QUANTUM SIMULATION OF LATTICE GAUGE THEORIES WITH COLD ATOMS

Benni Reznik
Tel-Aviv University

In collaboration with E. Zohar (Tel-Aviv) and J. Ignacio Cirac, (MPQ)

Lattice 2013, July 30, Gutenberg University, Mainz

OUTLINE

QUANTUM SIMULATION

COLD ATOMS IN OPTICAL LATTICES

HAMILTONIAN LGT

Q. SIMULATION: LGT

- REQUIREMENTS
- EXACT AND EFFECTIVE LOCAL GAUGE INVARIANCE
- LINKS AND PLAQUETTES – Examples: cQED, Z(N), SU(2)

CURRENT EXPERIMENTS AND LGT SIMULATIONS

OUTLOOK.

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have

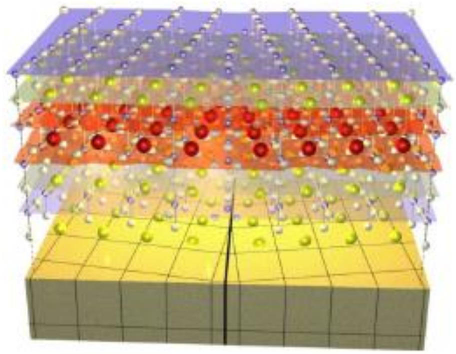
be understood very well in analyzing the situation. And I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy. Thank you.

International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982

QUANTUM SIMULATION

ANALOG

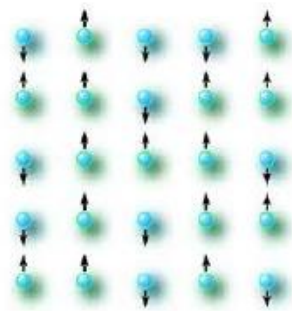
PHYSICAL SYSTEM



(Phenomenological) Hamiltonian

$$H = \ldots$$

QUANTUM SIMULATOR

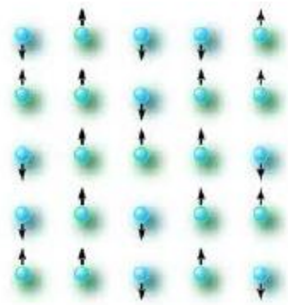


Physical Hamiltonian

$$H = \ldots$$

Example: Hubbard model in 2D: $H = -t \sum_{k,\sigma} c^{\dagger}_{k\sigma} c_{k\sigma} + V \sum_{k} n_{k\uparrow} n_{k\downarrow}$

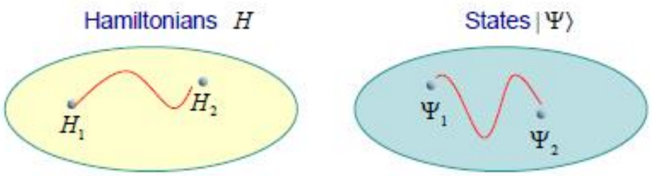
QUANTUM SIMULATION
ANALOG



$$H = \ldots$$

- **Questions:**
  - Dynamics: $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$
  - Ground state: $H|\Psi_0\rangle = E_0|\Psi_0\rangle$

Adiabatic algorithms



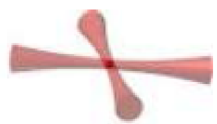
  - Physical properties: $\langle \sigma_n \rangle, \langle \sigma_n \sigma_m \rangle, \ldots$

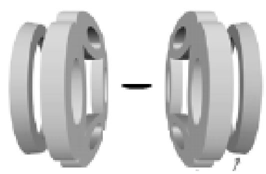
COLD ATOMS

- Control: External fields

Trapping:

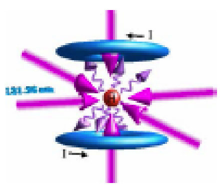
lasers — Magnetic fields



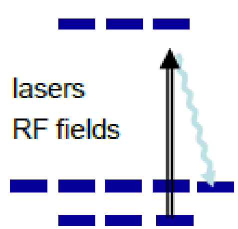


Cooling:

lasers — evaporation



Internal manipulation

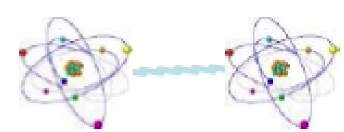


lasers
RF fields

purification
coherence
detection

Interactions

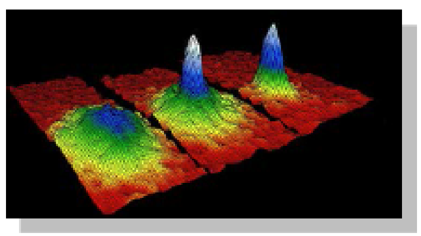
tune scattering length

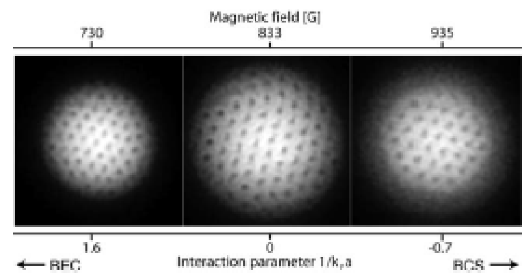


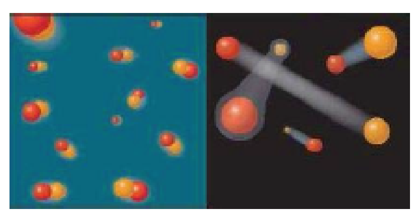
COLD ATOMS

- Many-body phenomena
  - Degeneracy: bosons and fermions (BE/FD statistics)
  - Coherence: interference, atom lasers, four-wave mixing, …
  - Superfluidity: vortices
  - Disorder: Anderson localization
  - Fermions: BCS-BEC

+ many other phenomena







COLD ATOMS

- Cold atoms are described by simple quantum field theories:

$$H = \int \Psi_{\sigma}^{\dagger} \left(-\nabla^2 + V(r) \right) \Psi_{\sigma} + u_{\sigma_i} \int \Psi_{\sigma_1}^{\dagger} \Psi_{\sigma_2}^{\dagger} \Psi_{\sigma_3} \Psi_{\sigma_4}$$

  - We can have bosons or fermions (or both).
  - We can different internal states (spin).
  - The external potential, V, and interaction coefficients, u, can be engineered using lasers, and electric and magnetic fields.
  - In certain limits, one obtains effective theories that are interesting in other fields of Physics.

$\Downarrow$

Quantum Simulations

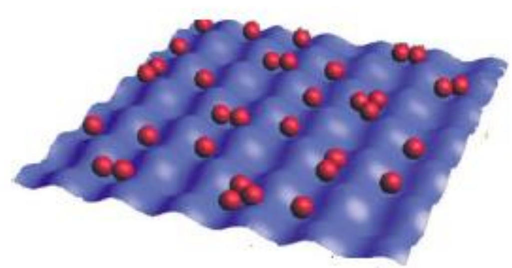
COLD ATOMS
OPTICAL LATTICES

- Laser standing waves: dipole-trapping

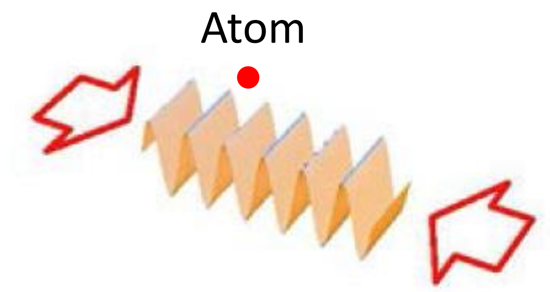
VOLUME 81, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1998

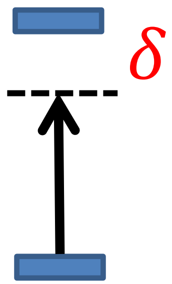
Cold Bosonic Atoms in Optical Lattices

D. Jaksch,[1,2] C. Bruder,[1,3] J. I. Cirac,[1,2] C. W. Gardiner,[1,4] and P. Zoller[1,2]



COLD ATOMS
OPTICAL LATTICES

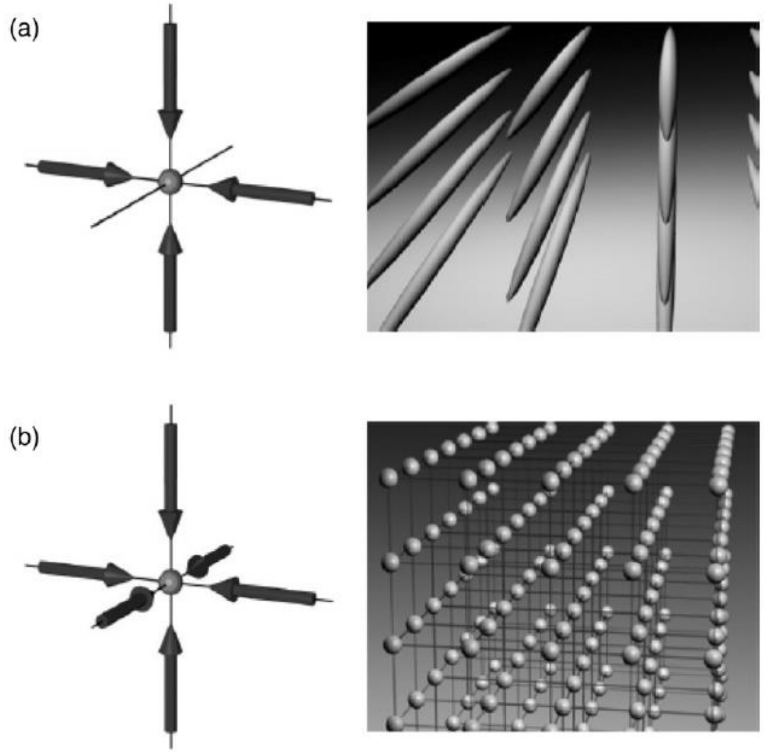




In the presence $\boldsymbol{E}(r,t)$ the atoms has a time dependent dipole moment $d(t) = \alpha(\omega)\,\boldsymbol{E}(r,t)$ of some non resonant excited states.
Stark effect:

$$\mathrm{V}(\mathrm{r}) \equiv \Delta\mathrm{E}(\mathrm{r}) = \alpha(\omega)\langle\,\boldsymbol{E}(r,t)\,\boldsymbol{E}(r,t)\rangle/\delta$$

COLD ATOMS
OPTICAL LATTICES



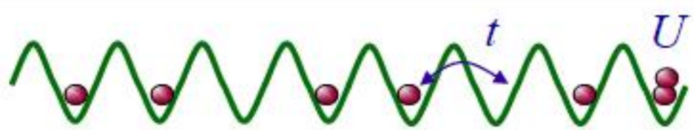
(a) 2d array of effective 1d traps
(b) 3d square lattice

M. Lewenstein et. al, Advances in Physics, 2010.

COLD ATOMS
OPTICAL LATTICES

- Laser standing waves: dipole-trapping

$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r)\right)\Psi_\sigma + u_{\sigma_i}\int \Psi_{\sigma_1}^\dagger \Psi_{\sigma_2}^\dagger \Psi_{\sigma_3} \Psi_{\sigma_4}$$



Lattice theory: Bose/Fermi-Hubbard model

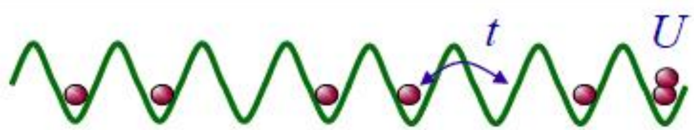
$$H = -t\sum_n \left(a_n^\dagger a_{n+1} + h.c\right) + U\sum_n a_n^{\dagger 2} a_n^2$$

COLD ATOMS

OPTICAL LATTICES

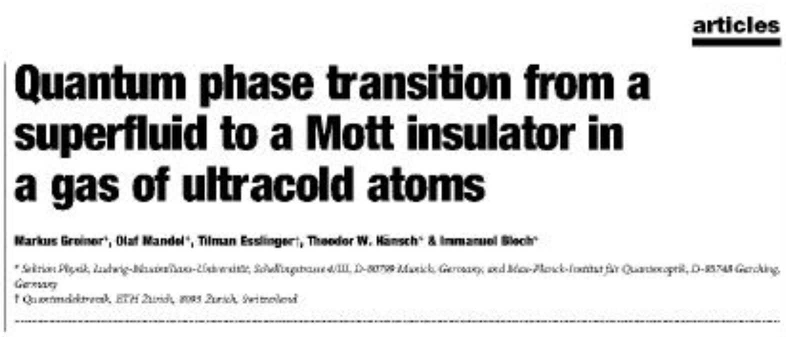
- Laser standing waves: dipole-trapping

$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r) \right) \Psi_\sigma + u_{\sigma_i} \int \Psi_{\sigma_1}^\dagger \Psi_{\sigma_2}^\dagger \Psi_{\sigma_3} \Psi_{\sigma_4}$$



Lattice theory: Bose/Fermi-Hubbard model

$$H = -t \sum_n \left(a_n^\dagger a_{n+1} + h.c \right) + U \sum_n a_n^{\dagger 2} a_n^2$$

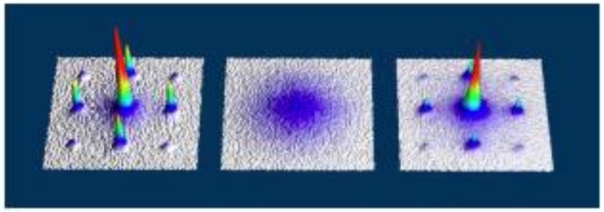


articles

Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

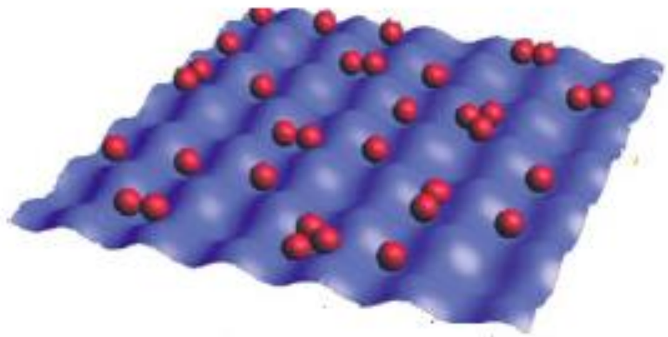
Markus Greiner*, Olaf Mandel*, Tilman Esslinger†, Theodor W. Hänsch* & Immanuel Bloch*

** Sektion Physik, Ludwig-Maximilians-Universität, Schellingstrasse 4/III, D-80799 Munich, Germany, and Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany*
† Quantenelektronik, ETH Zürich, 8093 Zurich, Switzerland



COLD ATOMS

QUANTUM SIMULATIONS



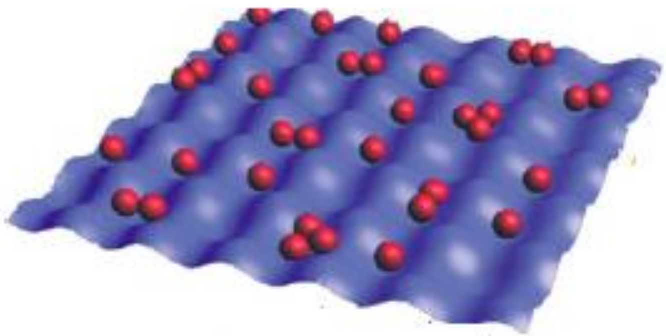
- Bosons/Fermions: $$H = -\sum_{\substack{\langle n,m\rangle \\ \sigma,\sigma'}} \left(t_{\sigma,\sigma'} a^{\dagger}_{n,\sigma} a_{m,\sigma'} + h.c \right) + \sum_{\substack{n \\ \sigma,\sigma'}} U_{\sigma,\sigma'} a^{\dagger}_{n,\sigma} a^{\dagger}_{n,\sigma'} a_{n,\sigma'} a_{n,\sigma}$$

- Spins: $$H = -\sum_{\substack{\langle n,m\rangle \\ \sigma,\sigma'}} \left(J_x S^x_n S^x_m + J_y S^y_n S^y_m + J_z S^z_n S^z_m \right) + \sum_{\substack{n \\ \sigma,\sigma'}} B_n S^z_n$$

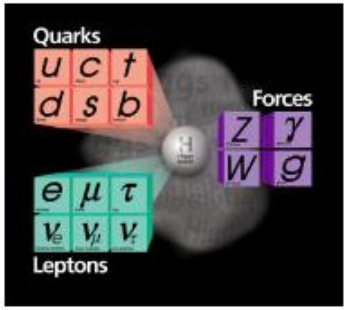
$\Rightarrow$ CONDENSED MATTER PHYSICS

COLD ATOMS

QUANTUM SIMULATIONS



HIGH ENERGY PHYSICS?



LATTICE GAUGE THEORIES
HAMILTONIAN FORMULATION

LATTICE GAUGE THEORIES
HAMILTONIAN FORMULATION

PHYSICAL REVIEW D VOLUME 11, NUMBER 2 15 JANUARY 1975

Hamiltonian formulation of Wilson's lattice gauge theories

John Kogut*
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Leonard Susskind†
Belfer Graduate School of Science, Yeshiva University, New York, New York
and Tel Aviv University, Ramat Aviv, Israel
and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

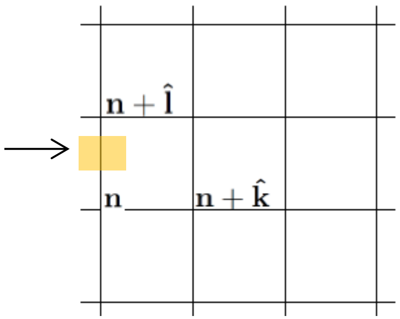
(Received 9 July 1974)

Wilson's lattice gauge model is presented as a canonical Hamiltonian theory. The structure of the model is reduced to the interactions of an infinite collection of coupled rigid rotators. The gauge-invariant configuration space consists of a collection of strings with quarks at their ends. The strings are lines of non-Abelian electric flux. In the strong-coupling limit the dynamics is best described in terms of these strings. Quark confinement is a result of the inability to break a string without producing a pair.

LATTICE GAUGE THEORIES
HAMILTONIAN FORMULATION

Gauge group elements:

U^r is an element of the gauge group (in the representation r), on each link



Left and right generators:

$$[L_a, U^r] = T_a^r U^r \quad ; \quad [R_a, U^r] = U^r T_a^r$$

$$[L_a, L_b] = -i f_{abc} L_c \quad ; \quad [R_a, R_b] = i f_{abc} R_c \quad ; \quad [L_a, R_b] = 0$$

$$\sum_a L_a L_a = \sum_a R_a R_a \equiv \sum_a E_a E_a$$

Gauge transformation:

$$U^r_{\mathbf{n},k} \rightarrow V^r_{\mathbf{n}} U^r_{\mathbf{n},k} V^{\dagger r}_{\mathbf{n}+\hat{\mathbf{k}}}$$

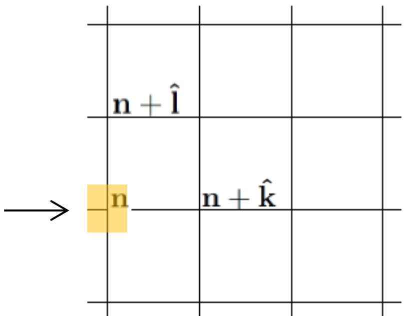
Generators:

$$(G_{\mathbf{n}})_a = \mathrm{div}_{\mathbf{n}} E_a = \sum_k \left((L_{\mathbf{n},k})_a - \left(R_{\mathbf{n}-\hat{\mathbf{k}},k}\right)_a \right)$$

LATTICE GAUGE THEORIES
HAMILTONIAN FORMULATION

Matter:

$$\psi_{\mathbf{n}} = (\psi_{\mathbf{n},a}) = \begin{pmatrix} \psi_{\mathbf{n},1} \\ \psi_{\mathbf{n},2} \\ \cdots \end{pmatrix}$$



Gauge transformation:

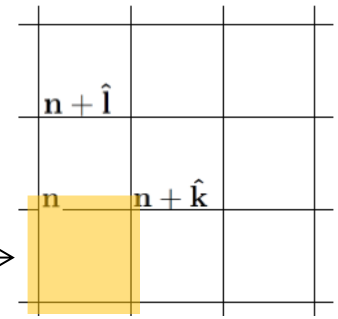
$$\psi_{\mathbf{n}} \rightarrow V^{r}_{\mathbf{n}} \psi_{\mathbf{n}}$$

LATTICE GAUGE THEORIES
HAMILTONIAN FORMULATION

Gauge field dynamics (Kogut-Susskind Hamiltonian):

$$H_E = \frac{g^2}{2} \sum_{\mathbf{n},k,a} \left(E_{\mathbf{n},k}\right)_a \left(E_{\mathbf{n},k}\right)_a$$

$$H_B = -\frac{1}{g^2} \sum_{\text{plaquettes}} \left(\mathrm{Tr} \left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$$

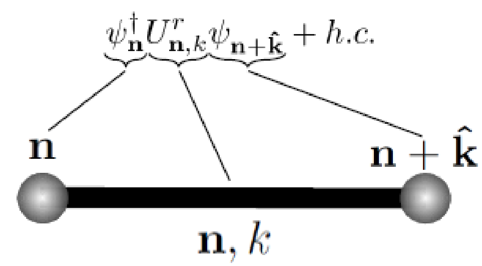


Strong coupling limit: *g* >> 1
Weak coupling limit: *g* << 1

Matter dynamics:

$$H_M = \sum_{\mathbf{n}} M_{\mathbf{n}} \psi_{\mathbf{n}}^\dagger \psi_{\mathbf{n}}$$

$$H_{int} = \epsilon \sum_{\mathbf{n},k} \left(\psi_{\mathbf{n}}^\dagger U_{\mathbf{n},k}^r \psi_{\mathbf{n}+\hat{\mathbf{k}}} + h.c. \right)$$



LATTICE GAUGE THEORIES
EXAMPLE – cQED

Compact QED (U(1)):

$$U_{\mathbf{n},k} = e^{i\phi_{\mathbf{n},k}}$$

$$[E_{\mathbf{n},k}, \phi_{\mathbf{m},l}] = -i\delta_{\mathbf{nm}}\delta_{kl}$$

Magnetic energy

$$H_{cQED} = \frac{g^2}{2}\sum_{\mathbf{n},k} E^2_{\mathbf{n},k} - \frac{1}{g^2}\sum_{\mathbf{n}} \cos\left(\phi_{\mathbf{n},1} + \phi_{\mathbf{n}+\hat{\mathbf{1}},2} - \phi_{\mathbf{n}+\hat{\mathbf{2}},1} - \phi_{\mathbf{n},2}\right)$$

Mass term

Electric energy

$$+ \epsilon\sum_{\mathbf{n},k}\left(\psi^\dagger_{\mathbf{n}} e^{i\phi_{\mathbf{n},k}}\psi_{\mathbf{n}+\hat{\mathbf{k}}} + \psi^\dagger_{\mathbf{n}+\hat{\mathbf{k}}} e^{-i\phi_{\mathbf{n},k}}\psi_{\mathbf{n}}\right) + M\sum_{n}(-1)^n\psi^\dagger_n\psi_n$$

Gauge-Matter interaction

QUANTUM SIMULATION
LATTICE GAUGE THEORIES

E. Zohar, BR, PRL **107**, 275301 (2011)

E. Zohar, I. Cirac, BR, PRL **109**, 125302 (2012)

E. Zohar, BR, NJP **15**, 043041 (2013)

E. Zohar, I. Cirac, BR, PRL **110** 055302 (2013)

E. Zohar, I. Cirac, BR, PRL **110** 125304 (2013)

E. Zohar, I. Cirac, BR, PRA (2013) arxiv 1303.5040

QUANTUM SIMULATION
LATTICE GAUGE THEORIES

E. Zohar, BR, PRL **107**, 275301 (2011)

E. Zohar, I. Cirac, BR, PRL **109**, 125302 (2012)

E. Zohar, BR, NJP **15**, 043041 (2013)

E. Zohar, I. Cirac, BR, PRL **110** 055302 (2013)

E. Zohar, I. Cirac, BR, PRL **110** 125304 (2013)

Detailed account → E. Zohar, I. Cirac, BR, PRA (2013) arxiv 1303.5040

QUANTUM SIMULATION
GAUGE THEORIES

- **Continuum fields**

Thirring, Gross-Neveu, I. Cirac, P. Maraner and J. K. Pachos, PRL, **105**, 19403 (2010)

Continuum QED, E. Kapit and E. Mueller, Phys. Rev. A **83**, 033625 (2011)

Abelian local gauge symmetry

2+1, (3+1), U(1) Kogut-Susskind

Pure gauge – E. Zohar and B. Reznik, PRL **107**, 275301 (2011)

Pure gauge – truncated – E. Zohar, J. I. Cirac and B. R., PRL **109**, 125302 (2012)

Truncated with dynamic matter – E. Zohar, J. I. Cirac and B. R. PRL **110**, 055302 (2013)

- **Full cQED** – E. Zohar, J. I. Cirac and B. Reznik, arxiv 1303.5040, Phys. Rev. A

2+1 U(1) gauge magnets

L. Tagliacozzo, A. Celi, A. Zamora and M. Lewenstein, Ann. Phys. **330**, 160 (2013) (digital s.)

1+1 U(1) link models

Cold Atoms – D. Banerjee, M. Dalmonte, M. Mueller, E. Rico, P. Stebler, U.-J. Wiese and P. Zoller, PRL **109**, 175302 (2012) (and 2+1 in the strong coupling)

Superconducting qubits – D. Marcos, P. Rabl, E. Rico, P. Zoller, arxiv 1306.1674

Ions – P. Hauke, D. Marcos, M. Dalmote and P. Zoller, arxiv 1306.2162

- Gauge symmetry connected with angular momentum conservation.

QUANTUM SIMULATION
GAUGE THEORIES

Discrete groups: 2+1, 3+1 Z(N)

L. Tagliacozzo, A. Celi, A. Zamora and M. Lewenstein,
Ann. Phys. **330**, 160 (2013) , (Z(2) digital s.)

- E. Zohar, J. I. Cirac and B. Reznik, arxiv 1303.5040, Phys. Rev. A

Non-abelian

Kogut-Susskind SU(2) Yang Mills , 1+1, (2+1) D

- E. Zohar, J. I. Cirac and B. Reznik, Phys. Rev. Lett. **110**, 25304 (2013)
- E. Zohar, J. I. Cirac and B. Reznik, arxiv 1303.5040, Phys. Rev. A

Non-abelian link models, 1+1 D

- D. Banerjee, M.Bögli, M. Dalmonte, E. Rico, P. Stebler,
U.-J. Wiese, P. Zoller, Phys. Rev. Lett. **110** 125303 (2013)

SU(2) gauge magnets, 2+1, digital simulation

L. Tagliacozzo, A. Celi, P. Orland, M. Lewenstein, arxiv 1211.2704

Quantum computation

Scattering probabilities for scalar fields: S. P. Jordan, K. S. M. Lee and J. Preskill,
Science **336**, 1130 (2012)

- Gauge symmetry connected with angular momentum conservation.

Requirements: HEP models

- <u>Fields</u>

  Fermion Matter fields

  Bosonic gauge fields

- <u>Local gauge invariance</u>

  Exact, or low energy, effective

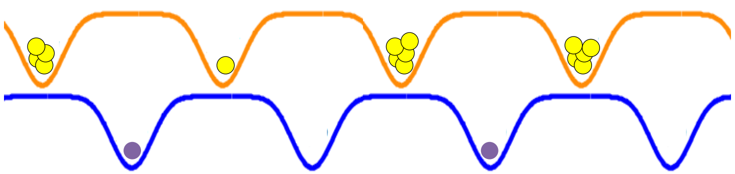
- <u>Relativistic invariance</u>

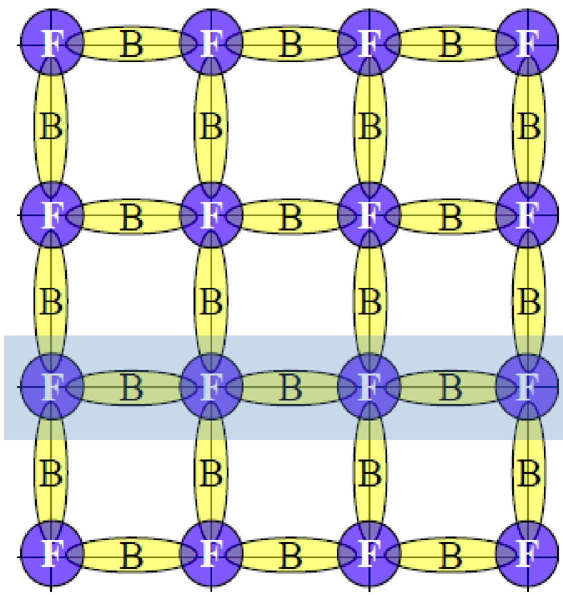
  Causal structure, in the continuum limit

QUANTUM SIMULATION
COLD ATOMS

- Fermion matter fields
- Bosonic gauge fields

Superlattices:





$$\psi_{\mathbf{n}} = (\psi_{\mathbf{n},a}) = \begin{pmatrix} \psi_{\mathbf{n},1} \\ \psi_{\mathbf{n},2} \\ \cdots \end{pmatrix} \longrightarrow$$ Atom internal levels

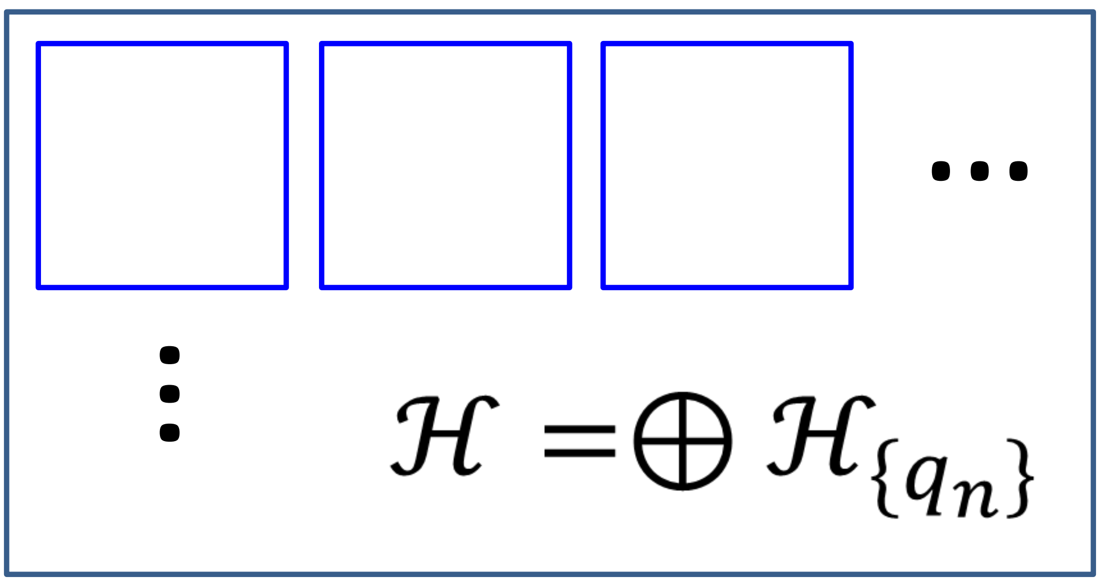
QUANTUM SIMULATION
LOCAL GAUGE INVARIANCE

- Generators of gauge transformations:

$$(G_{\mathbf{n}})_a = \mathrm{div}_{\mathbf{n}} E_a - Q_{\mathbf{n}}$$

$$G_{\mathbf{n}} \left| phys \right\rangle = q_{\mathbf{n}} \left| phys \right\rangle$$

$$[G_n, H] = 0$$



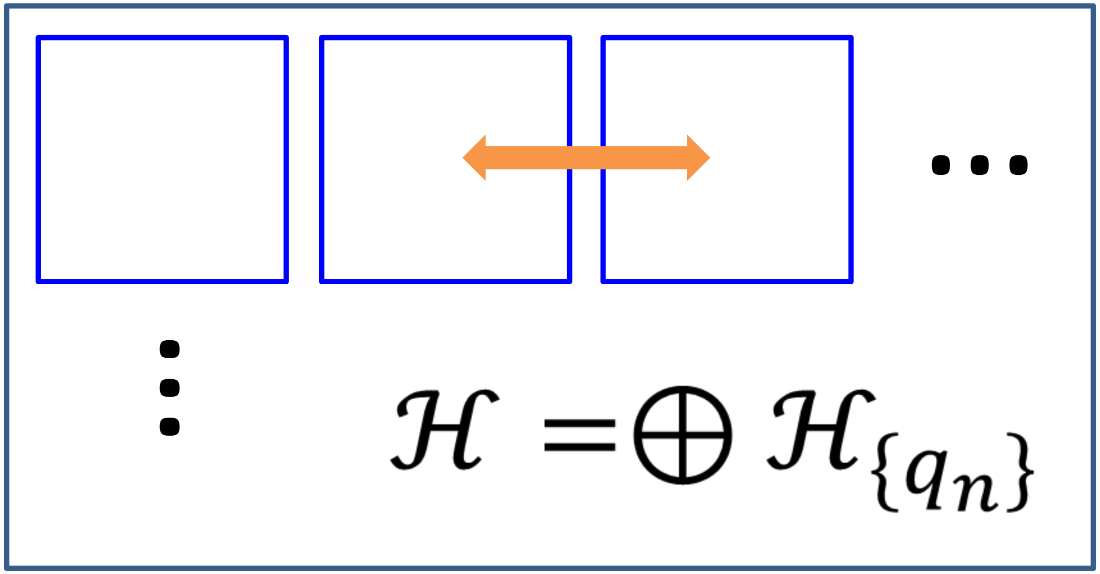
QUANTUM SIMULATION
LOCAL GAUGE INVARIANCE

- Generators of gauge transformations:

$$(G_{\mathbf{n}})_a = \mathrm{div}_{\mathbf{n}} E_a - Q_{\mathbf{n}}$$

$$G_{\mathbf{n}} \left| phys \right\rangle = q_{\mathbf{n}} \left| phys \right\rangle$$

$$[G_n , H] \cancel{=} 0$$



$$\mathcal{H} = \bigoplus \mathcal{H}_{\{q_n\}}$$

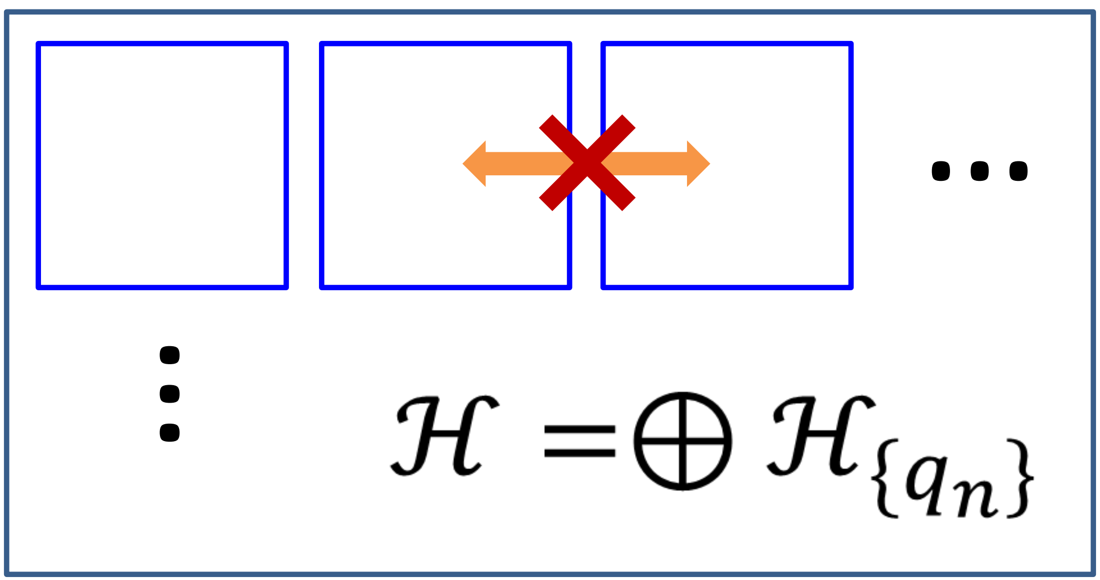
QUANTUM SIMULATION
LOCAL GAUGE INVARIANCE

- Generators of gauge transformations:

$$(G_{\mathbf{n}})_a = \mathrm{div}_{\mathbf{n}} E_a - Q_{\mathbf{n}}$$

$$G_{\mathbf{n}} \left|phys\right\rangle = q_{\mathbf{n}} \left|phys\right\rangle$$

$$[G_n, H] = 0$$



LOCAL GAUGE INVARIANCE

- Generators of gauge transformations:

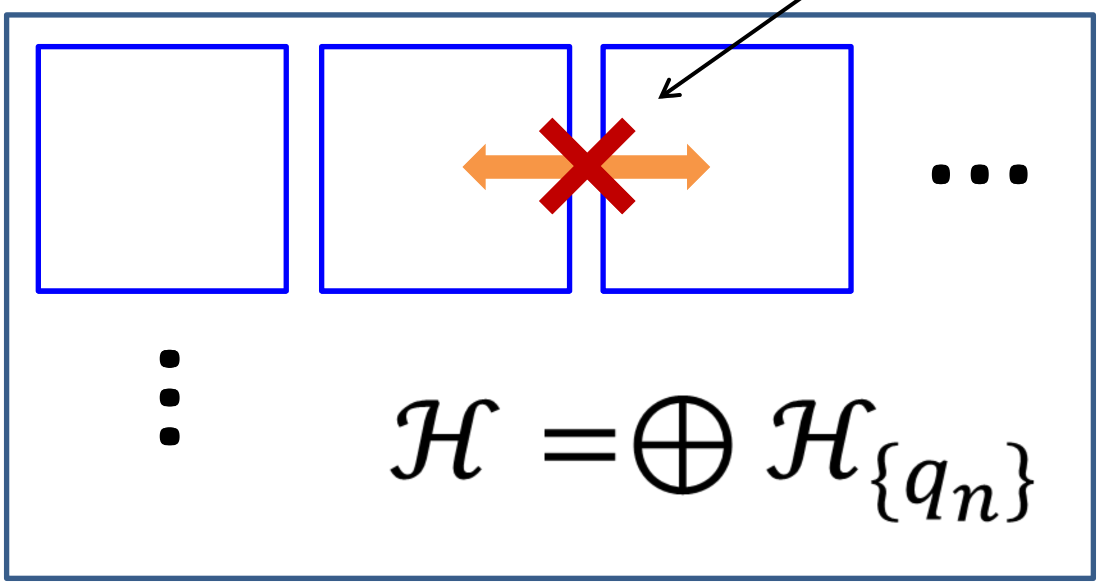
$$(G_{\mathbf{n}})_a = \mathrm{div}_{\mathbf{n}} E_a - Q_{\mathbf{n}}$$

$$G_{\mathbf{n}} \left| phys \right\rangle = q_{\mathbf{n}} \left| phys \right\rangle$$

$$[G_n, H] = 0$$

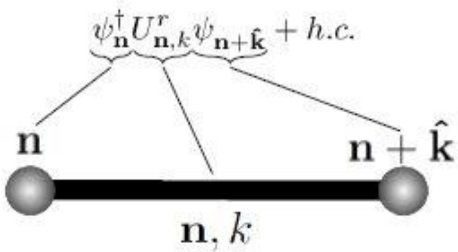
Gauge invariance –

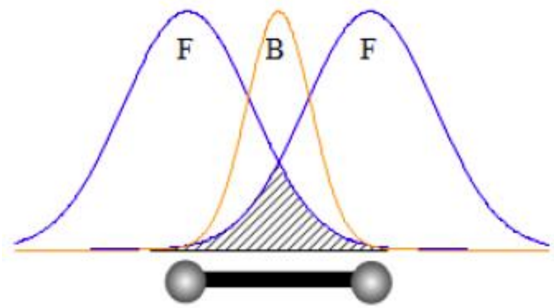
- **Low energy, effective**
- **Exact symmetry**



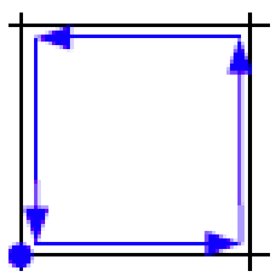
$$\mathcal{H} = \bigoplus \mathcal{H}_{\{q_n\}}$$

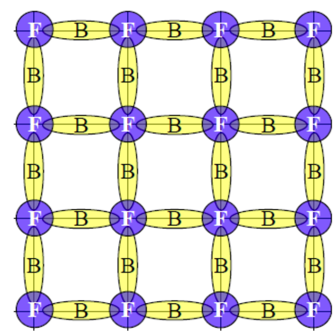
QUANTUM SIMULATION
LOCAL GAUGE INVARIANCE





- Links ↔ atomic scattering : gauge invariance is a <u>fundamental</u> symmetry



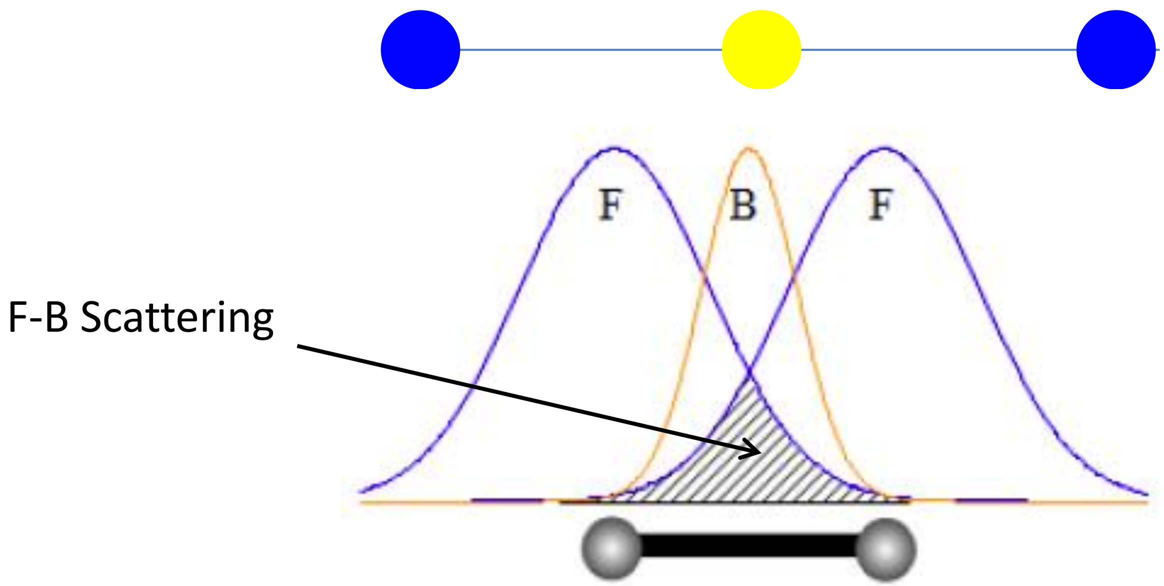


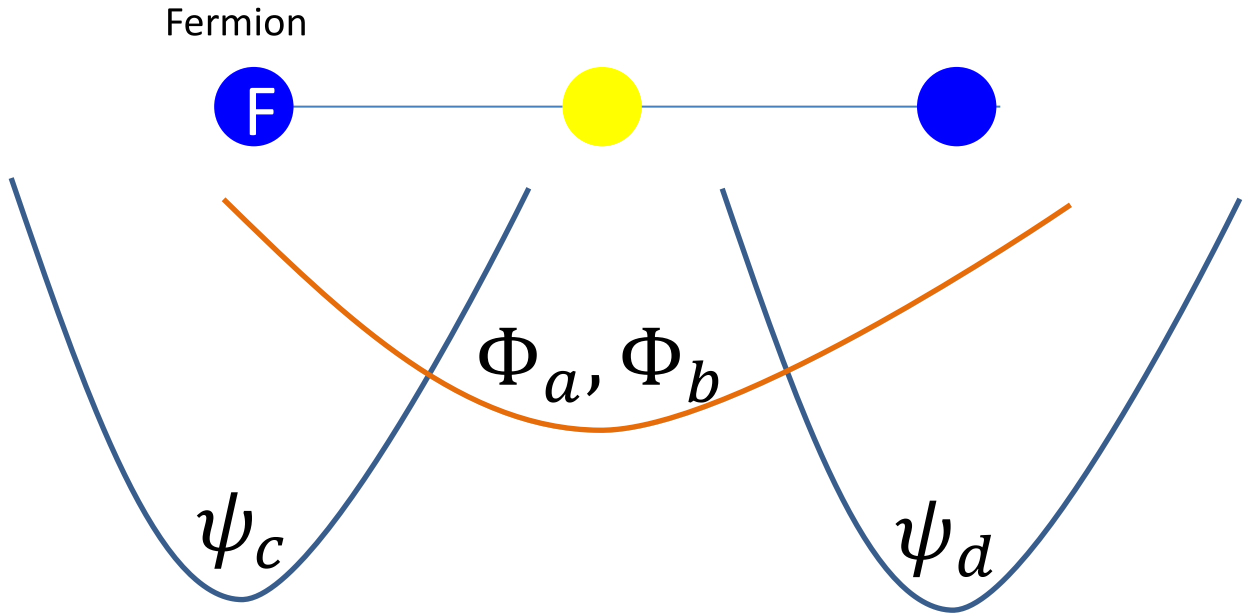
- Plaquettes ↔ gauge invariant links ↔ virtual loops of ancillary fermions.

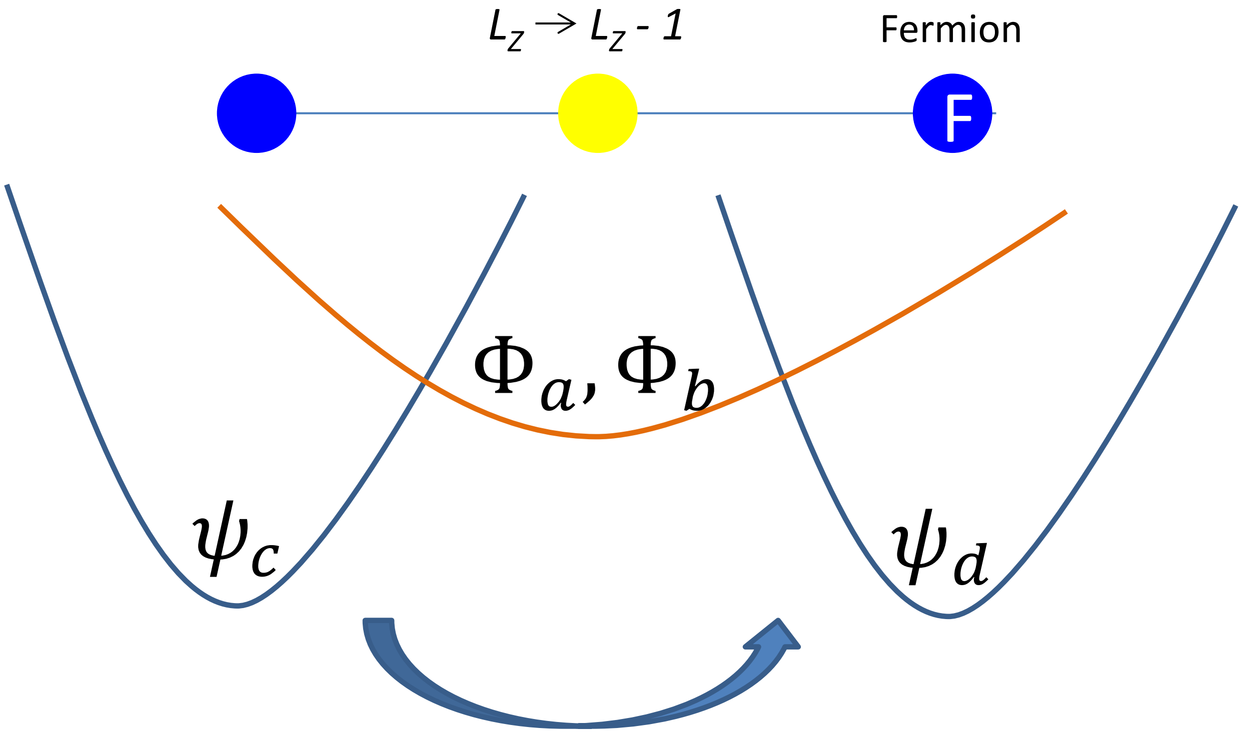
QUANTUM SIMULATION
LINKS

REALIZATION OF LINKS

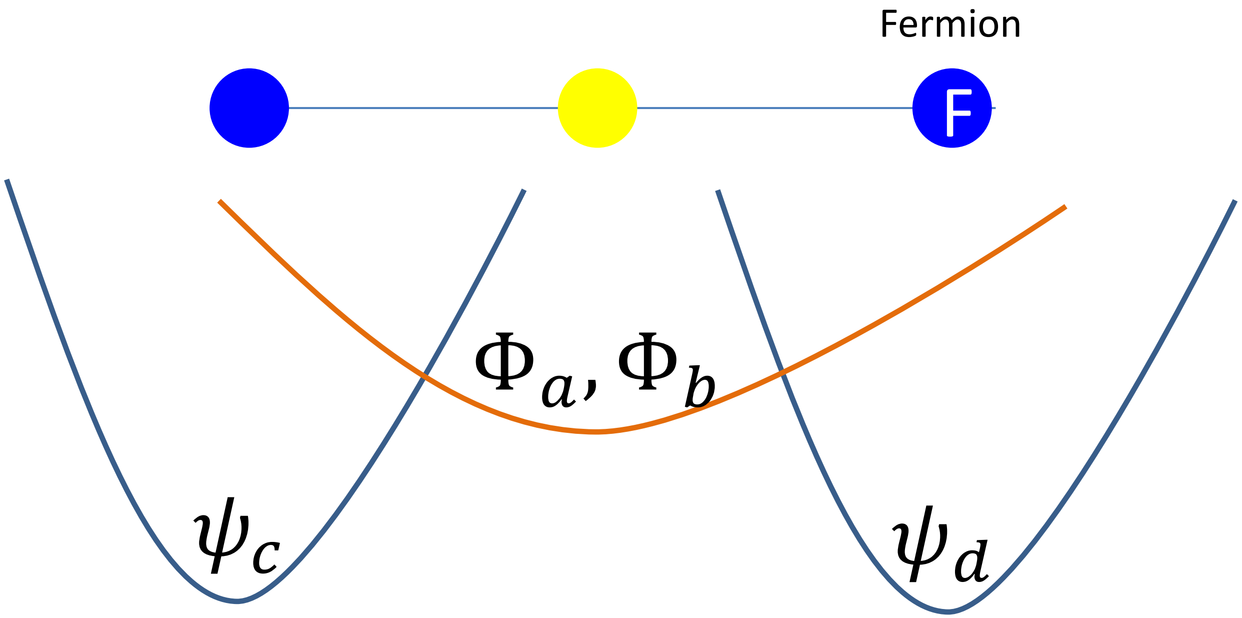
EXAMPLE : cQED

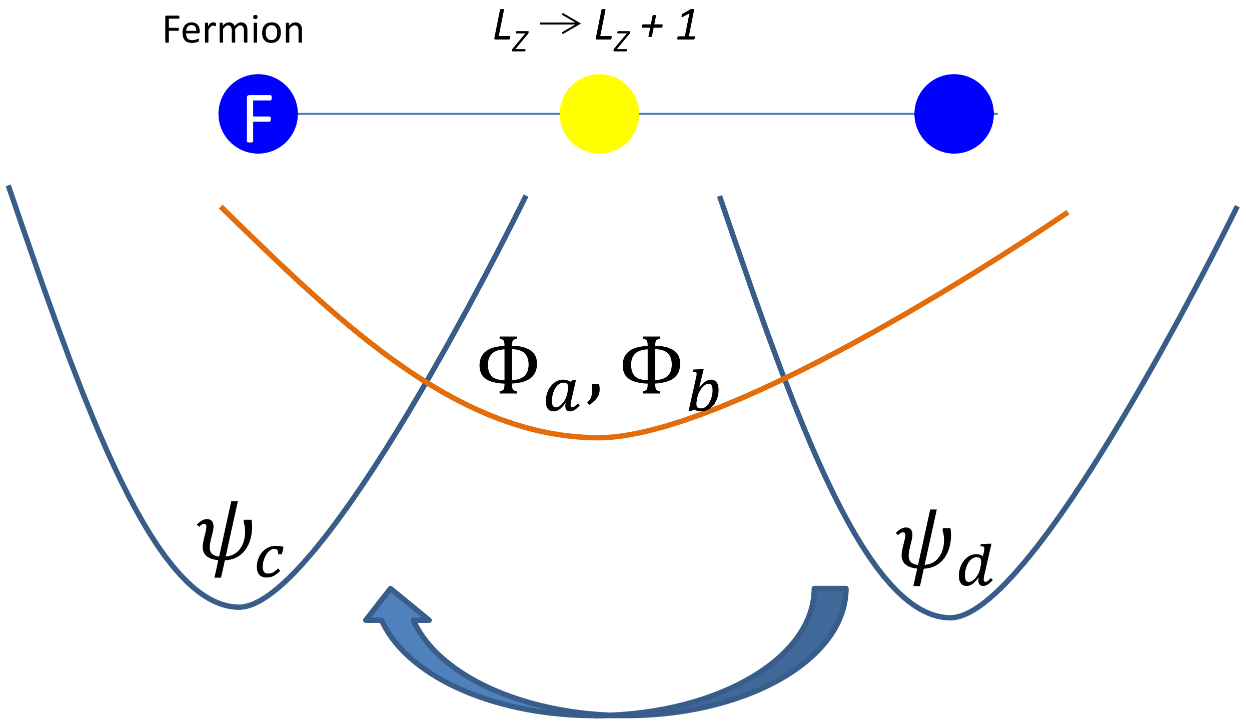



Fermion
F
Φ_a, Φ_b
ψ_c
ψ_d



$L_z \rightarrow L_z - 1$
Fermion
F
Φ_a, Φ_b
ψ_c
ψ_d

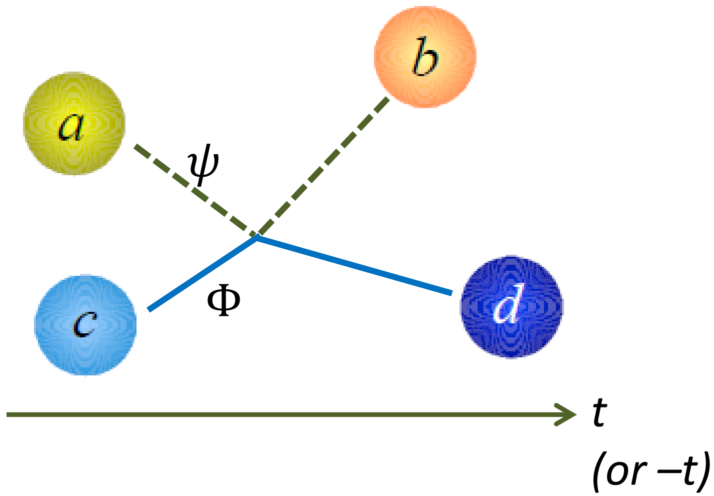

Fermion
F
Φ_a, Φ_b
ψ_c
ψ_d



Fermion
F
$L_z \rightarrow L_z + 1$
Φ_a, Φ_b
ψ_c
ψ_d

ANGULAR MOMENTUM CONSERVATION

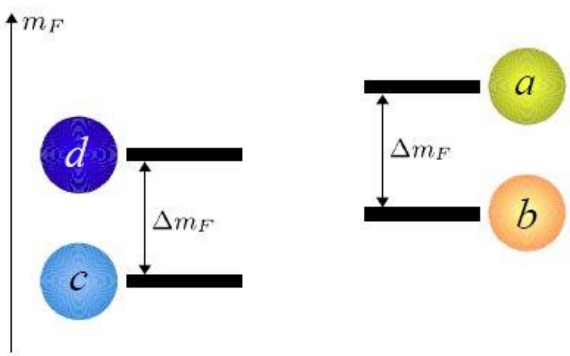
ATOMIC SCATTERING



Hyperfine angular momentum conservation in atomic scattering.

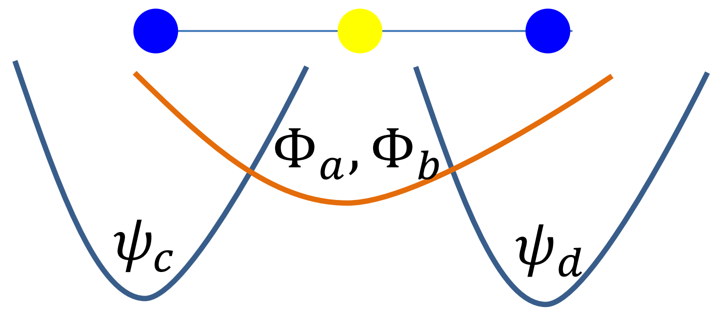
$$\int d^3\mathbf{x}\,\Psi^\dagger_\alpha(\mathbf{x})\,\Psi_\beta(\mathbf{x})\,\Phi^\dagger_\gamma(\mathbf{x})\,\Phi_\delta(\mathbf{x})$$

$$m_F(a) + m_F(c) = m_F(b) + m_F(d)$$

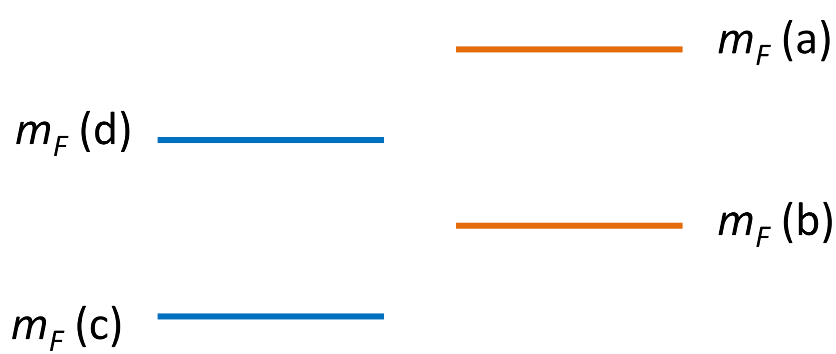


Fermionic atoms Bosonic atoms

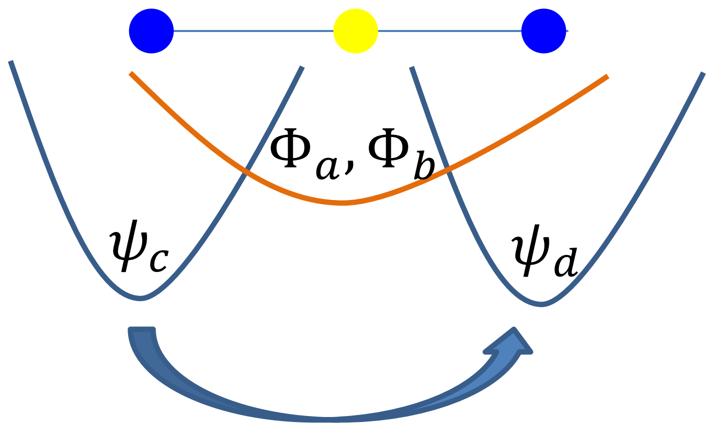
Angular Momentum conservation ↔ Local gauge invariance



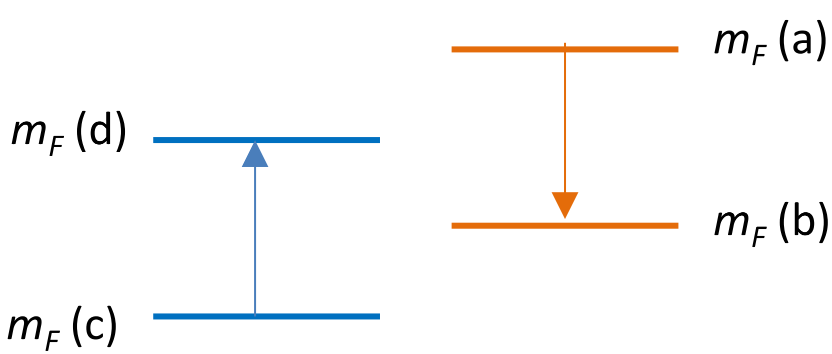
$$\psi_c{}^{\dagger}\Phi_a{}^{\dagger}\Phi_b\psi_d + \psi_d{}^{\dagger}\Phi_b{}^{\dagger}\Phi_a\psi_c$$



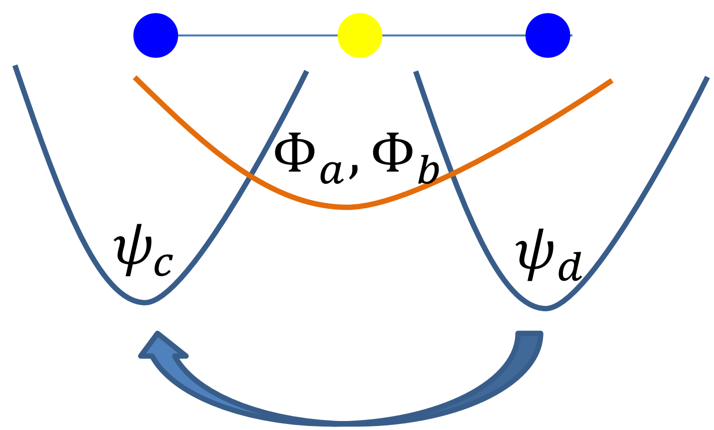
Angular Momentum conservation ↔ Local gauge invariance



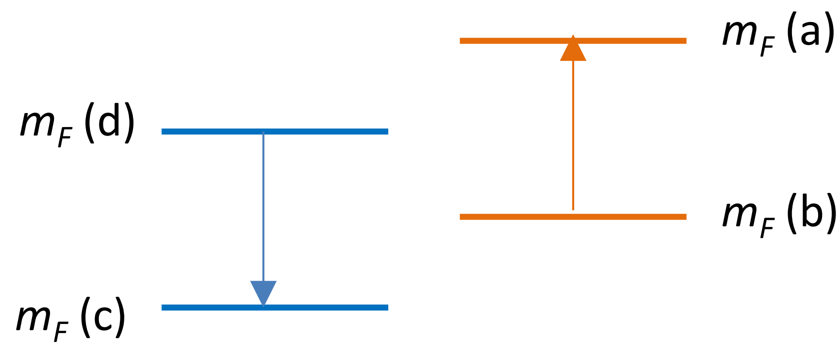
$$\psi_c{}^\dagger \Phi_a{}^\dagger \Phi_b \psi_d + \boxed{\psi_d{}^\dagger \Phi_b{}^\dagger \Phi_a \psi_c}$$



Angular Momentum conservation ↔ Local gauge invariance



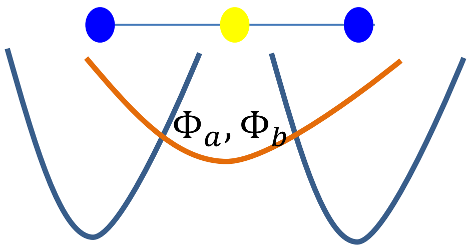
$$\psi_c{}^\dagger \Phi_a{}^\dagger \Phi_b \psi_d + \psi_d{}^\dagger \Phi_b{}^\dagger \Phi_a \psi_c$$



Gauge bosons: Schwinger algebra

$$L_+ = {\Phi_a}^\dagger \Phi_b \quad ; L_- = {\Phi_b}^\dagger \Phi_a$$

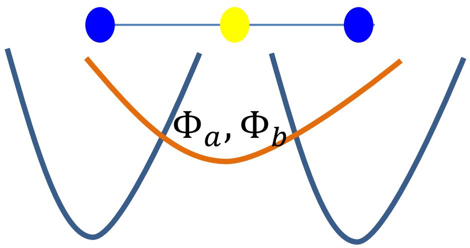
$$L_z = \frac{1}{2}(N_a - N_b) \quad ; l = \frac{1}{2}(N_a + N_b)$$



Gauge bosons: Schwinger algebra

$$L_+ = {\Phi_a}^\dagger \Phi_b \quad ; L_- = {\Phi_b}^\dagger \Phi_a$$

$$L_z = \frac{1}{2}(N_a - N_b) \quad ; l = \frac{1}{2}(N_a + N_b)$$



and thus what we have is

$$\psi_c^\dagger {\Phi_a}^\dagger \Phi_b \psi_d + \psi_d^\dagger {\Phi_b}^\dagger \Phi_a \psi_c$$

$$\Downarrow$$

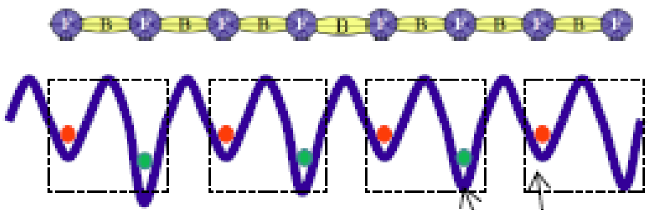
$$\psi_c^\dagger L_+ \psi_d \sim \psi_c^\dagger e^{i\theta} \psi_d \quad \checkmark$$

where for large l , $m \ll l$

$$L_+ \sim e^{i(\phi_1 - \phi_2)} \equiv e^{i\theta}$$

QUANTUM SIMULATION

DYNAMICAL FERMIONS 1+1



$d_n \quad c_{n+1}$

$$H_M = M\sum_n (-1)^n \psi_n^\dagger \psi_n$$

internal states

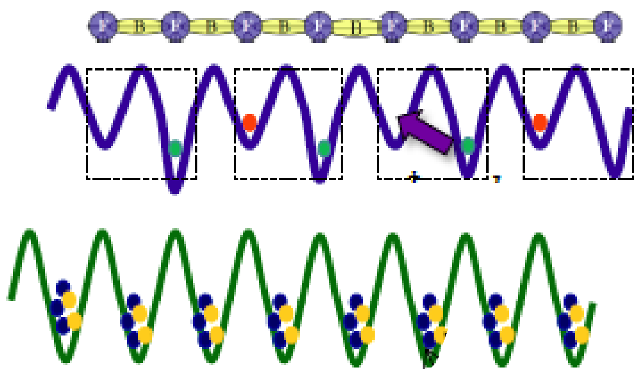
$$\{c_n, c_n^\dagger\} = \{d_n, d_n^\dagger\} = 1$$

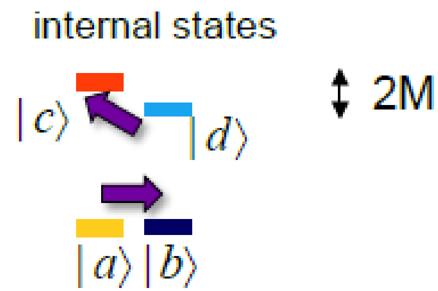
Staggered Fermions:

L. Susskind, Phys. Rev. D 16, 3031 (1977).

QUANTUM SIMULATION

SCHWINGER MODEL 1+1





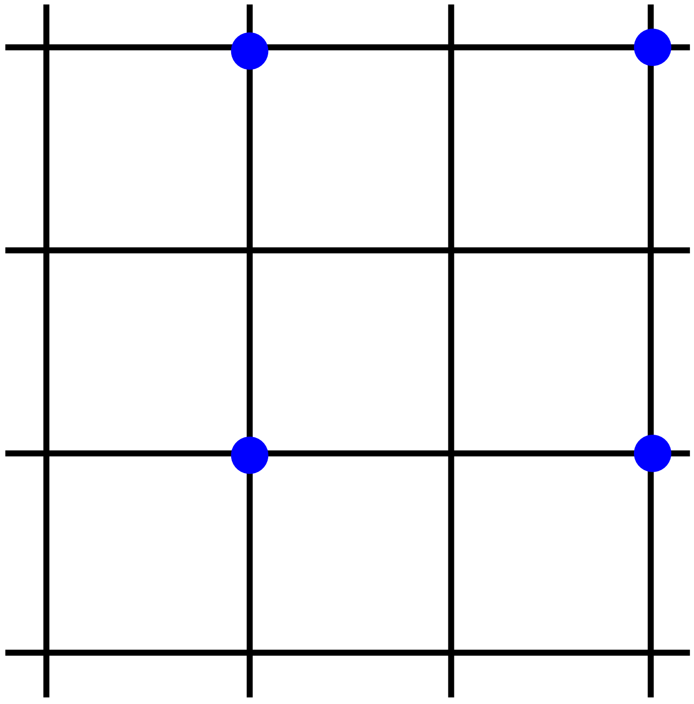
$$\frac{\epsilon}{\sqrt{\ell\left(\ell+1\right)}}\sum_{n}\left(\psi_n^{\dagger}L_{+,n}\psi_{n+1}+h.c.\right)$$

QUANTUM SIMULATION
PLAQUETTES

QUANTUM SIMULATION
PLAQUETTES

1d elementary link interactions – **already gauge invariant building blocks** of effective plaquettes

Auxiliary fermions := ●

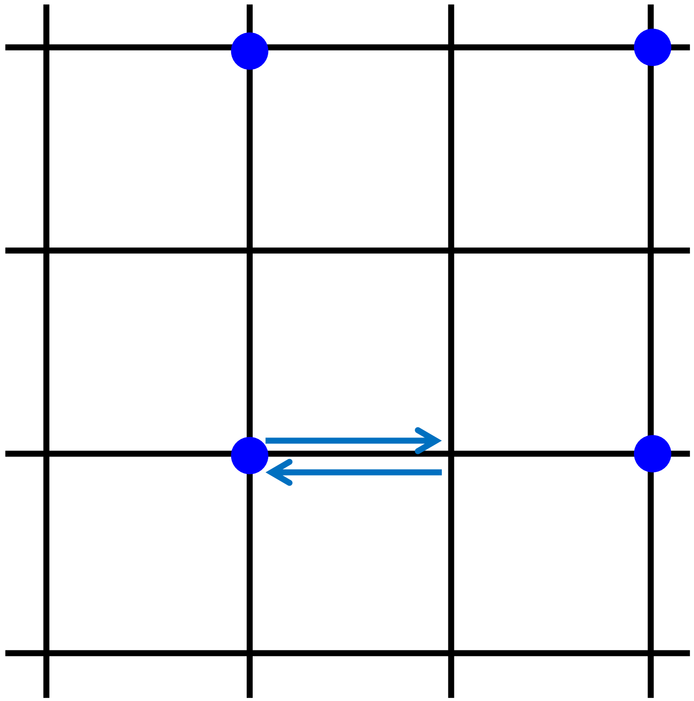


QUANTUM SIMULATION
PLAQUETTES

1d elementary link interactions – **already gauge invariant building blocks** of effective plaquettes

Auxiliary fermions

– virtual processes

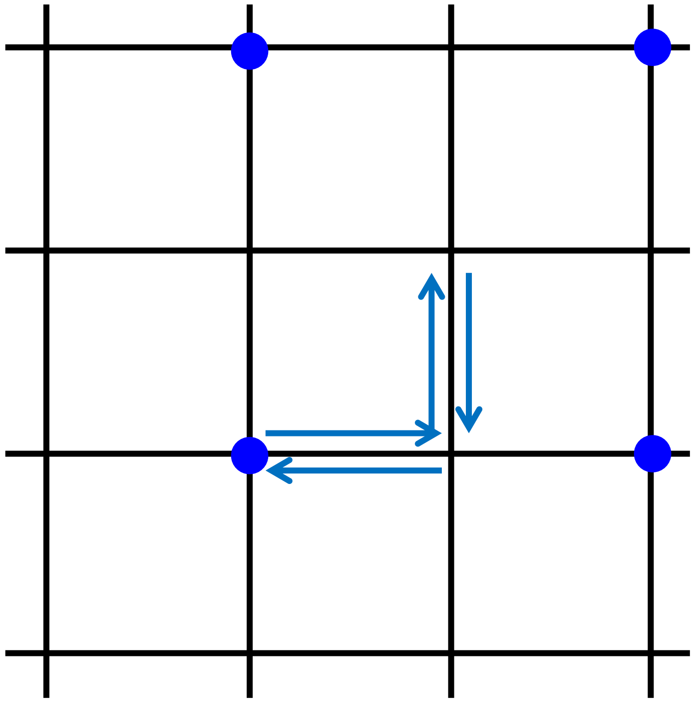


QUANTUM SIMULATION
PLAQUETTES

1d elementary link interactions – **already gauge invariant building blocks** of effective plaquettes

Auxiliary fermions

– virtual processes



QUANTUM SIMULATION
PLAQUETTES

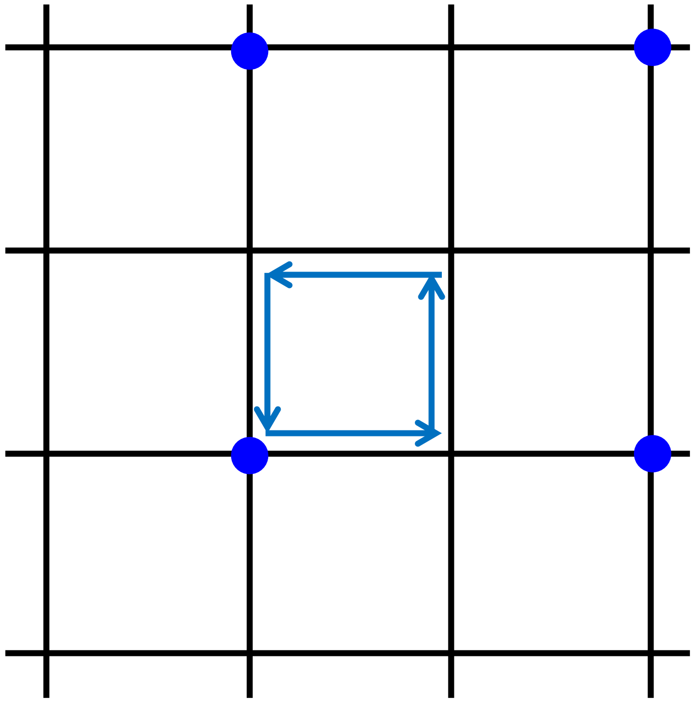
1d elementary link interactions – **already gauge invariant building blocks** of effective plaquettes

Auxiliary fermions

– virtual processes

- plaquettes.

$$\sum_{\text{plaquettes}} \left(\mathrm{Tr}\left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$$



QUANTUM SIMULATION
PLAQUETTES

1d elementary link interactions – **already gauge invariant building blocks** of effective plaquettes

Auxiliary fermions

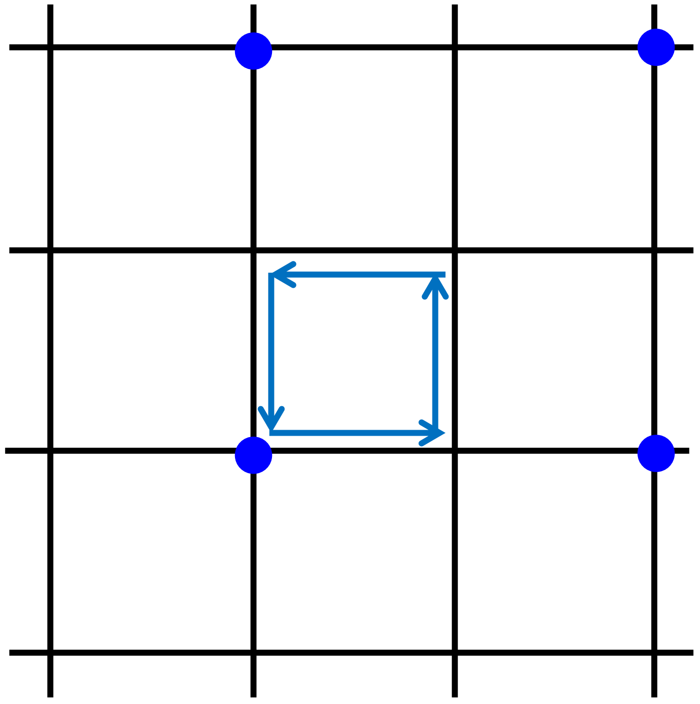
– virtual processes

- plaquettes.

$$\sum_{\text{plaquettes}} \left(\mathrm{Tr}\left(U_1 U_2 U_3^\dagger U_4^\dagger \right) + h.c. \right)$$

OKAY for: discrete, abelian

& non-abelian groups



QUANTUM SIMULATION

Example: U(1) PLAQUETTES

$$H_B = -\frac{2\epsilon^4}{\lambda^3}\sum_{\mathbf{n}}\left(U_{\mathbf{n},1}U_{\mathbf{n}+\hat{\mathbf{1}},2}U^{\dagger}_{\mathbf{n}+\hat{\mathbf{2}},1}U^{\dagger}_{\mathbf{n},2} + h.c.\right) =$$

$$-\frac{4\epsilon^4}{\lambda^3}\sum_{\mathbf{n}}\cos\left(\phi_{\mathbf{n},1} + \phi_{\mathbf{n}+\hat{\mathbf{1}},2} - \phi_{\mathbf{n}+\hat{\mathbf{2}},1} - \phi_{\mathbf{n},2}\right)$$

λ is the "energy penalty" of the auxiliary fermion
ϵ is the "link tunneling energy".

QUANTUM SIMULATION

Example: Z(N) PLAQUETTES

- Abelian *discrete* gauge theory: the gauge field degrees of freedom operate in a finite Hilbert space

$$P^N = Q^N = 1 \quad ; \quad P^\dagger Q P = e^{i\delta} Q$$

$$P \sim e^{iE}$$

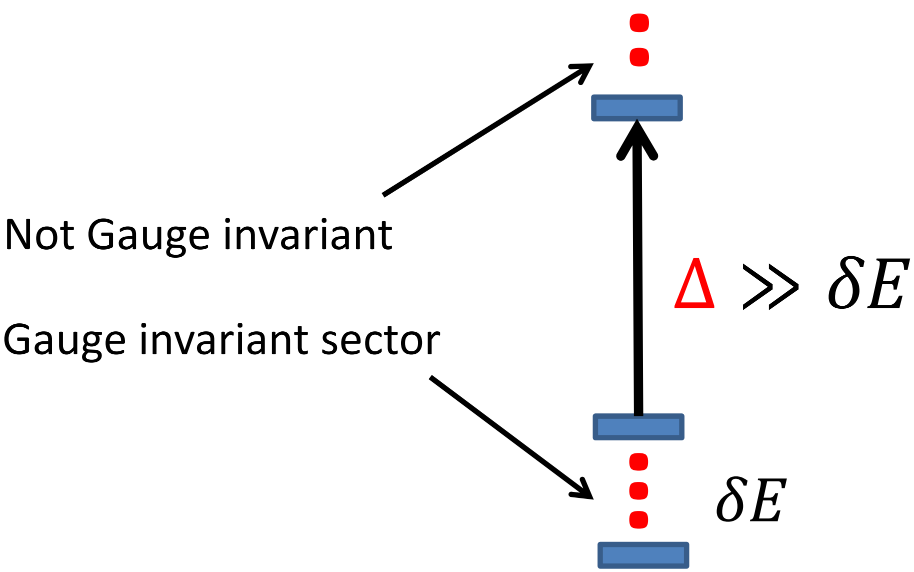
$$Q \sim e^{iA}$$

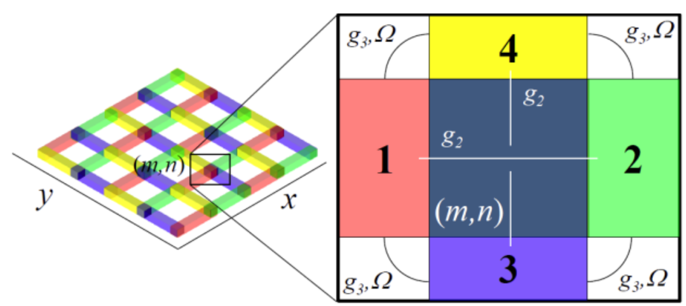
$$H_B = -\frac{4\epsilon^4}{\lambda^3} \sum_{\mathbf{n}} \left(Q_{\mathbf{n},1} Q_{\mathbf{n}+\hat{\mathbf{1}},2} Q^\dagger_{\mathbf{n}+\hat{\mathbf{2}},1} Q^\dagger_{\mathbf{n},2} + h.c. \right)$$

QUANTUM SIMULATION
PLAQUETTES – effective

<u>Gauss's law</u> is added as a constraint. Leaving the gauge invariant sector of Hilbert space costs too much Energy.

<u>Low energy sector with a effective gauge invariant Hamiltonian</u>.



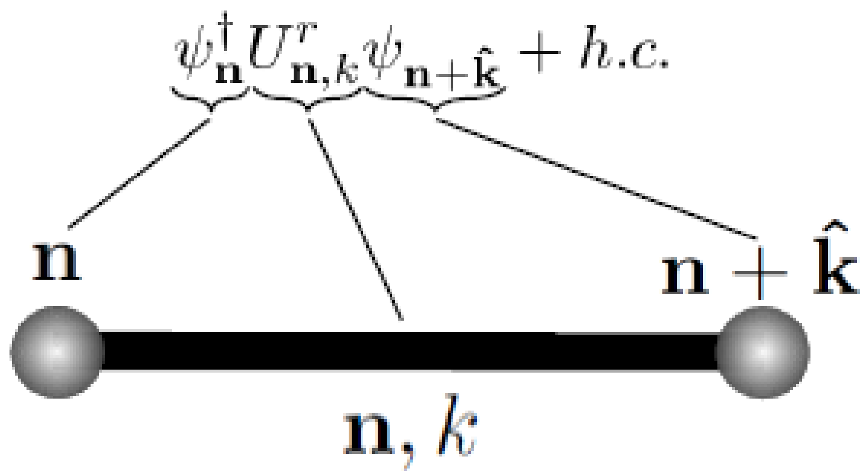


E. Zohar, B. R.,   Phys. Rev. Lett. 107, 275301 (2011)

NON ABELIAN MODELS
YANG MILLS

QUANTUM SIMULATIONS OF NONABELIAN MODELS

GENERAL STRUCTURE



U^r is an element of the gauge group (in the representation r)

$$\psi_{\mathbf{n}} = (\psi_{\mathbf{n},a}) = \begin{pmatrix} \psi_{\mathbf{n},1} \\ \psi_{\mathbf{n},2} \\ \cdots \end{pmatrix}$$

SCHWINGER REPRESENTATION: SU(2)
PRE-POTENTIAL APPROACH

On each link – $a_{1,2}$ bosons on the left, $b_{1,2}$ bosons on the right

$$L_a = \frac{1}{2}\sum_{k,l} a_k^\dagger \,(\sigma_a)_{lk}\, a_l \;;\; R_a = \frac{1}{2}\sum_{k,l} b_k^\dagger \,(\sigma_a)_{kl}\, b_l$$

$$[L_{n,a}, L_{n,b}] = -i\epsilon_{abc} L_{n,c}; \quad [R_{n,a}, R_{n,b}] = i\epsilon_{abc} R_{n,c}$$

In the fundamental representation -

$$U_L = \frac{1}{\sqrt{N_L+1}}\begin{pmatrix} a_1^\dagger & -a_2 \\ a_2^\dagger & a_1 \end{pmatrix} \;;\; U_R = \begin{pmatrix} b_1^\dagger & b_2^\dagger \\ -b_2 & b_1 \end{pmatrix}\frac{1}{\sqrt{N_R+1}}$$

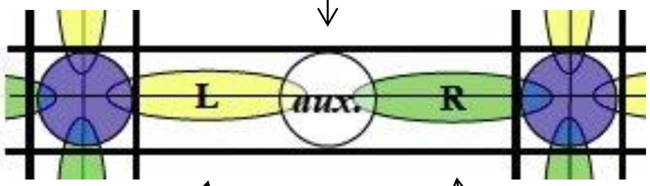
$$U = U_L U_R$$

M. Mathur, Journal of Physics A **38**, 10015 (2005)

SCHWINGER REPRESENTATION: SU(2)
REALIZATION

On each link – $a_{1,2}$ bosons on the left, $b_{1,2}$ bosons on the right

Ancillary Fermion state



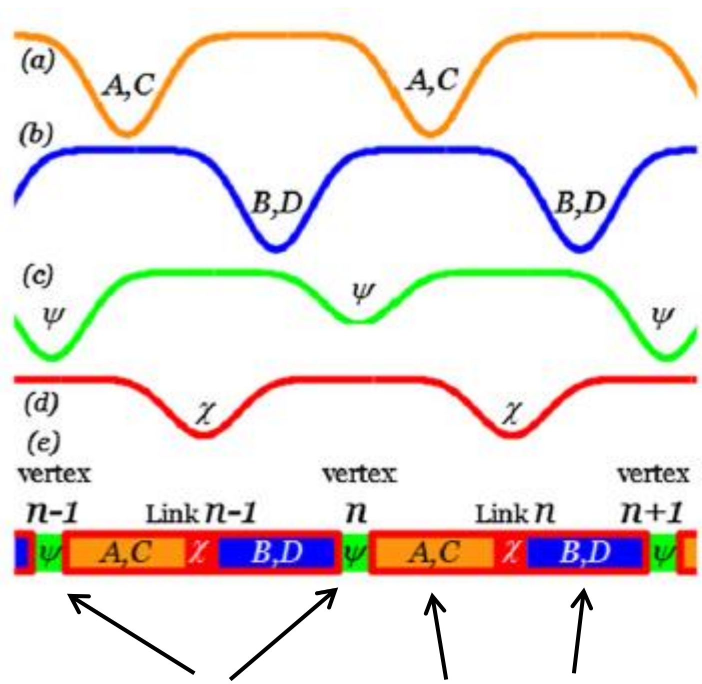
$$U_L = \frac{1}{\sqrt{N_L+1}}\begin{pmatrix} a_1^\dagger & -a_2 \\ a_2^\dagger & a_1 \end{pmatrix} \; ; \; U_R = \begin{pmatrix} b_1^\dagger & b_2^\dagger \\ -b_2 & b_1 \end{pmatrix}\frac{1}{\sqrt{N_R+1}}$$

$$\boxed{U = U_L U_R}$$

QUANTUM SIMULATION

EXAMPLE: SU(2) IN 1+1

Superlattices →

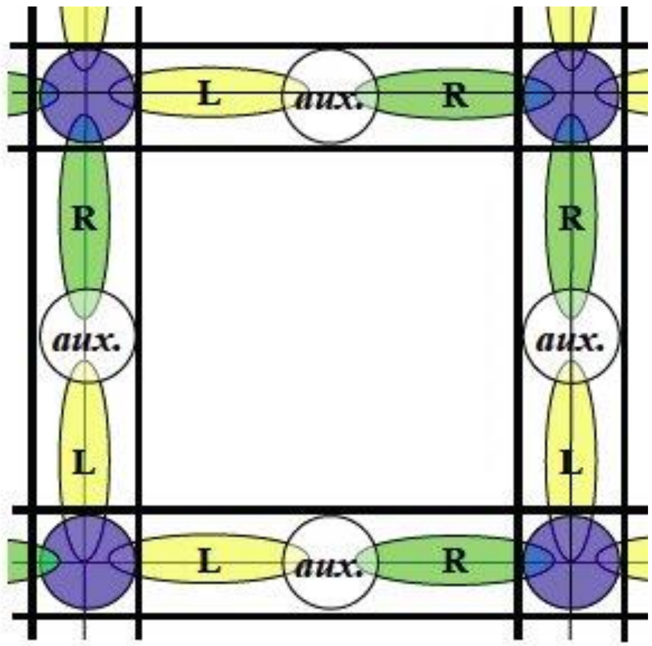


Fermions

L & R “gauge” bosons

$$\sum_{n,i,j}\left(\sqrt{N_{L,n}+1}\,\left(\psi_n^{\dagger}\right)_i\left(U_{L,n}\right)_{ij}\left(\chi_n\right)_j+\left(\chi_n^{\dagger}\right)_i\left(U_{R,n}\right)_{ij}\left(\psi_{n+1}\right)_j\sqrt{N_{R,n}+1}+h.c.\right)$$

QUANTUM SIMULATIONS OF NONABELIAN MODELS
GENERAL STRUCTURE



Each link has *left* and *right* degrees of freedom – forming together SU(N) elements. The “relative rotation” corresponds to the non-abelian charge on the link.

QUANTUM SIMULATION OF NON ABELIAN THEORIES
CHALLANGES

- The link's *J* quantum number (SU2) representation) is a dynamical

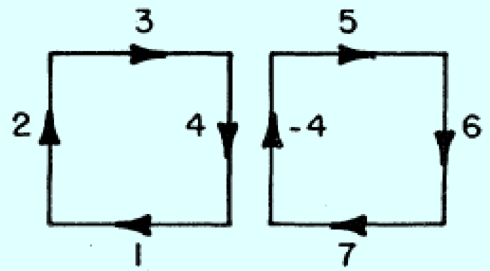


FIG. 6. Product of boxes with an overlapping link.

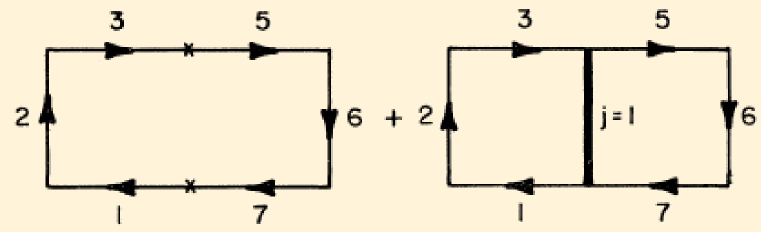


FIG. 7. Replacing the flux in link 4 of Fig. 6 by $j = 0$ and $j = 1$ flux lines.

Kogut and Susskind, PRD 1975

- How to obtain links enabling large values of *J*?
- How to obtain plaquettes when one truncates *J*?

So far only methods for *strong limit* simulations are known – including the 1+1 non-abelian generalization of the Schwinger model.

QUANTUM SIMULATIONS
MEASUREMENTS

QUANTUM SIMULATIONS

COLD ATOMS – EXPERIMENTS

PRL 103, 080404 (2009) PHYSICAL REVIEW LETTERS week ending 21 AUGUST 2009

Experimental Demonstration of Single-Site Addressability in a Two-Dimensional Optical Lattice

Peter Würtz,[1] Tim Langen,[1] Tatjana Gericke,[1] Andreas Koglbauer,[1] and Herwig Ott[1,2,*]
[1]*Institut für Physik, Johannes Gutenberg-Universität, 55099 Mainz, Germany*
[2]*Research Center OPTIMAS, Technische Universität Kaiserslautern, 67663 Kaiserslautern, Germany*
(Received 18 March 2009; published 21 August 2009)

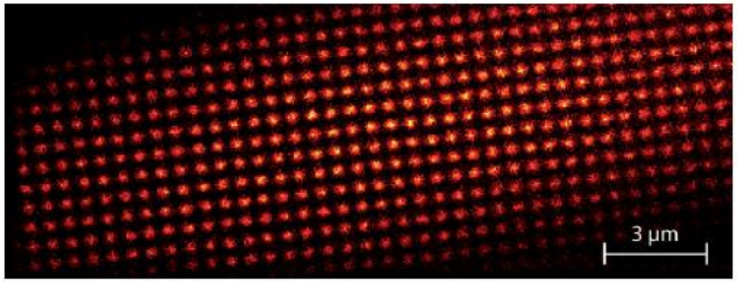


FIG. 1 (color online). Electron microscope image of a Bose-Einstein condensate in a 2D optical lattice with 600 nm lattice spacing (sum obtained from 260 individual experimental realizations). Each site has a tubelike shape with an extension of 6 μm perpendicular to the plane of projection. The central lattice sites contain about 80 atoms.

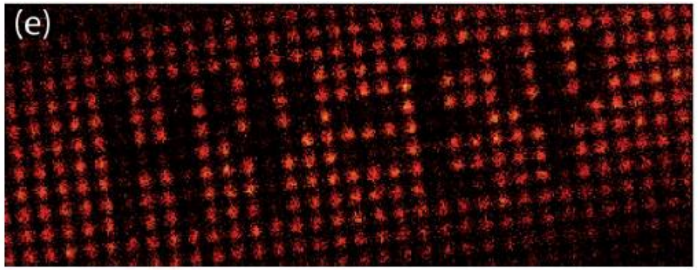


FIG. 2 (color online). Patterning a Bose-Einstein condensate in a 2D optical lattice with a spacing of 600 nm. Every emptied site was illuminated with the electron beam (7 nA beam current, 100 nm FWHM beam diameter) for (a),(b) 3, (c),(d) 2, and (e) 1.5 ms, respectively. The imaging time was 45 ms. Between 150 and 250 images from individual experimental realizations have been summed for each pattern.

QUANTUM SIMULATIONS
COLD ATOMS – EXPERIMENTS

nature | Vol 462 | 5 November 2009 | doi:10.1038/nature08482

A quantum gas microscope for detecting single atoms in a Hubbard-regime optical lattice

Waseem S. Bakr[1], Jonathon I. Gillen[1], Amy Peng[1], Simon Fölling[1] & Markus Greiner[1]

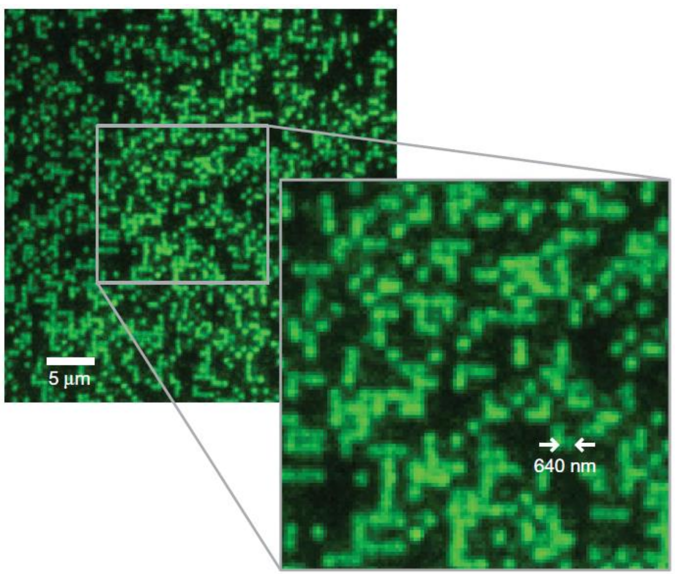


Figure 3 | Site-resolved imaging of single atoms on a 640-nm-period optical lattice, loaded with a high density Bose–Einstein condensate. Inset, magnified view of the central section of the picture. The lattice structure and the discrete atoms are clearly visible. Owing to light-assisted collisions and molecule formation on multiply occupied sites during imaging, only empty and singly occupied sites can be seen in the image.

mined through preparation and measurement. By implementing a high-resolution optical imaging system, single atoms are detected with near-unity fidelity on individual sites of a Hubbard-regime optical lattice. The lattice itself is generated by projecting a holographic mask through the imaging system. It has an arbitrary geometry, chosen to support both strong tunnel coupling between lattice sites and strong on-site confinement. Our approach can be

QUANTUM SIMULATIONS
COLD ATOMS – EXPERIMENTS

doi:10.1038/nature09827

Single-spin addressing in an atomic Mott insulator

Christof Weitenberg[1], Manuel Endres[1], Jacob F. Sherson[1]†, Marc Cheneau[1], Peter Schauß[1], Takeshi Fukuhara[1], Immanuel Bloch[1,2] & Stefan Kuhr[1]

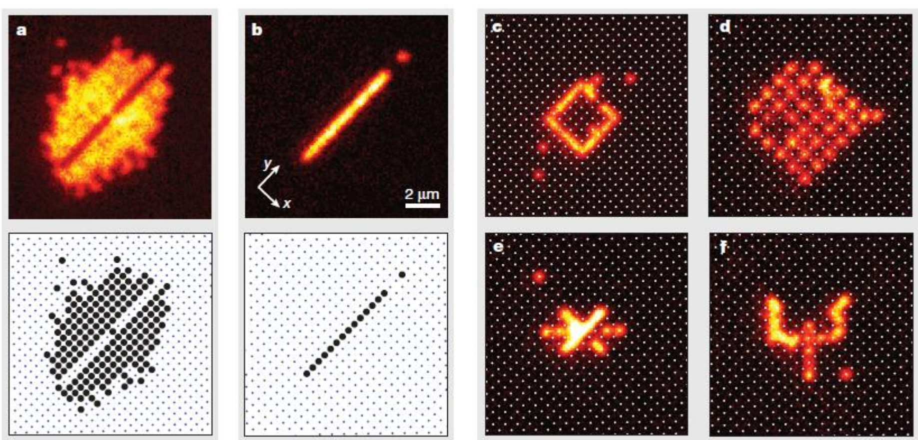
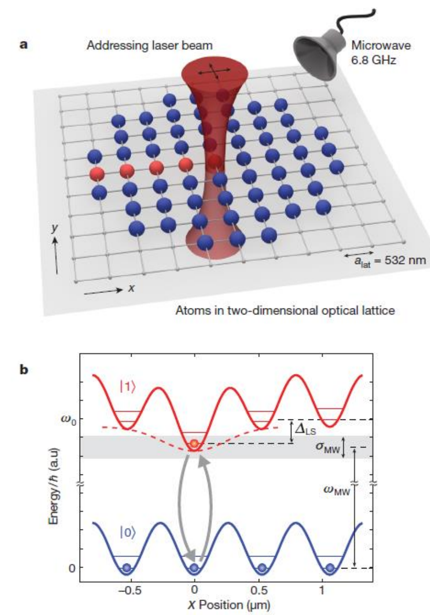
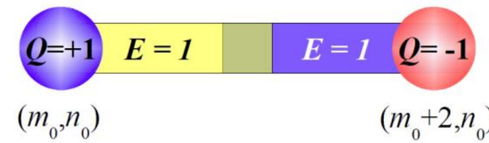


Figure 2 | Single-site addressing. **a**, Top, experimentally obtained fluorescence image of a Mott insulator with unity filling in which the spin of selected atoms was flipped from $|0\rangle$ to $|1\rangle$ using our single-site addressing scheme. Atoms in state $|1\rangle$ were removed by a resonant laser pulse before detection. Bottom, the reconstructed atom number distribution on the lattice. Each filled circle indicates a single atom; the points mark the lattice sites. **b**, Top, as for **a** except that a global microwave sweep exchanged the population in $|0\rangle$ and $|1\rangle$, such that only the addressed atoms were observed. Bottom, the reconstructed atom number distribution shows 14 atoms on neighbouring sites. **c–f**, As for **b**, but omitting the atom number distribution. The images contain 29 (**c**), 35 (**d**), 18 (**e**) and 23 (**f**) atoms. The single isolated atoms in **b**, **e** and **f** were placed intentionally to allow for the correct determination of the lattice phase for the feedback on the addressing beam position.

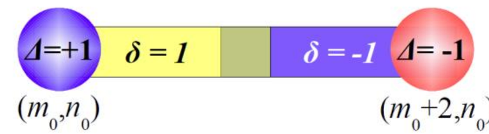


QUANTUM SIMULATIONS
MEASUREMENTS

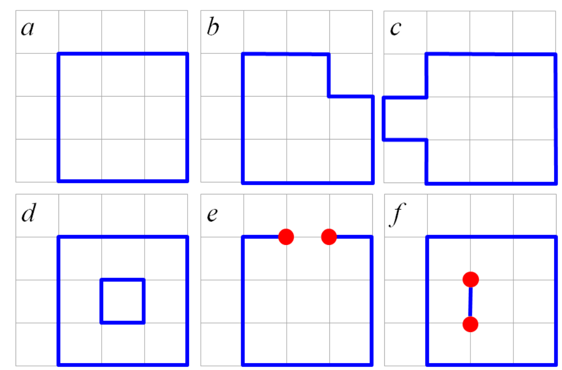
- Preparation of initial states
  - Various regimes and charge distributions
  (single addressing)



- Tunable parameters
  - (Feshbach resonances, tunneling rates, Raman lasers)



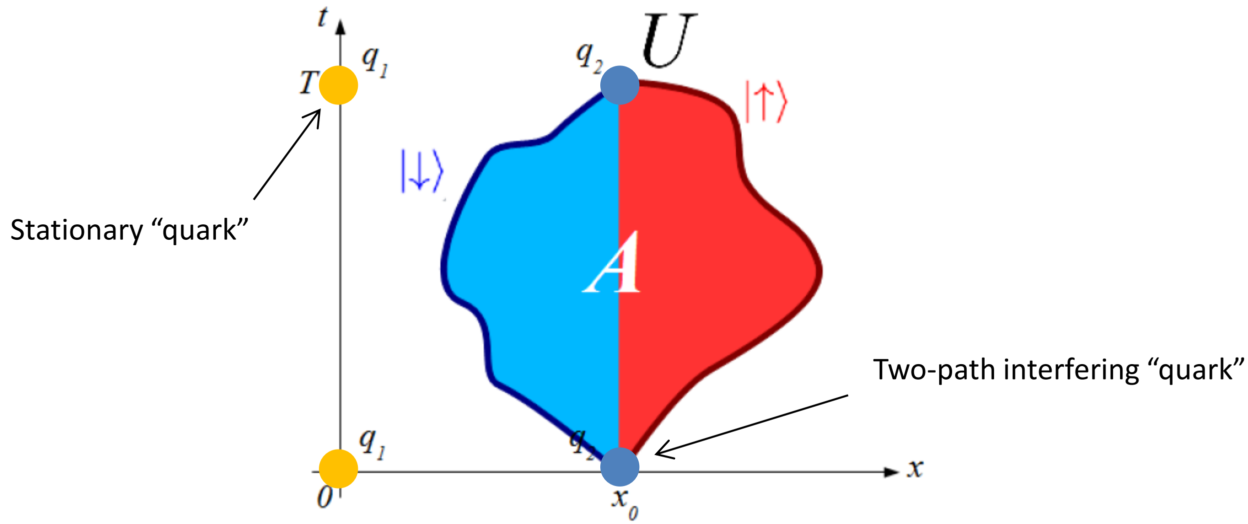
- Dynamical *real time* evolution
  - Confinement of dynamic charges
  - Flux tubes/loops breaking
  - Pair production, vacuum instability
  - ...



- Non-perturbative physics
- Probing phase transitions
- Finite chemical potential, Finite temperature,…
  Color superconductivity, Quark-Gluon plasma??

EXAMPLE
WILSON LOOP MEASUREMENTS

$$W(C) = P\left(e^{i\oint_C A_\mu dx^\mu}\right)$$



Detecting Wilson Loop's area law by interference of "Mesons".

This is equivalent to Ramsey Spectroscopy in quantum optics!

E. Zohar , BR, New J. Phys. 15 (2013) 043041

QUANTUM SIMUTIONS
OUR PROPOSALS

Theory		1+1 with matter	d+1 Pure	d+1 with matter
U(1) - cQED ✓		K.S. (or truncated)	K.S. (or truncated)	K.S. (or truncated)
Z(N)		✗	✓	✗
SU(2) – Yang Mills		Low energy states	Strong limit	Strong limit

K.S. := Kogut Susskind Hamiltonian LGT formalism

CONFINEMENT
TOY MODELS

- 1+1D: Schwinger's model.

- cQED: 2+1D: no phase transition
  Instantons give rise to confinement at $g < 1$ (Polyakov).
  (For T > 0: there is a phase transition also in 2+1D.)

- cQED: 3+1D: phase transition between a strong coupling confining phase, and a weak coupling coulomb phase.

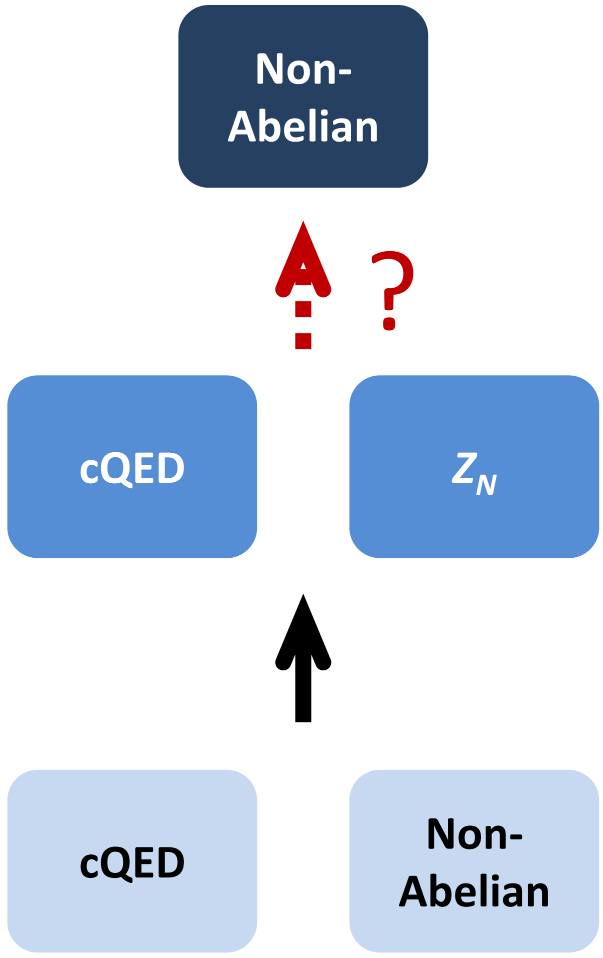
- Z(N): for $\mathrm{N} \geq N_c$: Three phases: electric confinement, magnetic confinement, and non confinement.

OUTLOOK

Non Abelian in Higher Dimensions

Plaquettes in 2+1 and 3+1
Abelian , cQED and Z(N)

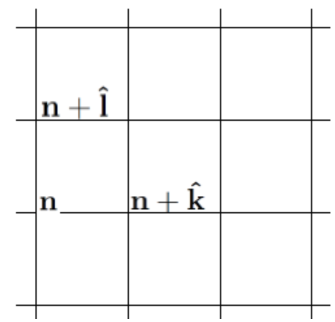
"Proof of principle" 1+1 toy models
Numerical comparison with DMRG

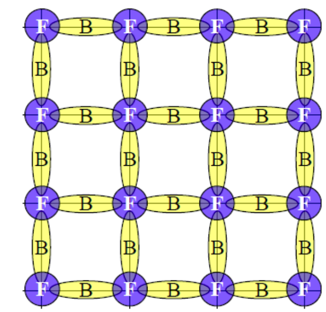


Decoherence, superlattices, scattering parameters control…

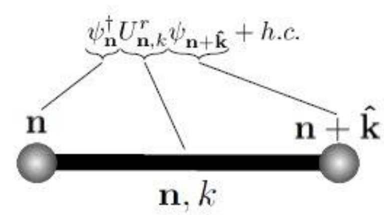
SUMMARY

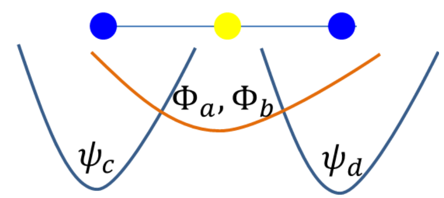
Lattice gauge theories can be mapped to an analog cold atom simulator.



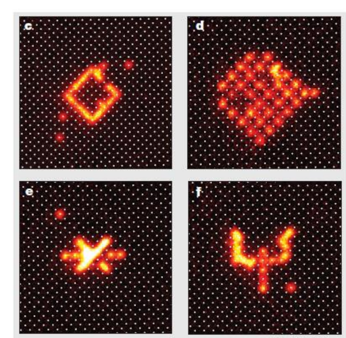


Atomic conservation laws give rise to exact gauge symmetry.

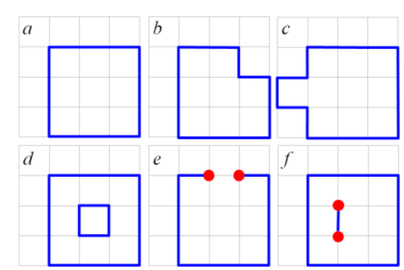




Near future experiments may be able to realize first steps in this direction, and offer a new types of LGT simulations.

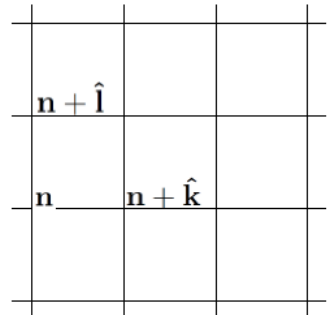


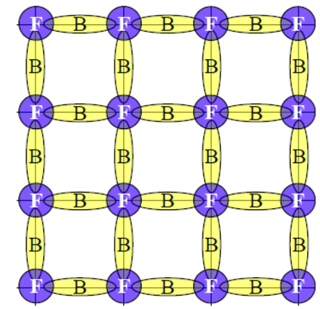
Weitenberg et. al., Nature, 2011



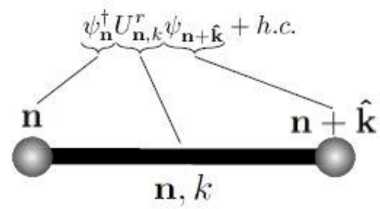
THANK YOU!

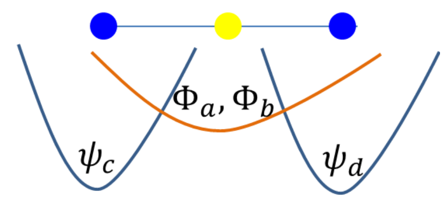
Lattice gauge theories can be mapped to an analog cold atom simulator.



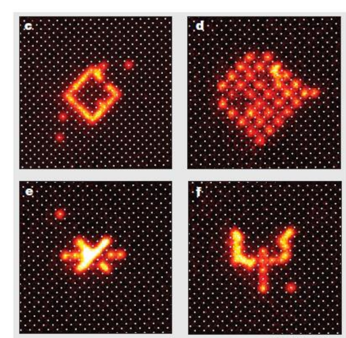


Atomic conservation laws give rise to exact gauge symmetry.

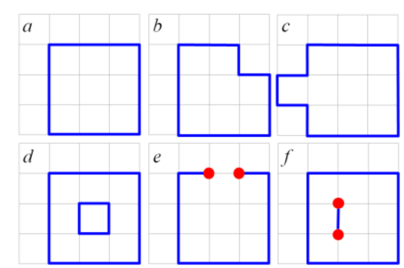




Near future experiments may be able to realize first steps in this direction, and offer a new types of LGT simulations.



Weitenberg et. al., Nature, 2011



Z_N Gauge theory

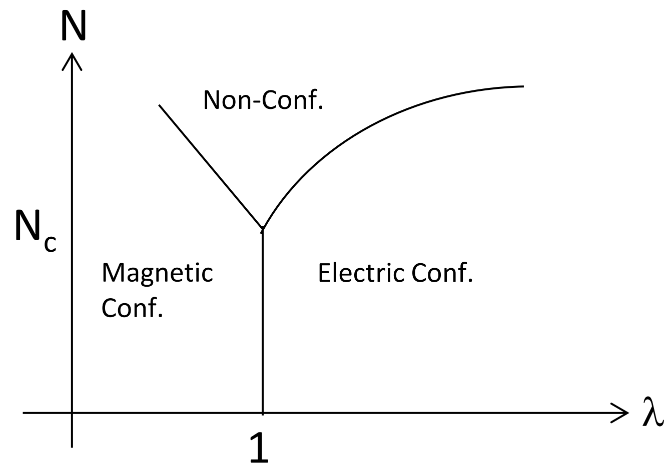
- Abelian *discrete* gauge theory: the gauge field degrees of freedom operate in a finite Hilbert space
- Three phases in 3+1 dimensions

$$P \sim e^{iE}$$

$$Q \sim e^{iA}$$

$$P^N = Q^N = 1 \quad ; \quad P^\dagger Q P = e^{i\delta} Q$$

;



Adapted from Horn et. al., PRD 19, 3715, 1979

Simulating Z_N Gauge theory

- Finite Hilbert spaces on links: one can realize *unitary* operators in the elementary link interactions, obtained using hybridized levels
- In a pure gauge theory, plaquettes are obtained similarly, using the “loop method”