QUANTUM SIMULATION OF LATTICE GAUGE THEORIES WITH COLD ATOMS

Benni Reznik
Tel-Aviv University

In collaboration with E. Zohar (Tel-Aviv) and J. Ignacio Cirac, (MPQ)

Lattice 2013, July 30, Gutenberg University, Mainz
OUTLINE

QUANTUM SIMULATION
COLD ATOMS IN OPTICAL LATTICES
HAMILTONIAN LGT

Q. SIMULATION: LGT
   — REQUIREMENTS
   — EXACT AND EFFECTIVE LOCAL GAUGE INVARIANCE
   — LINKS AND PLAQUETTES – Examples: cQED, Z(N), SU(2)

CURRENT EXPERIMENTS AND LGT SIMULATIONS
OUTLOOK.
Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don’t know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have

be understood very well in analyzing the situation. And I’m not happy with all the analyses that go with just the classical theory, because nature isn’t classical, dammit, and if you want to make a simulation of nature, you’d better make it quantum mechanical, and by golly it’s a wonderful problem, because it doesn’t look so easy. Thank you.

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982
(Phenomenological) Hamiltonian

\[ H = \ldots \]

Physical Hamiltonian

\[ H = \ldots \]

Example: Hubbard model in 2D:

\[ H = -t \sum_{k,\sigma} c_{k\sigma}^\dagger c_{k\sigma} + V \sum_{k} n_{k\uparrow} n_{k\downarrow} \]
QUANTUM SIMULATION
ANALOG

- Questions:
  - Dynamics: \(|\Psi(t)\rangle = e^{-iHt} |\Psi(0)\rangle\)
  - Ground state: \(H |\Psi_0\rangle = E_0 |\Psi_0\rangle\)

- Physical properties: \(\langle \sigma_n \rangle, \langle \sigma_n \sigma_m \rangle, \ldots\)

- Adiabatic algorithms
  - Hamiltonians \(H\)
  - States \(|\Psi\rangle\)
COLD ATOMS

Control: External fields

Trapping:
lasers  Magnetic fields

Cooling:
lasers  evaporation

Internal manipulation
lasers  RF fields
 purification coherence detection

Interactions
tune scattering length
COLD ATOMS

- Many-body phenomena
  - Degeneracy: bosons and fermions (BE/FD statistics)
  - Coherence: interference, atom lasers, four-wave mixing, …
  - Superfluidity: vortices
  - Disorder: Anderson localization
  - Fermions: BCS-BEC + many other phenomena
Cold atoms are described by simple quantum field theories:

\[ H = \int \Psi^\dagger (\mathbf{\nabla}^2 + V(r)) \Psi + u_{\sigma_1} \int \Psi^\dagger_{\sigma_1} \Psi^\dagger_{\sigma_2} \Psi_{\sigma_3} \Psi_{\sigma_4} \]

- We can have bosons or fermions (or both).
- We can have different internal states (spin).
- The external potential, \( V \), and interaction coefficients, \( u \), can be engineered using lasers, and electric and magnetic fields.
- In certain limits, one obtains effective theories that are interesting in other fields of Physics.

Quantum Simulations
Laser standing waves: dipole-trapping

Cold Bosonic Atoms in Optical Lattices

D. Jaksch, C. Bruder, J.I. Cirac, C. W. Gardiner, and P. Zoller
In the presence $E(r, t)$ the atoms has a time dependent dipole moment $d(t) = \alpha(\omega)E(r, t)$ of some non resonant excited states.

Stark effect:

$$V(r) \equiv \Delta E(r) = \alpha(\omega)\langle E(r, t) E(r, t) \rangle / \delta$$
COLD ATOMS
OPTICAL LATTICES

(a) 2d array of effective 1d traps
(b) 3d square lattice

M. Lewenstein et. al, Advances in Physics, 2010.
COLD ATOMS
OPTICAL LATTICES

- Laser standing waves: dipole-trapping

\[ H = \int \psi_{\sigma}^{\dagger} \left( -\nabla^2 + V(r) \right) \psi_{\sigma} + u_{\sigma_1} \int \psi_{\sigma_1}^{\dagger} \psi_{\sigma_2}^{\dagger} \psi_{\sigma_3} \psi_{\sigma_4} \]

\[ H = -t \sum_n (a_n^{\dagger} a_{n+1} + h.c.) + U \sum_n a_n^{\dagger} a_n^2 \]

Lattice theory: Bose/Fermi-Hubbard model
COLD ATOMS
OPTICAL LATTICES

- Laser standing waves: dipole-trapping

\[ H = \int \Psi^\dagger ( -\nabla^2 + V(r) ) \Psi + \hbar g \int \Psi^\dagger \Psi^\dagger \Psi \Psi + \Psi^\dagger \Psi \Psi^\dagger \Psi \Psi \]

Lattice theory: Bose/Fermi-Hubbard model

\[ H = -t \sum_n (a_n^\dagger a_{n+1} + a_n a_{n+1}^\dagger) + U \sum_n a_n^\dagger a_n \]

Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

articles
COLD ATOMS

QUANTUM SIMULATIONS

- Bosons/Fermions:
  \[ H = - \sum_{\langle n,m \rangle_{\sigma,\sigma'}} \left( t_{\sigma,\sigma'} a_{n,\sigma}^\dagger a_{m,\sigma'} + h.c. \right) + \sum_{n,\sigma,\sigma'} U_{\sigma,\sigma'} a_{n,\sigma}^\dagger a_{n,\sigma'} a_{n,\sigma} a_{n,\sigma'} \]

- Spins:
  \[ H = - \sum_{\langle n,m \rangle_{\sigma,\sigma'}} \left( J_x S_n^x S_m^x + J_y S_n^y S_m^y + J_z S_n^z S_m^z \right) + \sum_{n,\sigma,\sigma'} B_n S_n^z \]

CONDENSED MATTER PHYSICS
COLD ATOMS
QUANTUM SIMULATIONS

HIGH ENERGY PHYSICS?
LATTICE GAUGE THEORIES
HAMILTONIAN FORMULATION
Hamiltonian formulation of Wilson’s lattice gauge theories

John Kogut*
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Leonard Susskind†
Belfer Graduate School of Science, Yeshiva University, New York, New York
and Tel Aviv University, Ramat Aviv, Israel
and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York
(Received 9 July 1974)

Wilson’s lattice gauge model is presented as a canonical Hamiltonian theory. The structure of the model is reduced to the interactions of an infinite collection of coupled rigid rotators. The gauge-invariant configuration space consists of a collection of strings with quarks at their ends. The strings are lines of non-Abelian electric flux. In the strong-coupling limit the dynamics is best described in terms of these strings. Quark confinement is a result of the inability to break a string without producing a pair.
Gauge group elements:

$U^r$ is an element of the gauge group (in the representation $r$), on each link

Left and right generators:

$[L_a, U^r] = T_a^r U^r$ ; $[R_a, U^r] = U^r T_a^r$

$[L_a, L_b] = -i f_{abc} L_c$ ; $[R_a, R_b] = i f_{abc} R_c$ ; $[L_a, R_b] = 0$

$\sum_a L_a L_a = \sum_a R_a R_a \equiv \sum_a E_a E_a$

Gauge transformation:

$U^r_{n,k} \rightarrow V^r_n U^r_{n,k} V^{\dagger r}_{n+\hat{k}}$

Generators:

$(G_n)_a = \text{div}_n E_a = \sum_k \left( (L_{n,k})_a - (R_{n-k,k})_a \right)$
Matter:

\[ \psi_n = (\psi_n, a) = \begin{pmatrix} \psi_{n,1} \\ \psi_{n,2} \\ \vdots \end{pmatrix} \]

Gauge transformation:

\[ \psi_n \rightarrow V_{n}^T \psi_n \]
Gauge field dynamics (Kogut-Susskind Hamiltonian):

\[ H_E = \frac{g^2}{2} \sum_{n,k,a} (E_{n,k})_a (E_{n,k})_a \]

\[ H_B = -\frac{1}{g^2} \sum_{\text{plaquettes}} \left( \text{Tr} \left( U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right) \]

Strong coupling limit: \( g \gg 1 \)
Weak coupling limit: \( g \ll 1 \)

Matter dynamics:

\[ H_M = \sum_n M_n \psi_n^\dagger \psi_n \]

\[ H_{int} = \varepsilon \sum_{n,k} \left( \psi_n^\dagger U_{n,k}^r \psi_{n+\hat{k}} + h.c. \right) \]
Compact QED ($\mathbb{U}(1)$):

$U_{n,k} = e^{i\phi_{n,k}}$

$[E_{n,k}, \phi_{m,l}] = -i\delta_{nm}\delta_{kl}$

$$H_{cQED} = \frac{g^2}{2} \sum_{n,k} E_{n,k}^2 - \frac{1}{g^2} \sum_n \cos \left( \phi_{n,1} + \phi_{n+1,2} - \phi_{n+2,1} - \phi_{n,2} \right)$$

$$+ \epsilon \sum_{n,k} \left( \psi_n^\dagger e^{i\phi_{n,k}} \psi_{n+k} + \psi_{n+k}^\dagger e^{-i\phi_{n,k}} \psi_n \right) + M \sum_n (-1)^n \psi_n^\dagger \psi_n$$
QUANTUM SIMULATION
LATTICE GAUGE THEORIES

E. Zohar, BR, PRL 107, 275301 (2011)
E. Zohar, I. Cirac, BR, PRL 109, 125302 (2012)
E. Zohar, BR, NJP 15, 043041 (2013)
E. Zohar, I. Cirac, BR, PRL 110 055302 (2013)
E. Zohar, I. Cirac, BR, PRL 110 125304 (2013)
E. Zohar, I. Cirac, BR, PRA (2013) arxiv 1303.5040
QUANTUM SIMULATION
LATTICE GAUGE THEORIES

E. Zohar, BR, PRL 107, 275301 (2011)
E. Zohar, I. Cirac, BR, PRL 109, 125302 (2012)
E. Zohar, BR, NJP 15, 043041 (2013)
E. Zohar, I. Cirac, BR, PRL 110 055302 (2013)
E. Zohar, I. Cirac, BR, PRL 110 125304 (2013)

Detailed account  E. Zohar, I. Cirac, BR, PRA (2013) arxiv 1303.5040
QUANTUM SIMULATION
GAUGE THEORIES

- Continuum fields

Abelian local gauge symmetry

2+1, (3+1), U(1) Kogut-Susskind
  Pure gauge – E. Zohar and B. Reznik, PRL 107, 275301 (2011)
  Full cQED – E. Zohar, J. I. Cirac and B. Reznik, arxiv 1303.5040, Phys. Rev. A

2+1 U(1) gauge magnets

1+1 U(1) link models
  Superconducting qubits – D. Marcos, P. Rabl, E. Rico, P. Zoller, arxiv 1306.1674
  Ions – P. Hauke, D. Marcos, M. Dalmote and P. Zoller, arxiv 1306.2162

- Gauge symmetry connected with angular momentum conservation.
**QUANTUM SIMULATION**

**GAUGE THEORIES**

**Discrete groups:** 2+1, 3+1 \(Z(N)\)

- L. Tagliacozzo, A. Celi, A. Zamora and M. Lewenstein,

**Non-abelian**

**Kogut-Susskind SU(2) Yang Mills**, 1+1, (2+1) D


**Non-abelian link models**, 1+1 D

- D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler,

**SU(2) gauge magnets**, 2+1, digital simulation

- L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, arxiv 1211.2704

**Quantum computation**

Scattering probabilities for scalar fields: S. P. Jordan, K. S. M. Lee and J. Preskill,
Science **336**, 1130 (2012)

- Gauge symmetry connected with angular momentum conservation.
Requirements: HEP models

- **Fields**
  - Fermion Matter fields
  - Bosonic gauge fields

- **Local gauge invariance**
  - Exact, or low energy, effective

- **Relativistic invariance**
  - Causal structure, in the continuum limit
QUANTUM SIMULATION
COLD ATOMS

- Fermion matter fields
- Bosonic gauge fields

Superlattices:

$$\psi_n = (\psi_{n,a}) = \begin{pmatrix} \psi_{n,1} \\ \psi_{n,2} \\ ... \end{pmatrix}$$

Atom internal levels
• Generators of gauge transformations:

\[
(G_n)_a = \text{div}_n E_a - Q_n \\
G_n |\text{phys}\rangle = q_n |\text{phys}\rangle \\
\left[ G_n, H \right] = 0
\]

\[
\mathcal{H} = \bigoplus \mathcal{H}_{\{q_n\}}
\]
• Generators of gauge transformations:

\[(G_n)_a = \text{div}_n E_a - Q_n\]

\[G_n |\text{phys}\rangle = q_n |\text{phys}\rangle\]

\[[G_n, H] \neq 0\]

\[\mathcal{H} = \bigoplus \mathcal{H}\{q_n\}\]
Generators of gauge transformations:

\[(G_n)_a = \text{div}_n E_a - Q_n\]

\[G_n |\text{phys}\rangle = q_n |\text{phys}\rangle\]

\[[G_n, H] = 0\]

\[\mathcal{H} = \bigoplus \mathcal{H}_{\{q_n\}}\]
• Generators of gauge transformations:

\[(G_n)_a = \text{div}_n E_a - Q_n\]

\[G_n \ket{\text{phys}} = q_n \ket{\text{phys}}\]

\[[G_n, H] = 0\]

Gauge invariance –
• Low energy, effective
• Exact symmetry

\[\mathcal{H} = \bigoplus \mathcal{H}\{q_n\}\]
• Links $\leftrightarrow$ atomic scattering: gauge invariance is a fundamental symmetry

• Plaquettes $\leftrightarrow$ gauge invariant links $\leftrightarrow$ virtual loops of ancillary fermions.
QUANTUM SIMULATION

LINKS
REALIZATION OF LINKS

EXAMPLE : cQED

F-B Scattering
$L_z \rightarrow L_z - 1$  

Fermion

$\Phi_a, \Phi_b$

$\psi_c$  

$\psi_d$
Fermion

$\Phi_a, \Phi_b$

$\psi_c, \psi_d$
Hyperfine angular momentum conservation in atomic scattering.

\[ \int d^3x \Psi^\dagger_\alpha (x) \Psi_\beta (x) \Phi^\dagger_\gamma (x) \Phi_\delta (x) \]

\[ m_F (a) + m_F (c) = m_F (b) + m_F (d) \]
Angular Momentum conservation ↔ Local gauge invariance

\[ \psi_c \Phi_a \Phi_b \psi_d + \psi_d \Phi_b \Phi_a \psi_c \]

\[ m_F (a) \]
\[ m_F (d) \]
\[ m_F (c) \]
\[ m_F (b) \]
Angular Momentum conservation $\leftrightarrow$ Local gauge invariance

$$\psi_c \Phi_a^\dagger \Phi_b^\dagger \psi_d + \psi_d^\dagger \Phi_b^\dagger \Phi_a \psi_c$$

$m_F (c)$

$m_F (d)$

$m_F (a)$

$m_F (b)$
Angular Momentum conservation ↔ Local gauge invariance
Gauge bosons: Schwinger algebra

\[ L_+ = \Phi_a \dagger \Phi_b \; ; \; L_- = \Phi_b \dagger \Phi_a \]
\[ L_z = \frac{1}{2} (N_a - N_b) \; ; l = \frac{1}{2} (N_a + N_b) \]
Gauge bosons: Schwinger algebra

\[ L_+ = \Phi_a^\dagger \Phi_b \quad ; \quad L_- = \Phi_b^\dagger \Phi_a \]
\[ L_z = \frac{1}{2} (N_a - N_b) \quad ; \quad l = \frac{1}{2} (N_a + N_b) \]

and thus what we have is

\[ \psi_c^\dagger \Phi_a^\dagger \Phi_b \psi_d + \psi_d^\dagger \Phi_b^\dagger \Phi_a \psi_c \]

\[ \psi_c^\dagger L_+ \psi_d \sim \psi_c^\dagger e^{i\theta} \psi_d \]

where for large \( l \), \( m \ll l \)
\[ L_+ \sim e^{i(\phi_1 - \phi_2)} \equiv e^{i\theta} \]
QUANTUM SIMULATION
DYNAMICAL FERMIONS 1+1

$H_M = M \sum_n (-1)^n \psi_n^\dagger \psi_n$

Staggered Fermions:
QUANTUM SIMULATION

SCHWINGER MODEL 1+1

\[
\frac{\varepsilon}{\sqrt{\ell (\ell + 1)}} \sum_n \left( \psi_n^\dagger L_{+,n} \psi_{n+1} + h.c. \right)
\]
QUANTUM SIMULATION
PLAQUETTES
1d elementary link interactions – *already gauge invariant building blocks* of effective plaquettes

Auxiliary fermions := \[ \bullet \]
1d elementary link interactions – already gauge invariant building blocks of effective plaquettes

Auxiliary fermions
– virtual processes
1d elementary link interactions – already gauge invariant 
building blocks of effective plaquettes

Auxiliary fermions 
– virtual processes
1d elementary link interactions – already \textit{gauge invariant building blocks} of effective plaquettes

Auxiliary fermions
– virtual processes
- plaquettes.

\[ \sum_{\text{plaquettes}} \left( \text{Tr} \left( U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right) \]
QUANTUM SIMULATION
PLAQUETTES

1d elementary link interactions – already gauge invariant building blocks of effective plaquettes

Auxiliary fermions
– virtual processes
- plaquettes.

$$\sum_{\text{plaquettes}} \left( \text{Tr} \left( U_1 U_2 U_3^{\dagger} U_4^{\dagger} \right) + h.c. \right)$$

OKAY for: discrete, abelian & non-abelian groups
QUANTUM SIMULATION

Example: U(1) PLAQUETTES

\[ H_B = -\frac{2\varepsilon^4}{\lambda^3} \sum_n \left( U_{n,1} U_{n+\hat{1},2} U^\dagger_{n+\hat{2},1} U^\dagger_{n,2} + h.c. \right) = \]

\[ -\frac{4\varepsilon^4}{\lambda^3} \sum_n \cos \left( \phi_{n,1} + \phi_{n+\hat{1},2} - \phi_{n+\hat{2},1} - \phi_{n,2} \right) \]

\( \lambda \) is the “energy penalty” of the auxiliary fermion
\( \varepsilon \) is the “link tunneling energy”.
Abelian discrete gauge theory: the gauge field degrees of freedom operate in a finite Hilbert space

\[ P^N = Q^N = 1 \quad ; \quad P^\dagger Q P = e^{i\delta} Q \]

\[ P \sim e^{iE} \]

\[ Q \sim e^{iA} \]

\[ H_B = -\frac{4\epsilon^4}{\lambda^3} \sum_n \left( Q_{n,1} Q_{n+\hat{1},2} Q_{n+\hat{2},1}^\dagger Q_{n,2}^\dagger + h.c. \right) \]
Gauss’s law is added as a constraint. Leaving the gauge invariant sector of Hilbert space costs too much Energy.

**Low energy sector with a effective gauge invariant Hamiltonian.**

\[ \Delta \gg \delta E \]

NON ABELIAN MODELS

YANG MILLS
QUANTUM SIMULATIONS OF NONABELIAN MODELS

GENERAL STRUCTURE

$U^r$ is an element of the gauge group (in the representation $r$)

$\psi_n = (\psi_n, a) = \begin{pmatrix} \psi_{n,1} \\ \psi_{n,2} \\ \vdots \end{pmatrix}$
SCHWINGER REPRESENTATION: SU(2) PRE-POTENTIAL APPROACH

On each link – \( a_{1,2} \) bosons on the left, \( b_{1,2} \) bosons on the right

\[
L_a = \frac{1}{2} \sum_{k,l} a_k^\dagger (\sigma_a)_{lk} a_l ; \quad R_a = \frac{1}{2} \sum_{k,l} b_k^\dagger (\sigma_a)_{kl} b_l
\]

\[
[L_{n,a}, L_{n,b}] = -i \epsilon_{abc} L_{n,c} ; \quad [R_{n,a}, R_{n,b}] = i \epsilon_{abc} R_{n,c}
\]

In the fundamental representation -

\[
U_L = \frac{1}{\sqrt{N_L + 1}} \begin{pmatrix} a_1^\dagger & -a_2 \\ -a_2 & a_1 \\ a_2 & a_1 \end{pmatrix} ; \quad U_R = \begin{pmatrix} b_1^\dagger & b_2^\dagger \\ b_2 & b_1 \end{pmatrix} \frac{1}{\sqrt{N_R + 1}}
\]

\[
U = U_L U_R
\]
On each link – $a_{1,2}$ bosons on the left, $b_{1,2}$ bosons on the right

Ancillary Fermion state

$$U_L = \frac{1}{\sqrt{N_L} + 1} \begin{pmatrix} a_1^\dagger & -a_2 \\ a_2 & a_1 \end{pmatrix}; \quad U_R = \begin{pmatrix} b_1^\dagger & b_2^\dagger \\ -b_2 & b_1 \end{pmatrix} \frac{1}{\sqrt{N_R} + 1}$$

$$U = U_L U_R$$
QUANTUM SIMULATION
EXAMPLE: SU(2) IN 1+1

Superlattices →

Fermions
L & R “gauge” bosons

\[
\sum_{n,i,j} \left( \sqrt{N_{L,n} + 1} (\psi_n^\dagger)_i (U_{L,n})_{ij} (\chi_n)_j + (\chi_n^\dagger)_i (U_{R,n})_{ij} (\psi_{n+1})_j \sqrt{N_{R,n} + 1} + h.c. \right)
\]
Each link has *left* and *right* degrees of freedom – forming together SU(N) elements. The “relative rotation” corresponds to the non-abelian charge on the link.
• The link’s $J$ quantum number (SU2) representation) is a dynamical

![Diagram](image1)

**FIG. 6.** Product of boxes with an overlapping link.
Kogut and Susskind, PRD 1975

![Diagram](image2)

**FIG. 7.** Replacing the flux in link 4 of Fig. 6 by $j = 0$ and $j = 1$ flux lines.

• How to obtain links enabling large values of $J$?
• How to obtain plaquettes when one truncates $J$?

So far only methods for *strong limit* simulations are known – including the 1+1 non-abelian generalization of the Schwinger model.
QUANTUM SIMULATIONS
MEASUREMENTS
Experimental Demonstration of Single-Site Addressability in a Two-Dimensional Optical Lattice

Peter Würtz,¹ Tim Langen,¹ Tatjana Gericke,¹ Andreas Koglbauer,¹ and Herwig Ott¹,²,*

¹Institut für Physik, Johannes Gutenberg-Universität, 55099 Mainz, Germany
²Research Center OPTIMAS, Technische Universität Kaiserslautern, 67663 Kaiserslautern, Germany
(Received 18 March 2009; published 21 August 2009)

FIG. 1 (color online). Electron microscope image of a Bose-Einstein condensate in a 2D optical lattice with 600 nm lattice spacing (sum obtained from 260 individual experimental realizations). Each site has a tubelike shape with an extension of 6 μm perpendicular to the plane of projection. The central lattice sites contain about 80 atoms.

FIG. 2 (color online). Patterning a Bose-Einstein condensate in a 2D optical lattice with a spacing of 600 nm. Every emptied site was illuminated with the electron beam (7 nA beam current, 100 nm FWHM beam diameter) for (a),(b) 3, (c),(d) 2, and (e) 1.5 ms, respectively. The imaging time was 45 ms. Between 150 and 250 images from individual experimental realizations have been summed for each pattern.
A quantum gas microscope for detecting single atoms in a Hubbard-regime optical lattice

Waseem S. Bakr¹, Jonathon I. Gillen¹, Amy Peng¹, Simon Fölling¹ & Markus Greiner¹

minded through preparation and measurement. By implementing a high-resolution optical imaging system, single atoms are detected with near-unity fidelity on individual sites of a Hubbard-regime optical lattice. The lattice itself is generated by projecting a holographic mask through the imaging system. It has an arbitrary geometry, chosen to support both strong tunnel coupling between lattice sites and strong on-site confinement. Our approach can be

Figure 3 | Site-resolved imaging of single atoms on a 640-nm-period optical lattice, loaded with a high density Bose–Einstein condensate. Inset, magnified view of the central section of the picture. The lattice structure and the discrete atoms are clearly visible. Owing to light-assisted collisions and molecule formation on multiply occupied sites during imaging, only empty and singly occupied sites can be seen in the image.
Single-spin addressing in an atomic Mott insulator

Christof Weitenberg, Manuel Endres, Jacob F. Sherson, Marc Cheneau, Peter Schauß, Takeshi Fukuhara, Immanuel Bloch & Stefan Kuhr

Figure 2 | Single-site addressing. a, Top, experimentally obtained fluorescence image of a Mott insulator with unity filling in which the spin of selected atoms was flipped from |0⟩ to |1⟩ using our single-site addressing scheme. Atoms in state |0⟩ were removed by a resonant laser pulse before detection. Bottom, the reconstructed atom number distribution on the lattice. Each filled circle indicates a single atom, the points mark the lattice sites. b, Top, as for a except that a global microwave sweep exchanged the population in |0⟩ and |1⟩, such that only the addressed atoms were observed. Bottom, the reconstructed atom number distribution shows 14 atoms on neighbouring sites. c–f, As for b, but omitting the atom number distribution. The images contain 29 (c), 35 (d), 18 (e) and 23 (f) atoms. The single isolated atoms in b, e and f were placed intentionally to allow for the correct determination of the lattice phase for the feedback on the addressing beam position.
QUANTUM SIMULATIONS

MEASUREMENTS

• Preparation of initial states
  – Various regimes and charge distributions
    (single addressing)

• Tunable parameters
  – (Feshbach resonances, tunneling rates,
    Raman lasers)

• Dynamical real time evolution
  – Confinement of dynamic charges
  – Flux tubes/loops breaking
  – Pair production, vacuum instability
  – ...

• Non-perturbative physics
• Probing phase transitions
• Finite chemical potential, Finite temperature,...
  Color superconductivity, Quark-Gluon plasma??
Detecting Wilson Loop’s area law by interference of “Mesons”.
This is equivalent to Ramsey Spectroscopy in quantum optics!

### QUANTUM SIMULATIONS

#### OUR PROPOSALS

<table>
<thead>
<tr>
<th>Theory</th>
<th>1+1 with matter</th>
<th>d+1 Pure</th>
<th>d+1 with matter</th>
</tr>
</thead>
<tbody>
<tr>
<td>U(1) - cQED</td>
<td>✓</td>
<td>K.S. (or truncated)</td>
<td>K.S. (or truncated)</td>
</tr>
<tr>
<td>Z(N)</td>
<td>✗</td>
<td>✓</td>
<td>✗</td>
</tr>
<tr>
<td>SU(2) – Yang Mills</td>
<td>Low energy states</td>
<td>Strong limit</td>
<td>Strong limit</td>
</tr>
</tbody>
</table>

K.S. := Kogut Susskind Hamiltonian LGT formalism
• 1+1D: Schwinger’s model.

• cQED: 2+1D: no phase transition
  Instantons give rise to confinement at $g < 1$ (Polyakov).
  (For $T > 0$: there is a phase transition also in 2+1D.)

• cQED: 3+1D: phase transition between a strong coupling
  confining phase, and a weak coupling coulomb phase.

• $\mathbb{Z}(N)$: for $N \geq N_c$: Three phases: electric confinement, magnetic confinement, and non confinement.
OUTLOOK

Non Abelian in Higher Dimensions

Plaquettes in 2+1 and 3+1
Abelian, cQED and Z(N)

“Proof of principle” 1+1 toy models
Numerical comparison with DMRG

Decoherence, superlattices, scattering parameters control...
Lattice gauge theories can be mapped to an analog cold atom simulator.

Atomic conservation laws give rise to exact gauge symmetry.

Near future experiments may be able to realize first steps in this direction, and offer a new types of LGT simulations.

Weitenberg et. al., Nature, 2011
Lattice gauge theories can be mapped to an analog cold atom simulator.

Atomic conservation laws give rise to exact gauge symmetry.

Near future experiments may be able to realize first steps in this direction, and offer a new types of LGT simulations.

Weitenberg et. al., Nature, 2011
Z$_N$ Gauge theory

- Abelian *discrete* gauge theory: the gauge field degrees of freedom operate in a finite Hilbert space
- Three phases in 3+1 dimensions

\[ P \sim e^{iE} \]
\[ Q \sim e^{iA} \]
\[ P^N = Q^N = 1 \quad ; \quad P^\dagger QP = e^{i\delta} Q \]

Adapted from Horn et. al., PRD 19, 3715, 1979
Simulating $\mathbb{Z}_N$ Gauge theory

- Finite Hilbert spaces on links: one can realize unitary operators in the elementary link interactions, obtained using hybridized levels.
- In a pure gauge theory, plaquettes are obtained similarly, using the "loop method"