overline{\mathrm{MS}}}\right)\\ &= c_{00}(am_0)^2 + \alpha_s c_{10}(am_0)^2 + (am_0)^4\left[c_{01}l_\mu - 0.077(1)\right]\\ &+ \alpha_s(am_0)^4\Big[c_{11}l_\mu^2 + 0.1340(2)l_\mu + 0.406(15)\Big] + \ldots,\end{aligned}$$

where $l_\mu = \log(\mu a)$.

Details of lattice calculation

β	a/fm	m_π MeV	$L/a \times T/a$
5.80	0.15	310, 220, 134	16×48, 24×48, 32×48
6.00	0.12	315, 220, 134	24×64, 32×64, 48×64
6.30	0.09	310, 126	32×96, 64×96

- 2nd generation configurations produced by MILC collaboration.
- HISQ improved staggered quark action.
- 2+1+1 sea quarks.
- Physical point lattices included.

Not discussed here: HISQ 2+1 flavors from HOT collaboration, Valence HISQ on ASQTAD configurations, and cross-check from chiral susceptibility $\chi_f = \frac{\partial}{\partial m_f}(-\langle\overline{\psi}\psi_f\rangle)$

Computing the condensate

We used the following relation to compute the raw condensate.

$$-a^3 \langle \overline{\psi} \psi \rangle_0 = (am_q) \sum_t C_\pi(t)$$

where $C_\pi(t)$ is the correlator for the Goldstone pion.

- Diagrammatic proof (Sharpe & Kilcup Nucl.Phys. B283 (1987) 493) of the relation for staggered fermions.
- It is also easy to compute the condensate using noise sources.

Ratio

In the analysis introduce

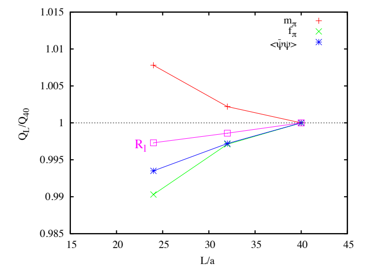
$$R_l = -\frac{4m_l\langle\overline{\psi}\psi_l\rangle}{(f_\pi^2 M_\pi^2)}$$

for light quarks and

$$R_s = -\frac{4m_s\langle\overline{\psi}\psi_s\rangle}{(f_{\eta_s}^2 M_{\eta_s}^2)}$$

for strange quarks, where η_s is the strange-strange fictitious pseudoscalar meson.
The R_l and R_s have reduced errors from mass tuning and finite size volumes.

Test finite volume effects



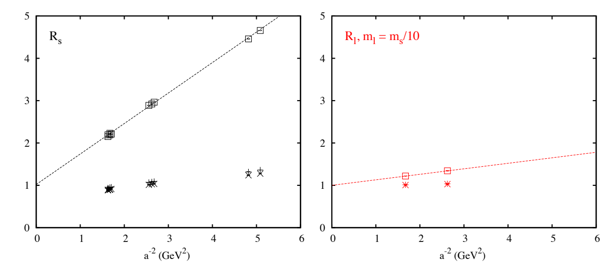
- At $\beta = 6.0$, $a = 0.12$ fm, m_π = 220 MeV
- The main analysis used $L = 32$.

Divergence

The physical $\overline{MS}$ condensate at the scale μ:

$$\langle m\overline{\psi}\psi\rangle_{NP,\overline{MS}}(\mu) = a^{-4}\left(a^4\langle m\overline{\psi}\psi\rangle_0 - \Delta_{PT}\right),$$

Graph shows condensate without subtraction (squares), tree level subtraction (plus) and one loop subtraction (cross).



Fit model

- One loop perturbation theory is not enough to remove all the divergence, so use Bayesian fitting techniques.

$$R_{q,0}(a, am_q) = R^{(q)}_{\mathrm{NP,phys}} + \delta R_{PT} + \delta R_{a^2} + \delta R_{\chi} + \delta R_{\mathrm{vol}}.$$

$R_{q,0}$ are the lattice results.

$R_{\mathrm{NP,phys}}$ physical result in the $\overline{MS}$ scheme at 2 GeV.

δR_{vol} finite volume effect $\delta R_{\mathrm{vol}} = ve^{-ML}$

R_{PT} Known tree and one loop results. Also $R_{PT,div} = a_n \frac{4\alpha_s^n (am_q)^2}{(af_\pi)^2 (aM_\pi)^2}$ and $\delta R_{PT,non-div} = c_n \frac{4\alpha_s^n (am_q)^4}{(af_\pi)^2 (aM_\pi)^2}$.

δR_{a^2} $\delta R_{a^2} = \sum_{i=1}^{2} d_i \left(\frac{\Lambda a}{\pi}\right)^{2i}$ with $\Lambda \approx 1$ GeV.

δR_{χ} includes valence and sea quark mass dependence.

Fit results

The physical results for R_q are

$$\begin{aligned} R_{l,phys} &= -\frac{4m_l\langle\overline{\psi}\psi_l\rangle_{\overline{MS}}(2\mathrm{GeV})}{(f_\pi^2 M_\pi^2)} \\ R_{s,phys} &= -\frac{4m_s\langle\overline{\psi}\psi_s\rangle_{\overline{MS}}(2\mathrm{GeV})}{(f_{\eta_s}^2 M_{\eta_s}^2)} \end{aligned}$$

The final fits had $\chi^2/dof \approx 0.8$ for 18 dof.

$$\begin{aligned} R_{s,phys} &= 0.574(86) \\ R_{l,phys} &= 0.985(5) \\ \frac{R_{s,phys}}{R_{l,phys}} &= 0.583(84). \end{aligned}$$

Summary of results

We take $m_s^{\overline{MS}}(2\,\mathrm{GeV})$ = 92.2(1.3) MeV (HPQCD, 1004.4285) and $m_s/m_l = 27.41(23)$ (MILC 0903.3598,0910.3618) These give:

$$\begin{aligned} \langle \bar{s}s \rangle^{\overline{MS}}(2\,\mathrm{GeV}) &= -0.0245(37)(3)\,\mathrm{GeV}^3 \\ &= -(290(15)\,\mathrm{MeV})^3 \\ \langle \bar{l}l \rangle^{\overline{MS}}(2\,\mathrm{GeV}) &= -0.0227(1)(4)\,\mathrm{GeV}^3 \\ &= -(283(2)\,\mathrm{MeV})^3, \end{aligned}$$

where the second error for each condensate in GeV^3 comes from the error in the quark masses.
The ratio of strange to light condensates

$$\frac{\langle \bar{s}s \rangle^{\overline{MS}}(2\,\mathrm{GeV})}{\langle \bar{l}l \rangle^{\overline{MS}}(2\,\mathrm{GeV})} = 1.08(16)(1),$$

Correctons to GMOR

Following Jamin hep-ph/0201174 define

$$R_l = -\frac{4m_l\langle\overline{\psi}\psi_l\rangle}{(f_\pi^2 M_\pi^2)} = 1 - \delta_\pi$$

From chiral perturbation theory where H_2^r and L_8^r are Gasser-Leutwyler coefficients.

$$\delta_\pi = 4\frac{M_\pi^2}{f_\pi^2}(2L_8^r - H_2^r)$$

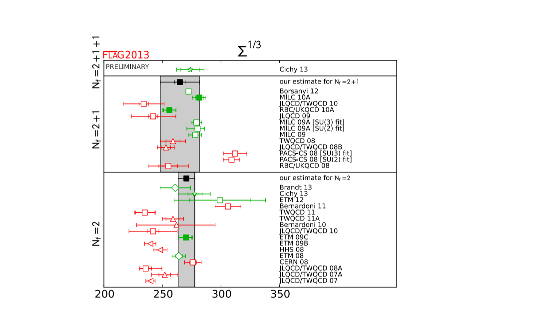
Group	method	δ_π
This work	lattice	0.015(5)
Jamin	sum rule	0.047(17)
Bordes et al.	sum rule	0.06(1)

Summary of chiral condensate

Compare $\langle \bar{l}l \rangle^{\overline{MS}}(2\,\mathrm{GeV}) = -(283(2)\,\mathrm{MeV})^3$

- $\Sigma_{n_f=2}$ = 270(7) MeV, 2013 FLAG review
- $\Sigma_{n_f=2+1}$ = 265(17) MeV, 2013 FLAG review

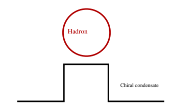
Summary of $\Sigma^{1/3}$ (2 GeV) from 2013 FLAG review.



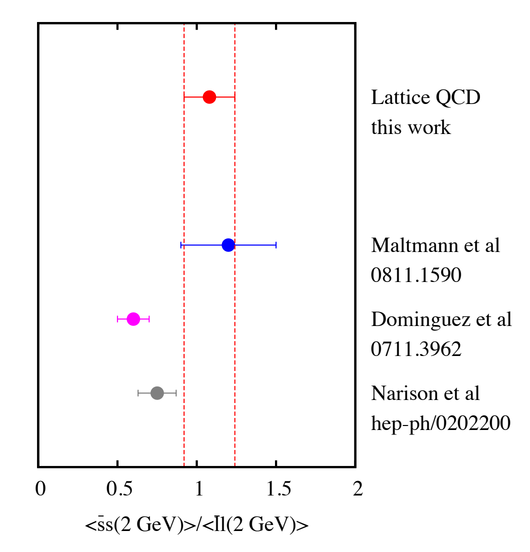
In medium condensate

- Brodsky and Shrock (0905.1151) have suggested that there is no vacuum chiral condensate, but the condensate is associated with a hadron.
- Some support from Schwinger-Dyson calculations and light front formalism (1202.2376).
- QCD condensates contribute $\sim 10^{45}$ observed vacuum energy.

$$\frac{f_\pi^2 M_\pi^2}{4} = -\frac{m_u + m_d}{2} \frac{(\langle 0|\overline{u}u + \overline{d}d|0\rangle)_\pi}{2}$$



Comparison of results to sum rules



Comment on sum rule determination

- The decay constant of the B_s meson depends on $\langle \bar{s}s \rangle$
- Other quantities can be used.

	f_{B_s}/f_B		$\langle s\bar{s}\rangle_{\overline{MS}}/\langle l\bar{l}\rangle_{\overline{MS}}$
Jamin, hep-ph/0201174	1.16(4)	$\Longrightarrow$	0.8 ± 0.3
Maltman, 0811.1590	1.21(4)	$\Longrightarrow$	1.2 ± 0.3

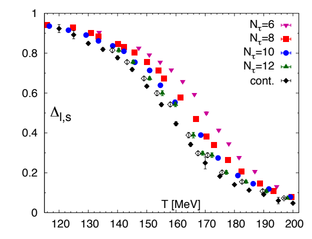
- The increase was from quenched QCD to n_f = 2+1 (chiral logs).
- The current average for f_{B_s}/f_B is 1.202(22) from lattice QCD (FLAG 2013).

Condensate at non-zero temperature

The HOTQCD collaboration (Bazavov et al. 1111.1710) uses the order parameter below at non-zero temperature.

$$\Delta_{l,s}(T) = \frac{\langle\bar{\psi}\psi\rangle_{l,\tau} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,\tau}}{\langle\bar{\psi}\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,0}}.$$

Perturbation theory shows higher order corrections small. Figure from arXiv:1301.3943, Bazavov and Petreczky

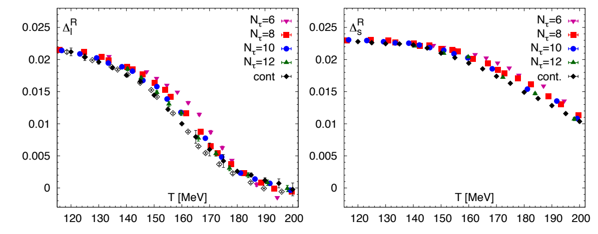


Condensate at non-zero temperature

Alternative order parameter at non-zero temperature

$$\Delta_q^R = d + 2m_s r_1^4(\langle\bar{\psi}\psi\rangle_{q,\tau} - \langle\bar{\psi}\psi\rangle_{q,0}), \quad q = l, s.$$

Figure from arXiv:1301.3943, Bazavov and Petreczky



Conclusions

- I have presented the first calculation of the mass dependence of the quark condensates (at T=0).

$$\frac{\langle \bar{s}s \rangle^{\overline{MS}}(2\,\mathrm{GeV})}{\langle \bar{l}l \rangle^{\overline{MS}}(2\,\mathrm{GeV})} = 1.08(16)(1)$$

Future directions

- It maybe possible to compute the next order in perturbation theory.
- Some people may prefer to develop non-perturbative formalism, but this is not easy because needs to be mass dependent.
- Perhaps a formalism could be developed using quark flow (1302.5246, Lüscher). The final answer needs to be converted to the $\overline{MS}$ scheme for use in continuum sum rules.