Review of Hadron Structure Calculations on a Lattice

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Thanks for your material!

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Outline

- Lattice QCD Gold-plated Observables
  nucleon axial charge, e/m radii, magnetic moment, quark momentum fraction and their systematic uncertainties
- Hadron Wave Functions
  Nucleon and resonance wave functions and distribution amplitudes
- Hadron Form Factors
  Vector & axial nucleon form factors,
  Delta axial form factors, Lambda electric form factor
  timelike vector and scalar pion form factors
- Decomposition of the Proton Spin
  contributions from light & strange quarks and glue
- Parton Distributions on a Lattice
  PDFs and TMDs

Lattice QCD Gold-Plated Observables

Isovector (u-d)

- axial charge
- Dirac & Pauli (or electric & magnetic) radii
- magnetic moment
- quark momentum fraction

✦ Best stochastic precision (forward or near-forward kinematics

✦ No disconnected diagrams

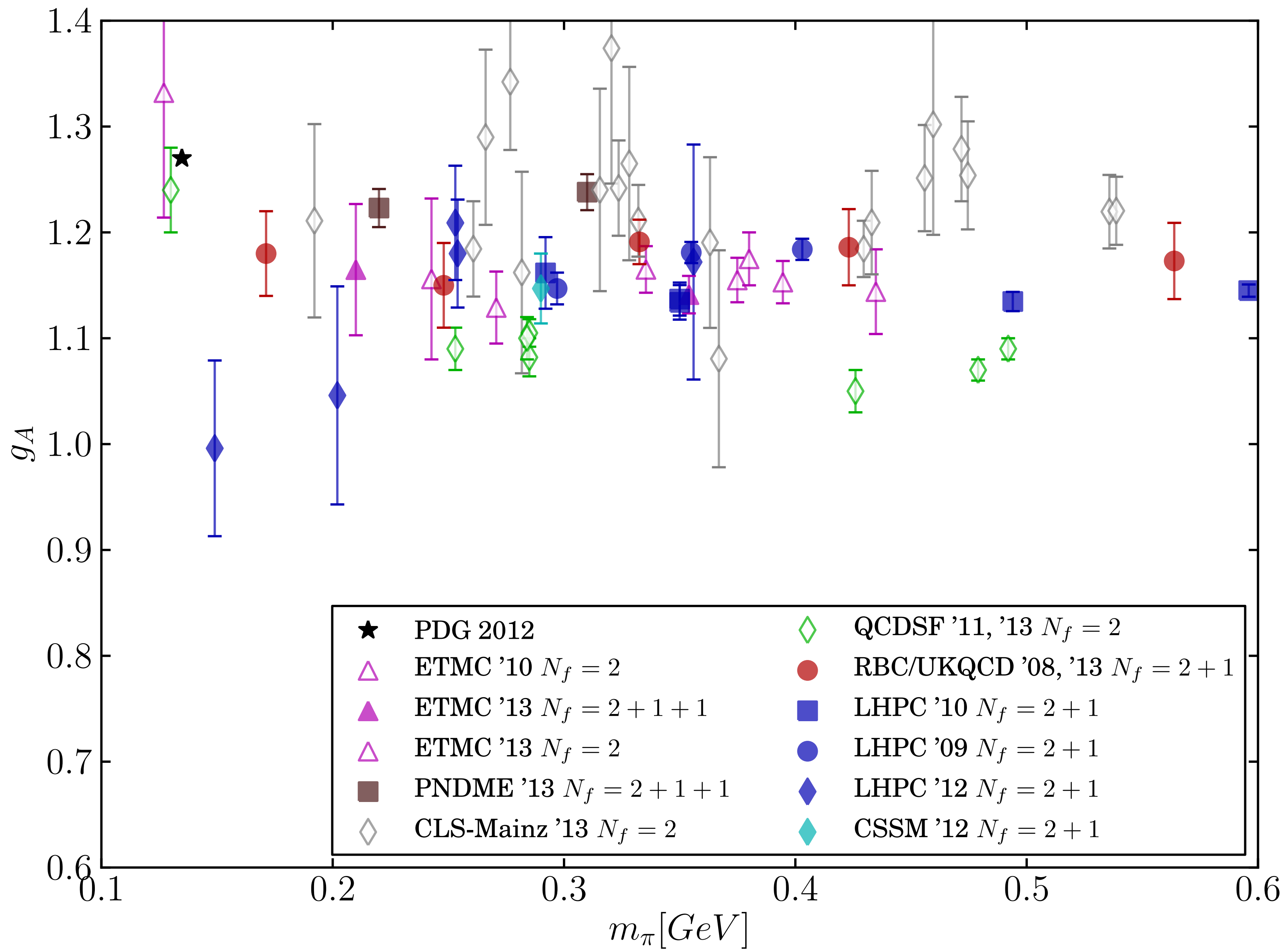
✦ (typically) simple renormalization

✦ Well-known experimentally

Drama of the Axial Charge

$$\langle N(p)|\bar{q}\gamma^\mu\gamma^5 q|N(p)\rangle = g_A\ \bar{u}_p\gamma^\mu\gamma^5 u_p\,,$$

Experiment (W.A.) [PDG'12] $g_A^{\rm ave} = 1.2701(25)$

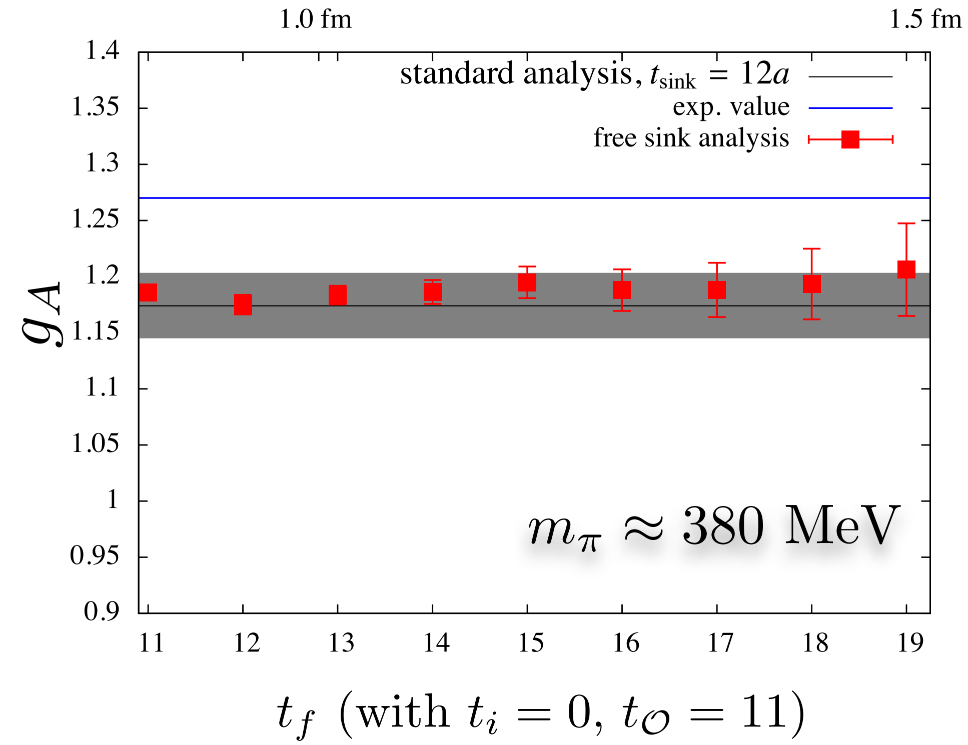


Many lattice calculations underestimated g_A by 10-15%

Nucleon Axial Charge: Excited State Effects?

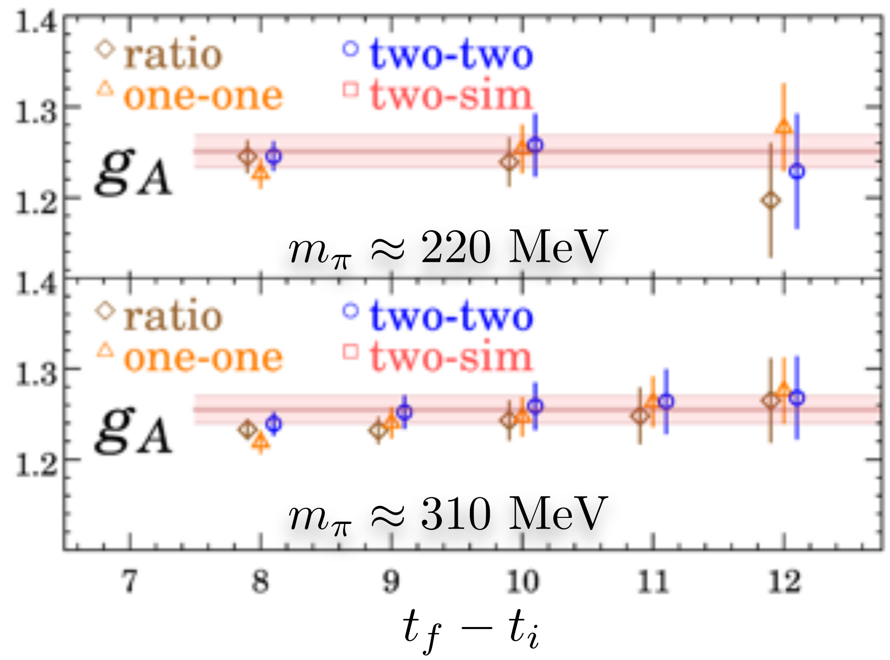
High-statistics study

[S.Dinter *et al* (ETMC) arXiv:1112.2931]



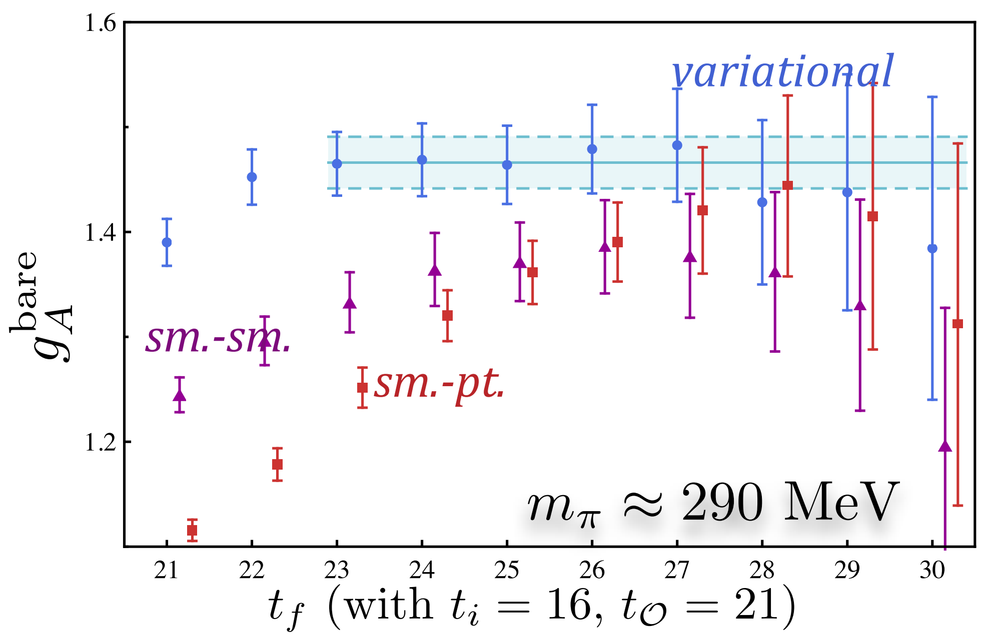
2-state fits

[H.W.Lin *et al* (PNDME) arXiv:1306.5435]



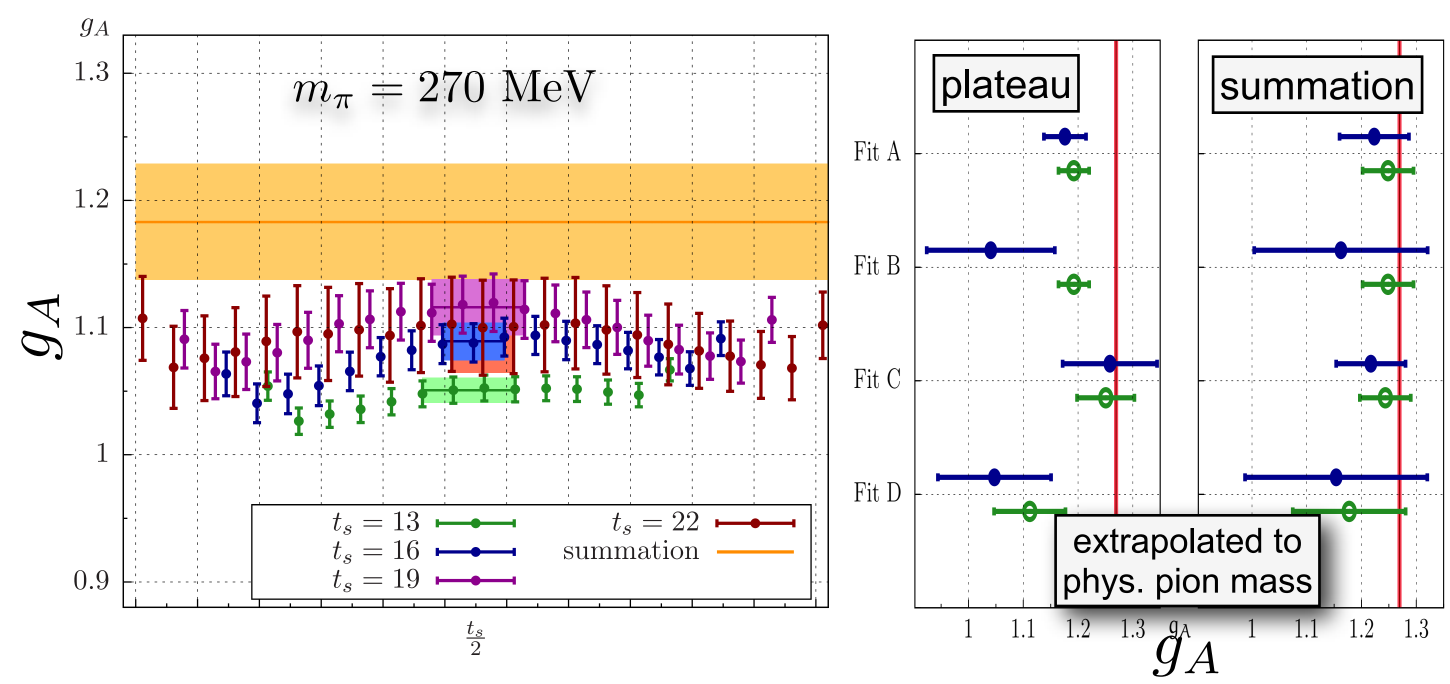
Variational method

[Ben Owen, et al. (CSSM) Phys.Lett. B 723 (2013) 217]



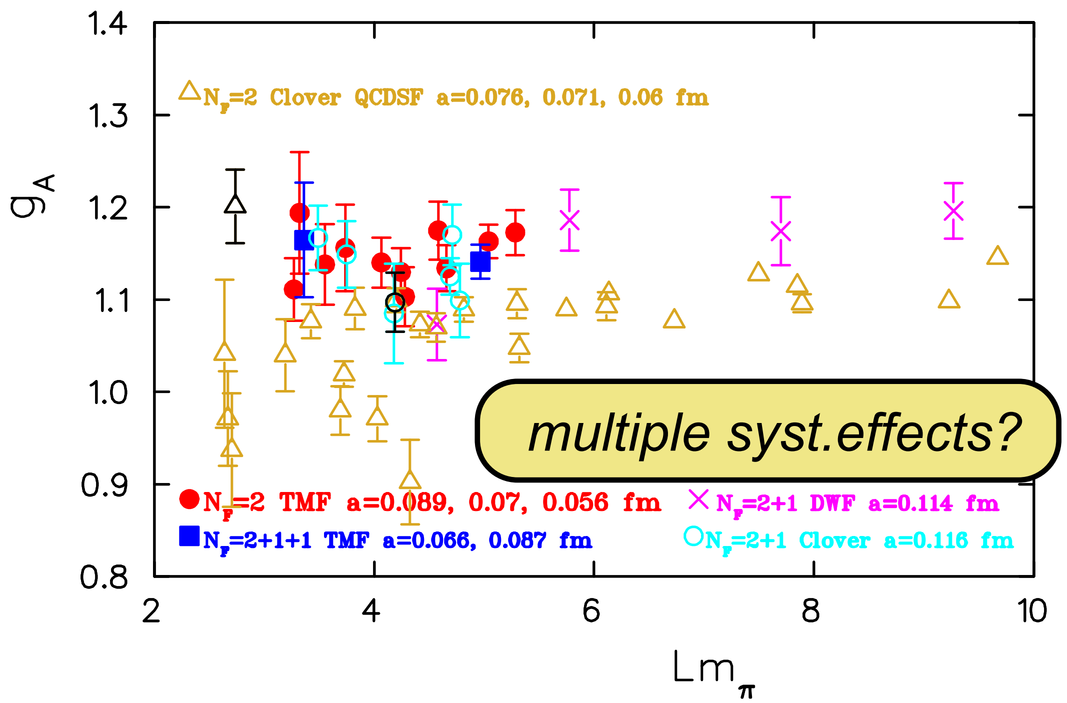
"Summation"

[T.D.Rae (CLS-Mainz); S.Capitani *et al,* Phys.Rev. D86 (2012) 074502]

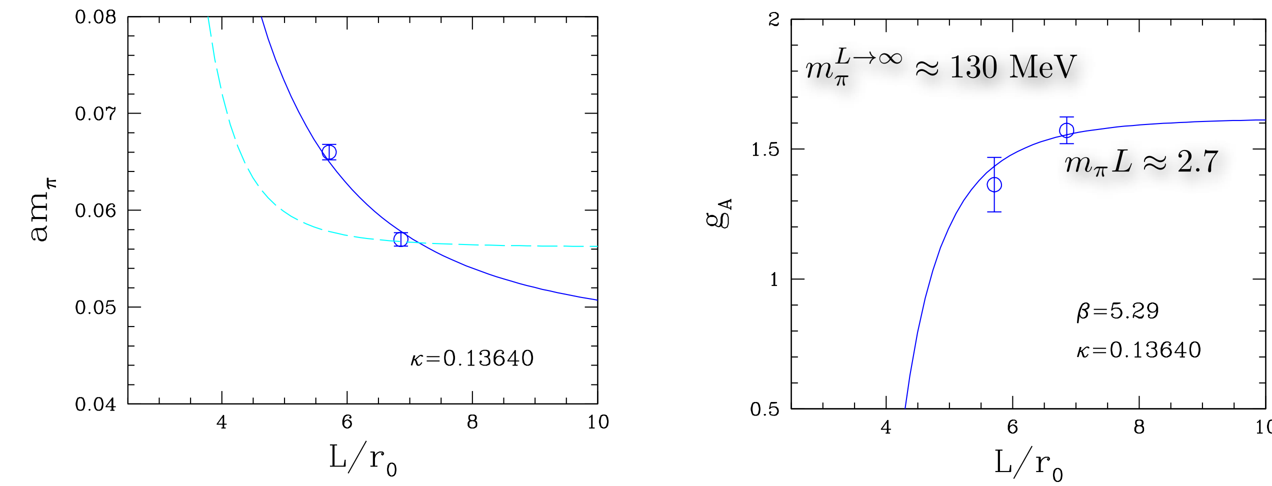


Nucleon Axial Charge: Lattice Size Effects?

[C.Alexandrou *et al*, 1303.5979]

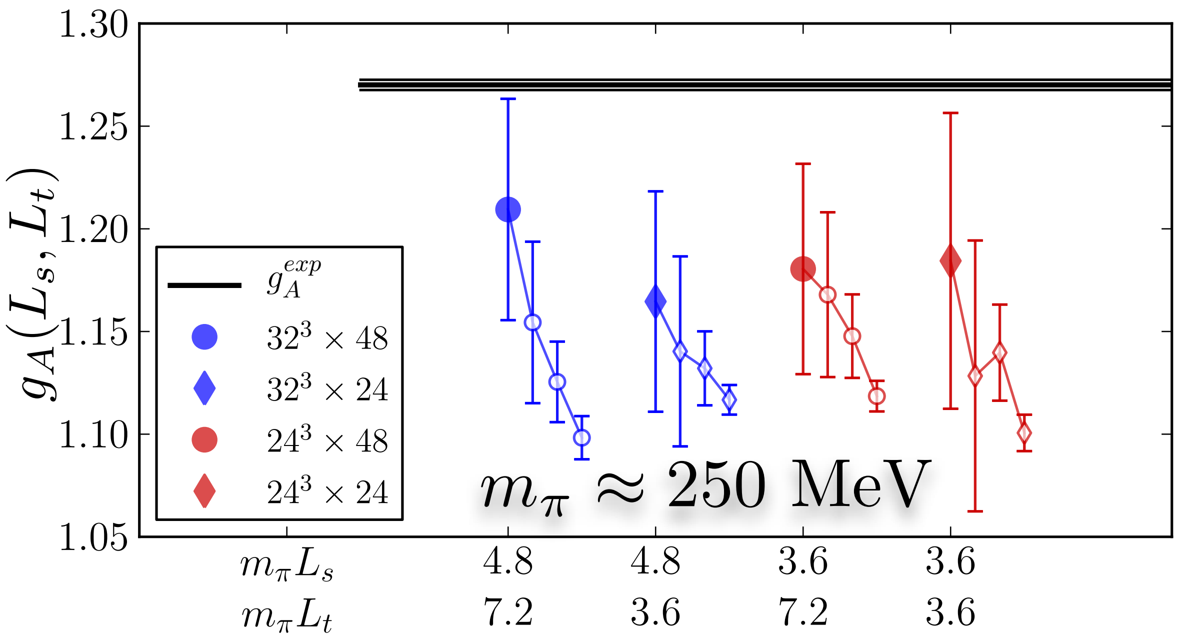


[R.Horsley *et al* (QCDSF), 1302.2233]



$$\delta g_A(m_\pi L \approx 2.7) \approx 3(\pm 3)\%$$

(L_s, L_t)-dependence with Wilson fermions [J.R.Green (LHPC), prelim.]



Fit $g_A(L_s, L_t) = g_A^\infty + Be^{-m_\pi L_s} + Ce^{-m_\pi L_t}$

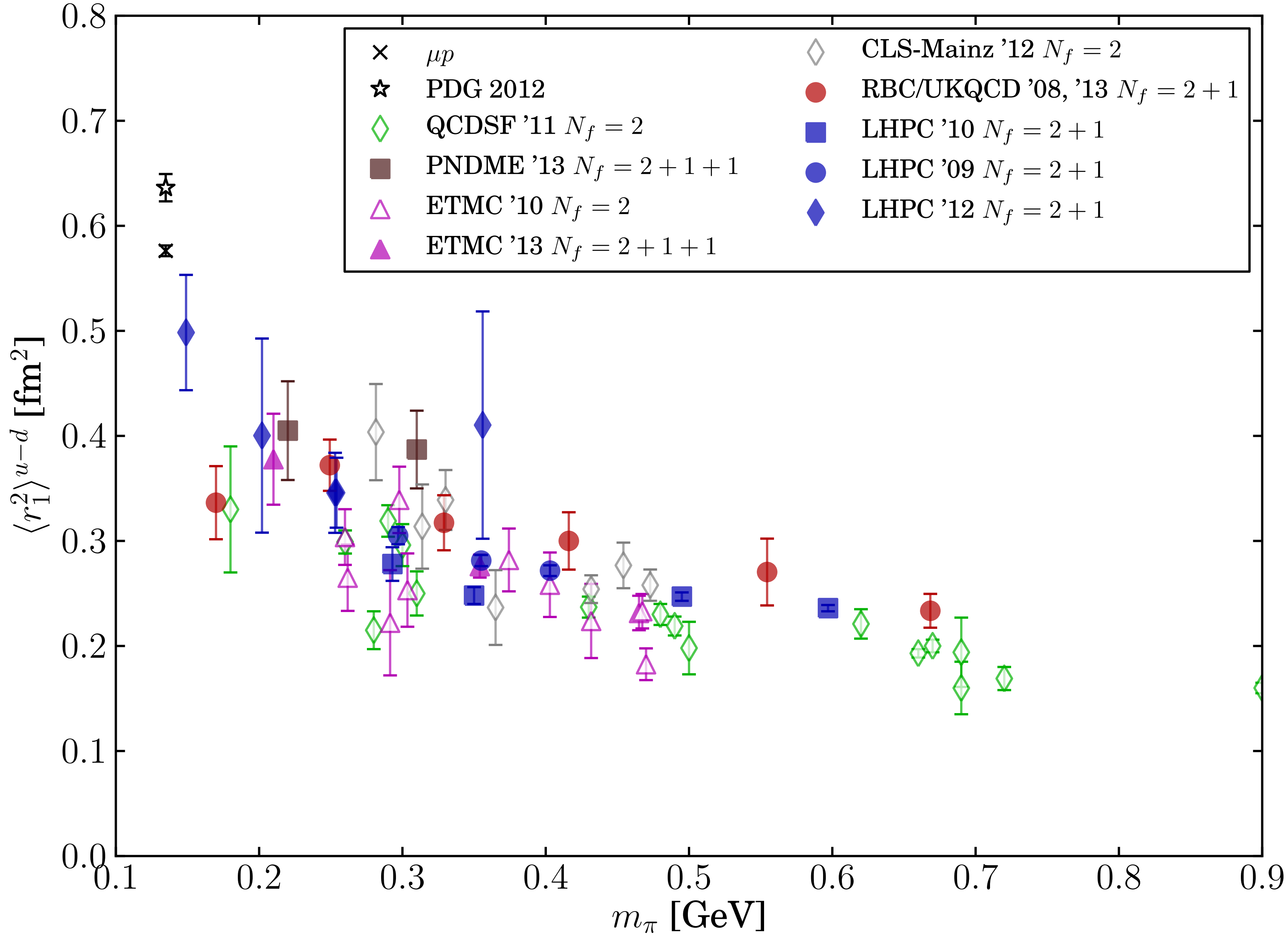
At $m_\pi \approx 250$ MeV :

$$g_A^{(m_\pi L_s=4)} - g_A^\infty = -0.009(54)$$

$$g_A^{(m_\pi L_t=4)} - g_A^\infty = -0.016(39)$$

Nucleon Dirac Radius

$$F_1^{u-d}(Q^2) \approx F(0)\left[1 - \frac{1}{6}Q^2\langle r_1^2\rangle^{u-d} + \mathcal{O}(Q^4)\right]$$



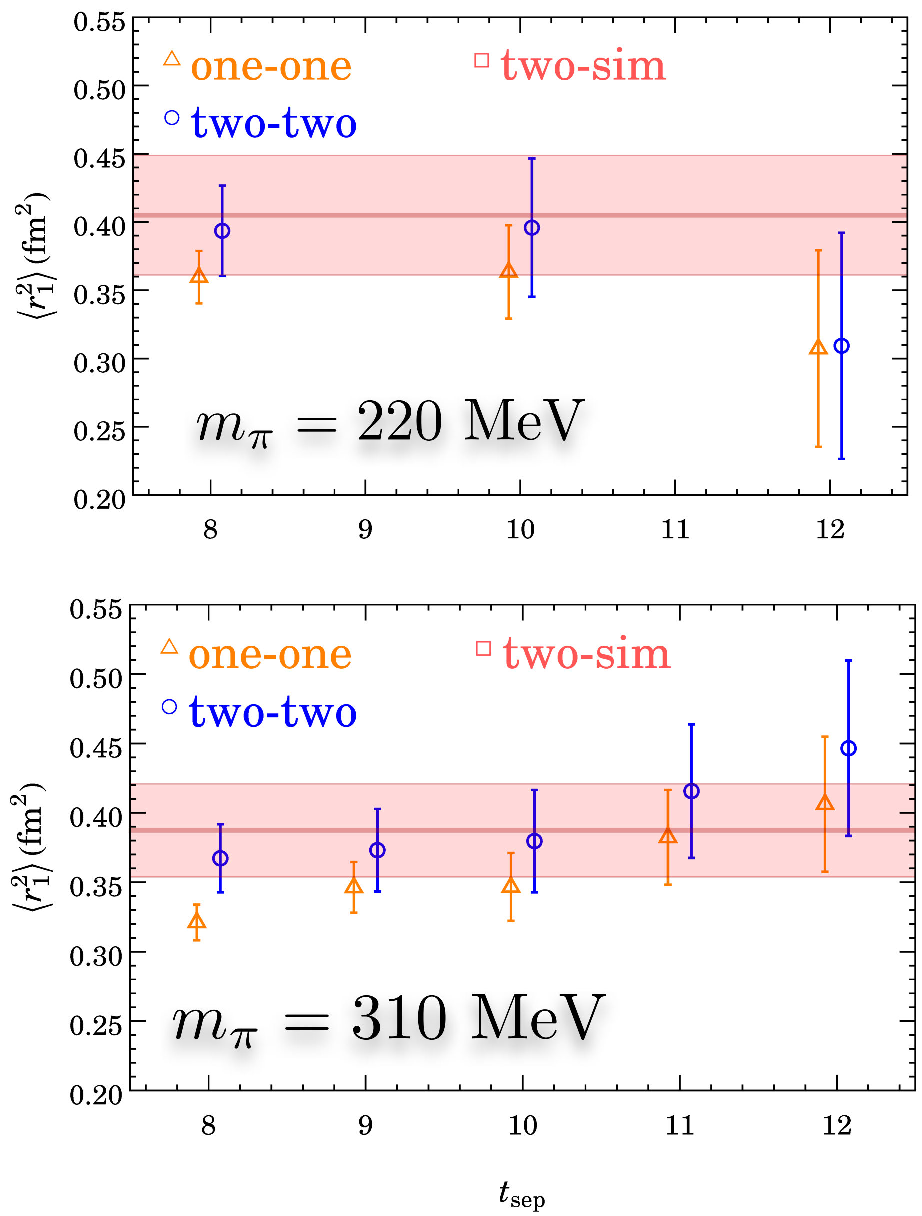
ChPT predicts divergence $\sim \log m_\pi^2$

Larger L_s, smaller $Q^2_{\rm min}$ are desirable

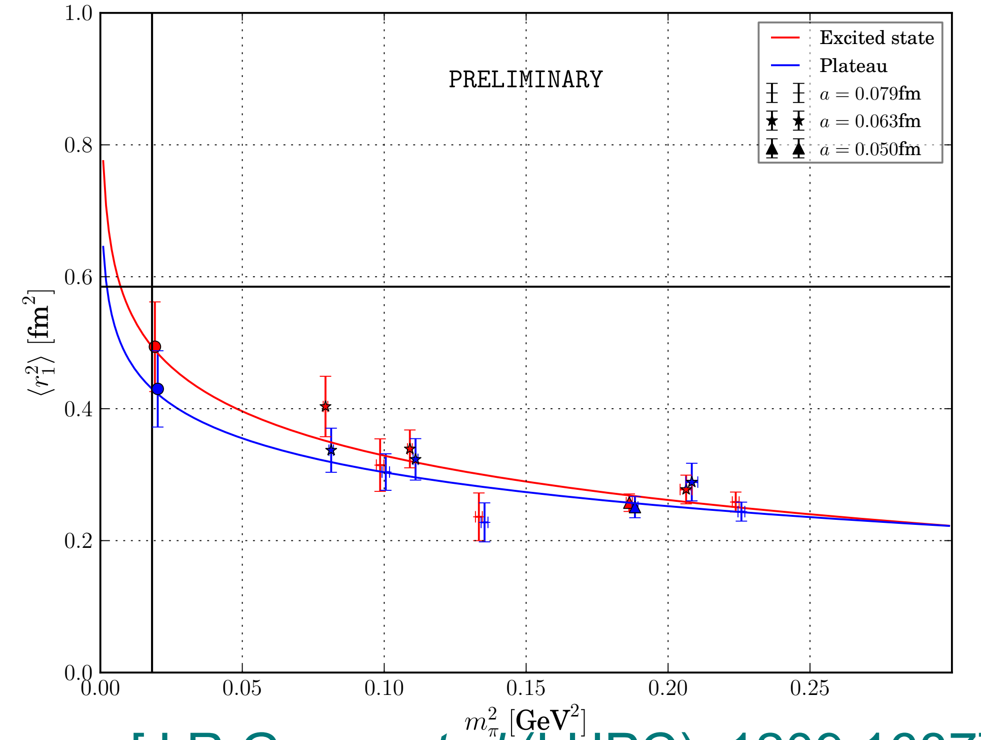
Dirac Radius: Excited States

2-state fits

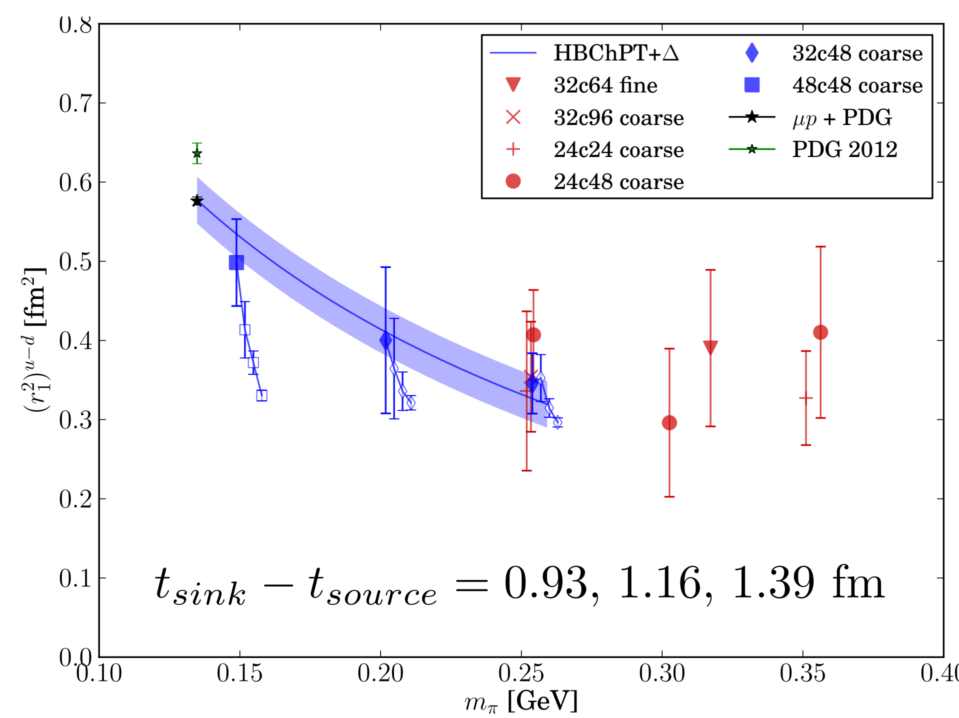
[H.W.Lin *et al* (PNDME) arXiv:1306.5435]



[T.D.Rae (CLS-Mainz)]



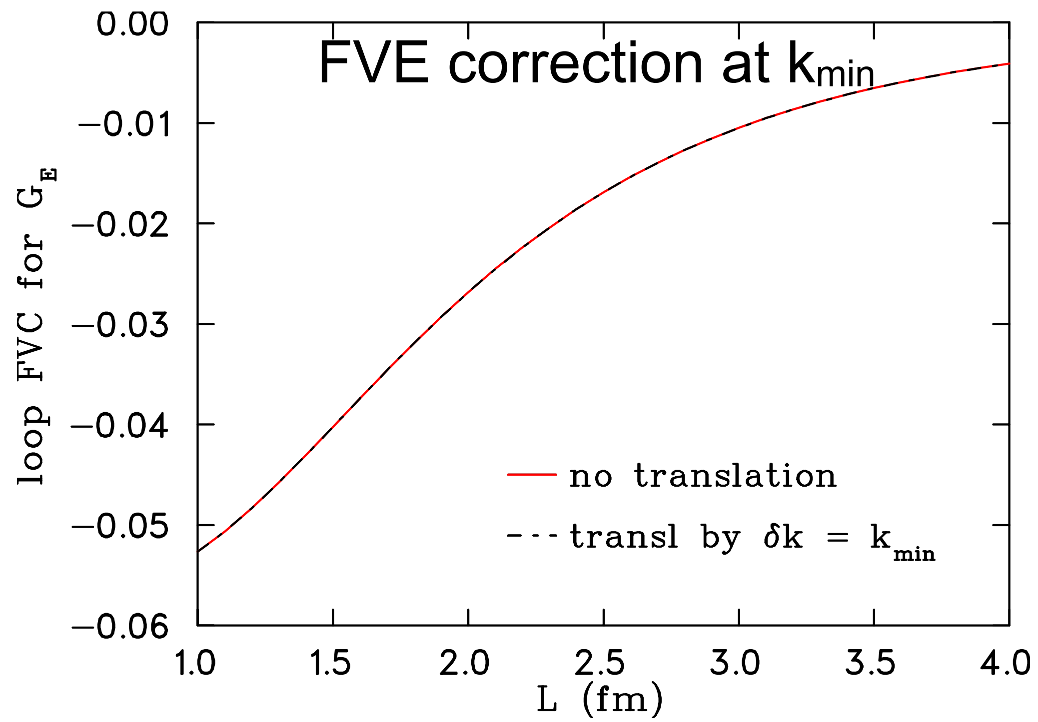
[J.R.Green *et al* (LHPC), 1209.1687]

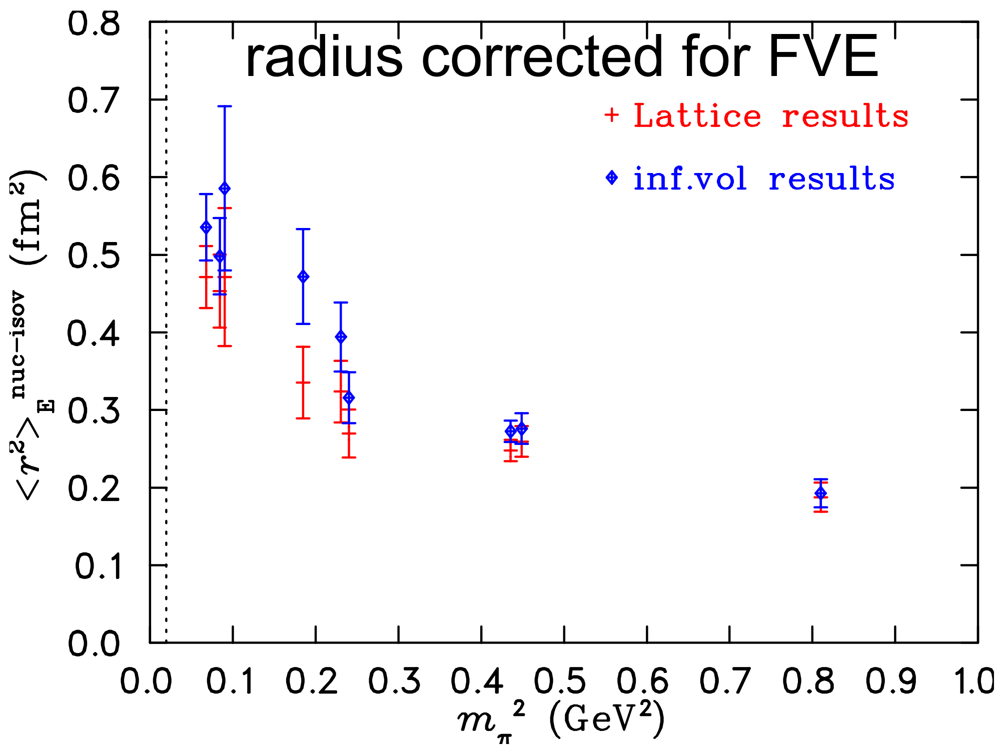


Excited states problem: Worse below 200 MeV?

Radius: Finite Volume Corrections

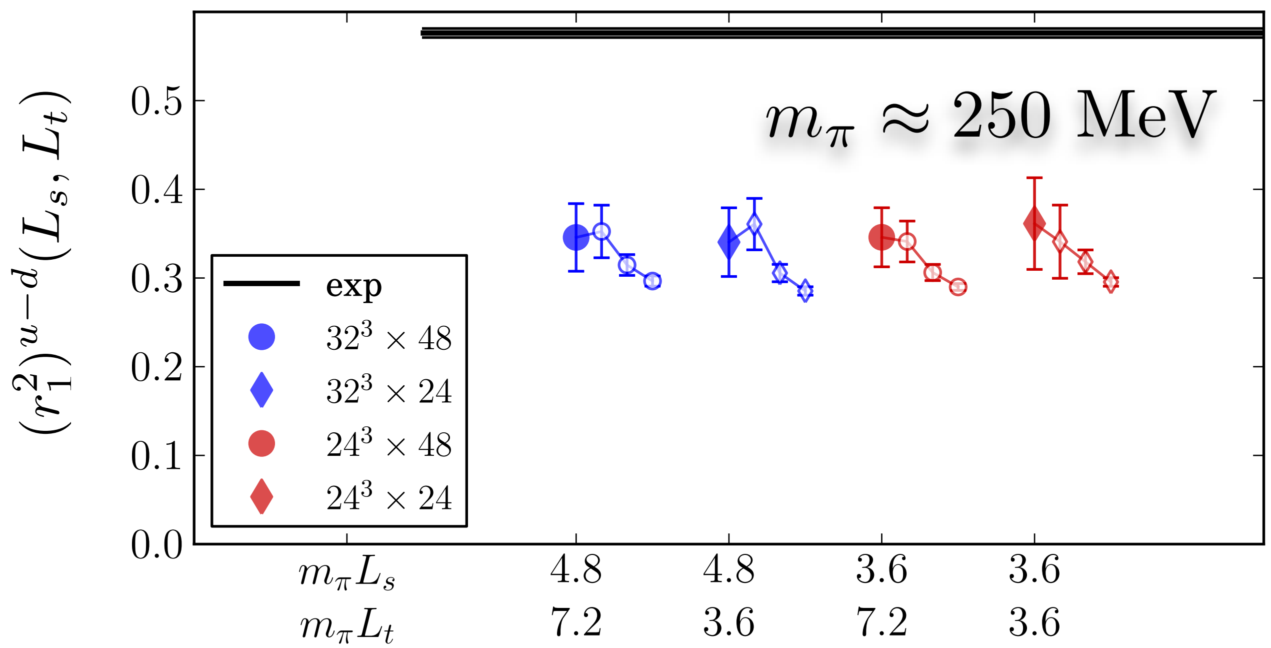
FVE corrections to nucleon electric radius
[J.M.Hall *et al,* arXiv:1210.6124 (to appear in PLB)]





(data : [S.Collins *et al*(QCDSF), 1106.3580])

(L_s, L_t)-dependence with Wilson fermions [J.R.Green (LHPC), prelim.]



Fit

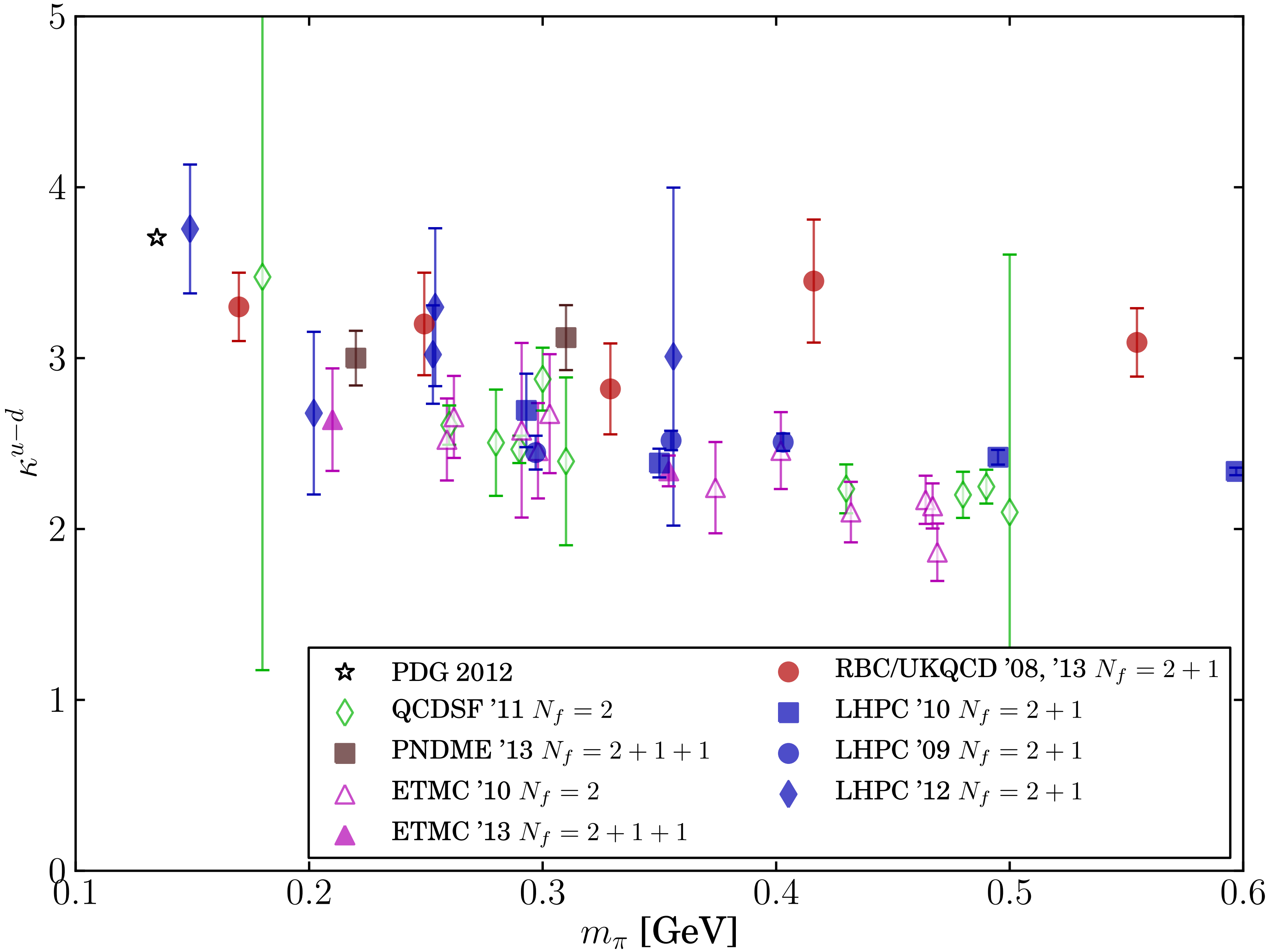
$$(r_1^2)^{u-d}(L_s, L_t) = (r_1^2)^{u-d}(\infty) + Be^{-m_\pi L_s} + Ce^{-m_\pi L_t}$$

$$\delta(r_1^2)^{u-d}\Big|_{m_\pi L_s=4} = 0.008(38)\ \mathrm{fm}^2$$

$$\delta(r_1^2)^{u-d}\Big|_{m_\pi L_t=4} = 0.003(28)\ \mathrm{fm}^2$$

Anomalous Magnetic Moment

$$\kappa_v = F_2^{u-d}(Q^2 = 0)$$



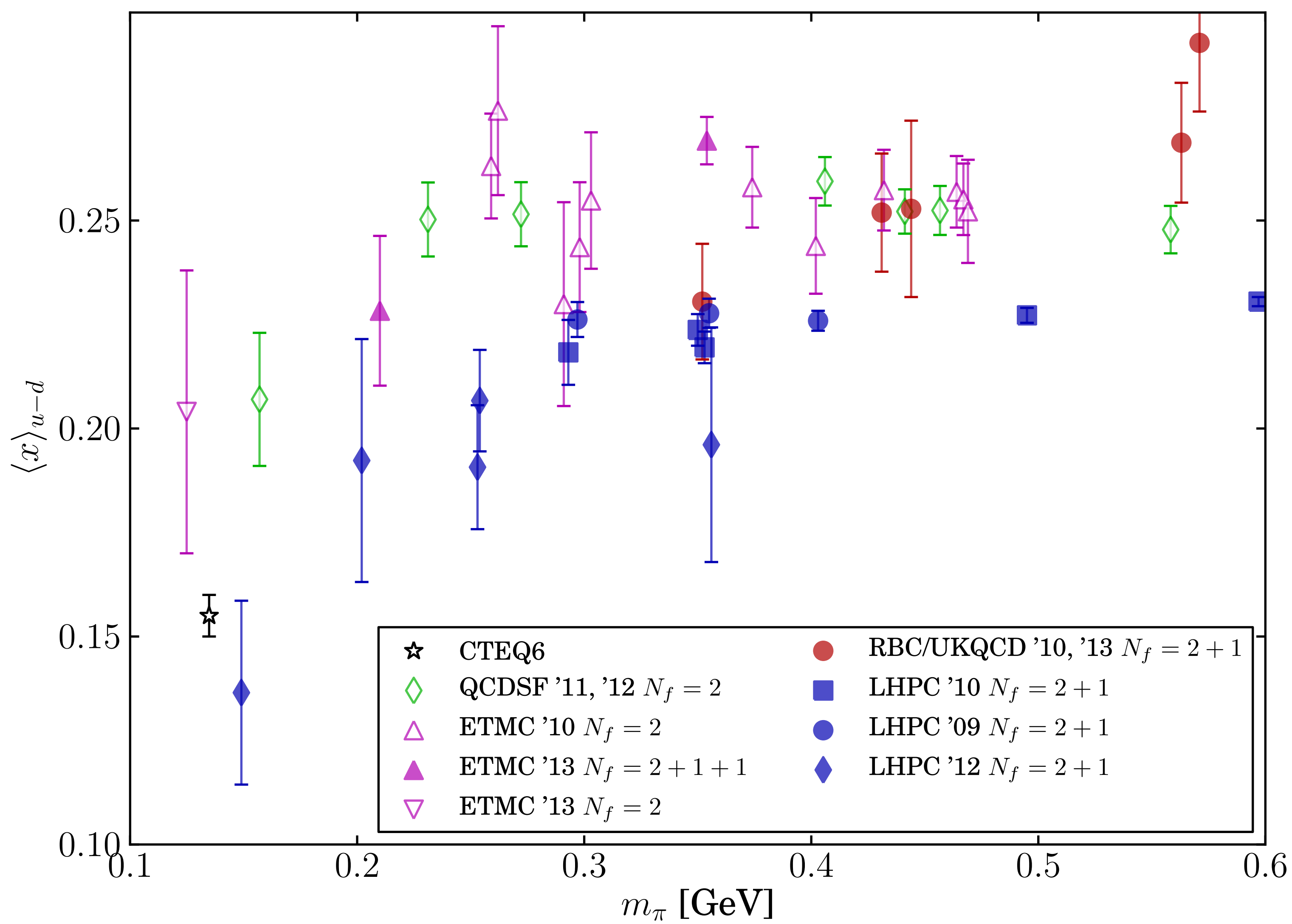
Larger L_s, smaller $Q^2_{\rm min}$ are desirable

Quark Momentum Fraction

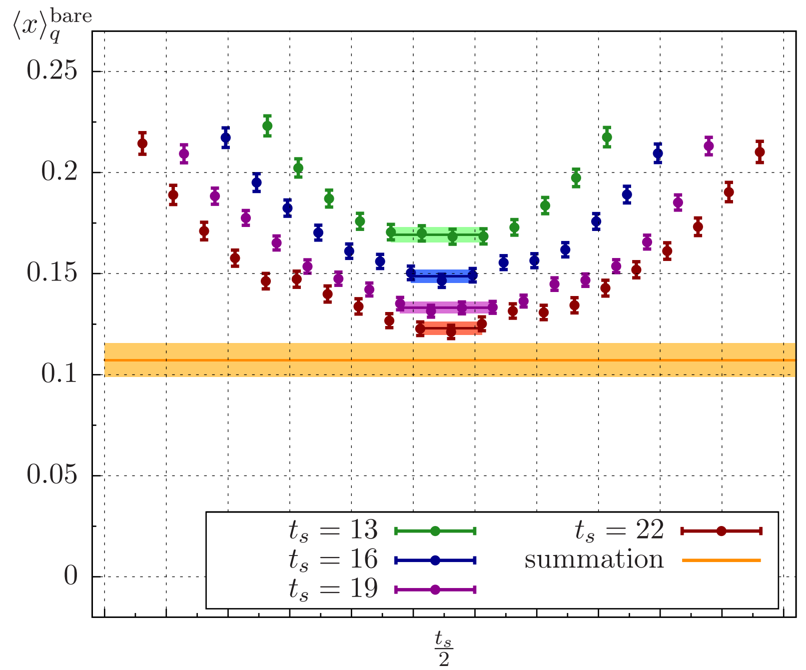
$$\langle x \rangle_{u-d} = \int dx\, x\, \big(u(x) + \bar{u}(x) - d(x) - \bar{d}(x)\big)$$

Phenomenology: $\langle x \rangle_{u-d}^{\overline{MS}(2\text{ GeV})} = 0.155(5)$

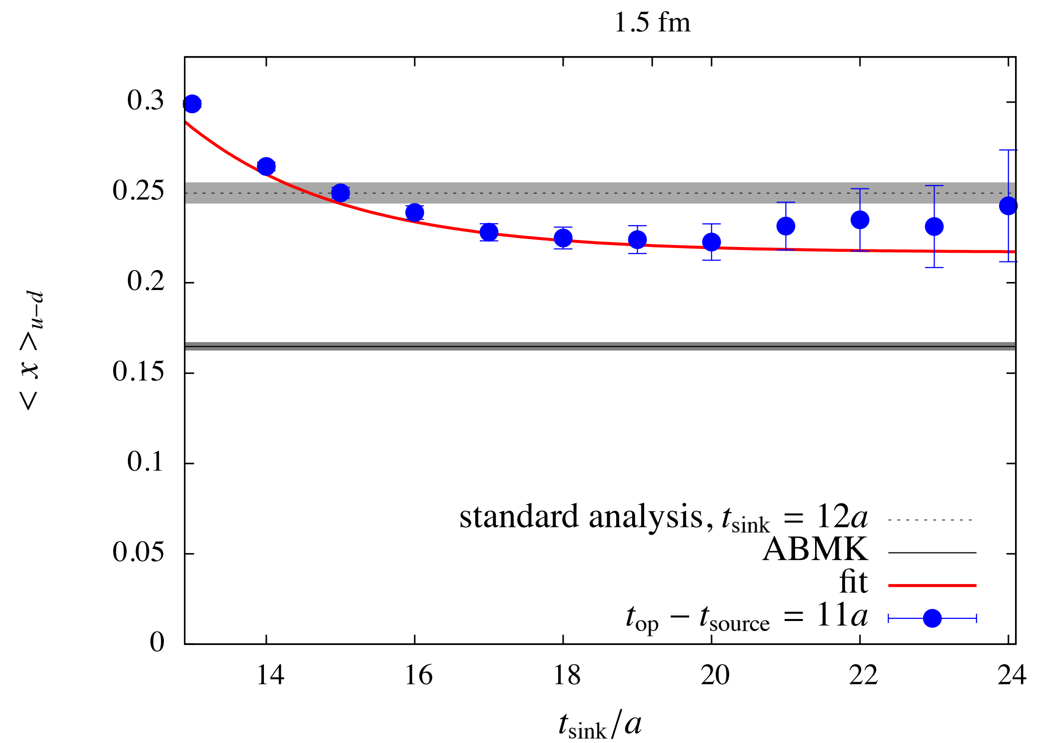
$$\langle N(p)|\bar{q}\gamma_{\{\mu}\overset{\leftrightarrow}{D}_{\nu\}}q|p\rangle = \langle x \rangle_q\, \bar{u}_p \gamma_{\{\mu}p_{\nu\}}u_p$$



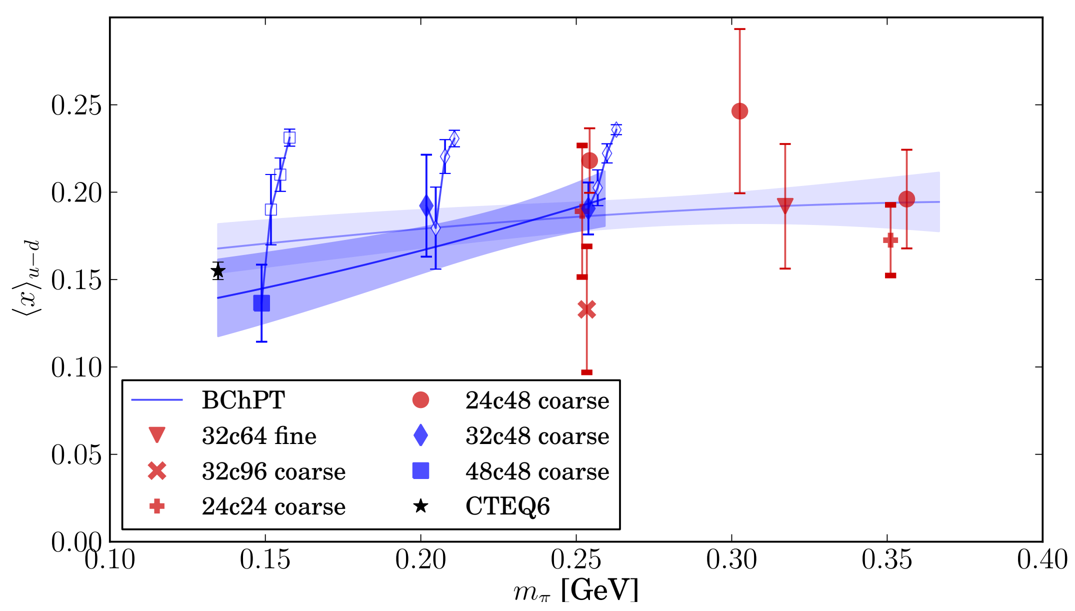
Quark Momentum Fraction: Excited States



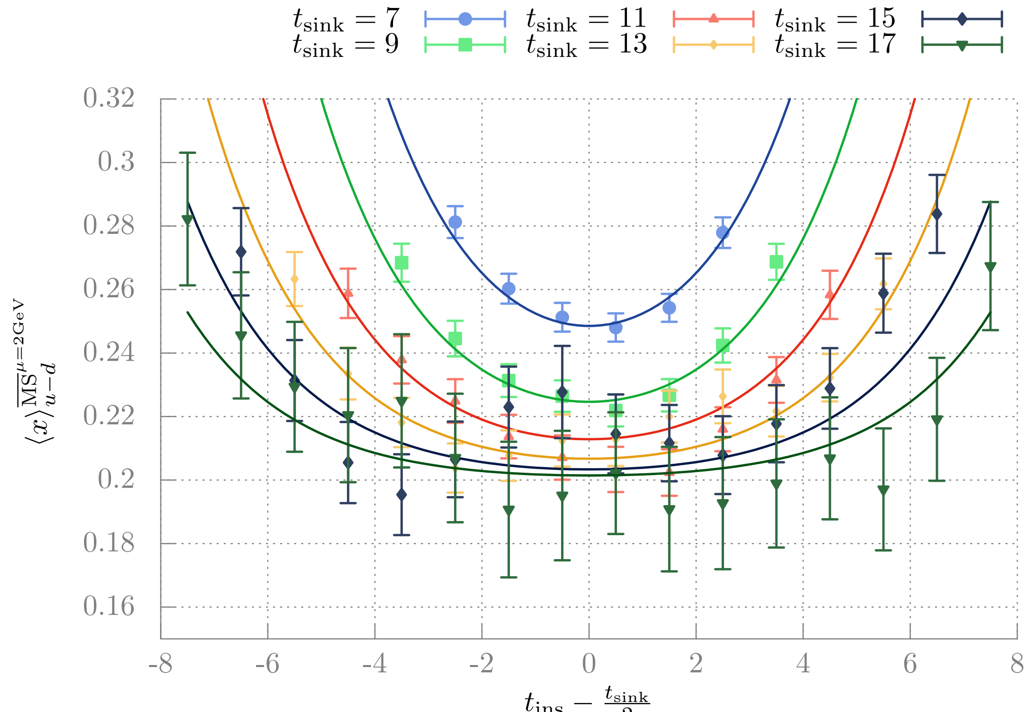
[T.D.Rae (CLS-Mainz)]



[S.Dinter *et at* (ETMC) arXiv:1112.2931]



[J.R.Green *et at* (LHPC) arXiv:1209.1687]



[S.Collins *et al* (U.Regensburg), Sec.3B

(Sub)Summary: Gold-plated observables

- ★ (finally) Exciting developments at the physical pion mass
- ★ Removing excited states is necessary in most cases
- ★ Agreement is reassuring, but much more work is required to ensure quality control.

Hadron Wave Functions

- Wave functions of the Roper state and n=2 radial nucleon excitation
- LC Wave functions (distribution amplitudes) of the nucleon and negative parity excitations

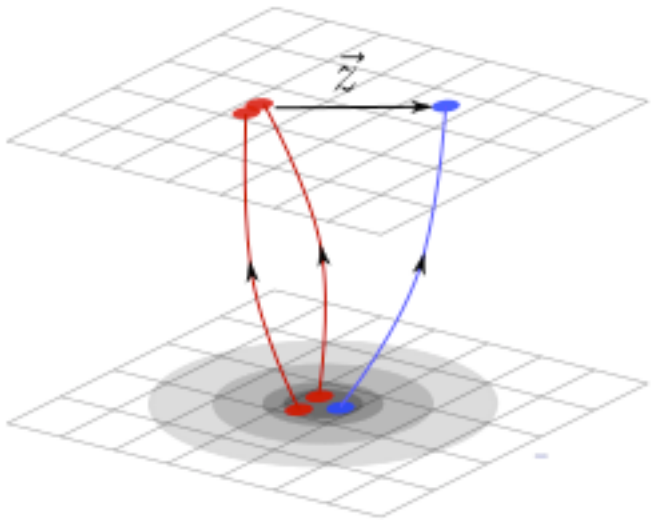
Nucleon & Radial Resonance Wave Funcions

[D.Roberts *et al* (CSSM), arXiv:1304.0325 (to appear in Phys.Lett.B)]

Variational method in a basis of 4 nucleon operators

$$\chi_1^{(\mathcal{S})}(\vec{x}) = \epsilon^{abc}\left[\left(\tilde{u}^{Ta}_{(\mathcal{S})}\, C\gamma_5\, \tilde{d}^{b}_{(\mathcal{S})}\right)\tilde{u}^{c}_{(\mathcal{S})}\right]_{\vec{x}}$$

with varying smearing radius *S* = *0.21, 0.32, 0.54* and *0.78* fm and find energy eigenvectors



Calculate w.f. of *d*-quark w.r.t. 2 *u* quarks:

$$\chi_1(\vec{x},\vec{z}) = \epsilon^{abc}\left(u^{Ta}(\vec{x})\, C\gamma_5\, d^b(\vec{x}+\vec{z})\right) u^c(\vec{x})$$

$$\psi^d_\alpha(\vec{p},t;\vec{z}) = \mathrm{const}\cdot\sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}}\langle\, \chi_1(\vec{x},\vec{z},t)\, \chi_1^{(\mathcal{S})}(t)\,\rangle v_\alpha^{(\mathcal{S})}$$

(assuming Landau gauge)

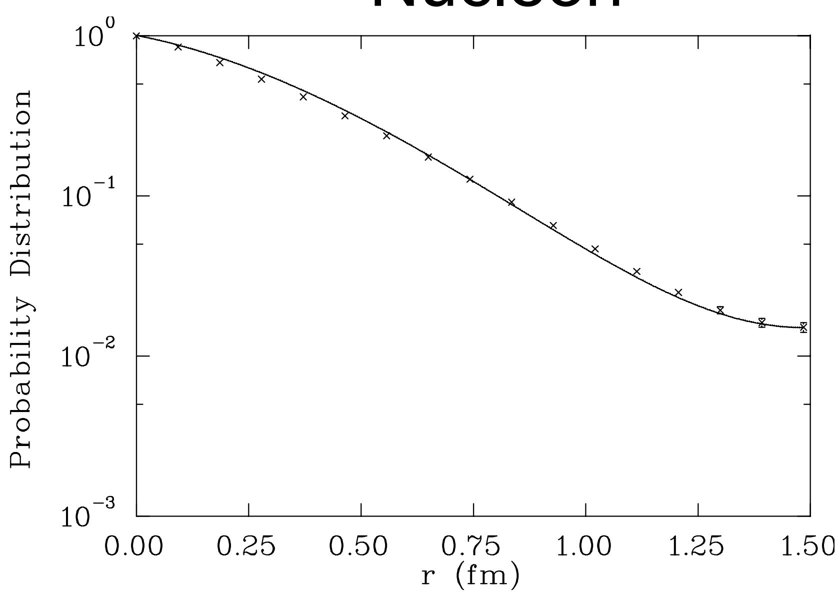
Nf=2+1 dynamical *O(a)*-improved Wilson fermions, $m_\pi = 156\ \mathrm{MeV}$

Nucleon & Radial Resonance Wave Funcions

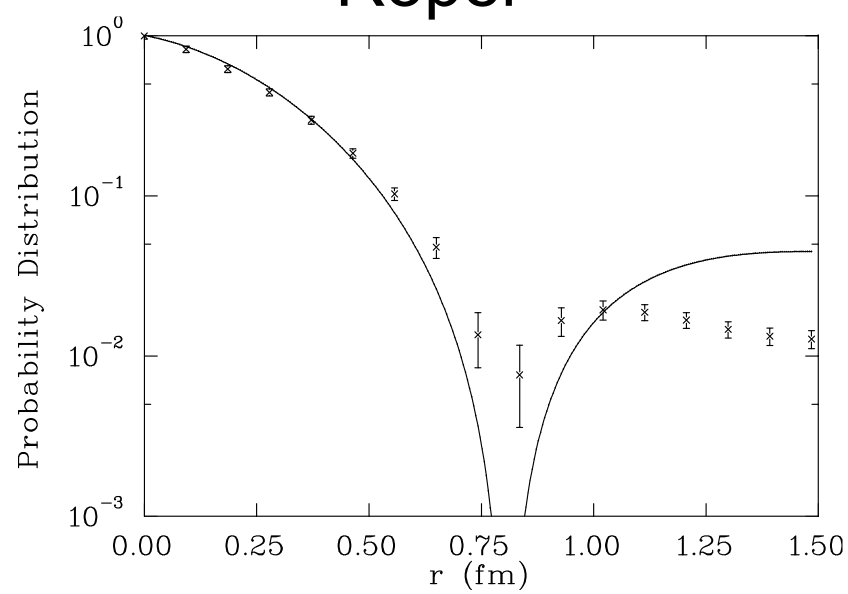
$m_\pi = 156\ \mathrm{MeV}$

[D.Roberts *et al* (CSSM), arXiv:1304.0325 (to appear in Phys.Lett.B)]

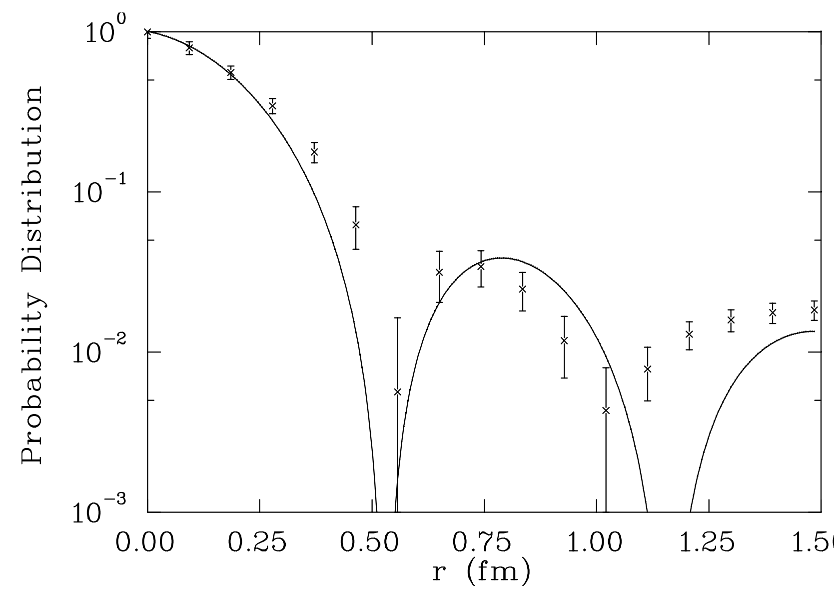
n = 0
Nucleon

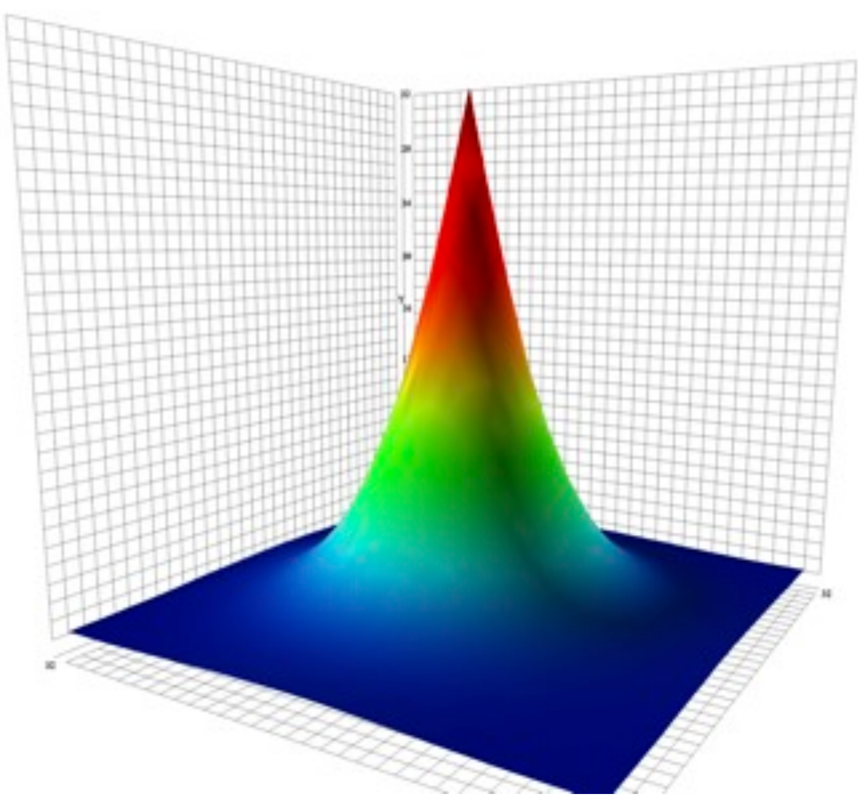


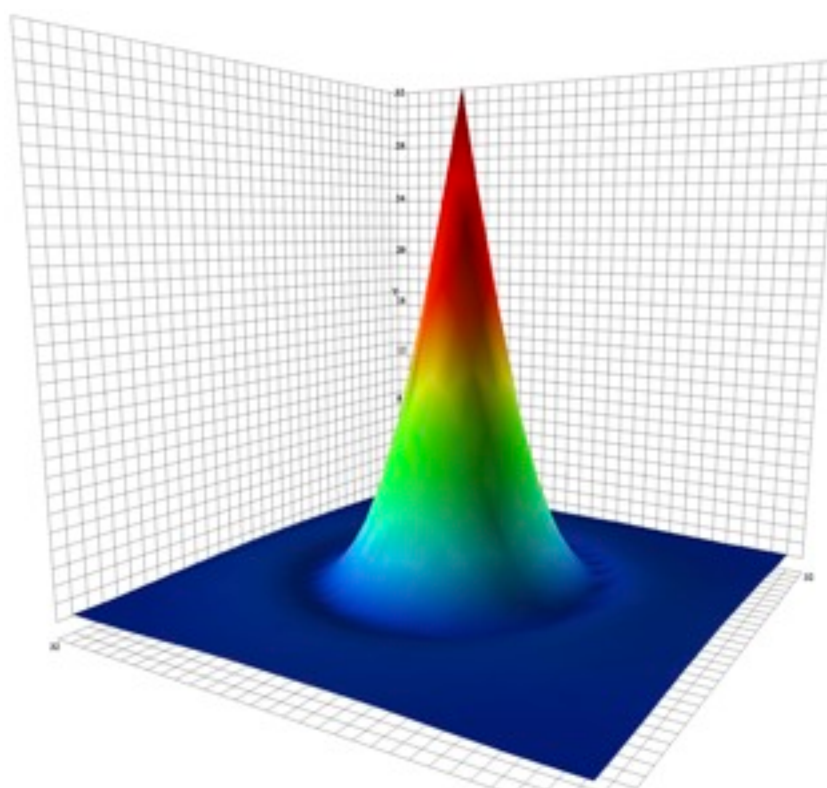
n = 1
“Roper”

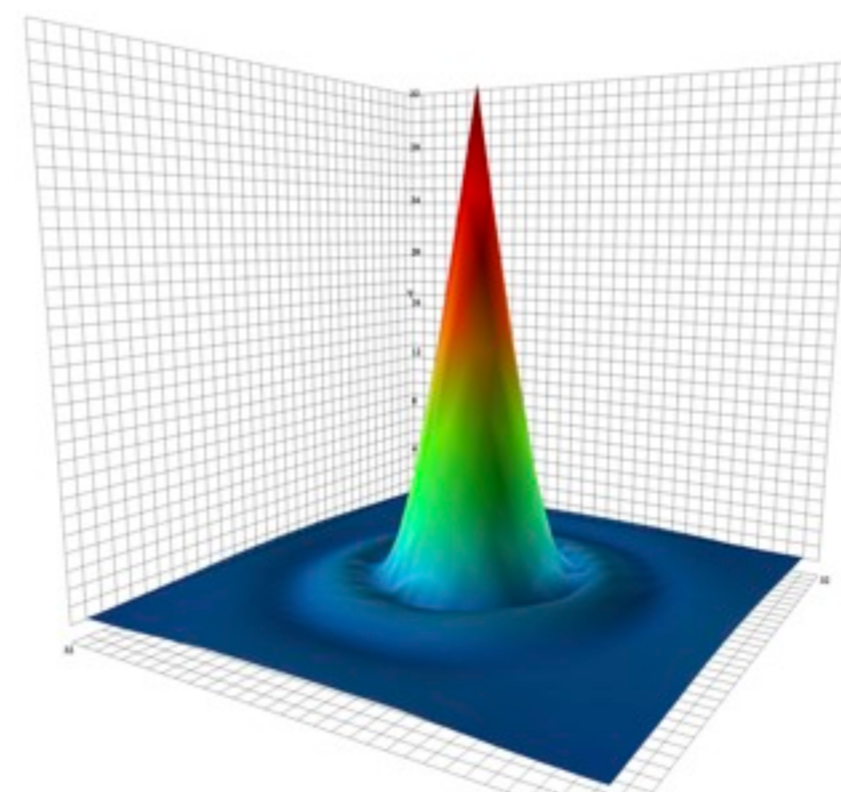


n = 2









Nucleon and N* Distribution Amplitudes

[R.Schiel (QCDSF) Sec.3B]

LC Fock valence state of a Baryon

$$|N^{(*)},\uparrow\rangle = \text{const}\int \frac{[dx]\,\varphi^{(*)}(x_i)}{2\sqrt{24x_1x_2x_3}}\{|\mathbf{u}^\uparrow(x_1)\mathbf{u}^\downarrow(x_2)\mathbf{d}^\uparrow(x_3)\rangle - |\mathbf{u}^\uparrow(x_1)\mathbf{d}^\downarrow(x_2)\mathbf{u}^\uparrow(x_3)\rangle\}$$

$$\varphi(x_i;\mu^2) = 120x_1x_2x_3\Big\{1 + c_{10}(x_1-2x_2+x_3)\Big(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\Big)^{\frac{8}{3\beta_0}} + c_{11}(x_1-x_3)\Big(\frac{\alpha_S(\mu)}{\alpha_S(\mu_0)}\Big)^{\frac{20}{9\beta_0}} + \ldots\Big\}$$

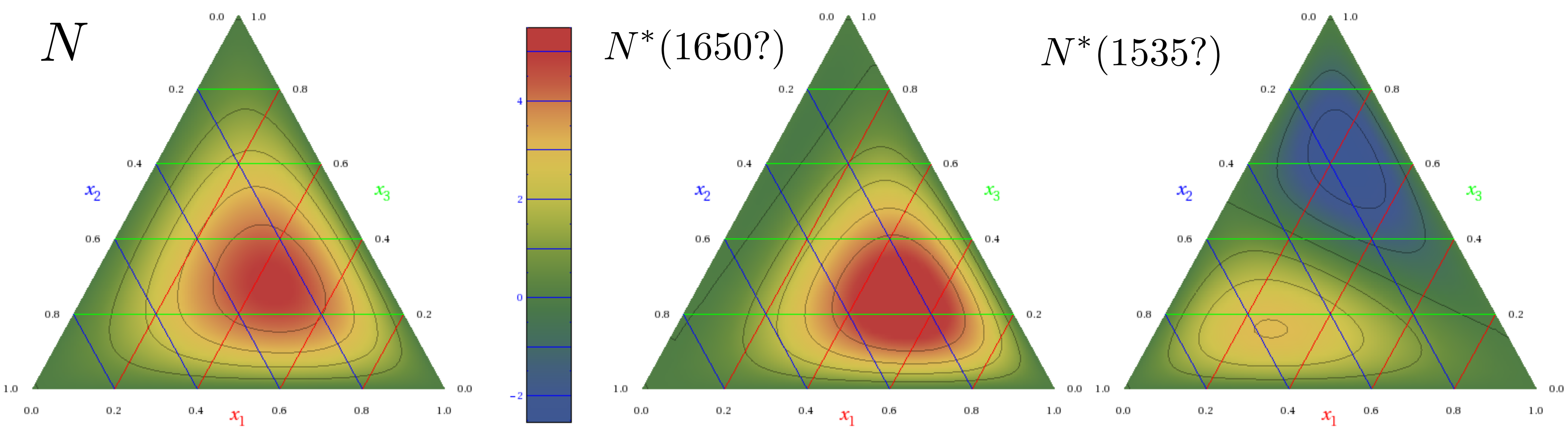
Compute moments of DA on a lattice: $\langle\mathcal{O}_{\alpha\beta\gamma}(x)N(0)\rangle \longrightarrow \langle\Omega|\mathcal{O}_{\alpha\beta\gamma}(x)|N\rangle$

$\{\mathcal{O}(x)\}$: local 3-quark operators with up to 2 derivatives

$$\varphi^{lmn} = \int [dx]\,x_1^l x_2^m x_3^n\,\varphi(x_1,x_2,x_3)$$

$$\{c_{1j},\,c_{2j}\} \longleftrightarrow \{\varphi^{lmn}\,|\,l+m+n=1,2\}$$

$$m_\pi = 290\text{ MeV},\ a = 0.072\text{ fm}$$



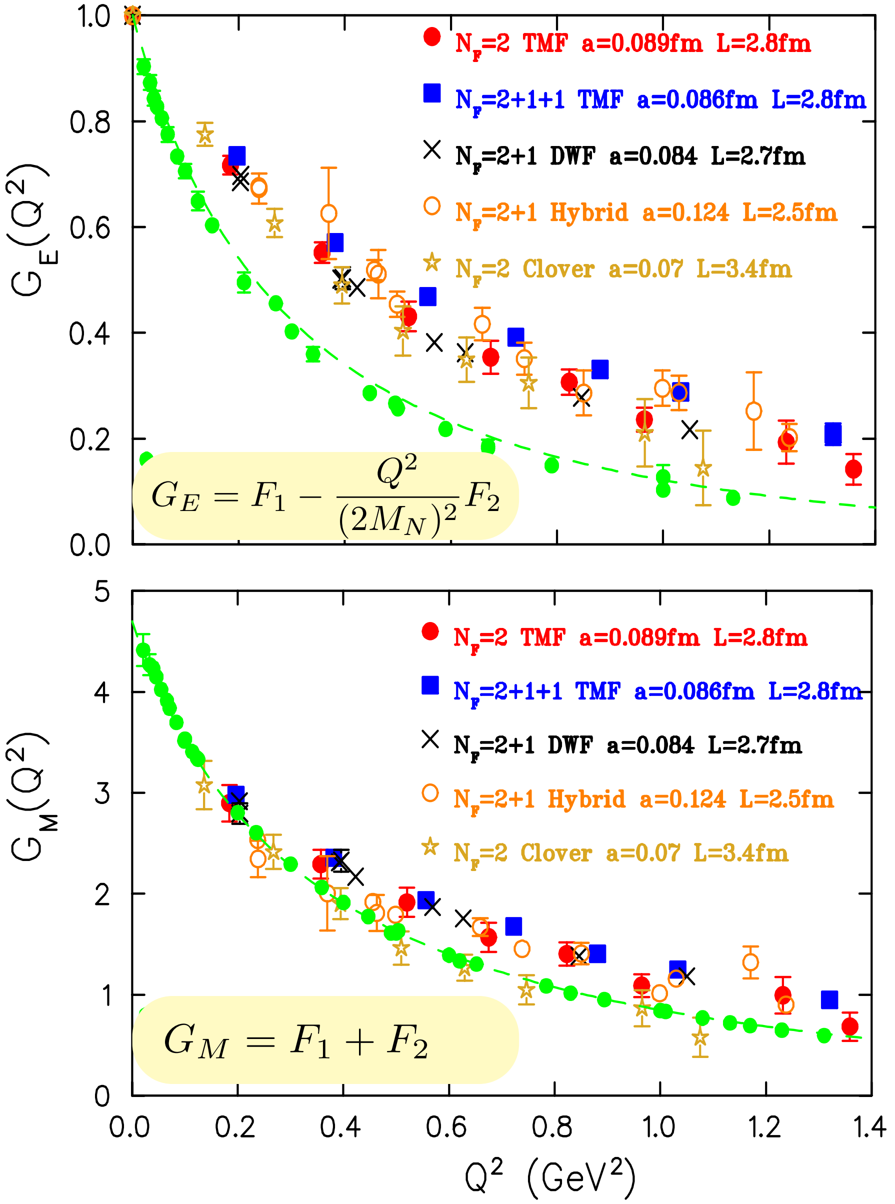
$$x_1 + x_2 + x_3 = 1$$

Select Hadron Form Factor Results

- Vector form factors of the nucleon
- Axial form factors of the nucleon
- Strange quark contributions to the nucleon form factors
- Axial form factors of Delta(1232)
- Electric form factor of Lambda(1405)
- Timelike vector form factor of the pion
- Scalar form factor and radius of the pion

Nucleon Vector Form Factors (u-d)

$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\Big[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]U_P$$



m_π = 354 and 210 MeV

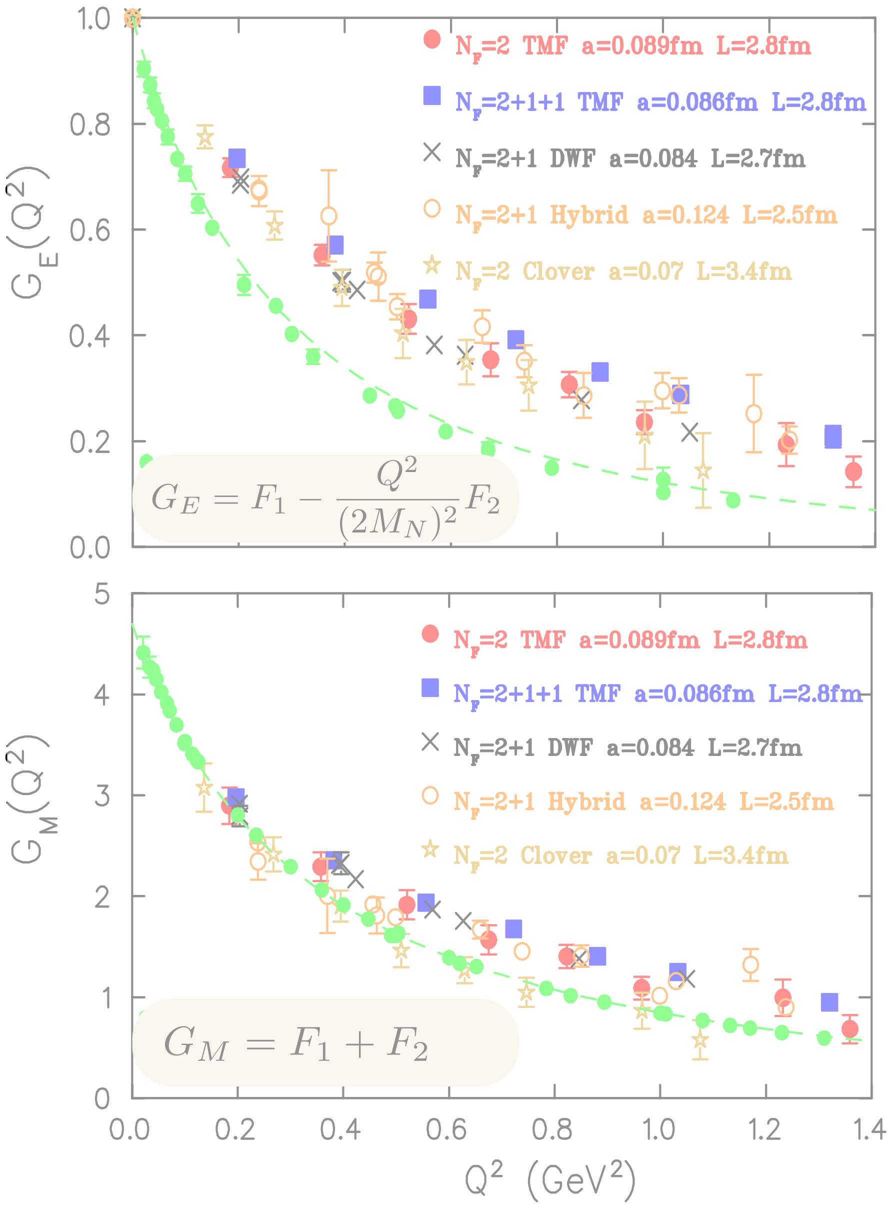
Nf=2+1+1 Twisted mass fermions

& earlier works: QCDSF, LHP, RBC

[C.Alexandrou et al (ETMC), arXiv:1303.5979]

Nucleon Vector Form Factors (u-d)

$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\left[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\right]U_P$$



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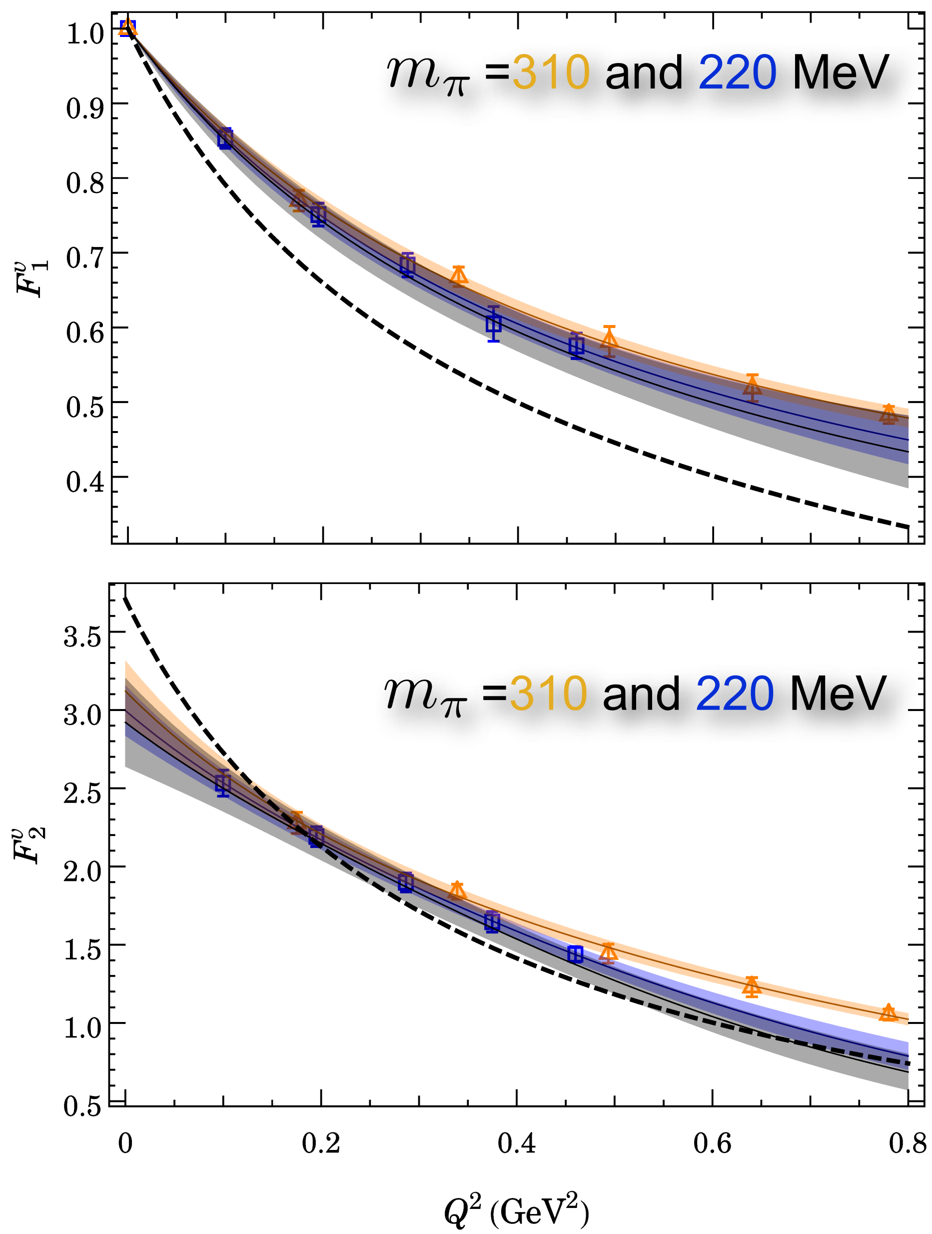


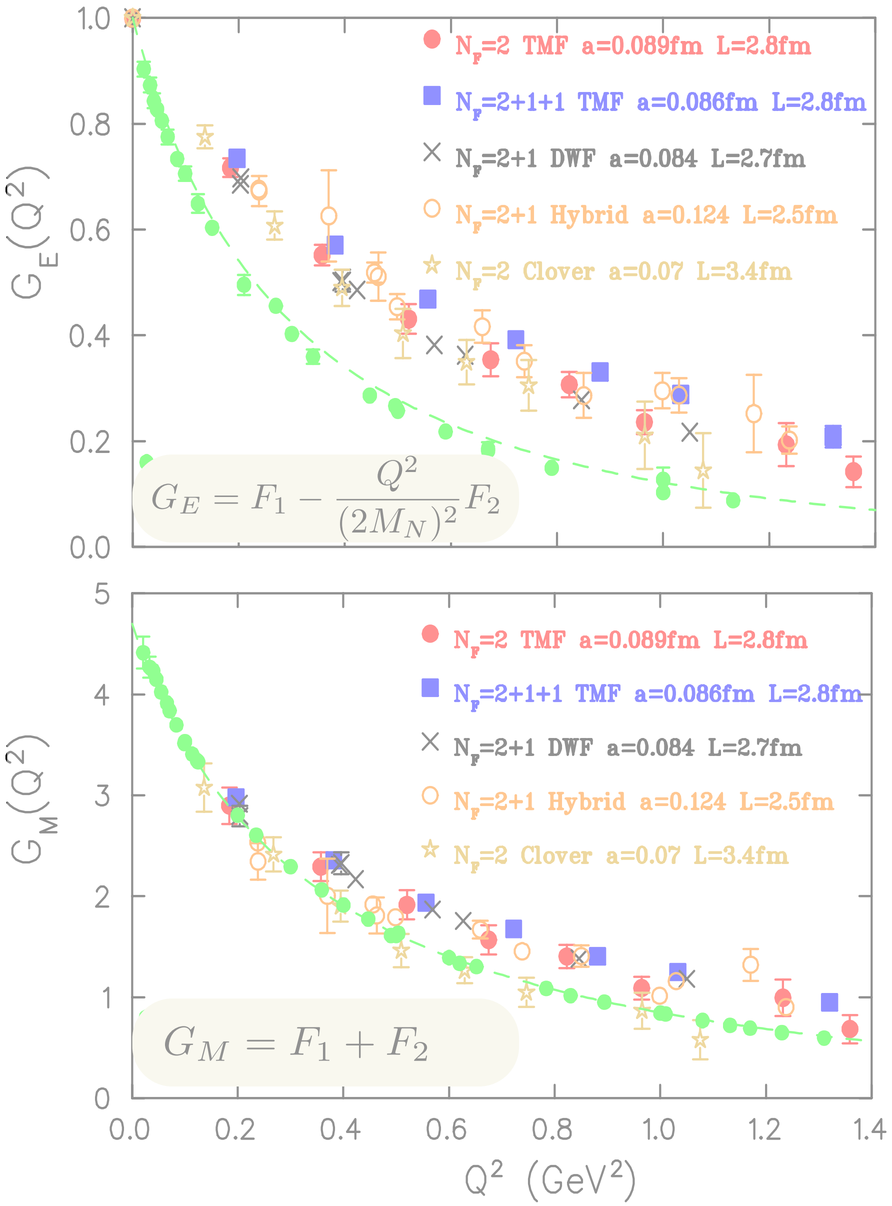
Nf=2+1+1 HISQ + Clover(v) fermions

2-state fits to suppress exc.states

[T.Bhattacharya et al (PNDME)]

Nucleon Vector Form Factors (u-d)

$$\langle P+q|\,\bar{q}\gamma^{\mu}q\,|P\rangle = \bar{U}_{P+q}\Big[F_1(Q^2)\,\gamma^{\mu} + F_2\,(Q^2)\,\frac{i\sigma^{\mu\nu}q_{\nu}}{2M_N}\Big]U_P$$



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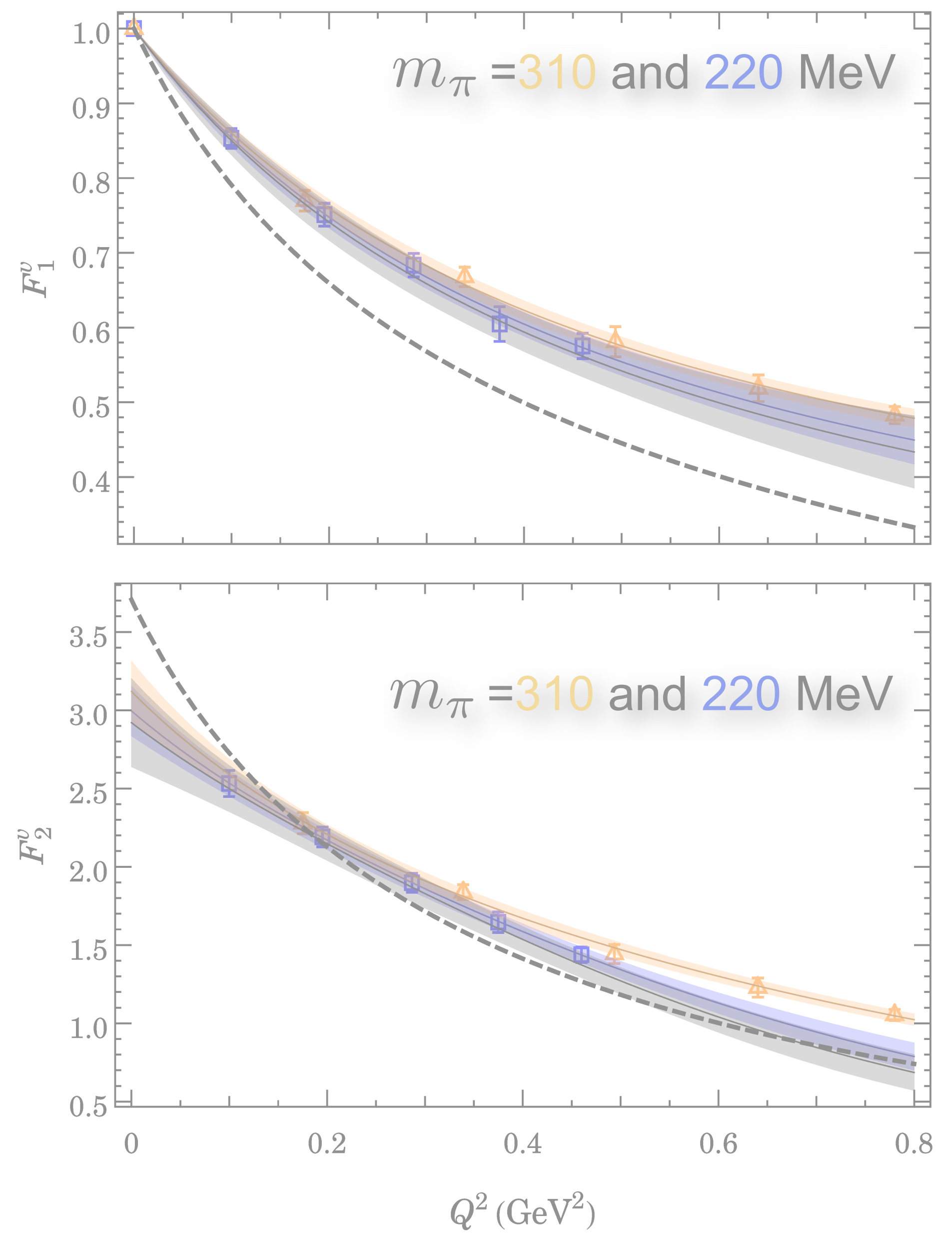
Nf=2+1+1 Twisted mass fermions & earlier works

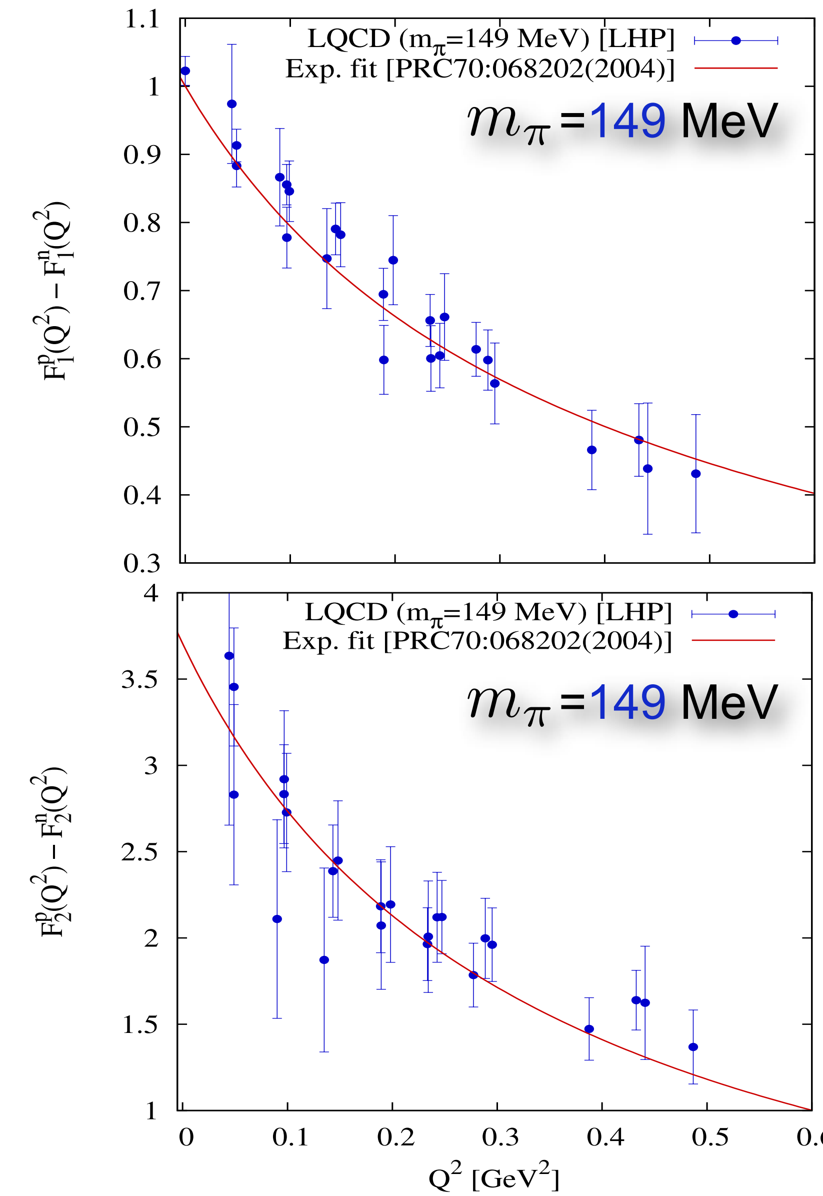
[C.Alexandrou et al (ETMC), arXiv:1303.5979]



Nf=2+1+1 HISQ + Clover(v) fermions

2-state fits to suppress exc.states

[T.Bhattacharya et al (PNDME)]

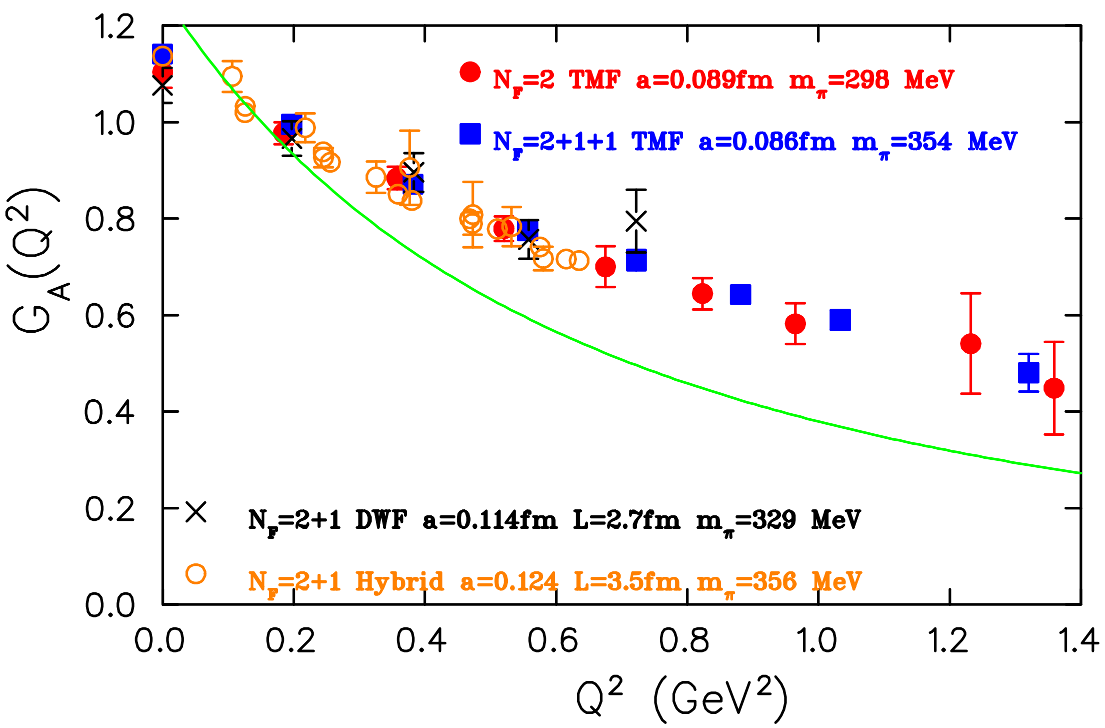


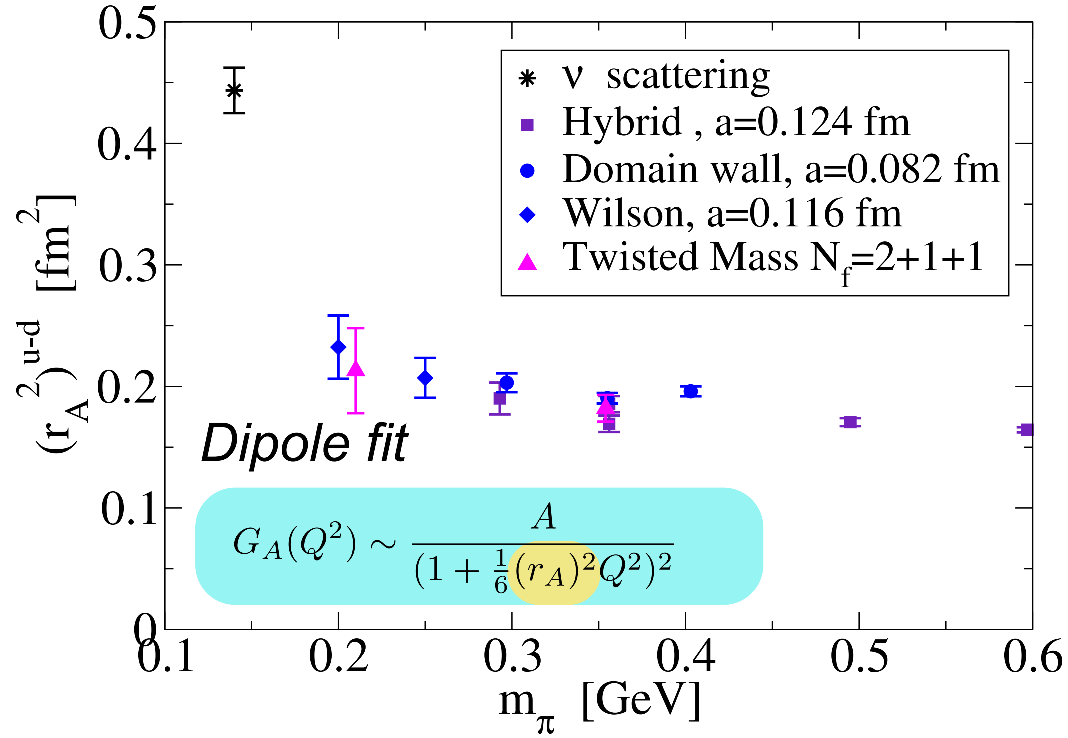
Nf=2+1 clover-imp.Wilson, "summation" to suppress excited states

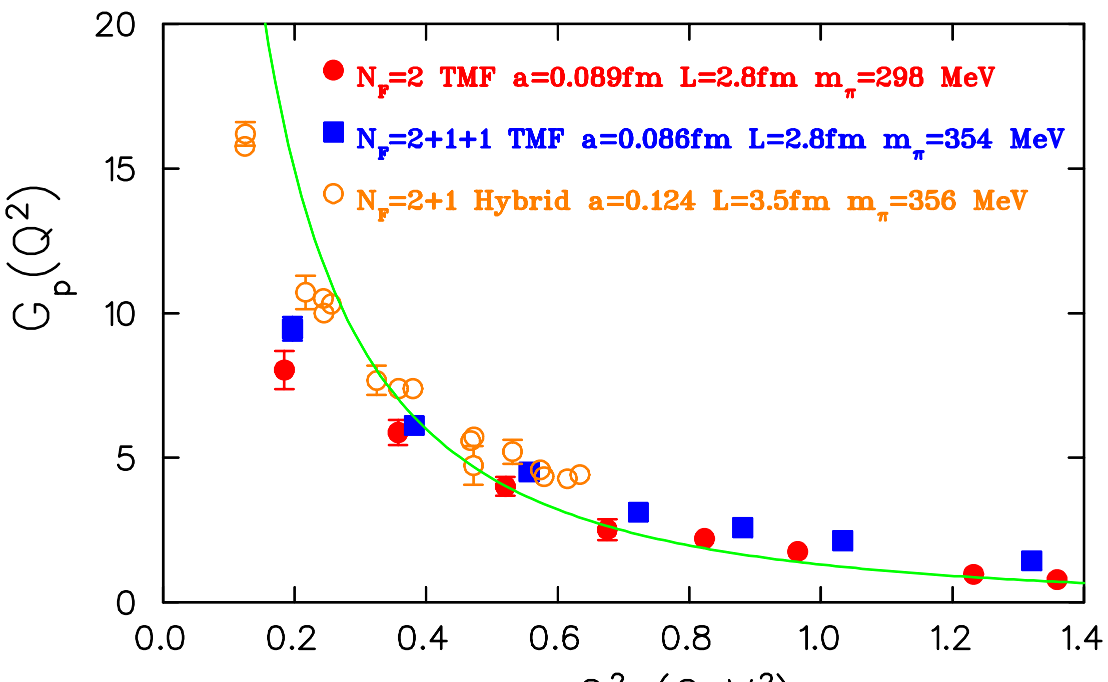
[J.R.Green et al (LHPC)]

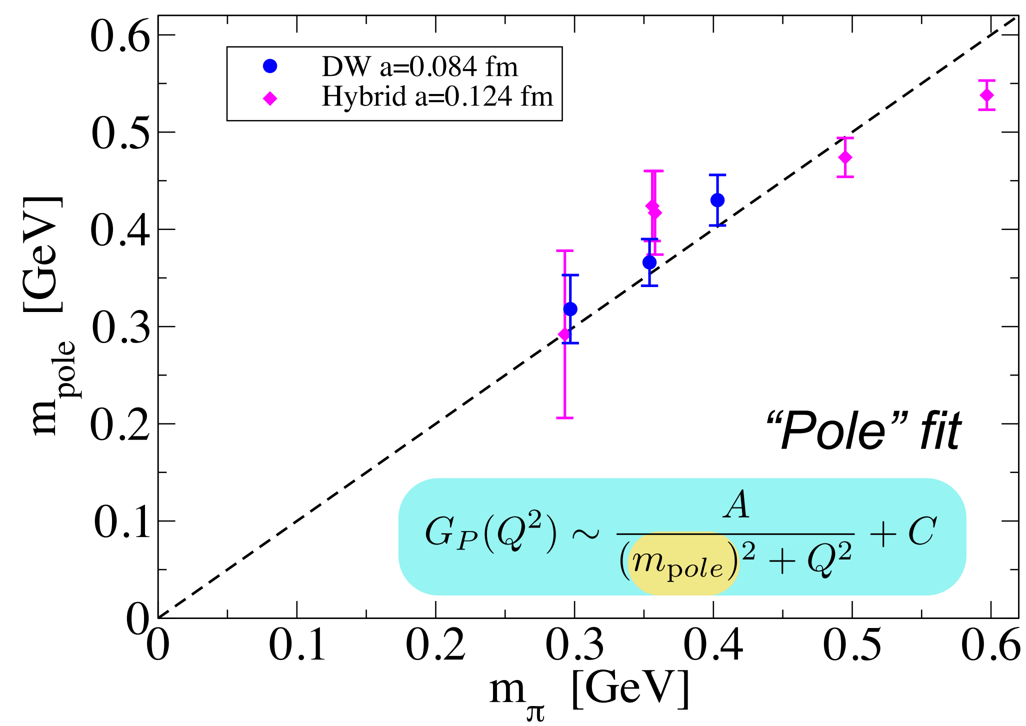
Nucleon Axial & Pseudoscalar Form Factors

$$\langle P+q|\,\bar{q}\gamma^\mu\gamma^5 q\,|P\rangle \;=\; \bar{U}_{P+q}\Big[G_A(Q^2)\,\gamma^\mu\gamma^5 + G_P(Q^2)\,\frac{\gamma^5 q^\mu}{2M_N}\Big]U_P$$



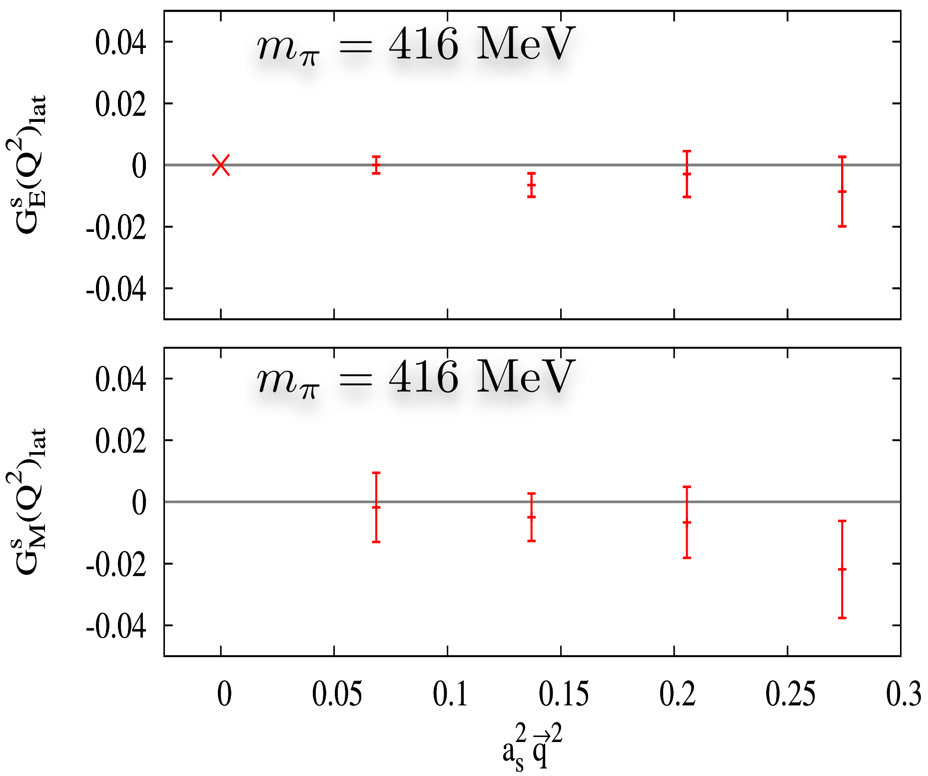


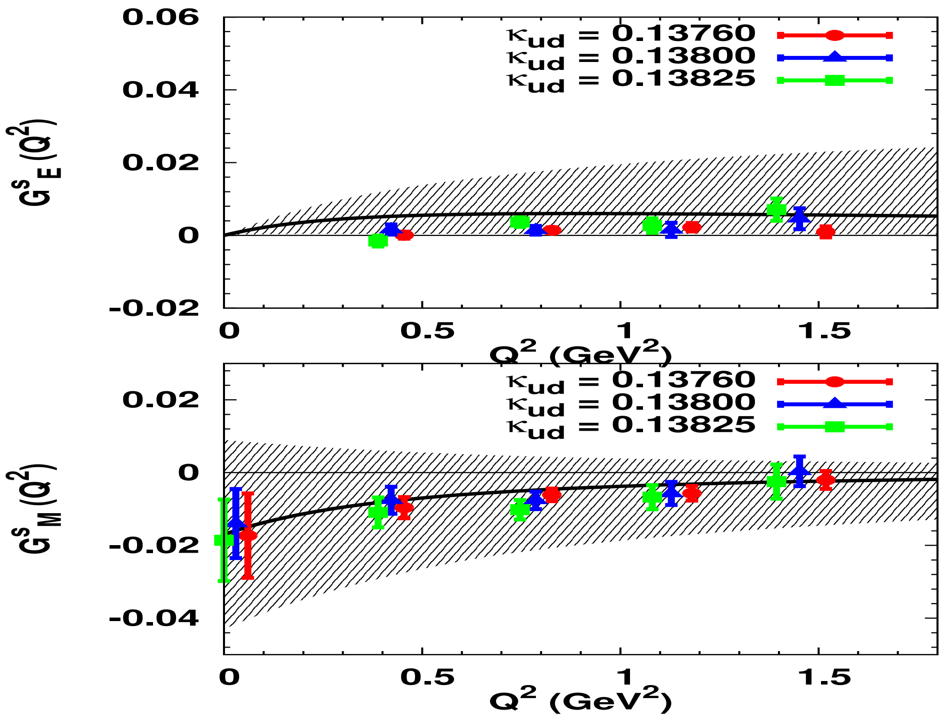




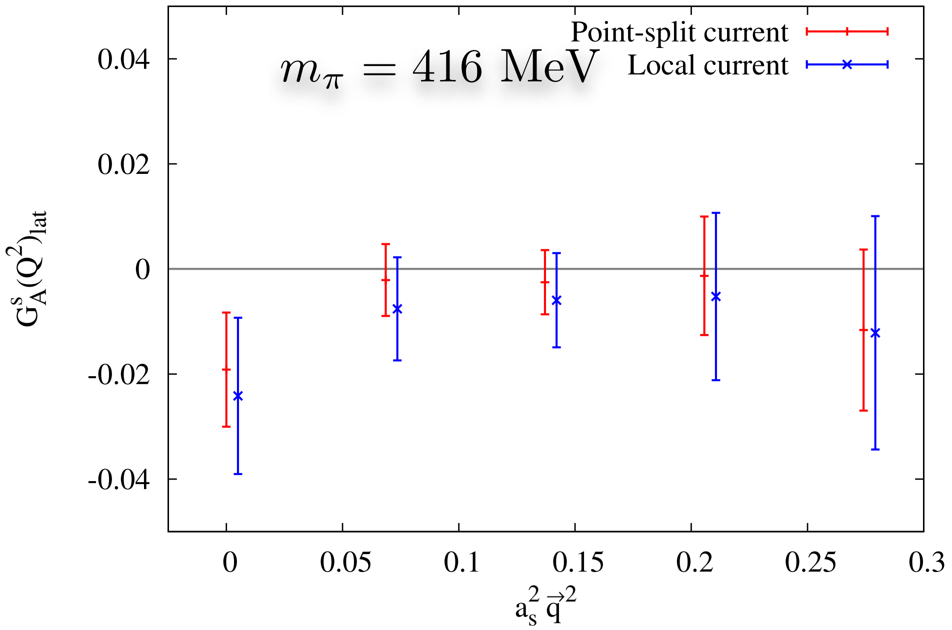
[C.Alexandrou (ETMC), 1303.5979]

Nucleon S-Quark VectorForm Factors





[T.Doi (ChiQCD), 1010.2834]



[R.Babich *et al,* (DISCO Collab.) Phys.Rev.D85, 054510]

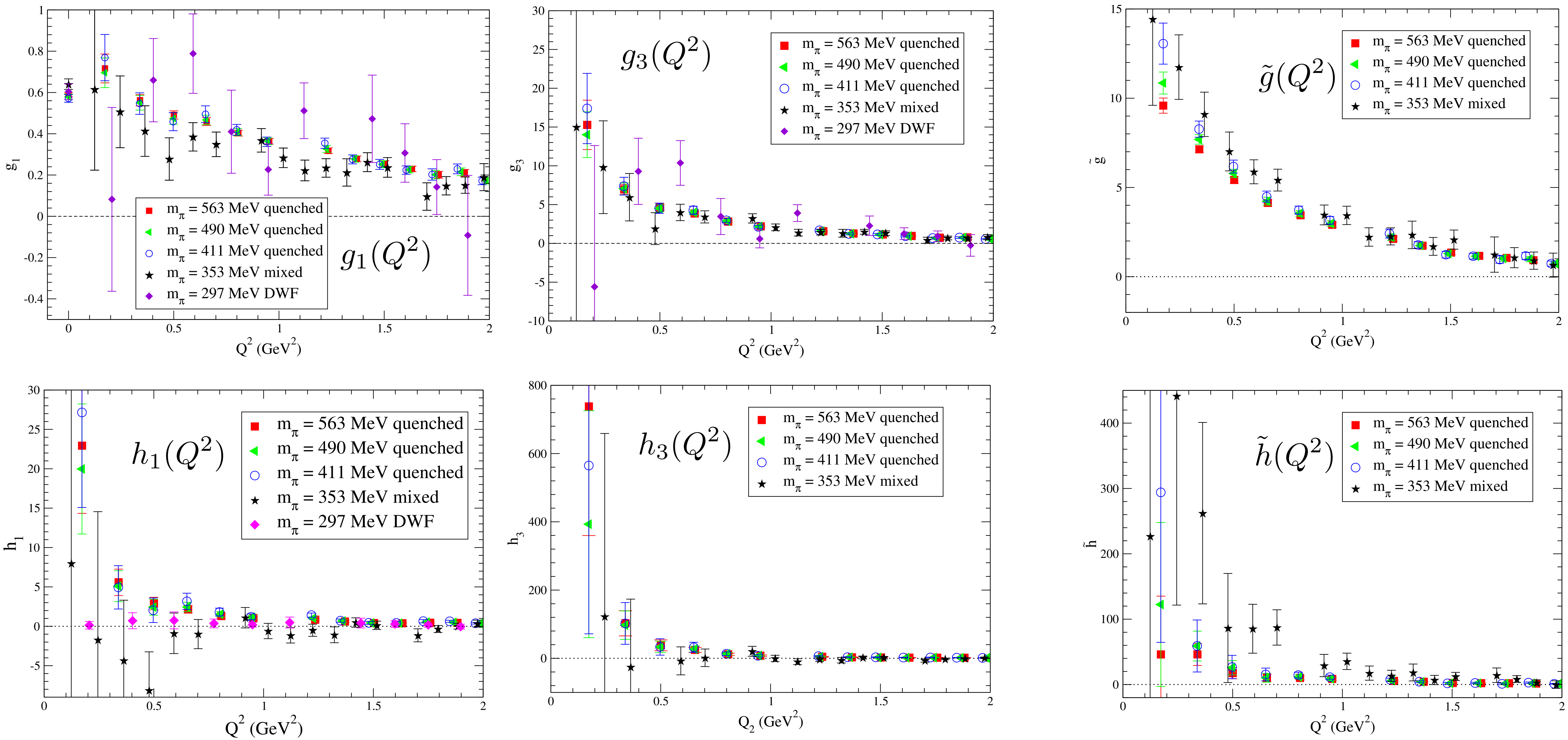
$$|G^s_{E,M,A}| \lesssim 1\% \text{ of } |G^{u/d}_{E,M,A}|$$

Delta(1232) Axial & Pseudoscalar Form Factors

[C.Alexandrou *et al* (ETMC), 1304.4614]

$$\langle \Delta^+ | \bar q \gamma^\mu \gamma^5 \tau^3 q | \Delta^+ \rangle = -\bar u_\sigma \Big[g^{\sigma\tau} \Big(g_1(Q^2)\gamma^\mu\gamma^5 + g_3(Q^2)\frac{q^\mu \gamma^5}{2M_\Delta}\Big) + \frac{q^\sigma q^\tau}{4M_\Delta^2}\Big(h_1(Q^2)\gamma^\mu\gamma^5 + h_3(Q^2)\frac{q^\mu\gamma^5}{2M_\Delta}\Big)\Big] u_\tau$$

$$\langle \Delta^+|\bar q\gamma^5\tau^3 q|\Delta^+\rangle = -\bar u_\sigma\Big[g^{\sigma\tau}\tilde g(Q^2)\gamma^5 + \frac{q^\sigma q^\tau}{4M_\Delta^2}\tilde h(Q^2)\gamma^5\Big]u_\tau$$

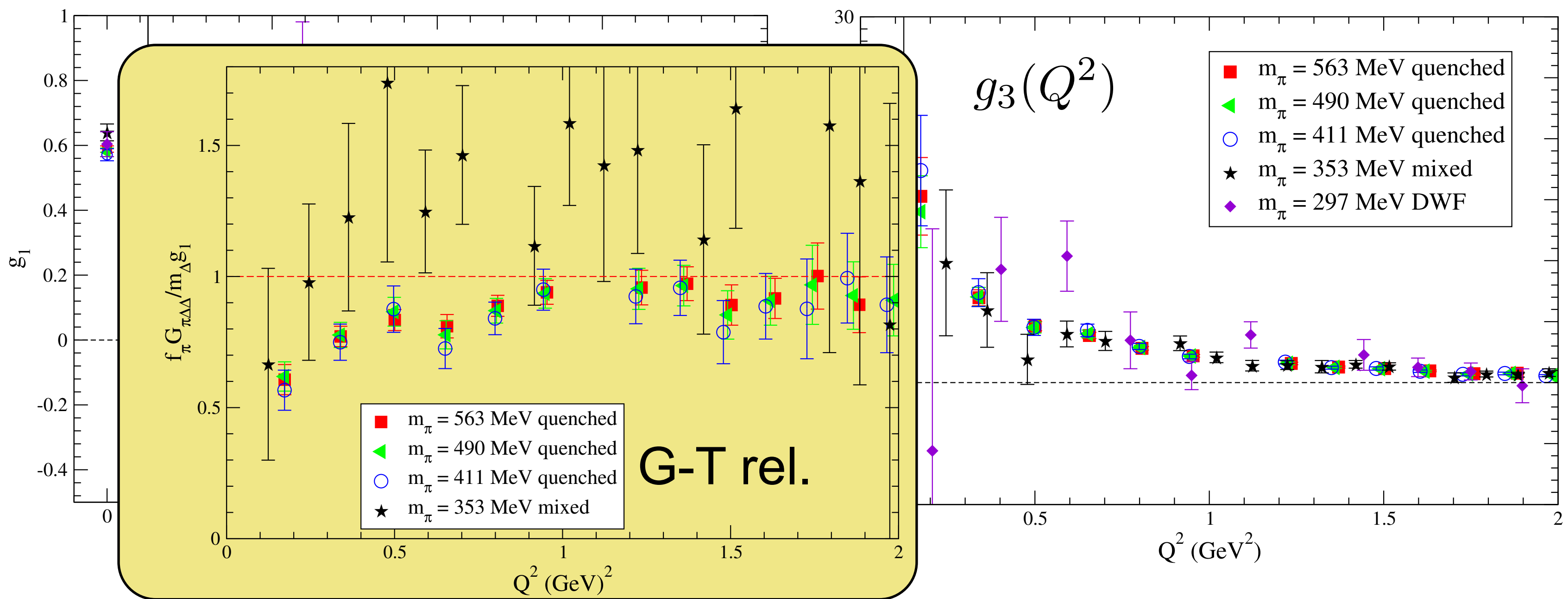


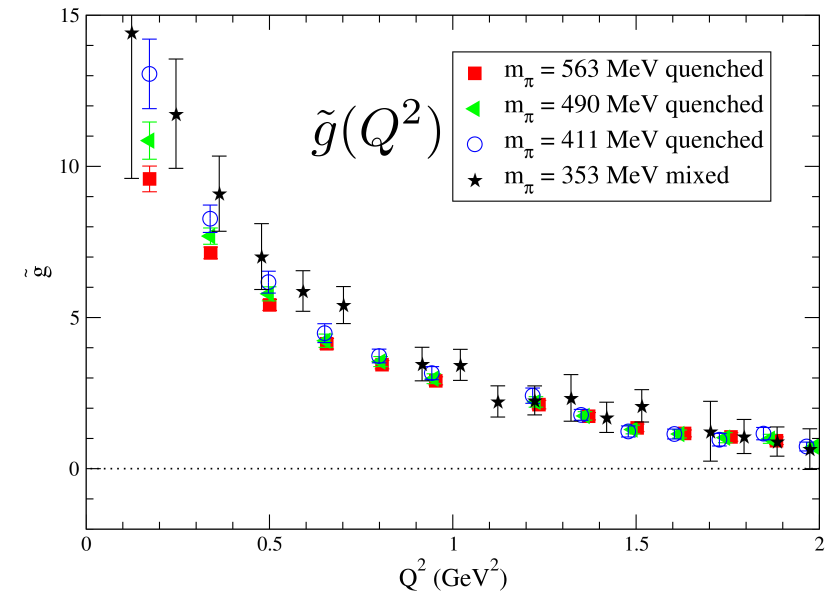
Delta(1232) Axial & Pseudoscalar Form Factors

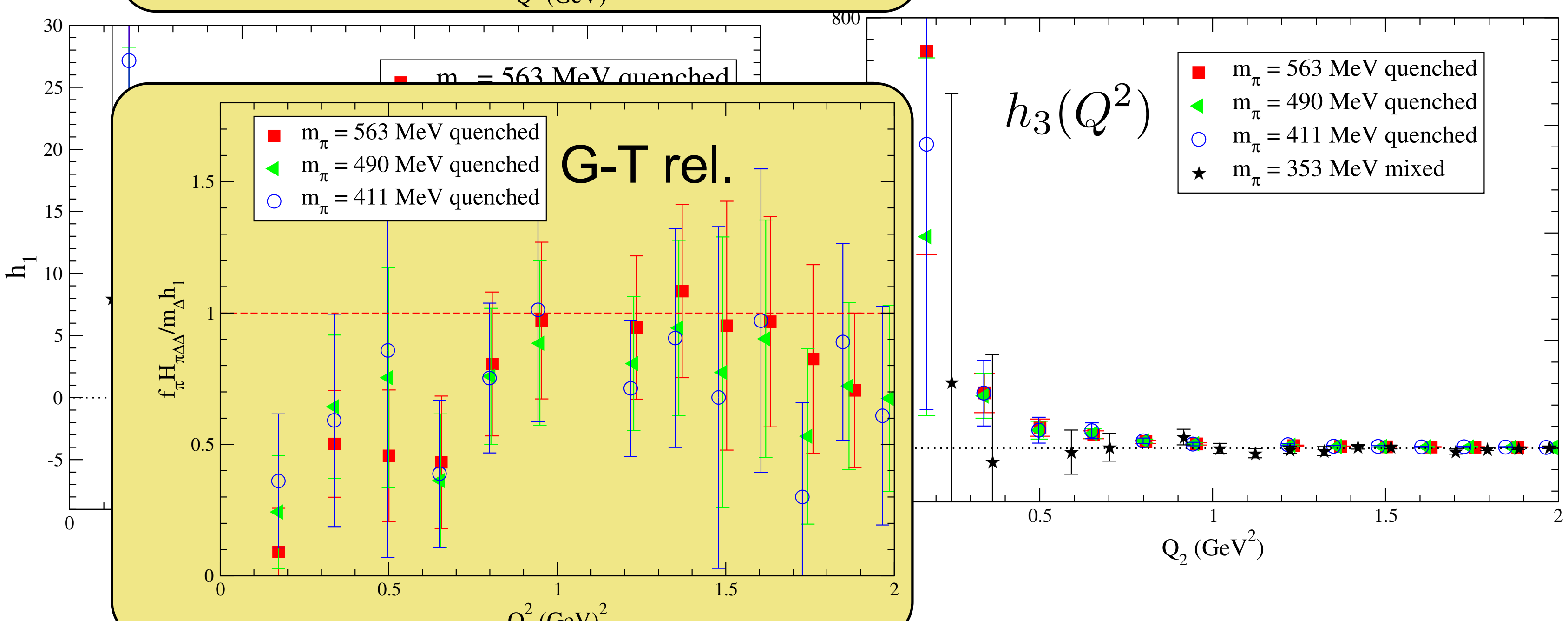
[C.Alexandrou *et al* (ETMC), 1304.4614]

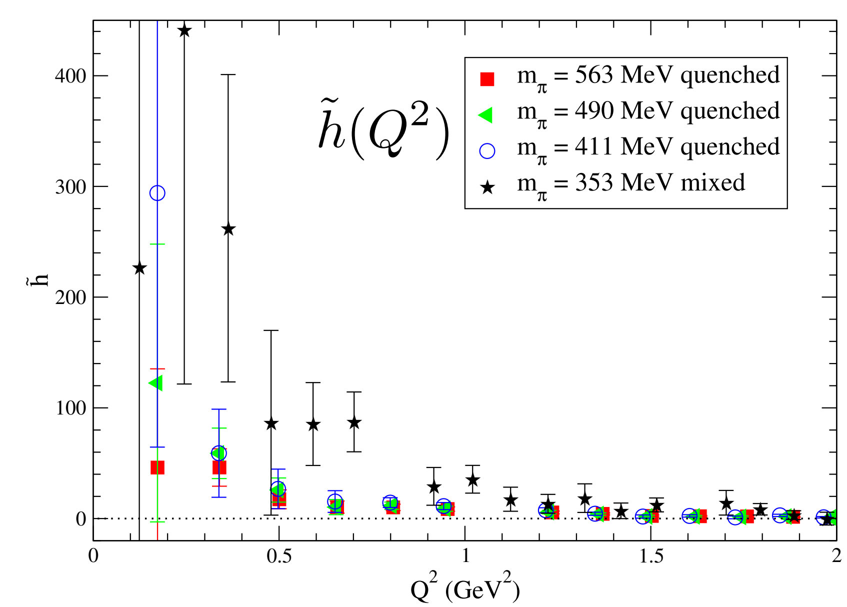
$$\langle\Delta^+|\bar q\gamma^\mu\gamma^5\tau^3 q|\Delta^+\rangle = -\bar u_\sigma\left[g^{\sigma\tau}\left(g_1(Q^2)\gamma^\mu\gamma^5 + g_3(Q^2)\frac{q^\mu\gamma^5}{2M_\Delta}\right) + \frac{q^\sigma q^\tau}{4M_\Delta^2}\left(h_1(Q^2)\gamma^\mu\gamma^5 + h_3(Q^2)\frac{q^\mu\gamma^5}{2M_\Delta}\right)\right]u_\tau$$

$$\langle\Delta^+|\bar q\gamma^5\tau^3 q|\Delta^+\rangle = -\bar u_\sigma\left[g^{\sigma\tau}\tilde g(Q^2)\gamma^5 + \frac{q^\sigma q^\tau}{4M_\Delta^2}\tilde h(Q^2)\gamma^5\right]u_\tau$$





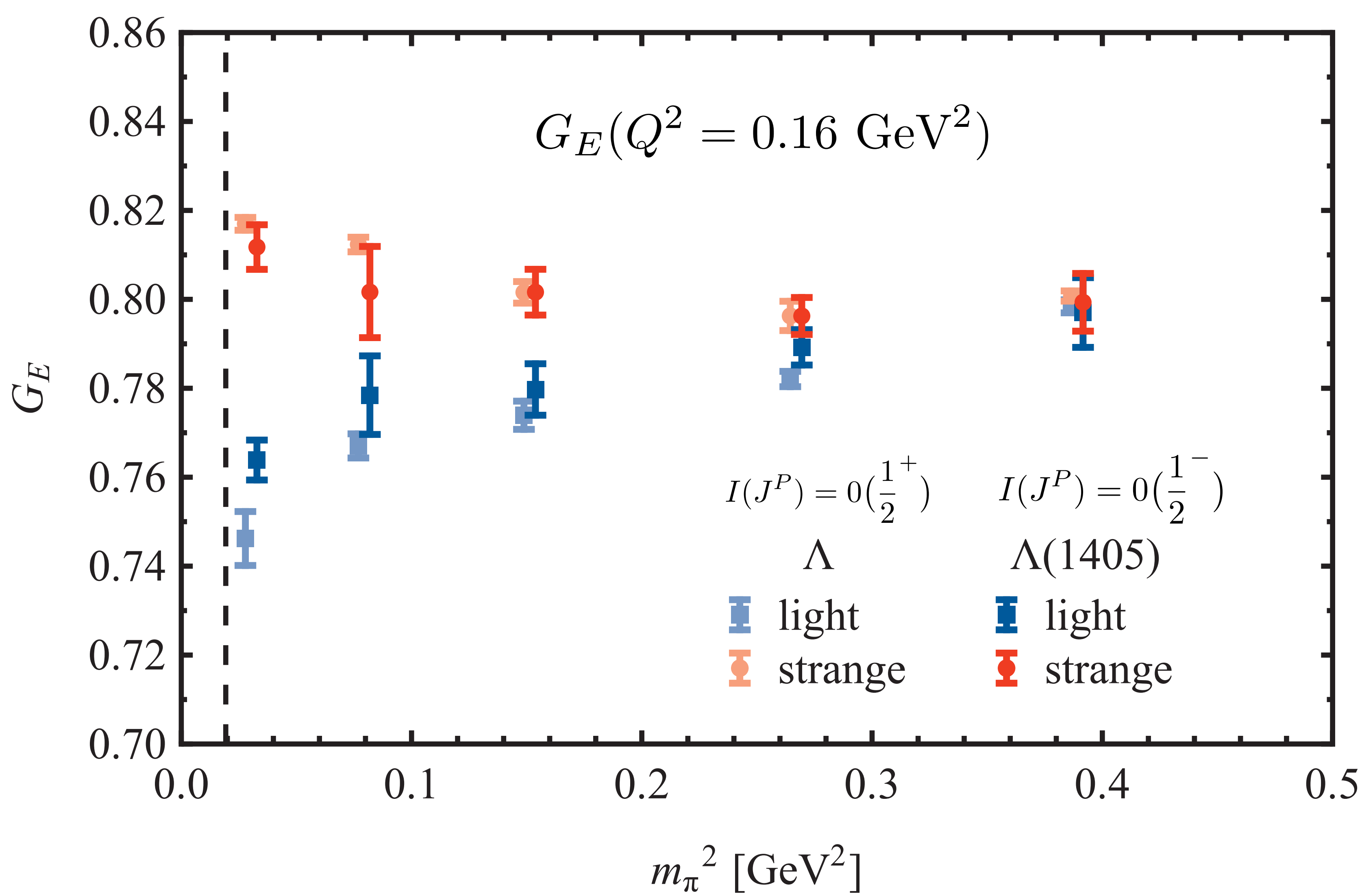




$\Lambda(1405)$ Electric Form Factor

[B.Menadue *et al*, (CSSM); Sec.8B(Thu)]

6x6 Variational analysis: 2 octets + 1 singlet $\otimes$ N=16,100 smearing



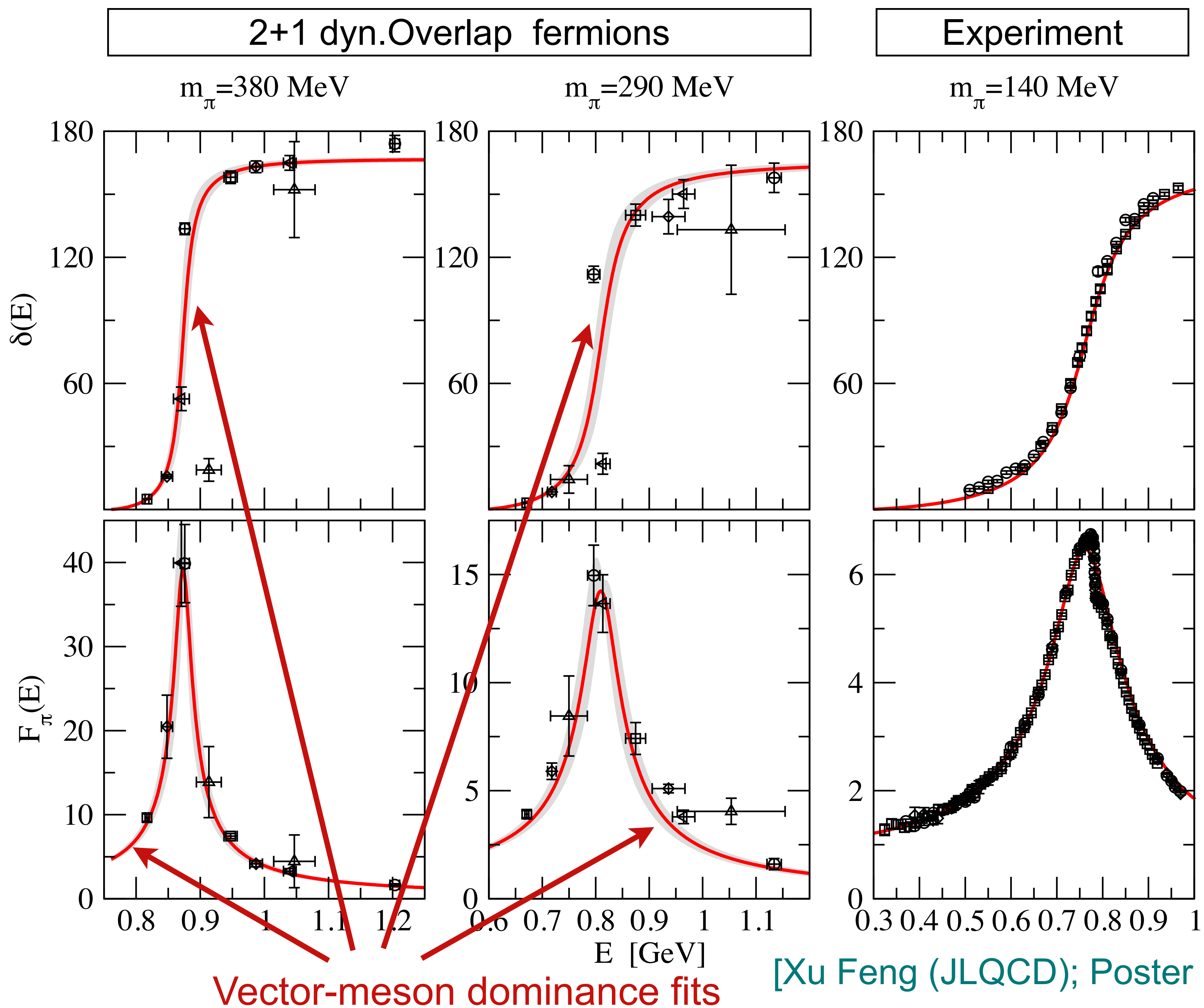
In $\Lambda(1405) \longleftrightarrow \bar{K}N$, virtual cloud of $\bar{K} = (s\,\bar{q}_{\mathrm{light}})$

enhances $\langle r^2 \rangle^s$ and *shrinks* $\langle r^2 \rangle^{u,d}$

Timelike Pion Form Factor

$$|\langle \Omega \,|\, J_\mu \,|\, (\pi^+\pi^-)_{l=1} \rangle|^2 \;\longrightarrow\; |F_\pi(t = E_{\pi\pi}^2)|^2$$

[H.B.Meyer, PRL 107:072002(2011); arxiv:1105.1892]

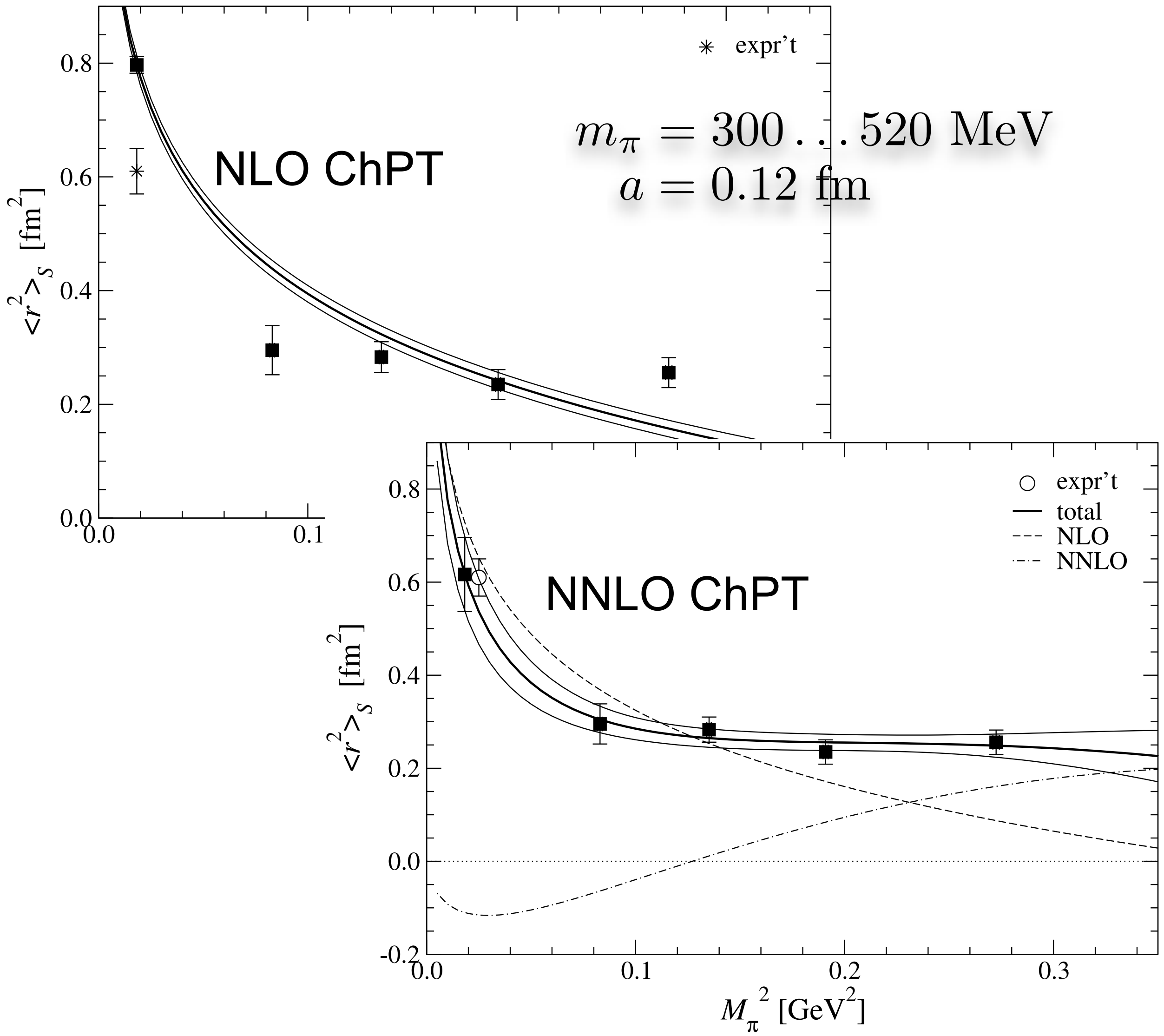


[Xu Feng (JLQCD); Poster sessn.]

Scalar Radius of the Pion

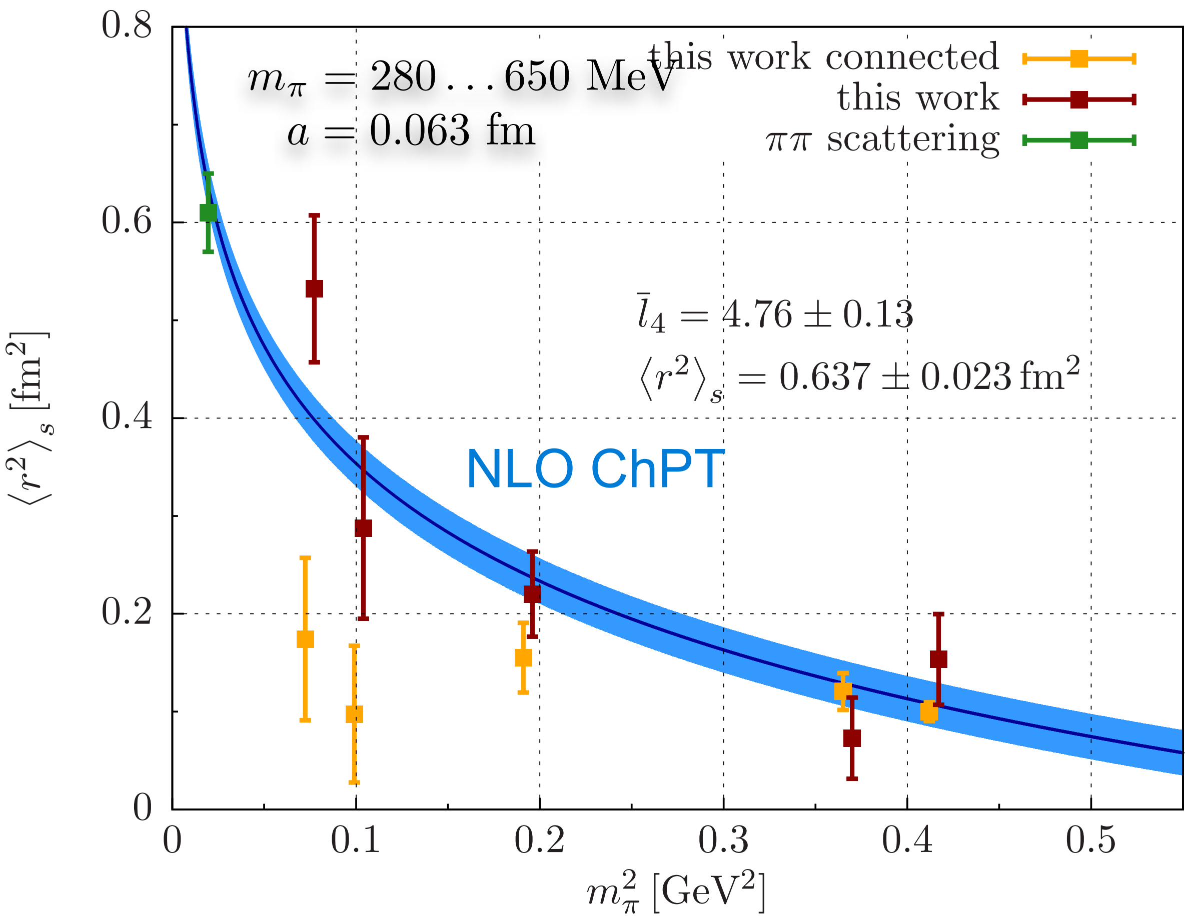
$$F_s(Q^2) = \langle \pi^+(p+q) | m_u \bar{u}u + m_d \bar{d}d | \pi^+(p) \rangle$$

[V.Guelpers, H.Wittig, G.von Hippel]



Nf=2 dyn.overlap
[S.Aoki *et al*(JLQCD & TWQCD) PRD80:034508]

Nf=2 *O(a)*-improved Wilson Fermions



Agreement with phenomenology
[Colangelo et al, Nucl.Phys.B603,125] :

$$\langle r^2 \rangle_s^\pi = 0.61(4)\ \mathrm{fm}^2$$

Large disconnected contributions

Origin of the Nucleon Spin

Proton spin puzzle:

1989 EMC experiment finds $$\Delta\Sigma = \sum_q (\Delta q + \Delta\bar{q}) = 0.2\ldots0.3$$

Spin sum rule:

$$J_{\rm glue} + \sum_q J_q = \frac{1}{2}\,,$$
$$J_q = \frac{1}{2}\Delta\Sigma_q + L_q$$

Quark Spin:

$$\langle N(p)|\bar{q}\gamma^\mu\gamma^5 q|N(p)\rangle = (\Delta\Sigma_q)\left[\bar{u}_p\gamma^\mu\gamma^5 u_p\right]$$

Angular momentum (J_q):

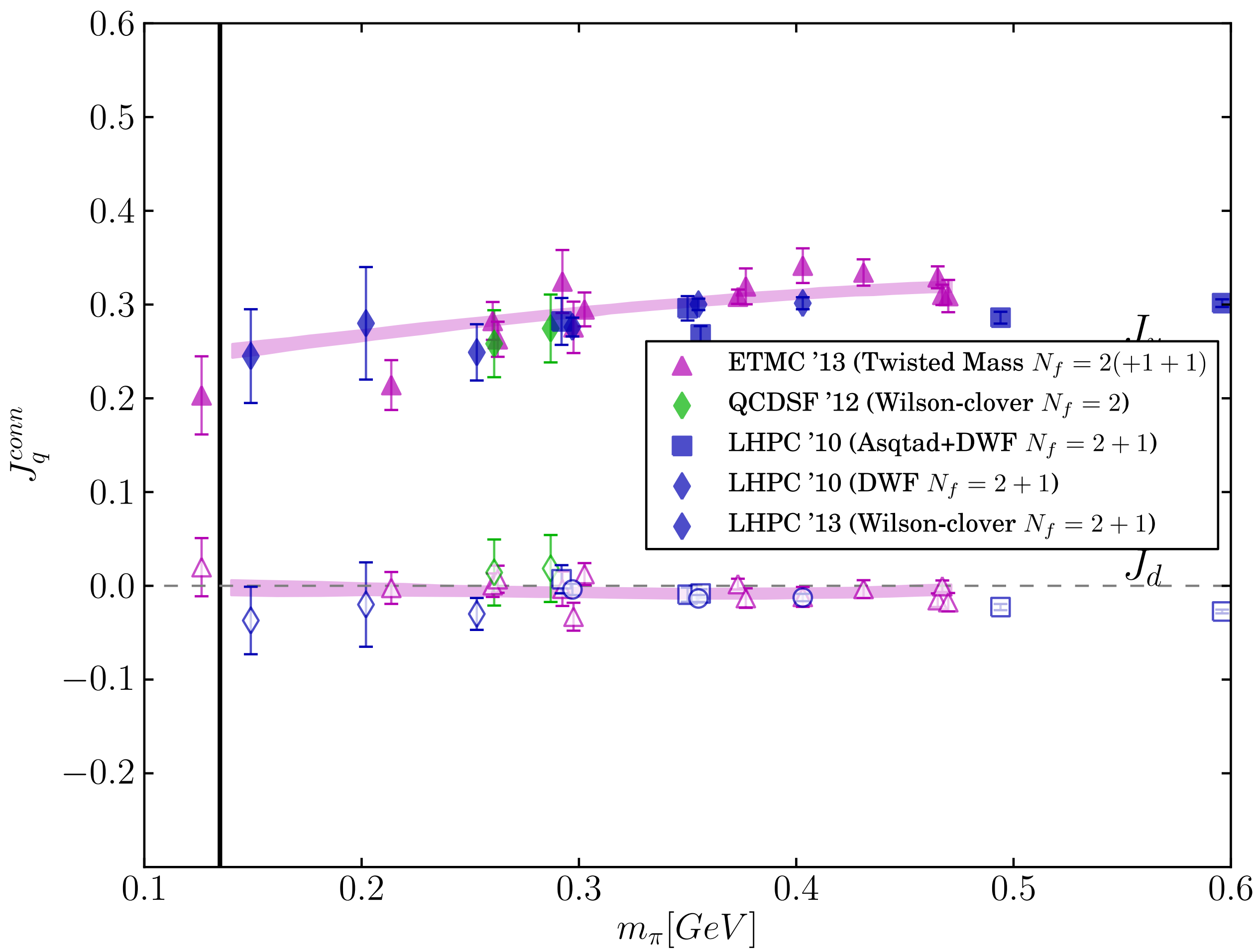
$$J_{q,glue} = \frac{1}{2}\left[A_{20}^{q,glue}(0) + B_{20}^{q,glue}(0)\right]$$

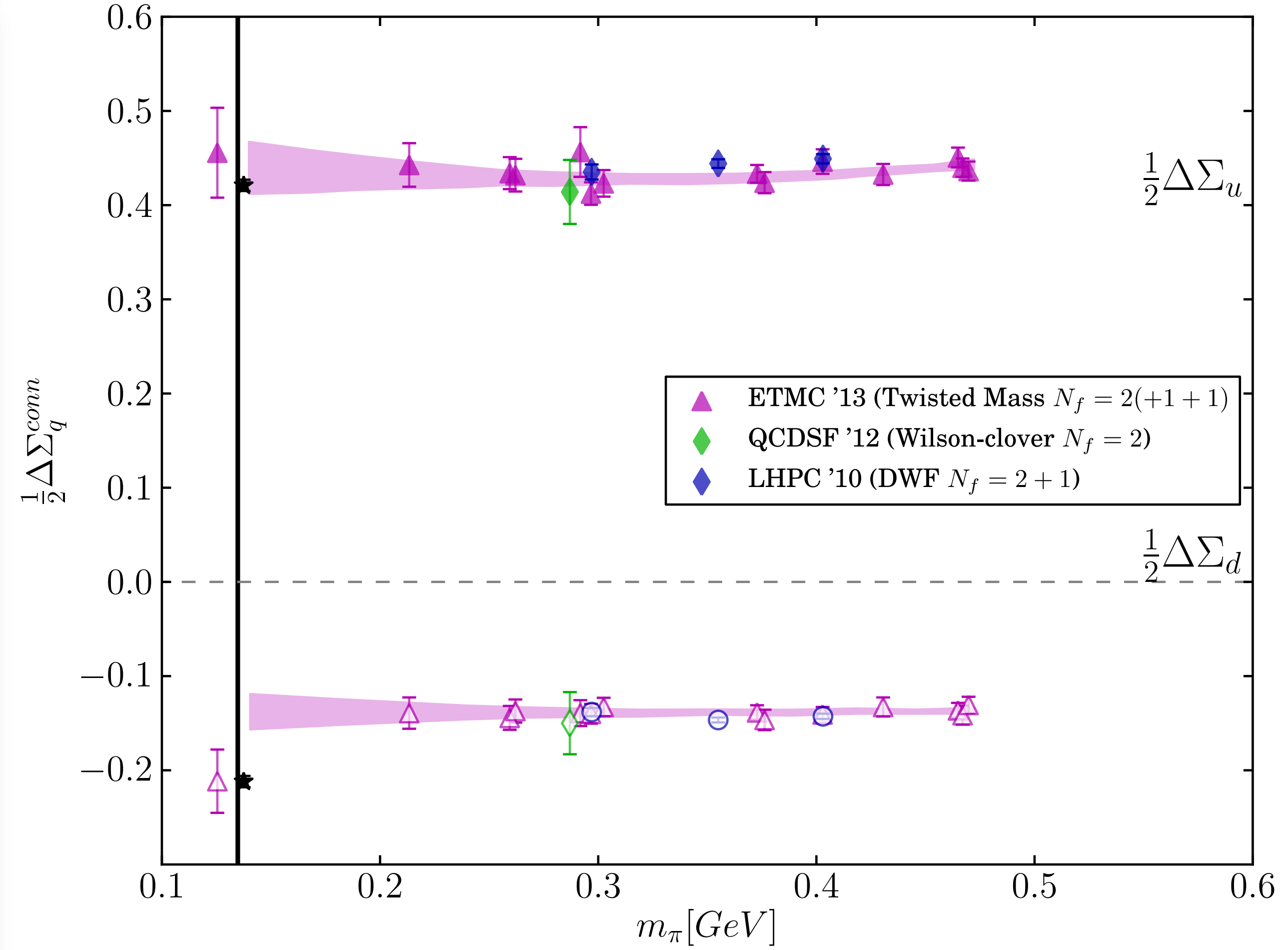
where A_{20} , B_{20} are E.-M. tensor form factors:

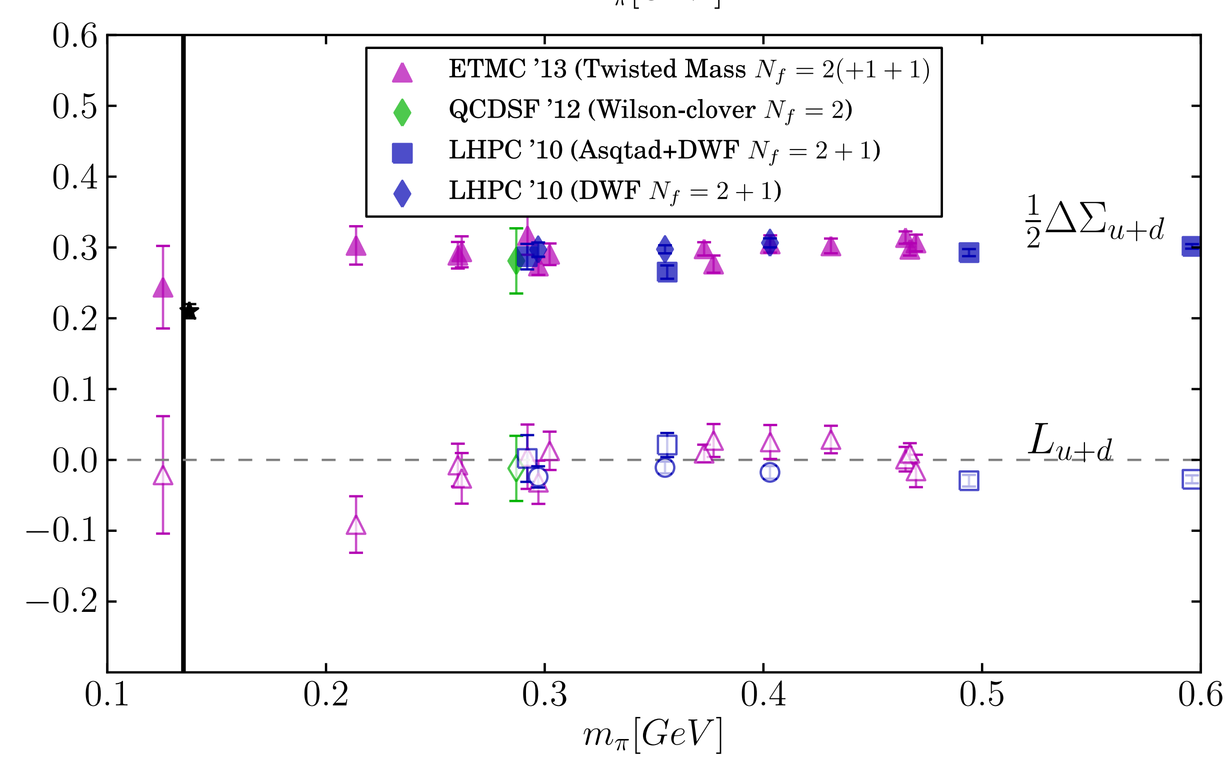
$$\langle N(p+q)|\,T_{\mu\nu}^{q,glue}\,|N(p)\rangle \to \left\{A_{20},\,B_{20},\,C_{20}\right\}(Q^2)$$

$$T_{\mu\nu}^q = \bar{q}\,\gamma_{\{\mu}\overset{\leftrightarrow}{D}_{\nu\}}\,q$$
$$T_{\mu\nu}^{\rm glue} = G_{\mu\lambda}^a G_{\nu\lambda}^a - \frac{1}{4}\delta_{\mu\nu}(G_{\mu\nu})^2$$

Quark Angular Momentum and Spin (Connected)







$$J_u \approx 40 - 50\% \quad *$$

$$|J_d| \lesssim 10\% \quad *$$

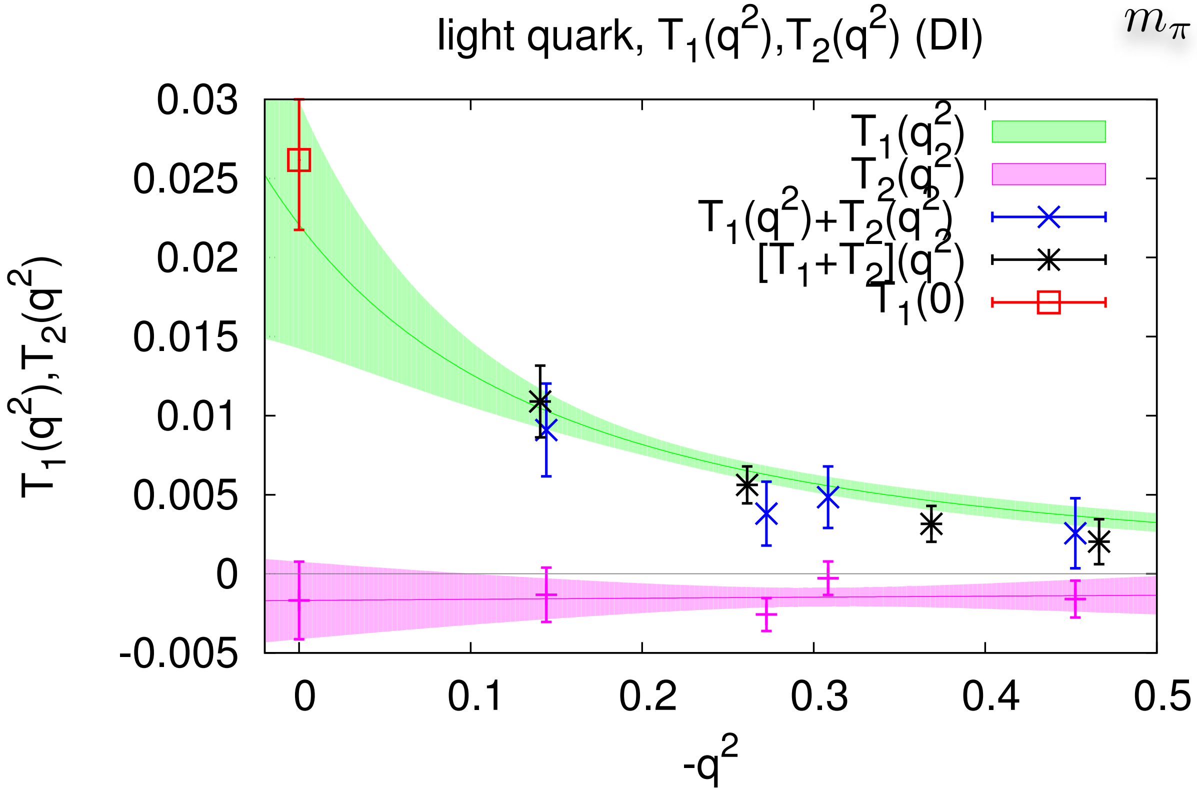
$$|L_{u+d}| \ll \frac{1}{2}\Delta\Sigma_{u+d} \quad *$$

(*) not including disconnected diagrams!

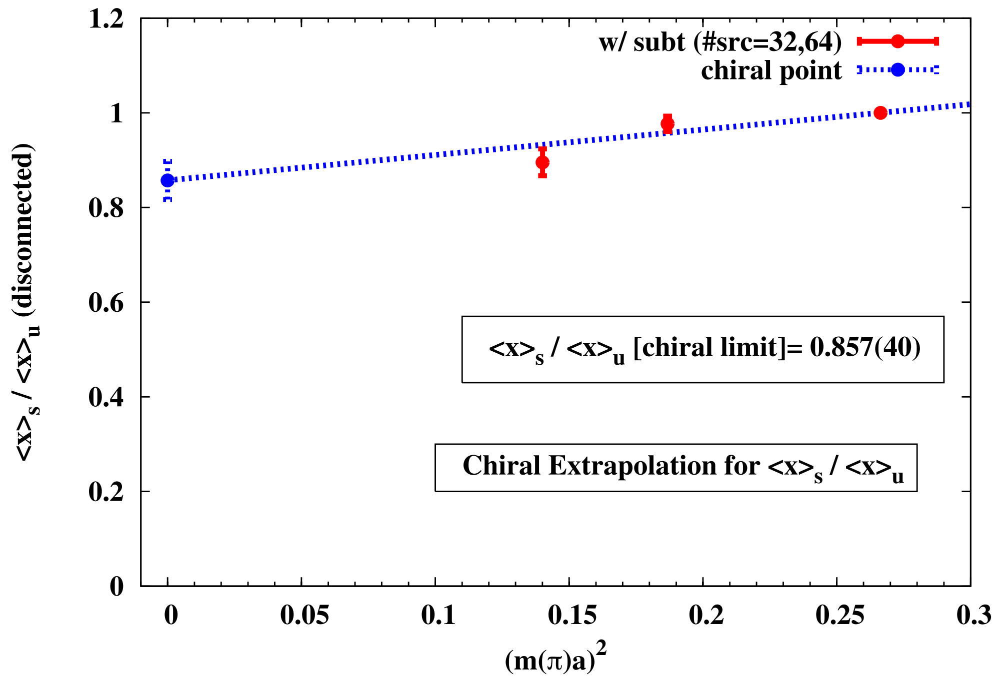
Disconnected Quark Angular Momentum

[K.F.Liu (ChiQCD), arXiv:1203.6388]

$m_\pi = 478 \ldots 650$ MeV



strange quark $\langle x \rangle$



$$\langle x \rangle_{u+d,(DI)} = 0.076(14)$$
$$2J_{u+d,(DI)} = 0.072(14)$$

$$\langle x \rangle_s = 0.024(6)$$
$$2J_s = 0.023(7)$$

(chiral extrapolation values)

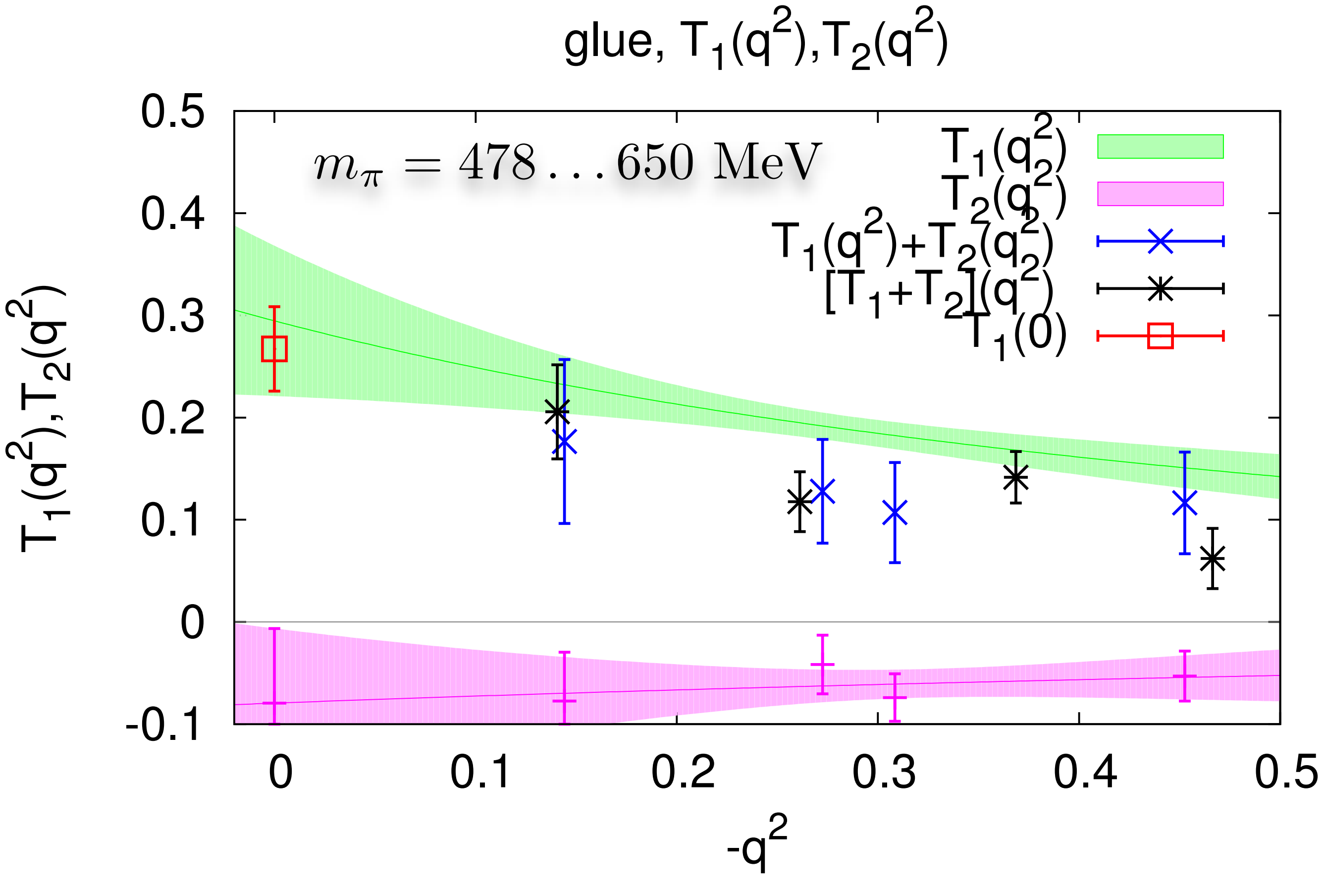
Gluon Momentum and Angular Momentum

[K.F.Liu (ChiQCD), arXiv:1203.6388]

(Quenched fermions)

Suppress UV fluctuations with the overlap operator:

$$\hat{G}_{\mu\nu} = \frac{1}{c_T a^2}\mathrm{Tr}_{\mathrm{spin}}\left[\sigma_{\mu\nu}D_{ov}(x,x)\right] + \mathcal{O}(a)$$



$$\langle x\rangle_{\mathrm{glue}} = T_1(0) = 0.313(56)$$

$$2J_{\mathrm{glue}} = T_1(0) + T_2(0) = 0.254(76)$$

[QCDSF (R. Horsley et al) Phys.Lett.B714:312]

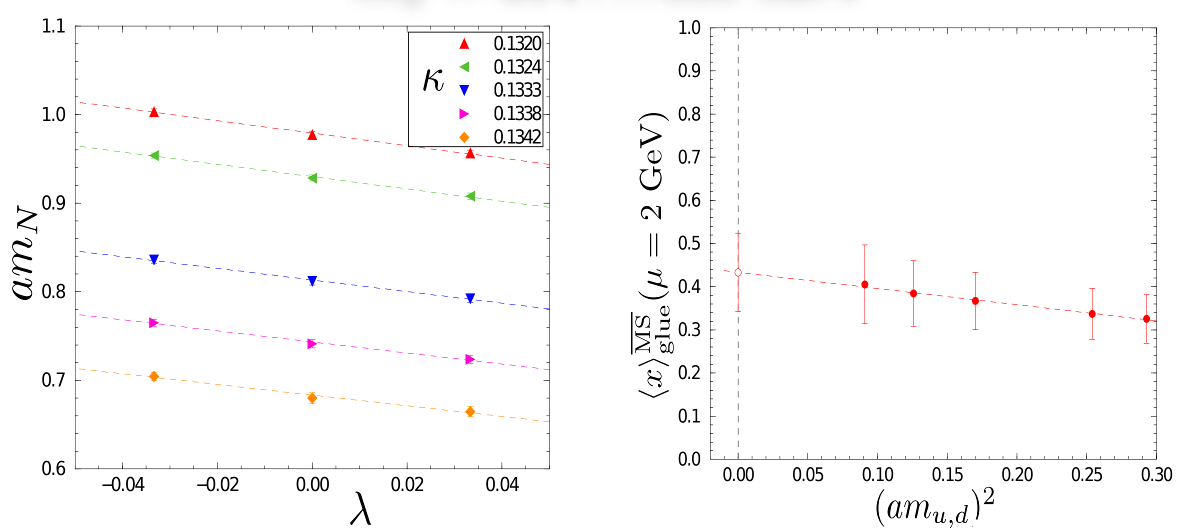
(Quenched fermions)

Background "field" :

$$S_{\mathrm{gauge}} \longrightarrow \underbrace{S_{\mathrm{gauge}}}_{\frac{1}{2}\left[(\mathcal{E}^a)^2+(\mathcal{B}^a)^2\right]} - \lambda a \cdot \underbrace{\left(T_{00} - \frac{1}{3}T_{ii}\right)}_{\frac{1}{2}\left[-(\mathcal{E}^a)^2+(\mathcal{B}^a)^2\right]}$$

$$\langle x\rangle_{\mathrm{glue}} = -\frac{2}{3m_N}\frac{\partial m_N}{\partial\lambda}$$

$m_\pi = 314\ldots555$ MeV

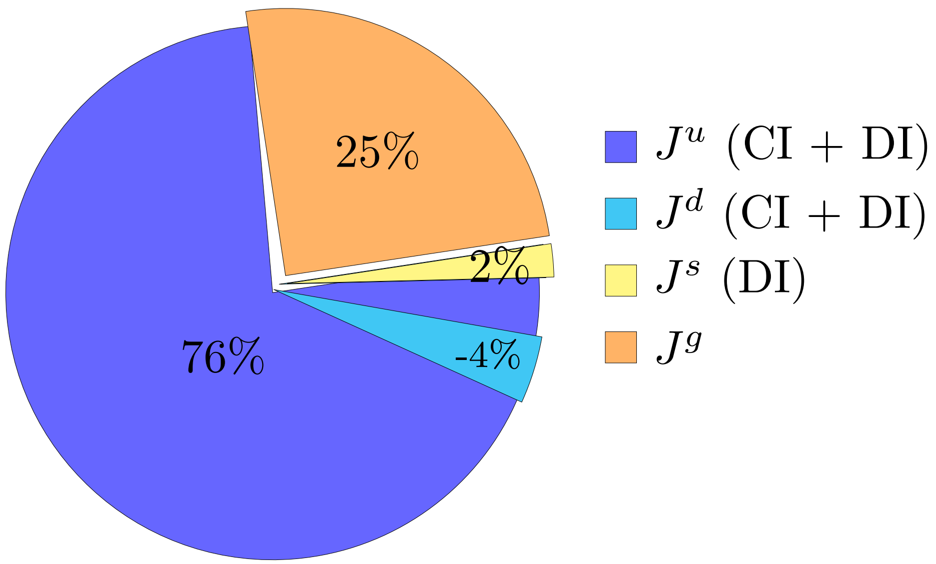


$$\langle x\rangle_{\mathrm{glue}} = 0.43(7)(5)$$

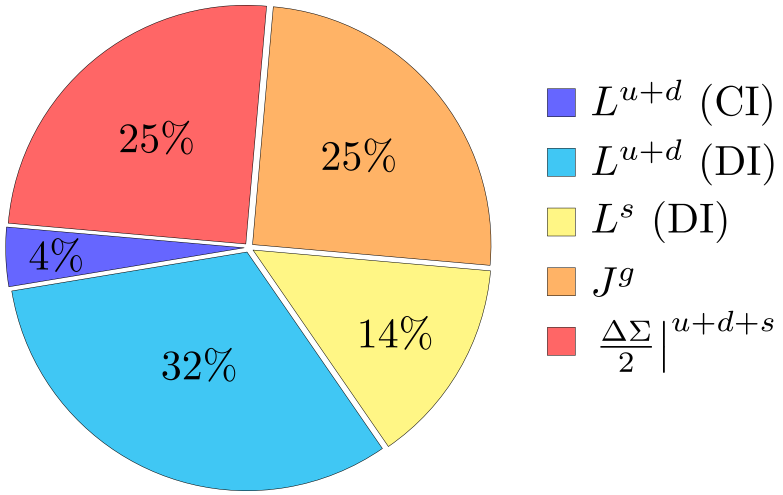
See also [C.Wiese *et al*, Sec.3B]

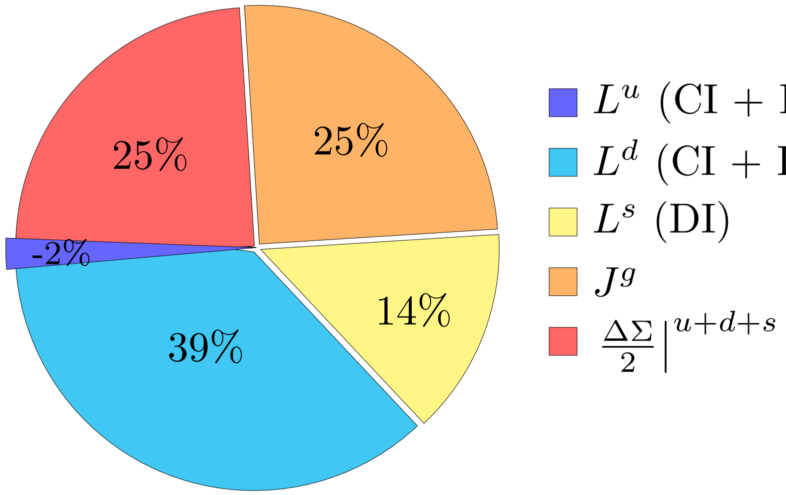
Angular momentum: Quenched studies

[K.F.Liu *et al*, '95]



$$2L^q = 2J^q - \Delta\Sigma^q$$

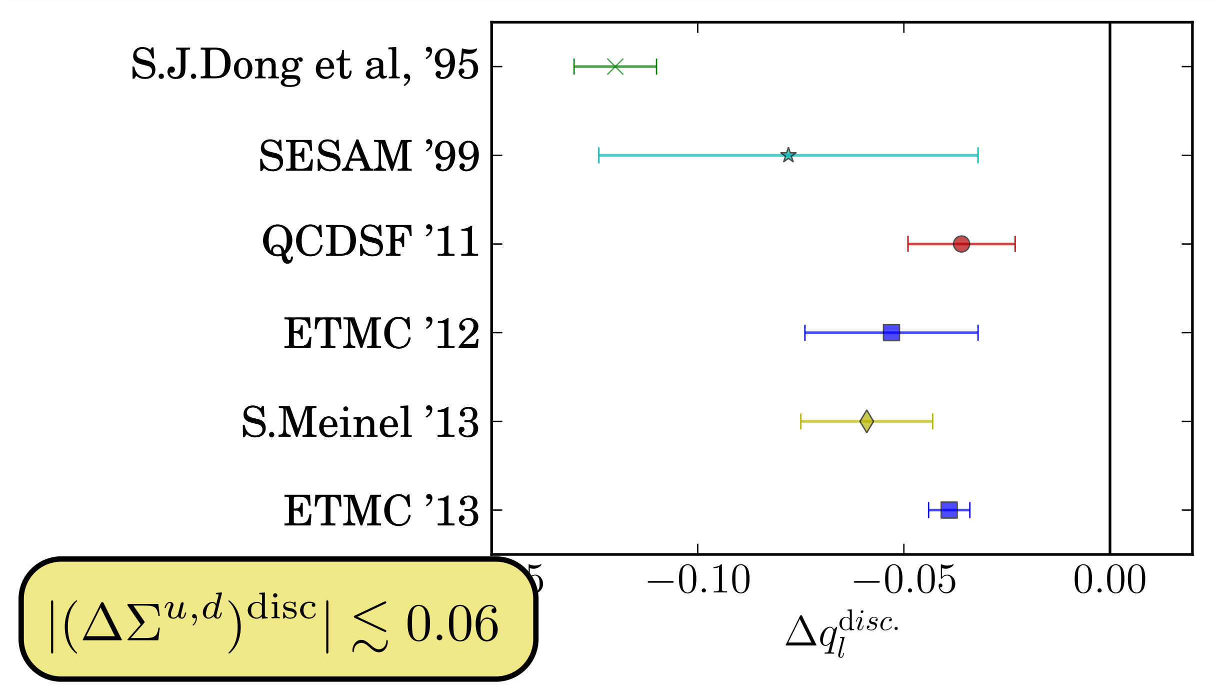


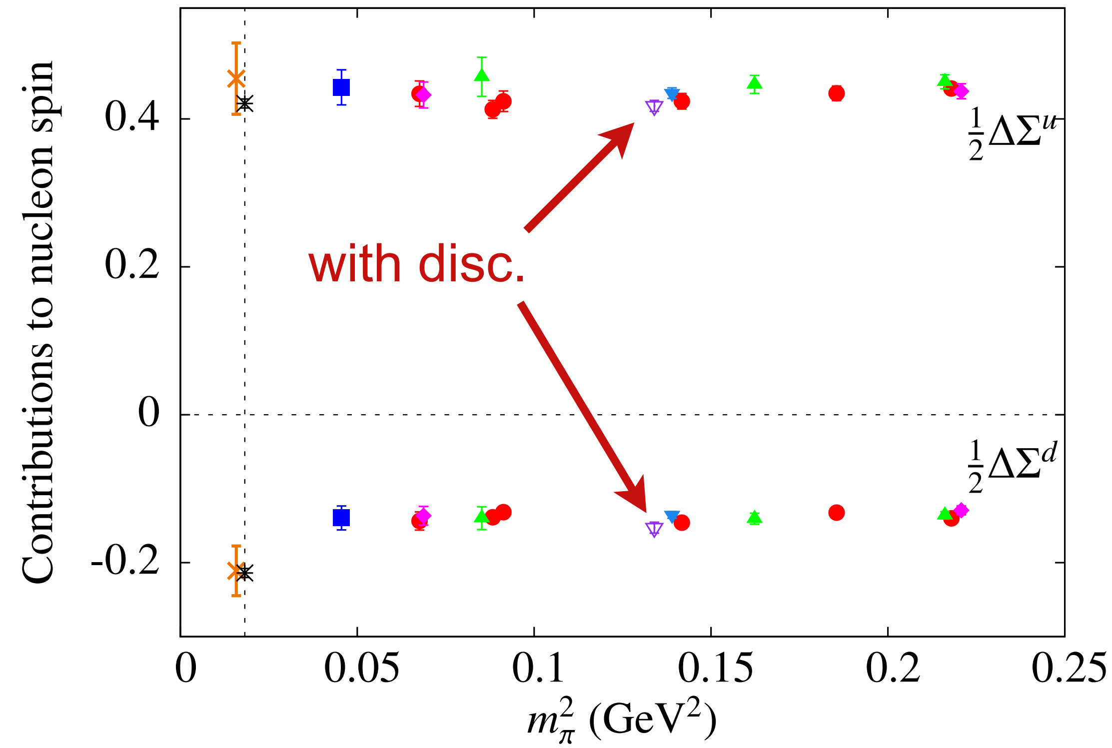


$$(\Delta\Sigma)^u_{\rm disc} = (\Delta\Sigma)^d_{\rm disc} \approx (\Delta\Sigma)^s_{\rm disc} \approx -0.12(1)$$

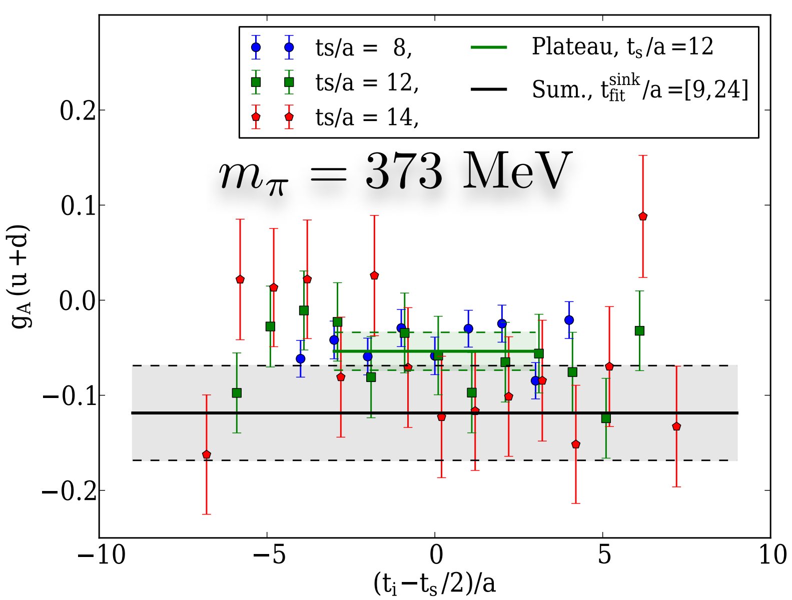
$$2L_{u+d} \approx 0.49 = 0.0\big|_{\rm conn} + 0.49\big|_{\rm disc}$$

(Disconnected) Light Quarks Spin

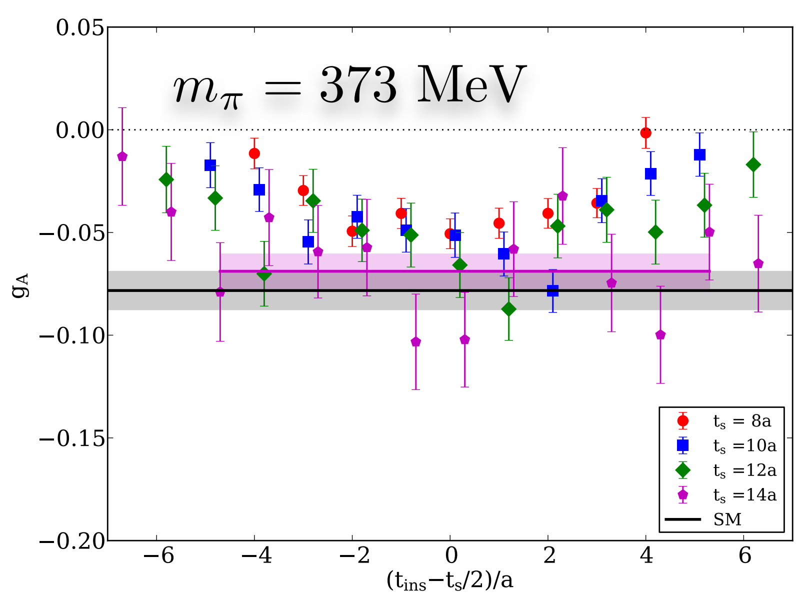




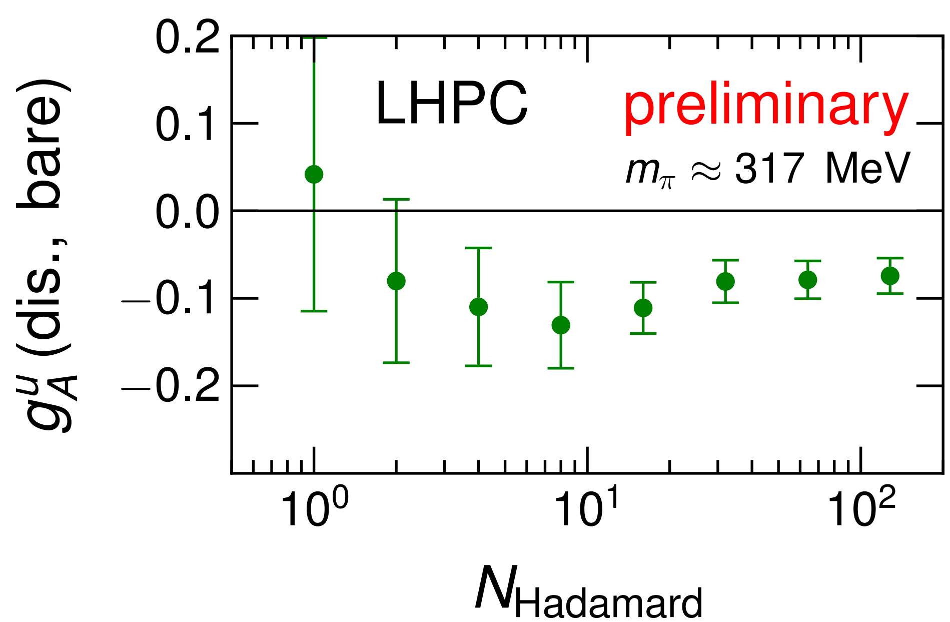
[C.Alexandrou et al (ETMC), 2013]



[C.Alexandrou et al (ETMC), 1211.0126]

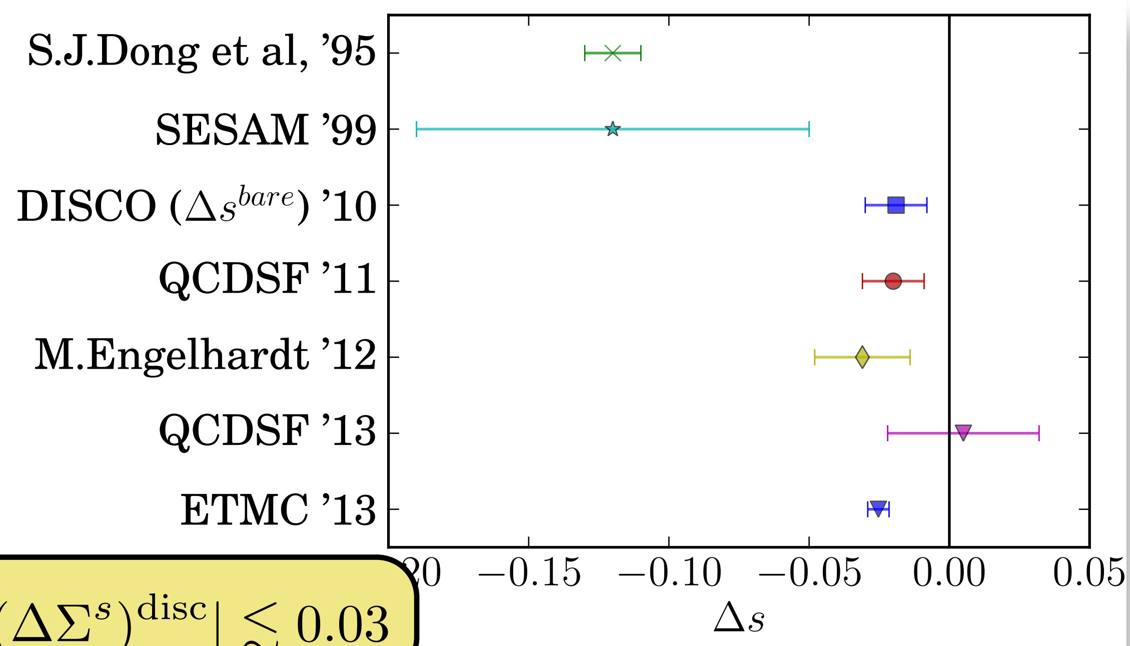


[C.Alexandrou et al (ETMC), 2013]



[S.Meinel '13 (LHPC)]
(Using hierarchical probing [K.Orginos 1302.4018])

Strange Quark Spin



$|(\Delta\Sigma^s)^{\rm disc}| \lesssim 0.03$

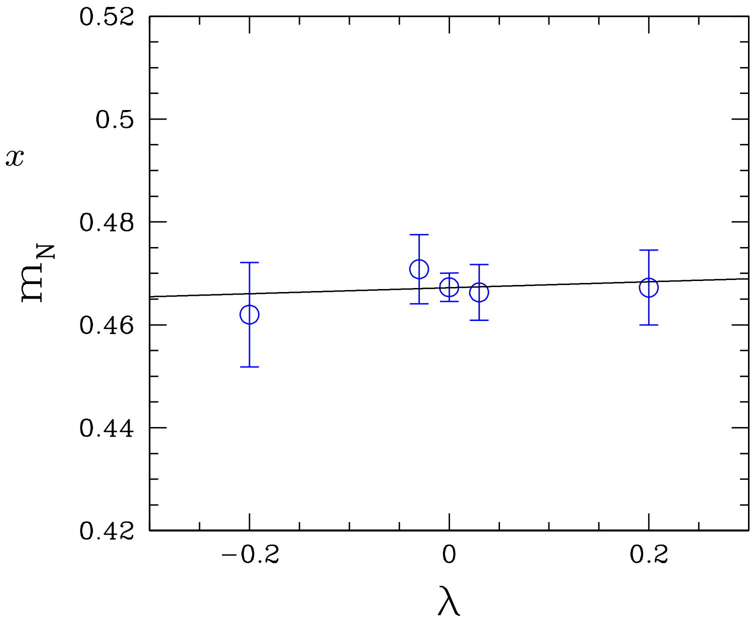
Background "field"

[QCDSF, '13]

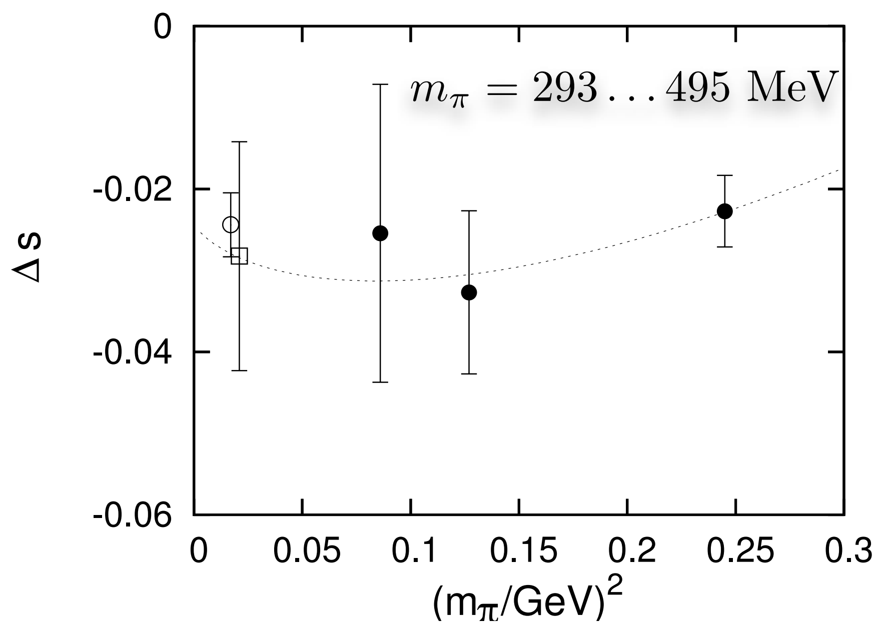
$$S = S_{\rm SLiNC} + \lambda \sum_x [\bar{s}\gamma_3\gamma_5 s]_x$$

$$\frac{\partial E_H}{\partial \lambda} = \langle N|\bar{s}\gamma_3\gamma_5 s|N\rangle$$

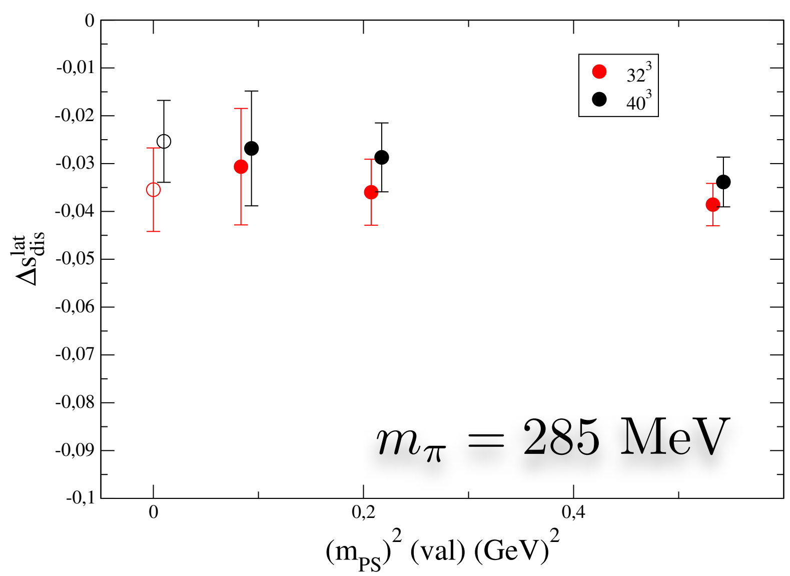
$$\Delta\Sigma^s = 0.005(24)$$



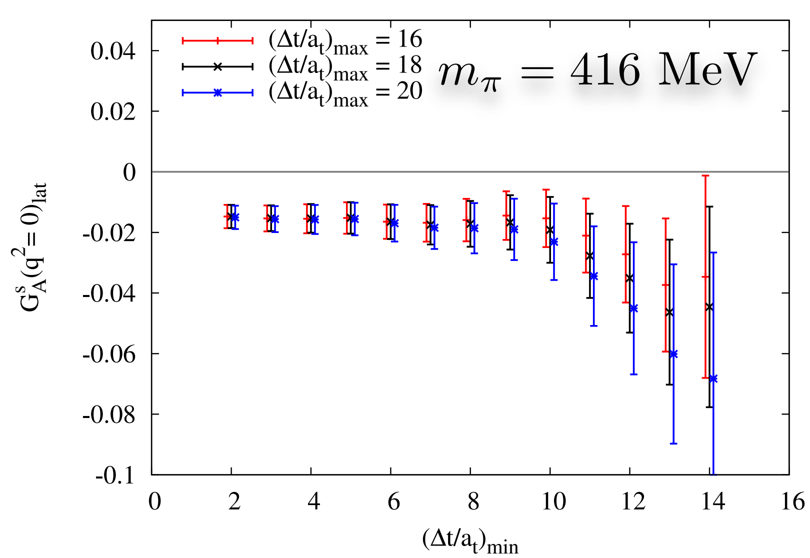
Stochastic estimation of the quark loop



[M.Engelhardt, Phys.Rev.D86, 114510]



[G.Bali *et al* (QCDSF) PRL 108, 222001]



[R.Babich *et al,* (DISCO Collab.) Phys.Rev.D85, 054510]

(Sub)Summary: Nucleon Spin

★ Quark spin from connected contractions agrees with phenomenology

★ Total quark orbital angular momentum is consistent with zero
(using only connected data for J_q and S_q)
although individual L_u and L_d are not zero

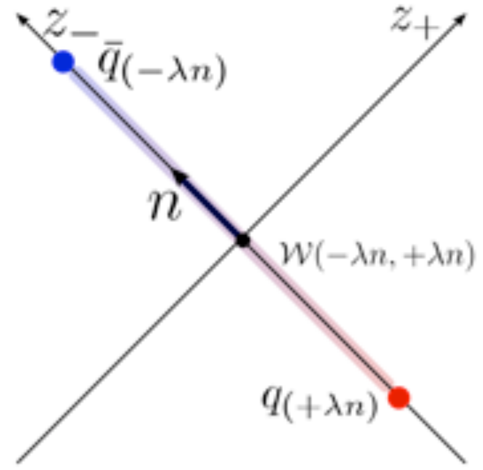
★ Older quenched calculations indicate L_{u+d} ~ 50%
(mostly due to disconnected contractions)

★ Newer dynamic fermion calculations yield much smaller values
and imply L_{u+d} ~ 20-30%

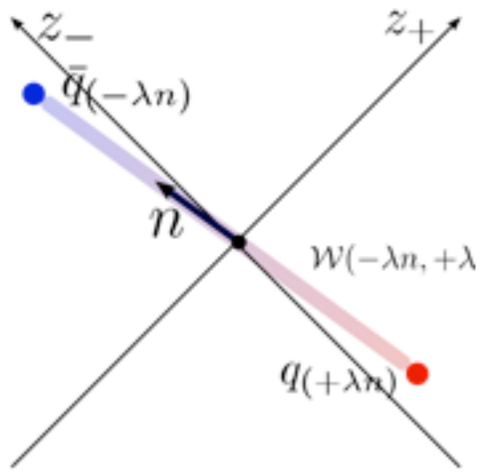
★ Need update for gluon angular momentum with dynamical fermions

Parton Distribution Functions on a Lattice

1. (TMD) PDFs = Quark-bilinear correlators separated by a light-cone shift

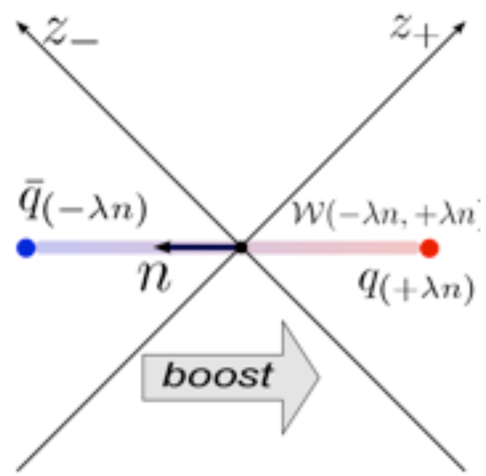


2. Relax the LC condition: slightly spacelike 4-vector *n*



3. Boost the system:
   Spatial separation is suitable for lattice QCD

4. Recover LC physics in $n \cdot P \to \infty$ limit



TMDs from Lattice: Formalism

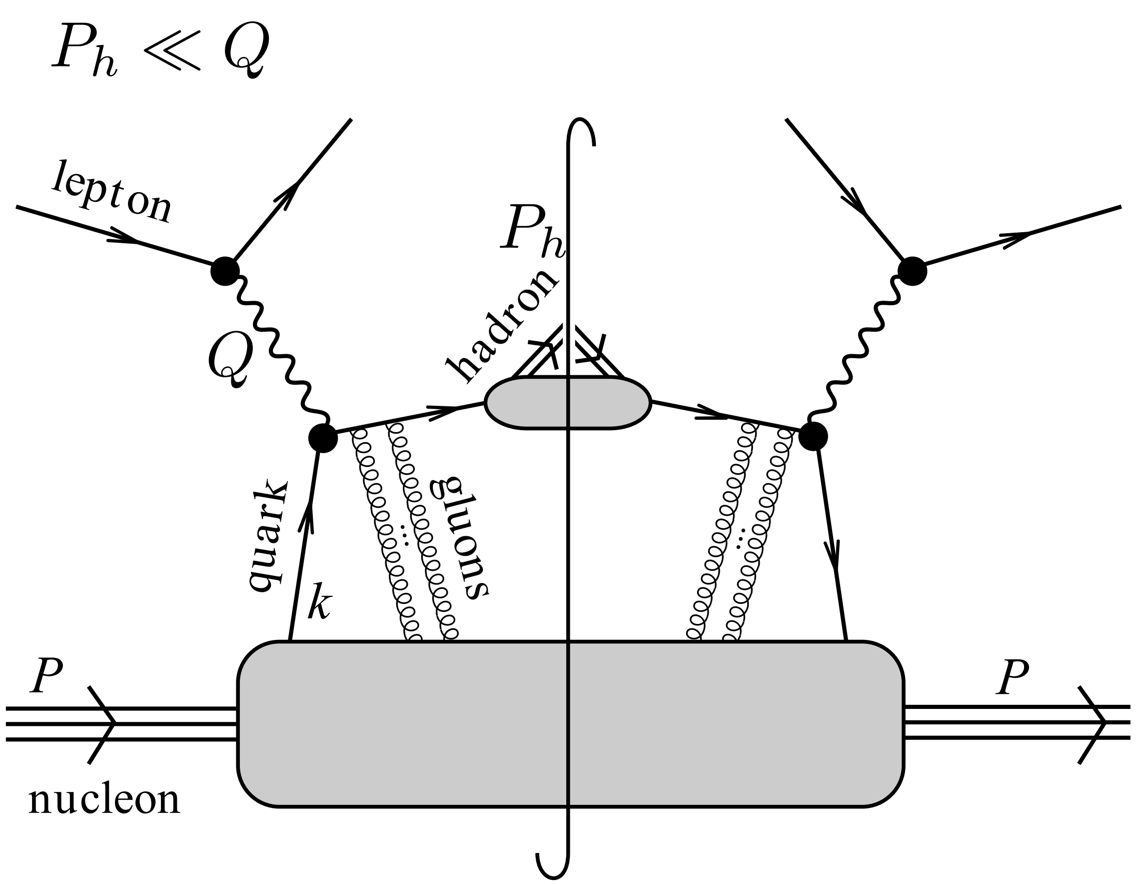
Transverse momentum-dependend (TMD) parton distributions

$$\tilde\Phi^{[\Gamma]}(x,\vec b_\perp;P,S,\dots)=\int\frac{db^-}{4\pi}\,e^{ix(b^-P^+)}\frac{\langle P,S|\,\bar q(0)\,\Gamma\,\mathcal{U}(\mathcal{C}_b)\,q(b)\,|P,S\rangle}{\{\text{soft factor}\}}$$

$$\Phi^{[\Gamma]}(x,\vec k_\perp;P,S,\dots)=\int\frac{d^2\vec b_\perp}{(2\pi)^2}\,\tilde\Phi^{[\Gamma]}(x,\vec b_\perp;P,S,\dots)$$

$\mathcal{C}_b$ is process-dependent

$P_h \ll Q$



$l+N(P)\longrightarrow l'+h(P_h)+X$

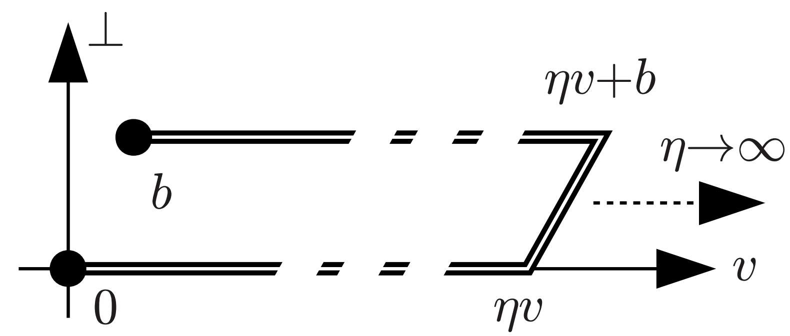
[M.Engelhardt, B.Mush, A.Shaefer, Ph.Hagler]

Gauge link structure:

In matrix element $\widetilde\Phi^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\dots)\equiv$

$\frac{1}{2}\langle P,S|\;\bar q(0)\;\Gamma\;\mathcal{U}[0,\dots,b]\;q(b)\;|P,S\rangle$

Staple-shaped gauge link $\mathcal{U}[0,\eta v,\eta v+b,b]$



incorporates SIDIS final state effects

LC limit: Collins-Soper parameter $\hat\zeta=\dfrac{P\cdot v}{m_N\,|v|}\to\infty$

TMDs from Lattice: T-odd momentum shift (1)

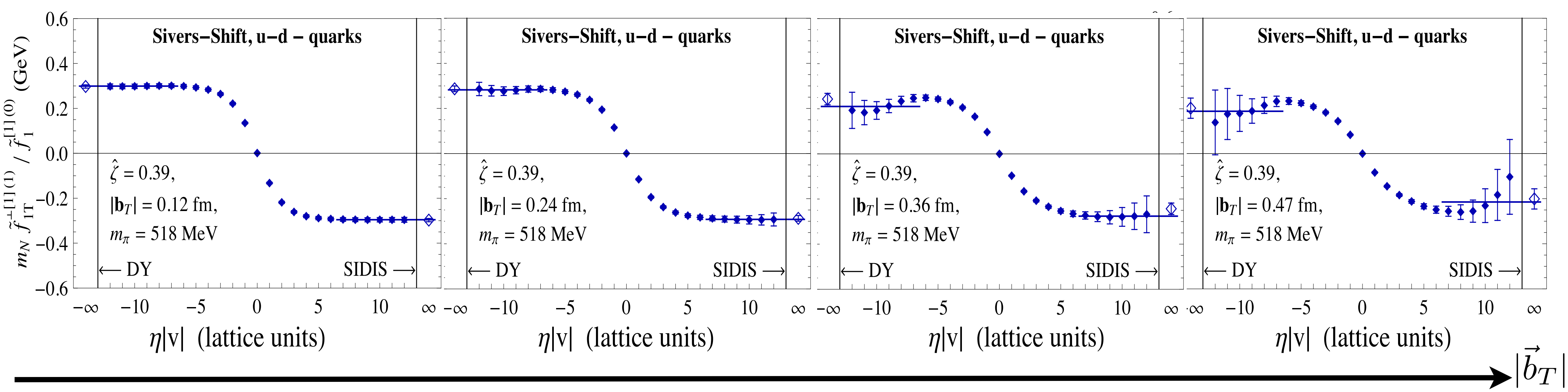
x-integrated TMDs (moments) with finite $\vec{b}_\perp^2 \neq 0$ as an UV-regulator

Sivers Shift: *avg. quark y-momentum in a transversely polarized proton*

$$\langle k_y \rangle^{Sivers}(\vec{b}_T^2) \equiv m_N \frac{\tilde{f}_{1T}^{\perp[1](1)}(\vec{b}_T^2)}{\tilde{f}_1^{[1](0)}(\vec{b}_T^2)} \xrightarrow{\vec{b}_T^2 \to 0} \frac{\int dx \int d^2\vec{k}_\perp \cdot k_y \cdot \Phi^{[\gamma^+]}(x, \vec{k}_\perp)}{\int dx \int d^2\vec{k}_\perp \cdot 1 \cdot \Phi^{[\gamma^+]}(x, \vec{k}_\perp)}$$

To compute an x-moment, specify kinematics: $\int dx \longrightarrow b \cdot P = 0$

To compute a k_y-moment, select Lorentz structure [B.Mush, Phys.Rev.D85, 094510]



[M.Engelhardt, B.Mush, A.Shaefer, Ph.Hagler]

TMDs from Lattice: T-odd momentum shift (1)

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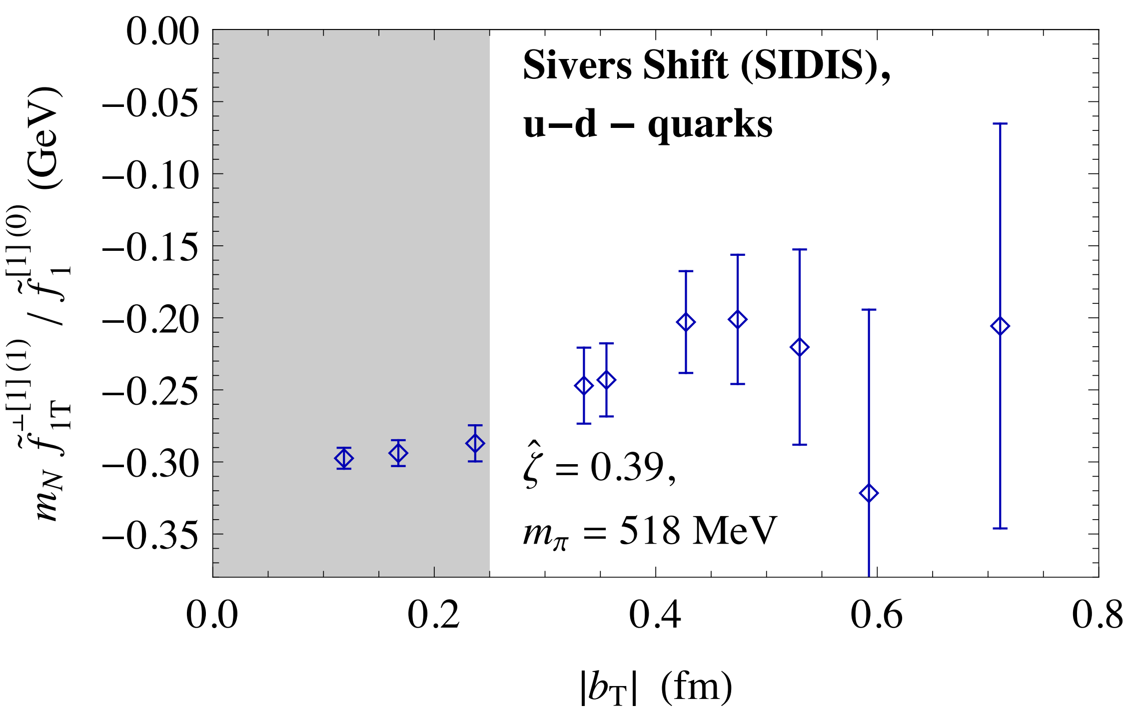


Transverse coordinate dependence

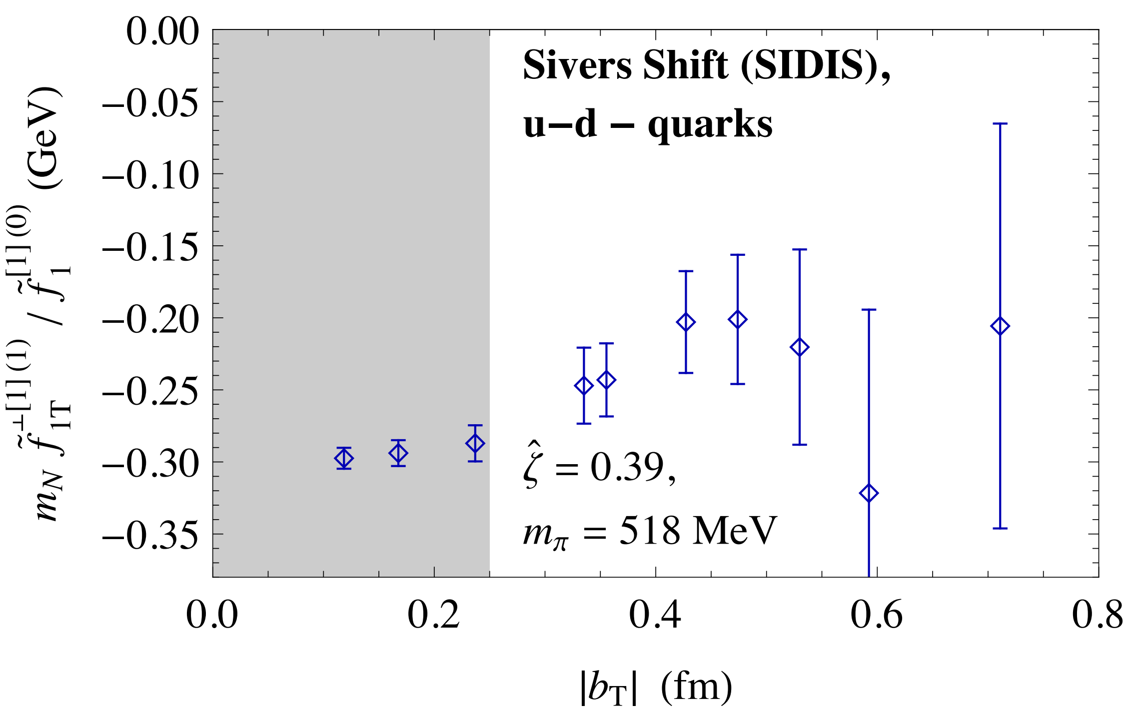
[M.Engelhardt, B.Mush, A.Shaefer, Ph.Hagler]

TMDs from Lattice: T-odd momentum shift (1)

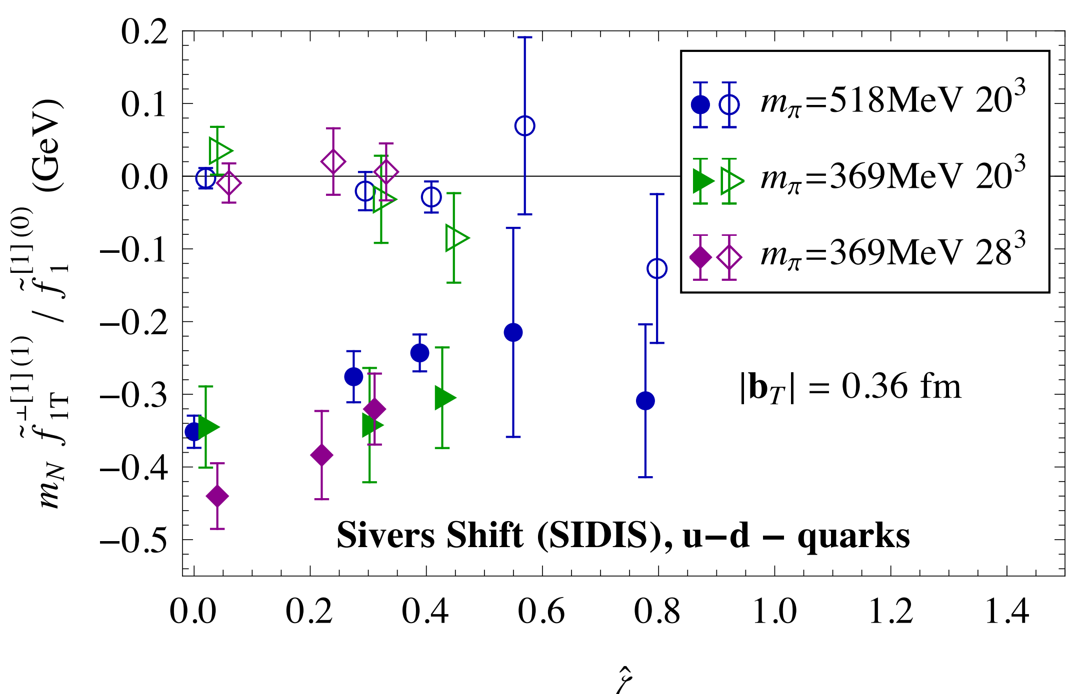
x-integrated TMDs (moments) with finite $\vec{b}_\perp^2 \neq 0$ as an UV-regulator

Sivers Shift: *avg. quark y-momentum in a transversely polarized proton*

$$\langle k_y \rangle^{Sivers}(\vec{b}_T^2) \equiv m_N \frac{\tilde{f}_{1T}^{\perp[1](1)}(\vec{b}_T^2)}{\tilde{f}_1^{[1](0)}(\vec{b}_T^2)} \xrightarrow{\vec{b}_T^2 \to 0} \frac{\int dx \int d^2\vec{k}_\perp \cdot k_y \cdot \Phi^{[\gamma^+]}(x,\vec{k}_\perp)}{\int dx \int d^2\vec{k}_\perp \cdot 1 \cdot \Phi^{[\gamma^+]}(x,\vec{k}_\perp)}$$



Transverse coordinate dependence



Light-cone limit: $\hat{\zeta} \to \infty$

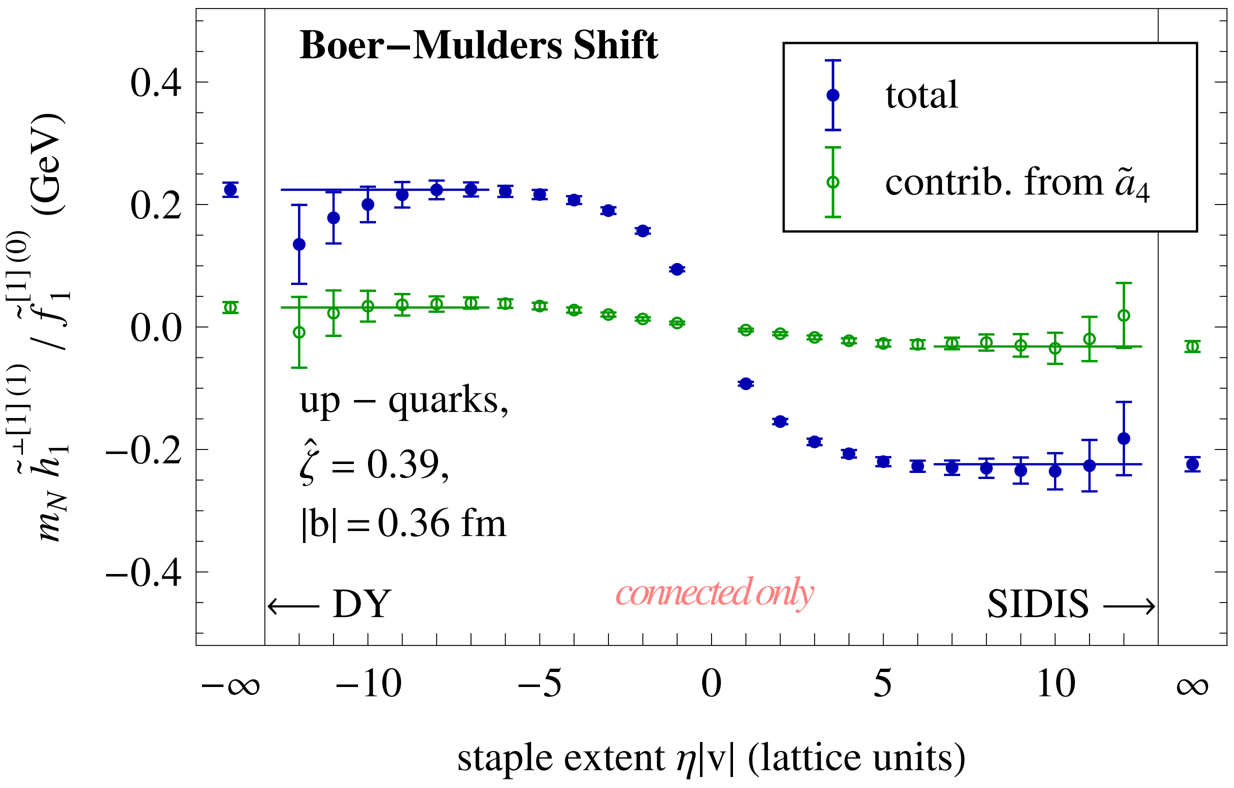
[M.Engelhardt, B.Mush, A.Shaefer, Ph.Hagler]

TMDs from Lattice: T-odd momentum shift (2)

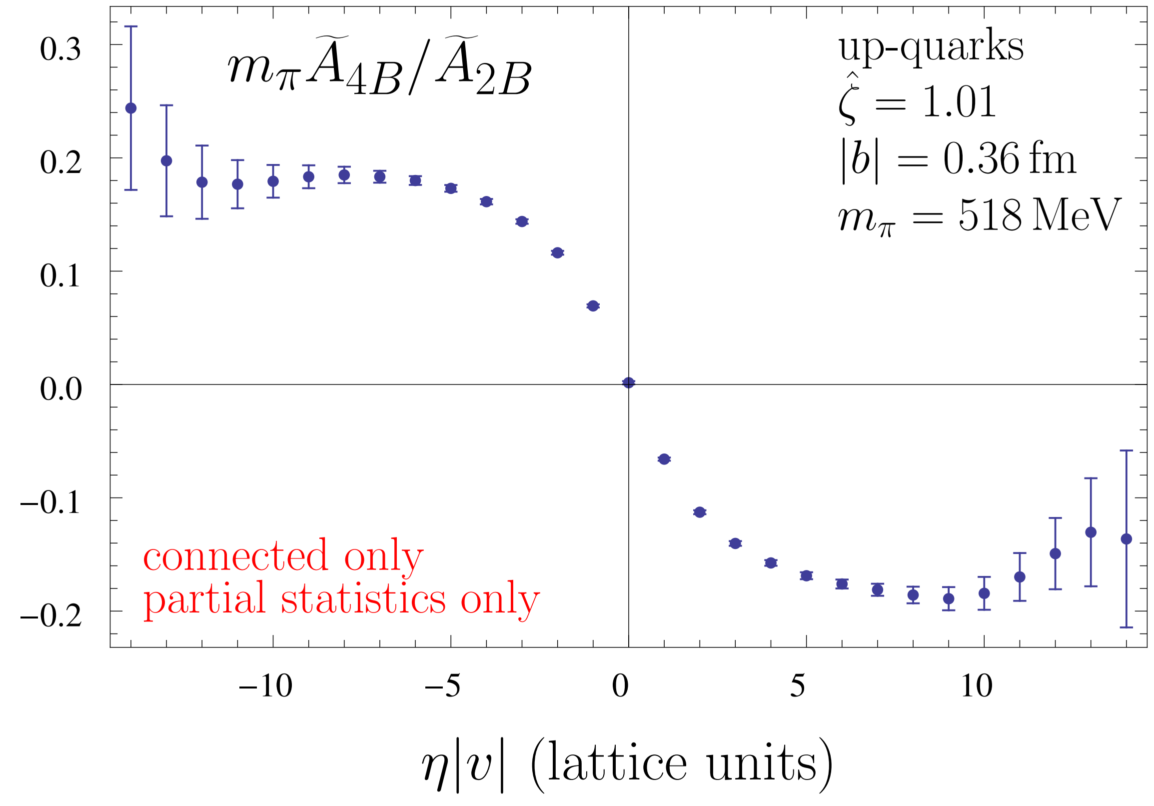
x-integrated TMDs (moments) with finite $\vec{b}_\perp^2 \neq 0$ as an UV-regulator

Boer-Mulders Shift: *avg. y-momentum of transv. polarized quarks in an unpolarized proton*

$$\langle k_y \rangle^{BM}(\vec{b}_T^2) \equiv m_N \frac{\tilde{h}_1^{\perp[1](1)}(\vec{b}_T^2)}{\tilde{f}_1^{[1](0)}(\vec{b}_T^2)} \xrightarrow{\vec{b}_T^2\to 0} \frac{\int dx \int d^2\vec{k}_\perp \cdot k_y \cdot \Phi^{[\sigma^{x,+}]}(x,\vec{k}_\perp)}{\int dx \int d^2\vec{k}_\perp \cdot 1 \cdot \Phi^{[\gamma^+]}(x,\vec{k}_\perp)}$$



proton



pion

[M.Engelhardt, B.Mush, A.Shaefer, Ph.Hagler]

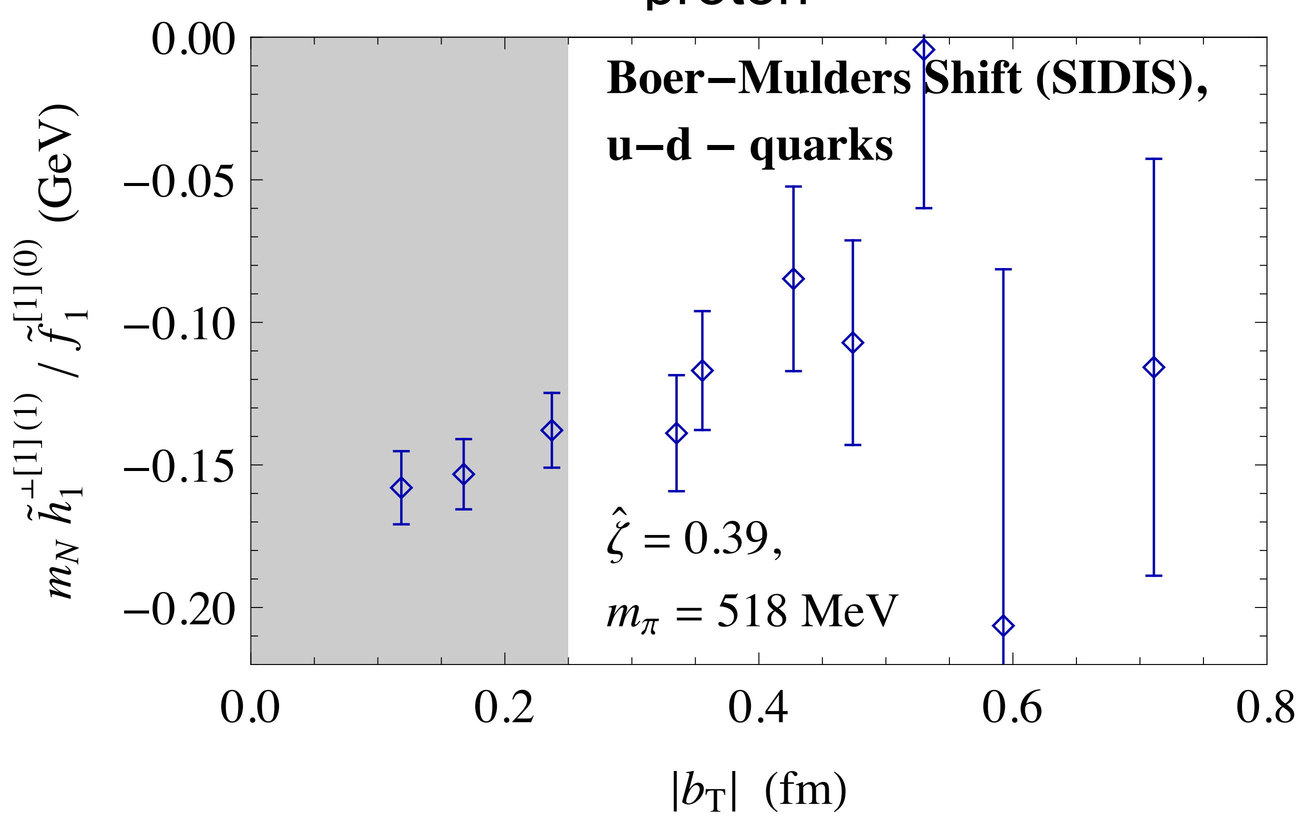
TMDs from Lattice: T-odd momentum shift (2)

x-integrated TMDs (moments) with finite $\vec{b}_\perp^2 \neq 0$ as an UV-regulator

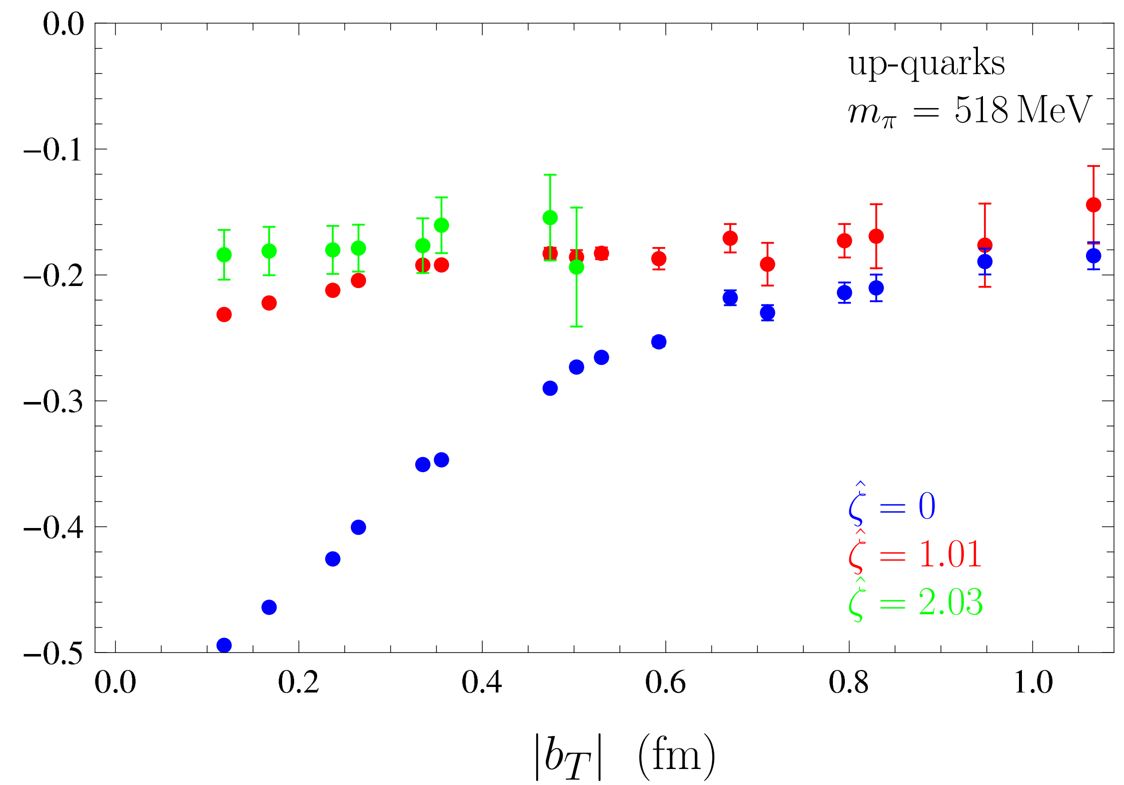
Boer-Mulders Shift: *avg. y-momentum of transv. polarized quarks in an unpolarized proton*

$$\langle k_y \rangle^{BM}(\vec{b}_T^2) \equiv m_N \frac{\tilde{h}_1^{\perp[1](1)}(\vec{b}_T^2)}{\tilde{f}_1^{[1](0)}(\vec{b}_T^2)} \xrightarrow{\vec{b}_T^2\to 0} \frac{\int dx \int d^2\vec{k}_\perp \cdot k_y \cdot \Phi^{[\sigma^{x,+}]}(x,\vec{k}_\perp)}{\int dx \int d^2\vec{k}_\perp \cdot 1 \cdot \Phi^{[\gamma^+]}(x,\vec{k}_\perp)}$$

proton



pion



Transverse coordinate dependence

[M.Engelhardt, B.Mush, A.Shaefer, Ph.Hagler]

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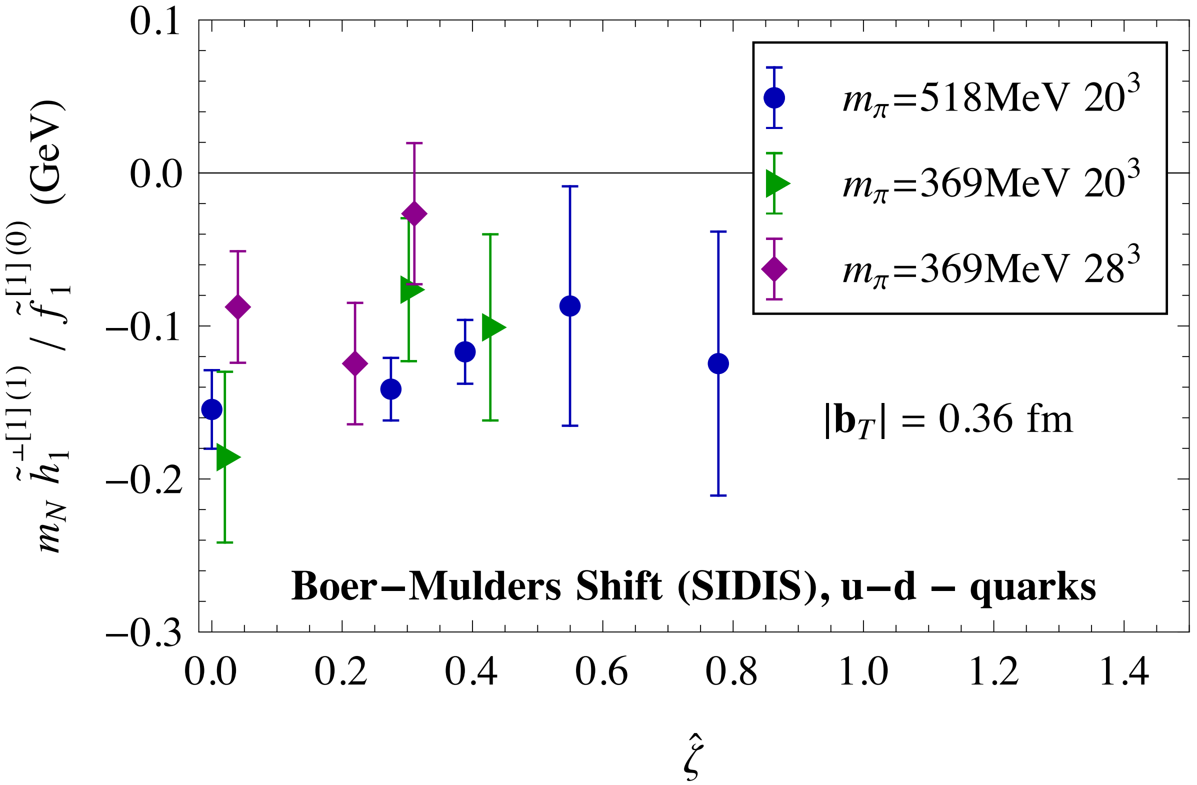
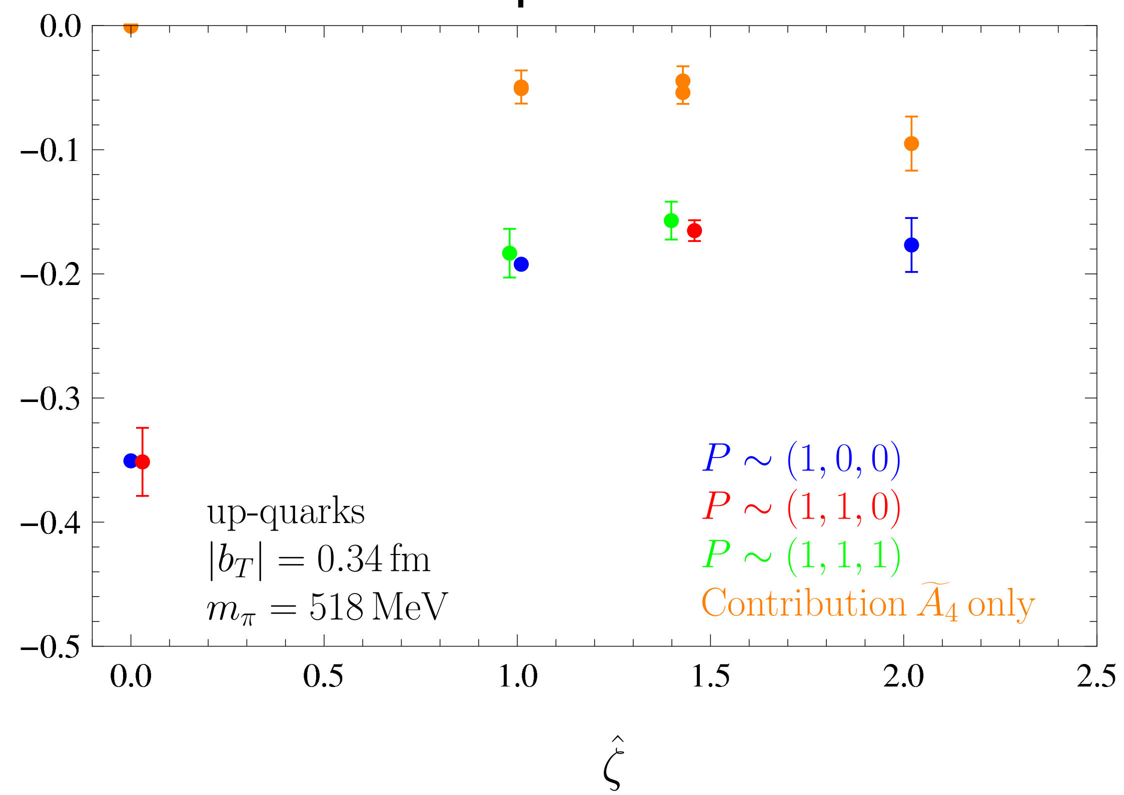


pion



Light-cone limit: $\hat{\zeta} \to \infty$

[M.Engelhardt, B.Mush, A.Shaefer, Ph.Hagler]

PDFs From Lattice: Spatial Quark Correlations

Definition of a parton distribution function:

$$q(x,\mu) = \int \frac{dx}{4\pi}\, e^{ix(z_- P_+)} \langle P|\bar{q}(z_-)\,\gamma^+ \exp\Big[-ig\int_0^{z_-} dt A_+(t)\Big]\, q(0)|P\rangle$$

Instead, boost the hadron and make gauge link spatial [X.-D. Ji, arXiv:1305.1539]

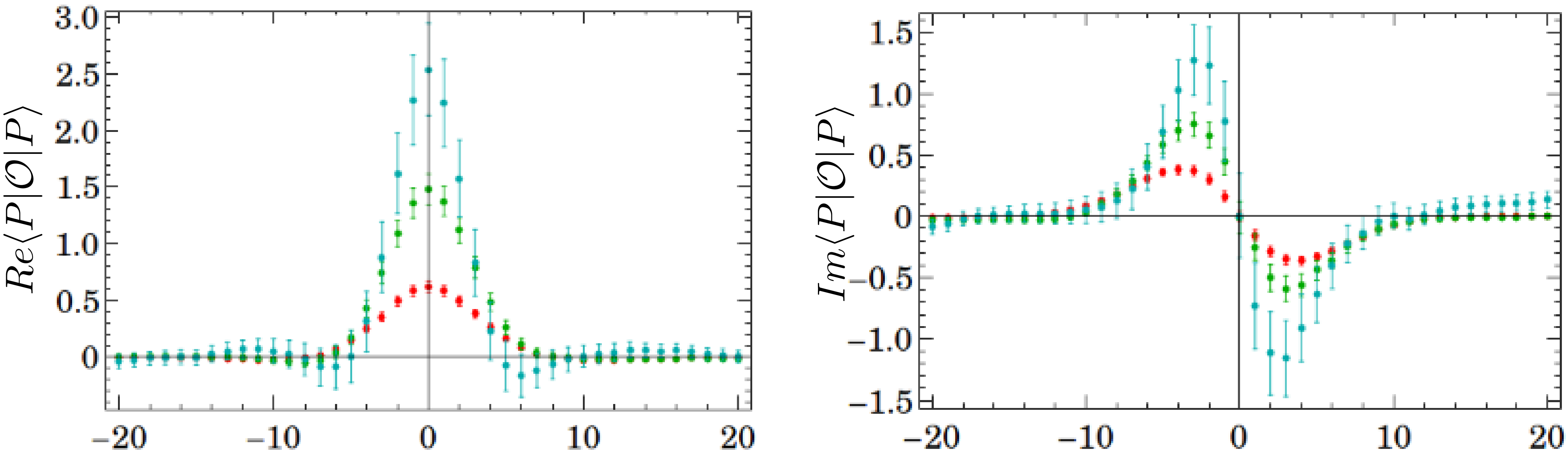
$$\tilde{q}(x,\mu,P_z) = \int \frac{dx}{4\pi}\, e^{ix(zP_z)} \langle P|\bar{q}(z)\,\gamma^+ \exp\Big[-ig\int_0^{z} dt A_z(t)\Big]\, q(0)|P\rangle + \mathcal{O}\Big(\frac{\Lambda^2_{\rm QCD}}{P_z^2}, \frac{M^2}{P_z^2}\Big)$$

Equivalent to “static” virtual photon $q^\mu = (0, \vec{Q})$ and boosted hadron $P_z = \frac{Q}{2x}$

PDFs From Lattice: Preliminary Results

$$\int \frac{dx}{4\pi}\, e^{ix(zP_z)}\, \langle P|\bar{q}(z)\,\gamma^z \exp\Big[-ig\int_0^z dt A_z(t)\Big]\, q(0)|P\rangle$$

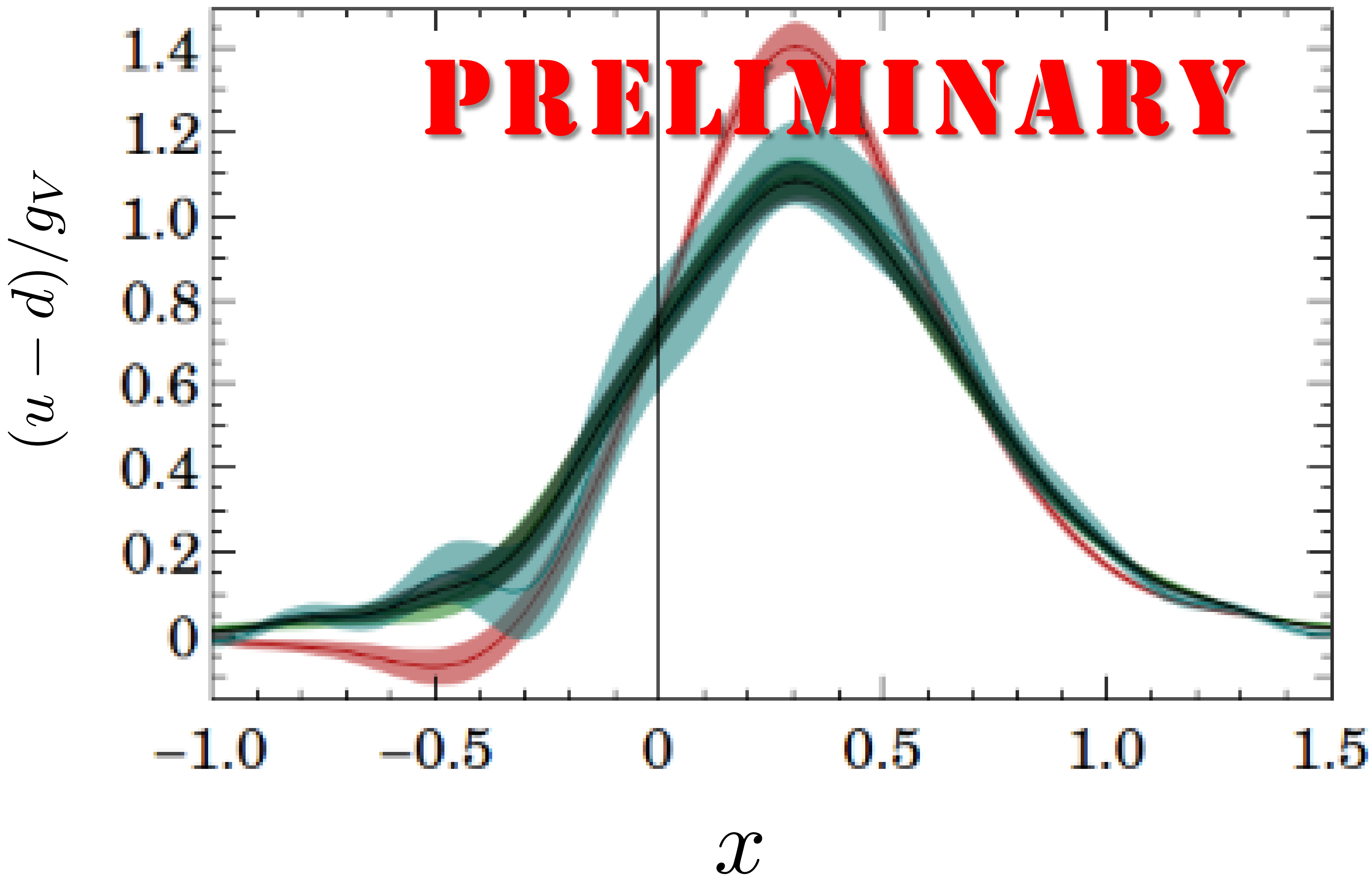
$$m_\pi = 310\,\mathrm{MeV} \qquad a = 0.12\,\mathrm{fm} \qquad P_z = \frac{2\pi}{L}\{1,\,2,\,3\}$$



[H.W.Lin, S.Cohen, J.-W.Chen, X.Ji]

PDFs From Lattice: Preliminary Results

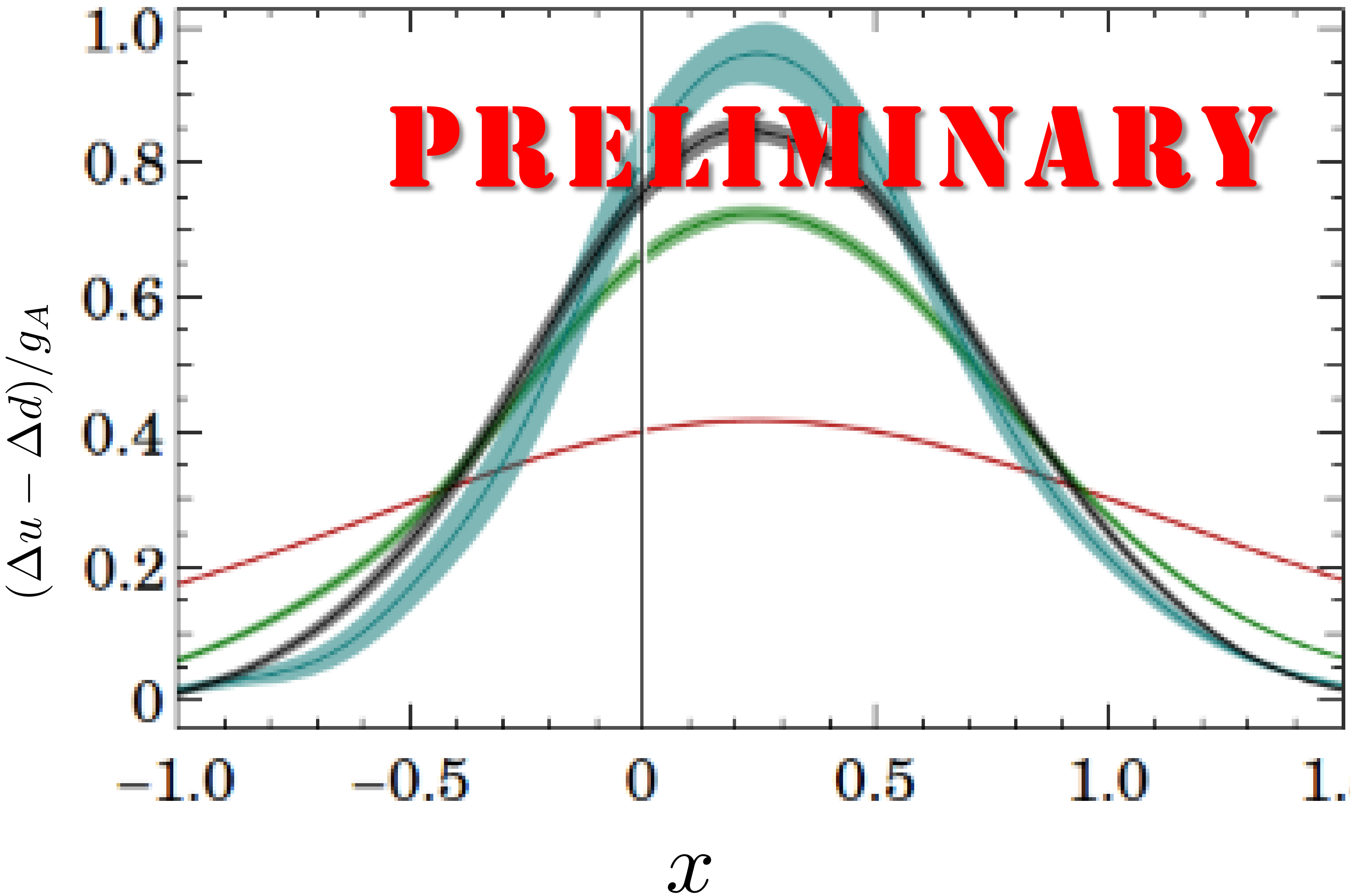
$$\int \frac{dx}{4\pi} e^{ix(zP_z)} \langle P|\bar{q}(z)\,\gamma^z \exp\Big[-ig\int_0^z dt A_z(t)\Big]\,q(0)|P\rangle$$



[H.W.Lin, S.Cohen, J.-W.Chen, X.Ji]

PDFs From Lattice: Preliminary Results

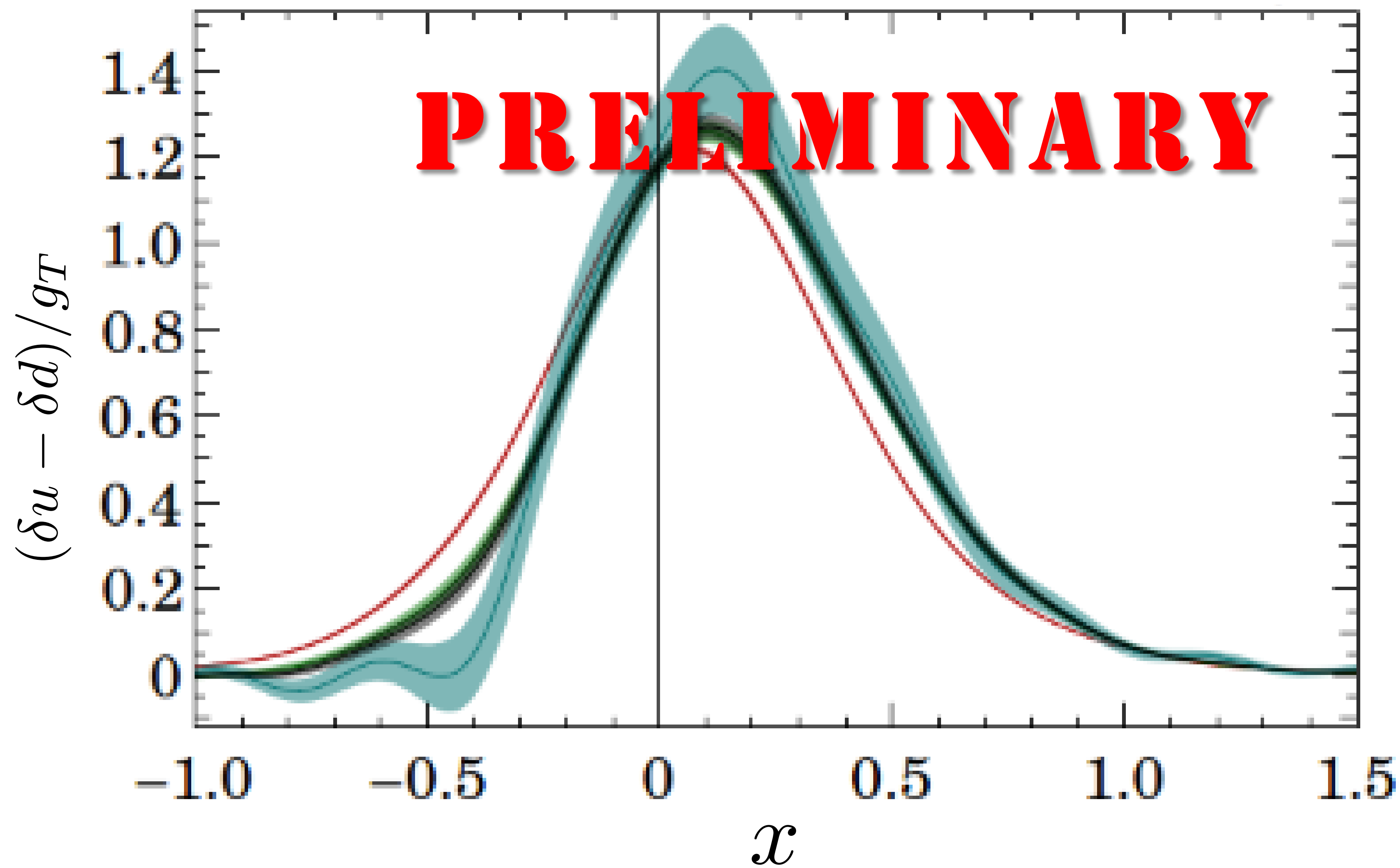
$$\int \frac{dx}{4\pi} e^{ix(zP_z)} \langle P|\bar{q}(z)\,\gamma^z\gamma^5 \exp\left[-ig\int_0^z dt A_z(t)\right] q(0)|P\rangle$$



[H.W.Lin, S.Cohen, J.-W.Chen, X.Ji]

PDFs From Lattice: Preliminary Results

$$\int \frac{dx}{4\pi} e^{ix(zP_z)} \langle P|\bar{q}(z)\,\sigma^{x,y} \exp\left[-ig\int_0^z dt A_z(t)\right] q(0)|P\rangle$$



[H.W.Lin, S.Cohen, J.-W.Chen, X.Ji]

Summary

- Encouraging Hadron Structure results at the physical pion mass
  axial charge, radius, vector form factors
  ... although clearing up systematic effects is still to be done
- Excited states require close attention
  variational methods look most promising
- Background field methods
  potential demonstrated for glue momentum fraction
- New approach to computing parton distribution functions on a lattice
  the first results look promising
  theory side needs more work