Determination of Light and Strange Quark Condensates

Craig McNeile
mcneile@uni-wuppertal.de

University of Wuppertal
Introduction

- I will present results for the first lattice QCD calculation of the quark condensates at the strange and light quark masses.

The talk will largely be based on the paper below

- Direct determination of the strange and light quark condensates from full lattice QCD. arXiv:1211.6577

S_f = \overline{\psi} M_f \psi

and

\langle \overline{\psi} \psi \rangle = \langle 0 | \overline{\psi}_f \psi_f | 0 \rangle = -\frac{1}{V} \langle \text{Tr} M_f (U)^{-1} \rangle_U,

The well-known Gell-Mann, Oakes, Renner (GMOR) relation:

\frac{f^2 M^2}{4} = \frac{m_u + m_d}{2} \langle 0 | \overline{u} u + \overline{d} d | 0 \rangle

connects the u/d quark masses times condensate to the square of the mass times decay constant.

- Goal extract \langle \overline{\psi} \psi \rangle for \( m_q > 0 \).
- Expect that as \( m \to \infty \) then \( \langle \overline{\psi} \psi \rangle \to 0 \).
- Is the strange quark heavy enough?
A rough count of chiral (zero quark mass) condensate calculations from lattice QCD using a variety of methods.

- $n_f=0$ 9 (Starting 1981)
- $n_f=2$ 10 (Starting 2007)
- $n_f=2+1$ 12 (Starting 2005)
- $n_f=2+1+1$ 1 (Starting 2012)

\[ \langle \bar{s}s \rangle \text{ and } \langle \bar{l}l \rangle \text{ applications} \]

- Some SUM rule calculations use \( \frac{\langle \bar{s}s \rangle}{\langle l\bar{l} \rangle} \) as input parameters. (see Jamin hep-ph/0201174 for discussion \(^1\)).
- Extract \( \langle \bar{s}s \rangle \) from kaons, baryon with valence strange quarks, or \( B_s \) meson.
- Sum rules have found \( \frac{\langle \bar{s}s \rangle}{\langle l\bar{l} \rangle} \) between 0.6 and 1.2.

Some values used in lattice QCD calculations.

<table>
<thead>
<tr>
<th>Group</th>
<th>application</th>
<th>( \frac{\langle \bar{s}s \rangle}{\langle l\bar{l} \rangle} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>JLQCD (1002.0371)</td>
<td>( \alpha_s )</td>
<td>1</td>
</tr>
<tr>
<td>HPQCD (1011.1208)</td>
<td>strange-heavy moments</td>
<td>0.7</td>
</tr>
<tr>
<td>Borsanyi et al. (1005.3508)</td>
<td>hadron resonance gas</td>
<td>0.8</td>
</tr>
</tbody>
</table>

\(^1\)The numbers have been updated in 0811.1590 Maltman
Back of the envelope calculation

- First attempt just use condensates at mass of strange quark and physical light quarks.
- 2+1+1 HISQ ensembles generated by MILC (details later).

Evidence of $1/a^2$ divergence.
The main problem

\[ -\langle \bar{\psi} \psi \rangle = \int_0^\Lambda \frac{d^4 k}{(2\pi)^4} \frac{12m}{k^2 + m^2} \]

\[ = \frac{3}{4 \pi^2} \left( m \Lambda^2 + m^3 \log \frac{m^2}{\Lambda^2 + m^2} \right) \]

- The \( m \Lambda^2 \) divergence depends on the regulator used.
- The \( m^3 \log(m/\Lambda) \) term is universal since it arises from infrared part of the integral.
Lattice perturbation theory

**Basic idea:** subtract $\frac{1}{a^2}$ using perturbation theory

**Figure:** Perturbative contribution to the quark condensate through $O(\alpha_s)$.

\[
-a^3 \langle \bar{\psi} \psi \rangle_{\text{PT,HISQ}} = a m_0 \left[ c_0 (a m_0) + c_1 (a m_0) \alpha_s + O(\alpha_s^2) \right]
\]
\[ \begin{align*} 
- a^3 \langle \bar{\psi} \psi \rangle_{\text{PT,HISQ}} &= am_0 \times \left[ c_0(am_0) + c_1(am_0) \alpha_s + O(\alpha_s^2) \right] \\
&= c_{00} + (am_0)^2 \left[ c_{01} \log(am_0) + c_{02} \right] \\
c_0(am_0) &= c_{10} + (am_0)^2 \left[ c_{11} \log^2(am_0) + c_{12} \log(am_0) + c_{13} \right] 
\end{align*} \]
• The condensate of strange or light quark extracted from the lattice needs to be compared against the condensates used in sum rule calculations.
• The extraction used a ratio of lattice to continuum perturbative factors (see appendix of arXiv:1211.6577 for some formalism).
• This also means that any issues with “renormalons” should cancel.

For convenience modern sum rule calculations don’t normal order the perturbation theory, as a consequence $m_q \langle \bar{\psi}_q \psi_q(\mu) \rangle$ depends slightly on the renormalisation scale.
Perturbative subtraction

\[-\langle \bar{\psi} \psi \rangle^{(\mu)}_{\text{PT,MS}} = \overline{m}^3(\mu) \times \left[ d_{01} l_m + d_{02} + \alpha_s \left( d_{11} l_m^2 + d_{12} l_m + d_{13} \right) + \ldots \right] \]

where \( l_m = \log(\overline{m}(\mu)/\mu) \).

\[-a^3 \langle \bar{\psi} \psi \rangle_{\text{PT,HISQ}} = a m_0 \times \left[ c_0 (a m_0) + c_1 (a m_0) \alpha_s + O(\alpha_s^2) \right], \]

\[\Delta_{\text{PT}} = -a^4 \left( \langle m_0 \bar{\psi} \psi \rangle_{\text{PT,HISQ}} - \langle \overline{m}(\mu) \bar{\psi} \psi \rangle_{\text{PT,MS}} \right) \]

\[= c_{00} (a m_0)^2 + \alpha_s c_{10} (a m_0)^2 + (a m_0)^4 \left[ c_{01} l_\mu - 0.077(1) \right] \]

\[+ \alpha_s (a m_0)^4 \left[ c_{11} l_\mu^2 + 0.1340(2) l_\mu + 0.406(15) \right] + \ldots, \]

where \( l_\mu = \log(\mu a) \).
Details of lattice calculation

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$a$/fm</th>
<th>$m_\pi$ MeV</th>
<th>$L/a \times T/a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.80</td>
<td>0.15</td>
<td>310, 220, 134</td>
<td>16$\times$48, 24$\times$48, 32$\times$48</td>
</tr>
<tr>
<td>6.00</td>
<td>0.12</td>
<td>315, 220, 134</td>
<td>24$\times$64, 32$\times$64, 48$\times$64</td>
</tr>
<tr>
<td>6.30</td>
<td>0.09</td>
<td>310, 126</td>
<td>32$\times$96, 64$\times$96</td>
</tr>
</tbody>
</table>

- 2nd generation configurations produced by MILC collaboration.
- HISQ improved staggered quark action.
- 2+1+1 sea quarks.
- Physical point lattices included.

Not discussed here: HISQ 2+1 flavors from HOT collaboration, Valence HISQ on ASQTAD configurations, and cross-check from chiral susceptibility $\chi_f = \frac{\partial}{\partial m_f}( - \langle \bar{\psi} \psi_f \rangle )$
Computing the condensate

We used the following relation to compute the raw condensate.

\[-a^3 \langle \bar{\psi} \psi \rangle_0 = (am_q) \sum_t C_{\pi}(t)\]

where $C_{\pi}(t)$ is the correlator for the Goldstone pion.

- It is also easy to compute the condensate using noise sources.
In the analysis introduce

\[ R_l = -\frac{4m_l\langle \bar{\psi}\psi_l \rangle}{(f_\pi^2 M_\pi^2)} \]

for light quarks and

\[ R_s = -\frac{4m_s\langle \bar{\psi}\psi_s \rangle}{(f_{\eta_s}^2 M_{\eta_s}^2)} \]

for strange quarks, where \( \eta_s \) is the strange-strange fictitious pseudoscalar meson. The \( R_l \) and \( R_s \) have reduced errors from mass tuning and finite size volumes.
Test finite volume effects

- At $\beta = 6.0$, $a = 0.12$ fm, $m_\pi = 220$ MeV
- The main analysis used $L = 32$. 
Divergence

The physical $\overline{MS}$ condensate at the scale $\mu$:

$$\langle m_{\bar{\psi}\psi} \rangle_{NP, \overline{MS}}(\mu) = a^{-4} \left( a^4 \langle m_{\bar{\psi}\psi} \rangle_0 - \Delta_{PT} \right),$$

Graph shows condensate without subtraction (squares), tree level subtraction (plus) and one loop subtraction (cross).
Fit model

- One loop perturbation theory is not enough to remove all the divergence, so use Bayesian fitting techniques.

\[ R_{q,0}(a, a m_q) = R_{NP,\text{phys}}^{(q)} + \delta R_{PT} + \delta R_{a^2} + \delta R_{\chi} + \delta R_{\text{vol}}. \]

- \( R_{q,0} \) are the lattice results.
- \( R_{NP,\text{phys}} \) physical result in the \( \overline{MS} \) scheme at 2 GeV.
- \( \delta R_{\text{vol}} \) finite volume effect \( \delta R_{\text{vol}} = v e^{-ML} \)
- \( R_{PT} \) Known tree and one loop results. Also
  \[ R_{PT,\text{div}} = a_n \frac{4 \alpha_s^2 (a m_q)^2}{(af_\pi)^2 (a M_\pi)^2} \]
  \[ \delta R_{PT,\text{non-div}} = c_n \frac{4 \alpha_s^2 (a m_q)^4}{(af_\pi)^2 (a M_\pi)^2}. \]
- \( \delta R_{a^2} \) includes valence and sea quark mass dependence.
- \( \delta R_{\chi} \) with \( \Lambda \approx 1 \) GeV.
The physical results for $R_q$ are

\[
R_{l,\text{phys}} = -\frac{4m_l\langle \bar{\psi}\psi \rangle_{\overline{\text{MS}}}(2\text{GeV})}{(f_\pi^2 M_\pi^2)}
\]

\[
R_{s,\text{phys}} = -\frac{4m_s\langle \bar{\psi}\psi \rangle_{\overline{\text{MS}}}(2\text{GeV})}{(f_{\eta_s}^2 M_{\eta_s}^2)}
\]

The final fits had $\chi^2/dof \approx 0.8$ for 18 dof.

\[
R_{s,\text{phys}} = 0.574(86)
\]

\[
R_{l,\text{phys}} = 0.985(5)
\]

\[
\frac{R_{s,\text{phys}}}{R_{l,\text{phys}}} = 0.583(84).
\]
Summary of results

We take $m_s^{\overline{MS}}(2 \text{ GeV}) = 92.2(1.3) \text{ MeV}$ (HPQCD, 1004.4285) and $m_s/m_l = 27.41(23)$ (MILC 0903.3598,0910.3618) These give:

\[
\langle \bar{s}s \rangle^{\overline{\text{MS}}}(2 \text{ GeV}) = -0.0245(37)(3) \text{ GeV}^3 \\
= -(290(15) \text{ MeV})^3
\]
\[
\langle \bar{l}l \rangle^{\overline{\text{MS}}}(2 \text{ GeV}) = -0.0227(1)(4) \text{ GeV}^3 \\
= -(283(2) \text{ MeV})^3,
\]

where the second error for each condensate in $\text{GeV}^3$ comes from the error in the quark masses.

The ratio of strange to light condensates

\[
\frac{\langle \bar{s}s \rangle^{\overline{\text{MS}}}(2 \text{ GeV})}{\langle \bar{l}l \rangle^{\overline{\text{MS}}}(2 \text{ GeV})} = 1.08(16)(1),
\]
Following Jamin hep-ph/0201174 define

\[ R_l = -\frac{4m_l \langle \bar{\psi} \psi \rangle}{f_\pi^2 M_\pi^2} = 1 - \delta_\pi \]

From chiral perturbation theory where \( H_2^r \) and \( L_8^r \) are Gasser-Leutwyler coefficients.

\[ \delta_\pi = 4 \frac{M_\pi^2}{f_\pi^2} (2L_8^r - H_2^r) \]

<table>
<thead>
<tr>
<th>Group</th>
<th>method</th>
<th>( \delta_\pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work</td>
<td>lattice</td>
<td>0.015(5)</td>
</tr>
<tr>
<td>Jamin</td>
<td>sum rule</td>
<td>0.047(17)</td>
</tr>
<tr>
<td>Bordes et al.</td>
<td>sum rule</td>
<td>0.06(1)</td>
</tr>
</tbody>
</table>
Summary of chiral condensate

Compare $\langle \bar{l}l \rangle^{\text{MS}}_{\text{MS}}(2 \text{ GeV}) = -(283(2) \text{ MeV})^3$

- $\sum_{n_f=2} = 270(7) \text{ MeV}$, 2013 FLAG review
- $\sum_{n_f=2+1} = 265(17) \text{ MeV}$, 2013 FLAG review

Summary of $\Sigma^{1/3}(2 \text{ GeV})$ from 2013 FLAG review.

### Table 1

<table>
<thead>
<tr>
<th>$N_f$</th>
<th>Calculation</th>
<th>Value (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Brandt 13</td>
<td>270</td>
</tr>
<tr>
<td></td>
<td>Cichy 13</td>
<td>265</td>
</tr>
<tr>
<td>2+1</td>
<td>ETM 12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bernardini 11</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TWOQCD 11</td>
<td></td>
</tr>
<tr>
<td></td>
<td>TWOQCD 11A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MLC 09 [SU(3)] fit</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MILC 09</td>
<td></td>
</tr>
<tr>
<td></td>
<td>RBC/UKQCD 10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>HQET-QCD/MSB 09</td>
<td></td>
</tr>
<tr>
<td></td>
<td>HQET-QCD/MSB 08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LOQCD two-loop fit</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>ETM 09B</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ETM 08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ETN 08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ENS 08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LOQCD two-loop fit</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LOQCD/MSB 08A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>HQET-QCD/MSB 09</td>
<td></td>
</tr>
<tr>
<td></td>
<td>HQET-QCD/MSB 07</td>
<td></td>
</tr>
</tbody>
</table>

### Diagram

![Diagram showing lattice results for the quark condensate](image)
In medium condensate

- Brodsky and Shrock (0905.1151) have suggested that there is no vacuum chiral condensate, but the condensate is associated with a hadron.
- Some support from Schwinger-Dyson calculations and light front formalism (1202.2376).
- QCD condensates contribute $\sim 10^{45}$ observed vacuum energy.

$$\frac{f_\pi^2 M_\pi^2}{4} = -\frac{m_u + m_d}{2} \left( \langle 0 | \bar{u}u + \bar{d}d | 0 \rangle \right)_\pi$$
Comparison of results to sum rules

\[ \frac{\langle s\bar{s}(2 \text{ GeV}) \rangle}{\langle l\bar{l}(2 \text{ GeV}) \rangle} \]

Lattice QCD
this work

Maltmann et al
0811.1590

Dominguez et al
0711.3962

Narison et al
hep-ph/0202200
• The decay constant of the $B_s$ meson depends on $\langle \bar{s}s \rangle$

• Other quantities can be used.

<table>
<thead>
<tr>
<th></th>
<th>$f_{B_s}/f_B$</th>
<th>$\langle \bar{s}s \rangle_{MS}/\langle \bar{b}b \rangle_{MS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jamin, hep-ph/0201174</td>
<td>1.16(4)</td>
<td>$0.8 \pm 0.3$</td>
</tr>
<tr>
<td>Maltman, 0811.1590</td>
<td>1.21(4)</td>
<td>$1.2 \pm 0.3$</td>
</tr>
</tbody>
</table>

• The increase was from quenched QCD to $n_f = 2+1$ (chiral logs).

• The current average for $f_{B_s}/f_B$ is 1.202(22) from lattice QCD (FLAG 2013).
The HOTQCD collaboration (Bazavov et al. 1111.1710) uses the order parameter below at non-zero temperature.

\[ \Delta_{l,s}(T) = \frac{\langle \bar{\psi}\psi \rangle_{I,\tau} - \frac{m_l}{m_s} \langle \bar{\psi}\psi \rangle_{s,\tau}}{\langle \bar{\psi}\psi \rangle_{I,0} - \frac{m_l}{m_s} \langle \bar{\psi}\psi \rangle_{s,0}}. \]

Perturbation theory shows higher order corrections small. Figure from arXiv:1301.3943, Bazavov and Petreczky
Condensate at non-zero temperature

Alternative order parameter at non-zero temperature

\[ \Delta^R_q = d + 2m_s r_1^4 (\langle \bar{\psi} \psi \rangle_{q,\tau} - \langle \bar{\psi} \psi \rangle_{q,0}), \quad q = l, s. \]

Figure from arXiv:1301.3943, Bazavov and Petreczky
Conclusions

• I have presented the first calculation of the mass dependence of the quark condensates (at T=0).

\[
\frac{\langle \bar{s}s\rangle^{\overline{MS}}(2 \text{ GeV})}{\langle \bar{l}l\rangle^{\overline{MS}}(2 \text{ GeV})} = 1.08(16)(1)
\]

Future directions

• It maybe possible to compute the next order in perturbation theory.
• Some people may prefer to develop non-perturbative formalism, but this is not easy because needs to be mass dependent.
• Perhaps a formalism could be developed using quark flow (1302.5246, Lüscher). The final answer needs to be converted to the \(\overline{MS}\) scheme for use in continuum sum rules.