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Isospin Breaking Effects in Lattice QCD

XXXI International Symposium on Lattice Field Theory
Mainz, Germany
30-07-2013

acknowledgments

let me start by thinking:

- LOC and IAC: for the opportunity of giving this talk!

- RM123 collaboration: *G.M. de Divitiis, P. Dimopoulos, R. Frezzotti, V. Lubicz, G. Martinelli, R. Petronzio, G.C. Rossi, F. Sanfilippo, S. Simula, N.T., C. Tarantino*

  for the enjoyable and fruitful collaboration: what I know on this subject comes from the work and the discussions with these nice people!

- speakers at the conference: S.Drury, B.Leder, J.Finkenrath, A.Portelli, G.Schierholz, D.Toussaint, J.Zanotti

  for their nice and interesting talks/posters on the subject!

- motivations
- isospin breaking on the lattice
- QCD vs. QCD+QED
- corrections to hadron masses
- QCD corrections to matrix elements
- conclusions

Duncan et al., Phys.Rev.Lett. 76(1996)

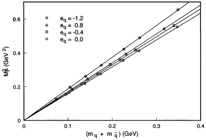
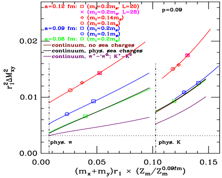
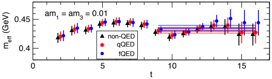


FIG. 1. The mass squared M_P^2 (in GeV^2) for neutral pseudoscalar meson vs lattice bare quark masses $m_q + m_{\bar{q}}$ (in GeV) is shown for various quark charges $e_q = 0.0, -0.4, 0.8$, and -1.2.

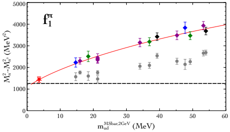
MILC, arXiv:1301.7137



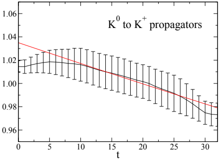
T.Izubuchi et al., Phys.Rev.Lett. 109(2012)



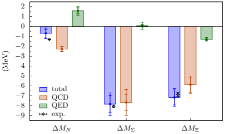
RM123, Phys.Rev. D87(2013)

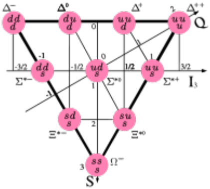


PACS-CS, Phys.Rev. D86(2012)



BMW arXiv:1306.2287





the two lightest quarks, the up and the down, have different masses and different electric charges, nevertheless

$$\frac{\hat{m}_d - \hat{m}_u}{\Lambda_{\mathrm{QCD}}} \ll 1 \; , \qquad (e_u - e_d) e_f \; \hat{\alpha}_{em} \ll 1$$

thanks to isospin symmetry:

- hadrons can be classified according to the representations of “angular momentum” algebra
- hadronic processes can be studied, separately, in the different isospin “channels”, for example

$$\pi\pi \underbrace{\longrightarrow \pi\pi\pi}_{\text{forbidden}} \, , \qquad\qquad [\; \pi\pi \;\; \longrightarrow \;\; \pi\pi \;]^{I=0,1,2}$$

- the neutral pion two-point function has no quark disconnected diagrams
- unquenched simulations with light Wilson fermions are possible (without reweighting)

$$\det\left(D[U] + m_{ud}\right) \; \det\left(D[U]^\dagger + m_{ud}\right) > 0$$

- etc. etc.

isospin breaking is a small effect but generates a rich phenomenology:

- chemistry: hydrogen is stable because the electron capture reaction $p + e \longrightarrow n + \nu$ is forbidden

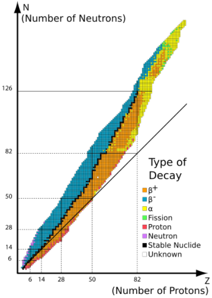
$$M_n - M_p = [M_n - M_p]^{\mathrm{QCD}} + \underbrace{[M_n - M_p]^{\mathrm{QED}}}_{<0} > M_e$$

- flavour content of the hadrons: the mixing angles between $\{\pi^0, \eta, \eta'\}$ and $\{\rho, \omega, \phi\}$ are very different, why?

- flavour content of the "new" X, Y, Z hadrons: $[\bar{c}\bar{c}][uu]$ would be a neutral state with definite flavour and isospin quantum numbers: a "pure" tetraquark!

A.Esposito, M.Papinutto, A.Pilloni, A.Polosa, N.T., arXiv:1307.2873

Y.Ikeda talk



concerning matrix elements relevant in flavour physics

FALG Eur.Phys.J.C71 (2011)
FALG2 http://itpwiki.unibe.ch/flag
A.Kastner,H.Neufeld Eur.Phys.J.C57 (2008)
V.Cirigliano,H.Neufeld Phys.Lett.B700 (2011)

$$F_+^{K\pi}(0) = 0.967(4) \quad \sim \quad 0.4\%$$

$$\left[\frac{F_+^{K^+\pi^0}(q^2)}{F_+^{K^0\pi^-}(q^2)} - 1\right]_{\mathrm{QCD}}^{\chi pt} = 0.029(4)$$

$$F_K/F_\pi = 1.194(5) \quad \sim \quad 0.4\%$$

$$\left[\frac{F_{K^+}/F_{\pi^+}}{F_K/F_\pi} - 1\right]_{\mathrm{QCD}}^{\chi pt} = -0.0022(6)$$

- QCD+QED is a renormalizable theory that can be put on the lattice
- the direct simulation is possible if each single determinant is positive
  G.Schierholz poster $m_{u,d} \sim m_s$
- but very expensive ...

$$\vec{g} = \left(e^2, g_s^2, m_u, m_d, m_s\right)$$

$$\langle \mathcal{O} \rangle^{\vec{g}} = \frac{\int dA e^{-S[A]} \; dU \; e^{-\beta S[U]} \; \det\left(D[U,A;\vec{g}]\right) \; \mathcal{O}[U,A;\vec{g}]}{\int dA e^{-S[A]} \; dU \; e^{-\beta S[U]} \; \det\left([U,A;\vec{g}]\right)}$$

$$\vec{g}^0 = \left(0, (g_s^0)^2, m_{ud}^0, m_{ud}^0, m_s^0\right)$$

$$\langle \mathcal{O} \rangle^{\vec{g}^0} = \frac{\int dU \; e^{-\beta^0 S[U]} \; \det\left(D[U;\vec{g}^0]\right) \; \mathcal{O}[U;\vec{g}^0]}{\int dU \; e^{-\beta^0 S[U]} \; \det\left(D[U;\vec{g}^0]\right)}$$

$$R[U,A;\vec{g}] = e^{-(\beta-\beta^0)S[U]} \frac{\det\left(D[U,A;\vec{g}]\right)}{\det\left(D[U;\vec{g}^0]\right)}$$

$$\langle \mathcal{O} \rangle^{\vec{g}} = \frac{\langle \; R \; O \; \rangle^{A,\vec{g}^0}}{\langle \; R \; \rangle^{A,\vec{g}^0}}$$

- much more practical to (re)use the gauge configurations generated in *isosymmetric QCD*
- this can be done by reweighting pure QCD ensembles
- the values of the bare parameters in the two theories depend upon the renormalization prescriptions, more to say later on this point...
- in the electroquenched approximation sea quarks are neutral w.r.t. QED:

$$R[U,A;\vec{g}] \longrightarrow 1$$

- QED is treated in the *non–compact* formulation: the gauge potential A_μ is the dynamical variable

$$S[A] = \frac{1}{4}\sum_{x;\mu,\nu}\left[\nabla^+_\mu A_\nu(x) - \nabla^+_\nu A_\mu(x)\right]^2$$

- the QED+QCD links are obtained by exponentiation

$$U_\mu(x) \quad \longrightarrow \quad e^{ie_f e A_\mu(x)}\, U_\mu(x)$$

Duncan,Eichten,Thacker, Phys.Rev.Lett. 76(1996)

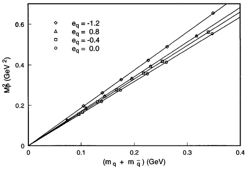


FIG. 1. The mass squared M_P^2 (in GeV^2) for neutral pseudoscalar meson vs lattice bare quark masses $m_q + m_{\bar q}$ (in GeV) is shown for various quark charges $e_q = 0.0, -0.4, 0.8$, and -1.2.

- by imposing periodic boundary conditions on the gauge field and a gauge fixing (here Feynman), one gets

$$\nabla^-_\mu A_\mu(x) = 0$$

$$\begin{aligned} S_{QED} &= \tfrac{1}{2}\textstyle\sum_x A_\mu(x)\left[-\nabla^-_\nu \nabla^+_\nu\right] A_\mu(x) \\ &= \tfrac{1}{2}\textstyle\sum_k \tilde{A}^\star_\mu(k)\,[2\sin(k_\nu/2)]^2\,\tilde{A}_\mu(k) \end{aligned}$$

- without additional prescriptions, the photon propagator is infrared divergent and the Gauss's law is inconsistent

$$\nabla^-_\mu F_{\mu\nu}(x) = J_\nu(x)$$

$$0 = \sum_{\vec{x}} \nabla^-_i E_i(t,\vec{x}) = e\sum_{\vec{x}} \delta^3(t,\vec{x}) = e$$

- QED is treated in the *non–compact* formulation: the gauge potential A_μ is the dynamical variable

$$S[A] = \frac{1}{4} \sum_{x;\mu,\nu} \left[\nabla^+_\mu A_\nu(x) - \nabla^+_\nu A_\mu(x)\right]^2$$

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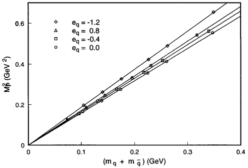
Duncan,Eichten,Thacker, Phys.Rev.Lett. 76(1996)



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- by subtracting the zero momentum mode, a residual gauge ambiguity, both problems are solved

$$\nabla^-_\mu \left[A_\mu(x) + c\right] = 0$$

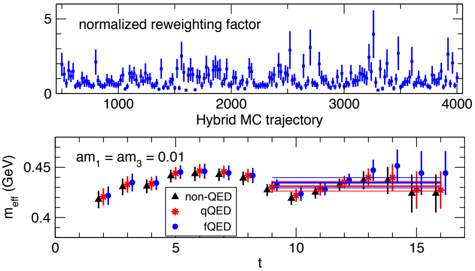
$$\mathrm{P}^\perp \phi(x) = \phi(x) - \tfrac{1}{V} \textstyle\sum_y \phi(y)$$

$$\begin{aligned} S_{QED} &= \tfrac{1}{2} \textstyle\sum_{k\neq 0} \tilde{A}^\star_\mu(k)\, [2\sin(k_\nu/2)]^2\, \tilde{A}_\mu(k) \\ &= \tfrac{1}{2} \textstyle\sum_x A_\mu(x) \left[-\nabla^-_\nu \nabla^+_\nu\right] \mathrm{P}^\perp A_\mu(x) \end{aligned}$$

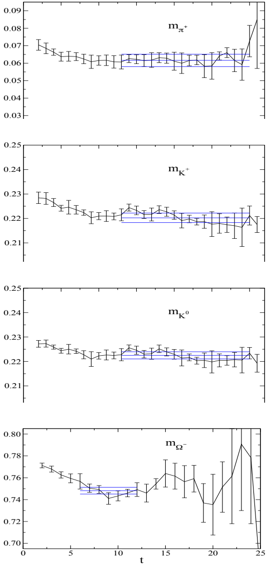
$$\longrightarrow \quad \mathrm{P}^\perp \left[\nabla^-_\mu F_{\mu\nu}(x) - J_\nu(x)\right] = 0$$

- it can be shown that this infrared regularization changes physical quantities by finite volume effects, no new ultraviolet divergences: (large) FVE are unavoidable, QED is a long range interaction!

T.Izubuchi et al., Phys.Rev.Lett. 109(2012)



PACS-CS, Phys.Rev. D86(2012)

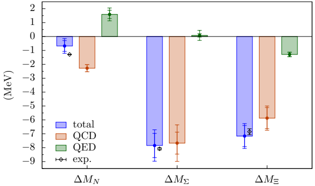


- by splitting the ratio of determinants into several factors (n^{th}–root trick, mass/charge preconditioning) the T.Izubuchi et al. and PACS-CS collaborations have been able, on volumes $L \sim 3$ fm, to take the fluctuations of the reweighting factor under control!
- for $\hat{m}_d \neq \hat{m}_u$ reweighting see also

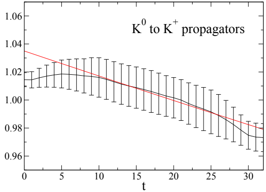
  J.Finkenrath F.Knechtli B.Leder, arXiv:1306.3962

  J.Finkenrath and B.Leder talks
- note: since isospin breaking effects are very small, the differences between isosymmetric QCD, electroquenched and full QED results may be smaller than the statistical fluctuations, back on this point later. . .
- **home message: $1 + 1 + 1$ QED+QCD lattice simulations are feasible!**

BMW arXiv:1306.2287, A.Portelli talk



PACS-CS, Phys.Rev. D86(2012)



- a (trivial?) statement: QCD and QCD+QED are two different theories

$(e_f e)^2$  $\longrightarrow$ $[m_f - m_f^0]$ 

$$J^\mu(x) J_\mu(0) \longrightarrow c_1(x) 1 + \sum_f \left[c_m^f(x) m_f + c_{cr}^f(x) \right] \bar{\psi}_f \psi_f + c_{g_s}(x) G_{\mu\nu} G^{\mu\nu} + \cdots$$

electromagnetic currents generate divergent contributions that redefine the vacuum energy, c_1, the quark masses, c_m^f, the quark critical masses (if chirality is broken), c_{cr}^f, and the strong coupling constant (the lattice spacing), c_g

- physics is QCD+QED: the PACS-CS collaboration, used

$$\left\{ M_{\pi^+}, M_{K^+}, M_{K^0}, M_{\Omega^-} \right\} \longrightarrow \left\{ \hat{m}_u, \hat{m}_d, \hat{m}_s, a \right\}$$

and, of course, the mass of the up and the mass of the down are different: that's it!

- on the other hand, it is interesting (and useful in practice) to define differences as $M_p^{QED+QCD} - M_p^{QCD}$: how?

J.Gasser, A.Rusetsky, I.Scimemi, Eur.Phys.J. C32 (2003)
RM123, Phys.Rev. D87(2013)

- the parameters $\vec{g}_0$ of isosymmetric QCD can eventually be fixed independently from $\vec{g}$ by performing "standard" QCD simulations, for example

$$\left\{M_{\pi^+}, M_{K^+}, M_{\Omega^-}\right\} \qquad \longrightarrow \qquad \left\{\hat{m}^0_{ud}, \hat{m}^0_s, a^0\right\}$$

- on the other hand, when simulations of the full theory are performed, one can use the following matching condition

$$\text{experiment} \;\longrightarrow\; g_i\;, \qquad \hat{g}_i(\mu^\star) = \hat{g}^0_i(\mu^\star)\;, \qquad g^0_i = \frac{Z_i(\mu^\star)}{Z^0_i(\mu^\star)}\, g_i \;\longrightarrow\; \text{IB}$$

- and define isospin breaking effects as $\boldsymbol{\Delta}\mathcal{O} = \mathcal{O}(\vec{g}) - \mathcal{O}(\vec{g}_0)$ and Leading Isospin Breaking (LIB) effects as

$$\boldsymbol{\Delta}\mathcal{O} \quad = \quad \left\{ e^2 \frac{\partial}{\partial e^2} + \left[g_s^2 - (g_s^0)^2\right] \frac{\partial}{\partial g_s^2} + [m_f - m_f^0] \frac{\partial}{\partial m_f} + [m_f^{cr} - m_0^{cr}] \frac{\partial}{\partial m_f^{cr}} \right\} \mathcal{O}$$

J.Gasser, A.Rusetsky, I.Scimemi, Eur.Phys.J. C32 (2003)
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- and define isospin breaking effects as $\boldsymbol{\Delta}\mathcal{O} = \mathcal{O}(\vec{g}) - \mathcal{O}(\vec{g}_0)$ and Leading Isospin Breaking (LIB) effects as

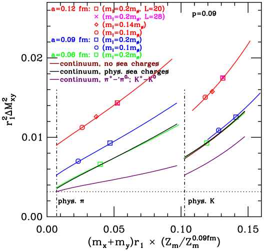
$$\boldsymbol{\Delta}\mathcal{O} = \left\{\hat{e}^2 \frac{\partial}{\partial \hat{e}^2} + \left[\hat{g}_s^2 - \left(\frac{Z_{g_s}}{Z^0_{g_s}} \hat{g}^0_s\right)^2\right] \frac{\partial}{\partial \hat{g}_s^2} + \left[\hat{m}_f - \frac{Z_{m_f}}{Z^0_{m_f}} \hat{m}^0_f\right] \frac{\partial}{\partial \hat{m}_f} + \Delta m_f^{cr} \frac{\partial}{\partial m_f^{cr}}\right\} \mathcal{O}$$

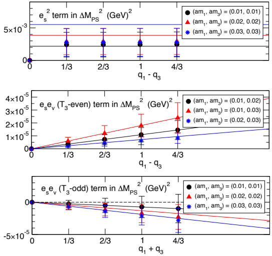
- the counter–terms in the perturbative expansion do arise because the renormalization constants (the bare parameters) of the two theories are different
- one could use a similar strategy to match the $n_f = 2 + x$ and $n_f = 2 + y$ theories in order to calculate quenching effects

...
T.Blum et al., Phys.Rev. D82(2010)
T.Izubuchi et al., Phys.Rev.Lett. 109(2012)
S.Drury talk

...
MILC, PoS LATTICE2012 137
MILC, arXiv:1301.7137
D.Toussaint talk

J.Bijnens, N.Danielsson Phys.Rev. D75(2007)
M.Hayakawa, S.Uno Prog.Theor.Phys. 120(2008)





- all the terms allowed by symmetries are present in χpt formulae

$$M_{12}^2 = \hat{B}(\hat{m}_1 + \hat{m}_2) + \hat{e}^2 \hat{C}(e_1 - e_2)^2 \ + \ [\ \cdots\ ]$$

that can be reexpressed in terms of the parameters of isosymmetric QCD by redefining the low energy constants

$$\hat{m}_i = (1 + \hat{e}^2 \delta_i)\hat{m}_i^0 \quad \longrightarrow \quad M_{12}^2 = \hat{B}(\hat{m}_1^0 + \hat{m}_2^0) + \hat{e}^2 \hat{C}(e_1 - e_2)^2 \ + \ \left[\ \cdots + \hat{e}^2 \hat{B}(\delta_1 \hat{m}_1^0 + \delta_2 \hat{m}_2^0)\ \right]$$

- the matching is somehow "automatic" but the separation prescription has to be specified when quoting results for the QED and QCD IB effects

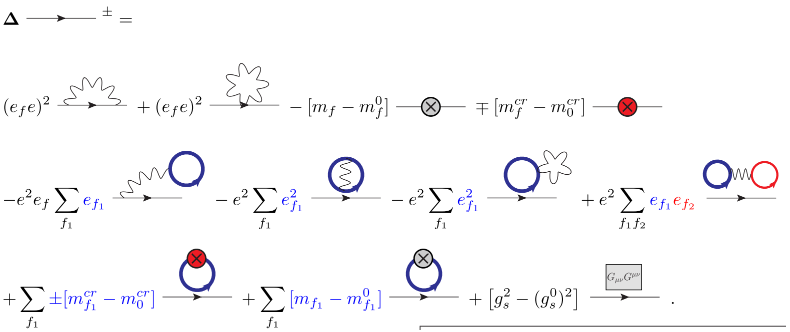
- note: whenever lattice data are fitted by neglecting $O[\hat{\alpha}_{em}(\hat{m}_d - \hat{m}_u)]$ terms, **one is actually calculating LIB effects**

RM123, JHEP 1204(2012)
RM123, Phys.Rev. D87(2013)

- LIB effects can be calculated directly by expanding the lattice path– integral w.r.t. $\hat{\alpha}_{em} \sim (\hat{m}_d - \hat{m}_u)/\Lambda_{QCD}$

$$\mathcal{O}(\vec{g}) = \frac{\langle\ R[U,A;\vec{g}]\ O[U,A;\vec{g}]\ \rangle^{A,\vec{g}^0}}{\langle\ R[U,A;\vec{g}]\ \rangle^{A,\vec{g}^0}} = \frac{\langle\ \left(1+\dot{R}+\cdots\right)\ \left(O+\dot{O}+\cdots\right)\ \rangle}{\langle\ 1+\dot{R}+\cdots\ \rangle} = \mathcal{O}(\vec{g}^0) + \mathbf{\Delta}\mathcal{O}$$

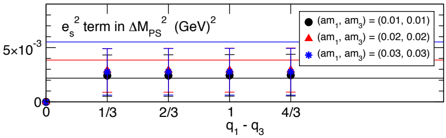
- the building blocks for the graphical notation, used here as a device to do calculations, are the corrections to the quark propagator



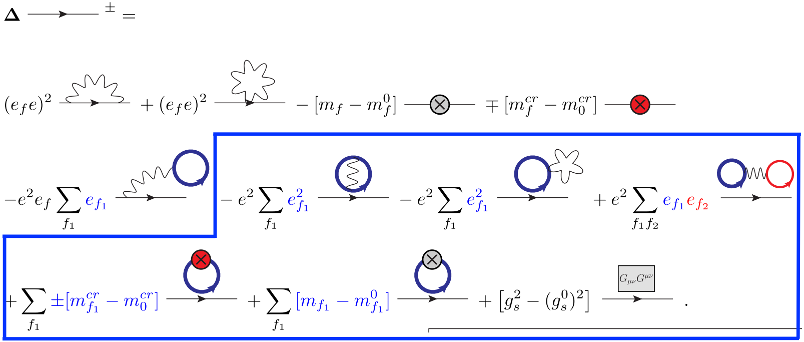
RM123, JHEP 1204(2012)
RM123, Phys.Rev. D87(2013)

$$\mathcal{O}(\vec{g}^0) + \Delta\mathcal{O} = \frac{\langle\ \left(1+\dot{R}+\cdots\right)\ \left(O+\dot{O}+\cdots\right)\ \rangle}{\langle\ 1+\dot{R}+\cdots\ \rangle}$$

T.Izubuchi et al., Phys.Rev.Lett. 109(2012)



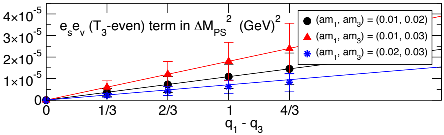
- isosymmetric vacuum polarization effects, those that do not "read" the charge of the valence quarks, are expected to be sizable (confirmed by T.Izubuchi et al.)



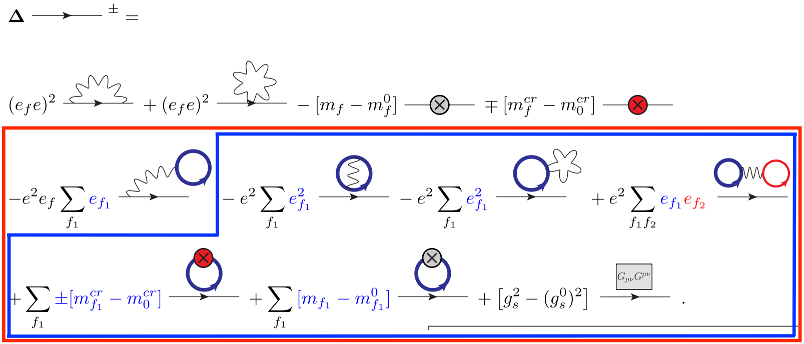
RM123, JHEP 1204(2012)
RM123, Phys.Rev. D87(2013)

$$\mathcal{O}(\vec{g}^0) + \boldsymbol{\Delta}\mathcal{O} = \frac{\langle \left(1 + \dot{R} + \cdots\right) \left(O + \dot{O} + \cdots\right) \rangle}{\langle 1 + \dot{R} + \cdots \rangle}$$

T.Izubuchi et al., Phys.Rev.Lett. 109(2012)



- vacuum polarization effects proportional to the charge of the valence quarks are a flavour $SU(3)$ breaking effect; can be estimated by the knowledge of the leading order χpt QED low energy constant

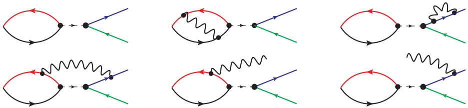


- consider a two-point correlator in the full theory ($m_u \neq m_d$ and $e_f \neq 0$)

$$C_{H\,H}(t;\vec{g}) = \langle\, \mathcal{O}_H(t)\; \mathcal{O}_H^{\dagger}(0)\,\rangle^{\vec{g}} \qquad \longrightarrow \qquad e^{M_H} = \frac{C_{H\,H}(t-1;\vec{g})}{C_{H\,H}(t;\vec{g})} + \text{ non leading exps.}$$

where $\mathcal{O}_H$ is an interpolating operator having the quantum numbers of a given hadron H

- if H is a charged particle, the correlator $C_{H\,H}(t;\vec{g})$ is not QED gauge invariant; for this reason it is not possible, in general, to extract physical informations directly from the residues of the different poles; to physical decay rates do contribute diagrams as



- on the other hand, the mass of the hadron is gauge invariant and *finite* in the continuum and infinite volume limits, provided that the parameters of the actions have been properly renormalized; it follows that the ratio $C_{H\,H}(t-1;\vec{g})/C_{H\,H}(t;\vec{g})$ is both gauge and renormalization group (RGI) invariant

- by applying the differential operator $\boldsymbol{\Delta}$ to full theory correlators one gets

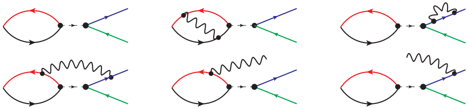
$$\frac{C_{H\,H}(t;\vec{g})}{C_{H\,H}(t;\vec{g}^0)} \quad = \quad 1 + \frac{\boldsymbol{\Delta} C_{H\,H}(t;\vec{g}^0)}{C_{H\,H}(t;\vec{g}^0)} + \cdots \; = \; c - t(M_H - M_H^0) + \ldots$$

- consider a two-point correlator in the full theory ($m_u \neq m_d$ and $e_f \neq 0$)

$$C_{HH}(t;\vec{g}) = \langle\, \mathcal{O}_H(t)\ \mathcal{O}_H^\dagger(0)\, \rangle^{\vec{g}} \qquad \longrightarrow \qquad e^{M_H} = \frac{C_{HH}(t-1;\vec{g})}{C_{HH}(t;\vec{g})} + \text{ non leading exps.}$$

where $\mathcal{O}_H$ is an interpolating operator having the quantum numbers of a given hadron H

- if H is a charged particle, the correlator $C_{HH}(t;\vec{g})$ is not QED gauge invariant; for this reason it is not possible, in general, to extract physical informations directly from the residues of the different poles; to physical decay rates do contribute diagrams as

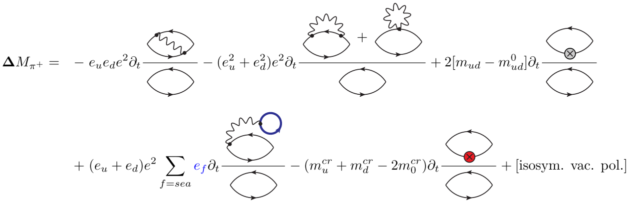


- on the other hand, the mass of the hadron is gauge invariant and *finite* in the continuum and infinite volume limits, provided that the parameters of the actions have been properly renormalized; it follows that the ratio $C_{HH}(t-1;\vec{g})/C_{HH}(t;\vec{g})$ is both gauge and renormalization group (RGI) invariant

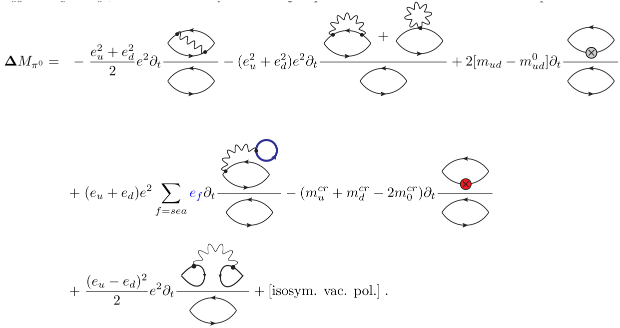
- by applying the differential operator $\boldsymbol{\Delta}$ to full theory correlators one gets

$$-\partial_t \frac{\boldsymbol{\Delta} C_{HH}(t;\vec{g}^0)}{C_{HH}(t;\vec{g}^0)} + \cdots \ = \ M_H - M_H^0$$

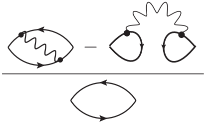
- in order to calculate the LIB corrections to M_{π^+} and, separately, to M_{π^0} one needs to determine the quark (critical) masses and the lattice spacing in the full theory



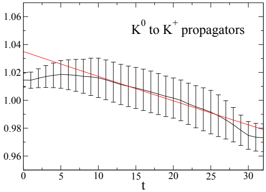
- since $M_{\pi^+} - M_{\pi^0}$ is already an isospin breaking effect, many terms cancel in the difference and one gets. . .



$$M_{\pi^+} - M_{\pi^0} = \frac{(e_u - e_d)^2}{2} e^2 \partial_t$$



PACS-CS, Phys.Rev. D86(2012)

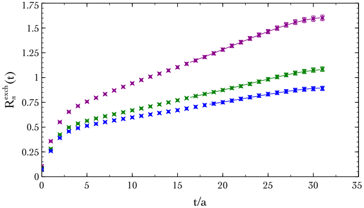


- there are no contributions proportional to $\hat{m}_d - \hat{m}_u$: the pion mass difference at this order is a pure QED effect
- note: sea quark effects are not neglected, they cancel in the difference!
- the electric charge does not renormalize at this order (a problem that *must* instead be faced at higher orders) and the previous expression is finite,

$$e^2 = \hat{e}^2 = 4\pi\hat{\alpha}_{em} = \frac{4\pi}{137}$$

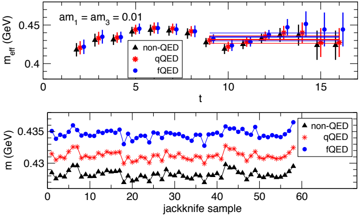
- it can be shown (Dashen's theorem, more to say later) that the disconnected diagram is of $O(\hat{\alpha}_{em}\hat{m}_{ud})$ and it can be considered, for physical quark masses, a higher order effect
- for all these reasons the pion mass difference can be considered a "clean" theoretical prediction and a benchmarking observable
- it can be computed as done by the PACS-CS collaboration in the case of the kaon mass difference or by calculating "directly" the diagrams (correlators) appearing in the formula ...

RM123, Phys.Rev. D87(2013)



 $\sim c. - t\Delta M_P$

T.Izubuchi et al., Phys.Rev.Lett. 109(2012)



- isospin breaking effects are small because very small coefficients multiply sizable hadronic matrix elements

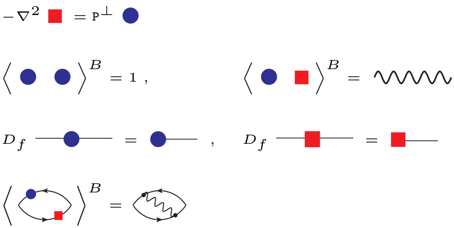
$$\langle B_\mu(x) B_\nu(y) \rangle^B = \delta_{\mu\nu}\ \delta(x-y)$$

$$\mathrm{P}^\perp \phi(x) = \phi(x) - \frac{1}{V}\sum_y \phi(y)$$

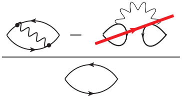
$$[-\nabla_\rho^- \nabla_\rho^+] C_\mu[B;x] = \mathrm{P}^\perp\ B_\mu(x)$$

$$\langle B_\mu(y) C_\nu[B;x] \rangle^B = D_{\mu\nu}^\perp(x-y)$$

- electromagnetic corrections can be calculated by introducing real $\mathbb{Z}_2$ noise vectors and two sequential quark propagator inversions



- **home message: leading isospin breaking effects can also be calculated by expanding the lattice path–integral**



- the point now is: QED is a long range unconfined interaction, how large are finite volume effects?

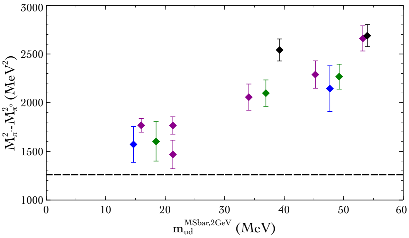
- for pseudoscalar meson masses these have been estimated in χpt coupled to electromagnetism

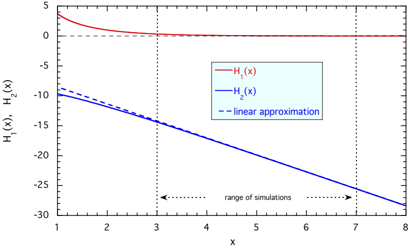
M.Hayakawa, S.Uno Prog.Theor.Phys. 120(2008)

$$\left[M^2_{\pi^+} - M^2_{\pi^0}\right](L) - \left[M^2_{\pi^+} - M^2_{\pi^0}\right](\infty)$$

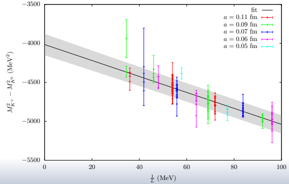
$$= \frac{\hat{e}^2}{4\pi L^2}\left[H_2(M_\pi L) - 4CH_1(M_\pi L)\right]$$

$$\sim -\frac{\hat{e}^2 2.8373\ldots}{4\pi}\left(\frac{M_\pi}{L} + \frac{2}{L^2}\right)$$

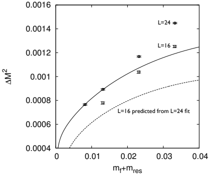




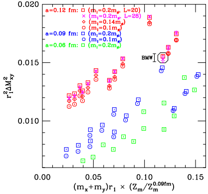
BMW prel. $\Delta M_K^2(L)$: A.Portelli talk



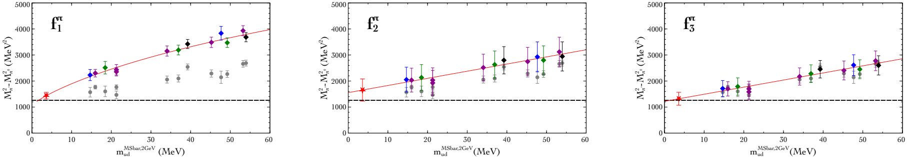
T.Blum et al., Phys.Rev. D82(2010)



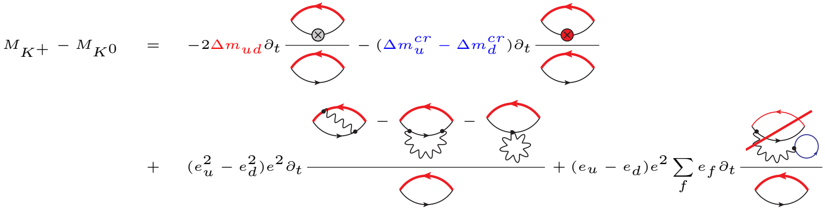
MILC, arXiv:1301.7137



RM123, Phys.Rev. D87(2013)



- RM123, BMW, MILC: cutoff effects are reasonably small
- the large (20–30%) finite volume effects predicted by χpt may be over-estimated and/or can be compensated by the chiral logs; BMW ($L \in [1.9, 6]$ fm) is on the way to settle the question...
- **home message: finite volume effects are the issue! who is surprised?**

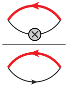
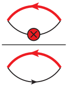


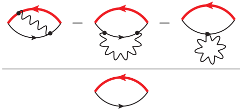
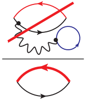
- the kaon mass difference can be used to determine $\Delta\hat{m}_{ud} = (\hat{m}_d - \hat{m}_u)/2$ and to separate QCD from QED isospin breaking effects; first note

$$\Delta m_{ud} = \frac{\hat{m}_d}{2Z_{m_d}} - \frac{\hat{m}_u}{2Z_{m_u}} = Z^0_{\bar{\psi}\psi}\Delta\hat{m}_{ud} + \frac{\hat{m}_{ud}}{\mathcal{Z}_{ud}}$$

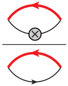
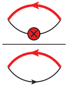
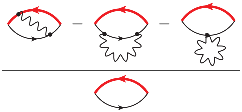
$$Z_{\bar{\psi}\psi} = \frac{1}{2Z_{m_d}} + \frac{1}{2Z_{m_u}} \longrightarrow Z^0_{\bar{\psi}\psi}$$

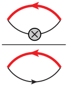
$$\frac{1}{\mathcal{Z}_{ud}} = \frac{1}{2Z_{m_d}} - \frac{1}{2Z_{m_u}} \longrightarrow \frac{(e_d^2 - e_u^2)e^2}{32\pi^2}\left[6\log(a\mu) + \text{ finite }\right] Z^0_{\bar{\psi}\psi}$$

$M_{K^+} - M_{K^0} \quad = \quad -2\Delta m_{ud}\,\partial_t$  $-\;(\Delta m_u^{cr} - \Delta m_d^{cr})\,\partial_t$ 

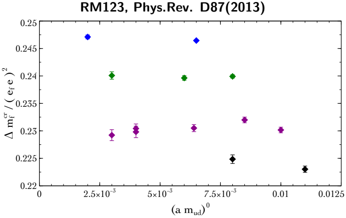
$+ \quad (e_u^2 - e_d^2)e^2\,\partial_t$  $+\;(e_u - e_d)e^2 \sum_f e_f\,\partial_t$ 

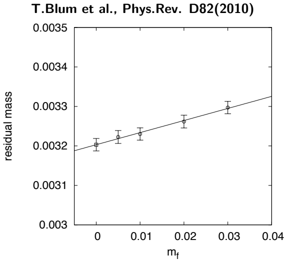
- the kaon mass difference can be used to determine $\Delta\hat{m}_{ud} = (\hat{m}_d - \hat{m}_u)/2$ and to separate QCD from QED isospin breaking effects; then

$QED = -\dfrac{2\hat{m}_{ud}}{\mathcal{Z}_{ud}}\,\partial_t$  $-\;(\Delta m_u^{cr} - \Delta m_d^{cr})\,\partial_t$  $+\;(e_u^2 - e_d^2)e^2\,\partial_t$ 

$QCD = -2\Delta\hat{m}_{ud} \left(Z^0_{\bar{\psi}\psi}\;\partial_t \right.$  $\left. \right)$

- the QCD contribution is finite and RGI; the QED contribution is finite only if both counter-terms are present, though the first has a very small numerical impact. what about the second?

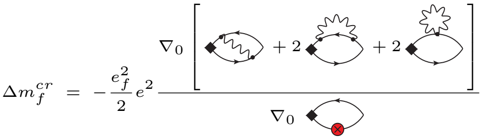




$$\left\langle \nabla_\mu \left[\bar\psi_f \gamma^\mu \tau^1 \psi_f\right](x) \; \left[\bar\psi_f \gamma^5 \tau^2 \psi_f\right](0) \right\rangle^{\vec g} = 0$$

$$\frac{\left\langle \sum_x J^a_{5q}(\vec x, t) \; P^a(0) \right\rangle^{\vec g}}{\left\langle \sum_x J^a_5(\vec x, t) \; P^a(0) \right\rangle^{\vec g}}$$

- with (here Twisted Mass) Wilson and DMW fermions, the shift of the (residual) critical masses of the quarks, a *linear divergent counter–term*, can be calculated by restoring the validity of chiral WT identities



- the electric charge operator is diagonal in flavour space

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad\qquad \hat{Q} = \frac{\hat{e}}{3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

- if the down and the strange have the same mass ($m_d = m_s \longrightarrow \hat{m}_d = \hat{m}_s$) we have

$$\hat{m}_d = \hat{m}_s \qquad \longrightarrow \qquad M_{\pi^+} = M_{K^+}$$

- in the chiral limit, to each flavour generator commuting with the electric charge corresponds a Godlstone's boson, even in the presence of electromagnetic interactions; in particular

$$\left[\hat{T}^a, \hat{Q}\right] = 0 \qquad \longrightarrow \qquad \partial^\mu \left[\bar{\psi} i\gamma_5\gamma_\mu \hat{T}^a \psi\right] = \partial^\mu A^a_\mu(x) = 0$$

$$\hat{m}_u = \hat{m}_d = 0 \qquad \longrightarrow \qquad M_{\pi^0} = 0$$

$$\hat{m}_u = \hat{m}_d = \hat{m}_s = 0 \qquad \longrightarrow \qquad M_{\pi^0} = M_{K^0} = 0$$

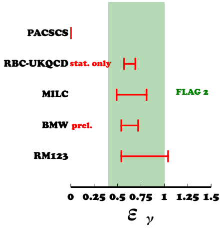
- note, since LIB corrections to the pion mass difference are a pure electromagnetic effect, one has

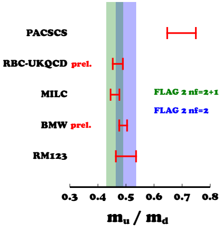
$$\varepsilon_\gamma = \underbrace{\frac{\left[M^2_{K^+} - M^2_{K^0}\right]^{QED} - \left[M^2_{\pi^+} - M^2_{\pi^0}\right]^{QED}}{M^2_{\pi^+} - M^2_{\pi^0}}}_{O(\hat{m}_s\, \hat{\alpha}_{em})} = \frac{\left[M^2_{K^+} - M^2_{K^0}\right]^{QED}}{\left[M^2_{\pi^+} - M^2_{\pi^0}\right]^{QED}} - 1 + O\left[\Delta\hat{m}_{ud}\, \hat{\alpha}_{em}\right]$$

- the value of ε_γ depends upon the renormalization prescription used to separate QED from QCD IB effects

$$\varepsilon_\gamma = \frac{\left[M^2_{K^+} - M^2_{K^0}\right]^{QED} - \left[M^2_{\pi^+} - M^2_{\pi^0}\right]^{QED}}{M^2_{\pi^+} - M^2_{\pi^0}}$$

- it is needed to calculate the light quark masses by starting from QCD ($\hat{m}_u \neq \hat{m}_d$) lattice simulations and using the QCD contribution to the kaon mass splitting as "experimental" input

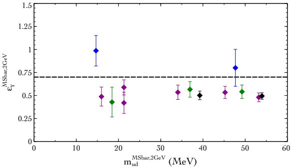


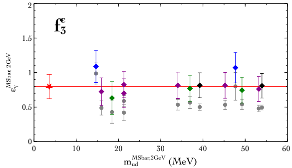


- the electroquenched uncertainty on ε_γ can be estimated, at present, by using χpt results, it is of the order of 10%

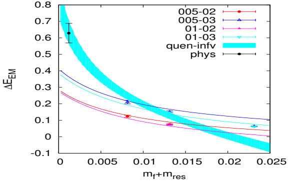
J.Bijnens, N.Danielsson Phys.Rev. D75(2007)
M.Hayakawa, S.Uno Prog.Theor.Phys. 120(2008)

RM123, Phys.Rev. D87(2013)

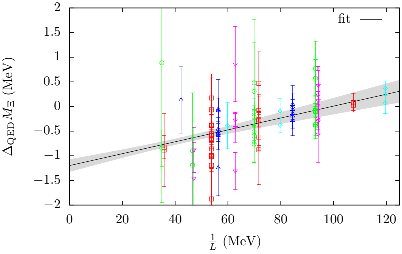


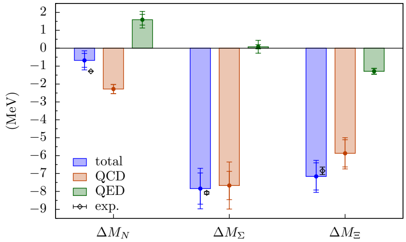


T.Blum et al., Phys.Rev. D82(2010)



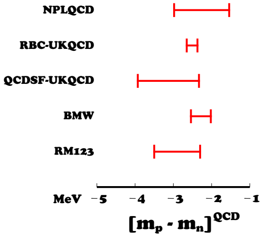
BMW arXiv:1306.2287, A.Portelli talk





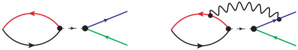
- the BMW collaboration has recently completed a calculation of the octet baryon mass splittings
- also in this case the dominant source of uncertainty may come from finite volume effects, they can be as large as 80%!!!
- the other uncertainties are still too large to draw conclusions
- once QED and QCD isospin breaking corrections have been separated, one can calculate only the (simpler/cheaper) QCD corrections $(\hat{m}_u \neq \hat{m}_d)\ldots$

see J.Zanotti talk

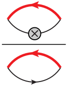
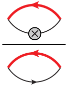


V.Cirigliano,H.Neufeld Phys.Lett.B700 (2011), RM123, Phys.Rev. D87(2013), HPQCD, arXiv:1303.1670

- the physical observable is the decay rate $\Gamma[K^+ \to \ell^+\nu(\gamma)]$; this is ultraviolet and infrared finite, gauge invariant, unambiguous
- it is *only neglecting electromagnetic corrections* that the hadronic and leptonic tensors can be factorized

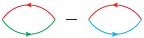


- using the matching prescription defined above and by considering the $\Delta\hat{m}_{ud}$ QCD corrections to kaons two point functions, we have the QCD RGI invariant formula

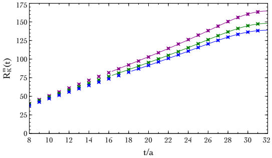
$-\Delta\hat{m}_{ud} \Big(Z^0_{\bar\psi\psi}$  $\Big) = -\left(\Delta\hat{m}^0_{ud} Z^0_{\bar\psi\psi}\right)$ 

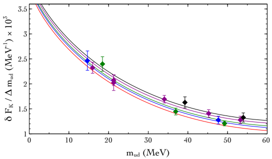
$$= \delta \left(\frac{G_K^2}{2M_K}\right)^{QCD} - t\Delta M_K^{QCD}$$

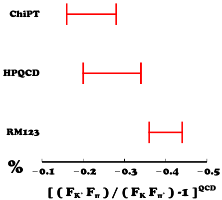
- without factorizing the *small coefficient* Δm^0_{ud}, one can also calculate "directly" the difference



- remember: pion two point functions do not get corrected at $O(\Delta m^0_{ud})$







- lattice calculation of QCD+QED isospin breaking effects on the hadron spectrum are feasible, even including the QED unquenching effects
- one can use reweighting and start from the full theory path integral. the fluctuations of the reweighting factor can be kept under control: shown by T.Izubuchi et al. and PACS-CS on volumes $L \sim 3$ fm, scaling with the volume?
- or expand the relevant correlators with respect to $\hat{m}_d - \hat{m}_u$ and $\hat{\alpha}_{em}$; here to avoid the electroquenched approximation disconnected diagrams have to be calculated
- cutoff effects are reasonably small, shown by the RM123, BMW and MILC collaborations that obtain results in the continuum limit for pseudoscalar and baryon mass splittings
- finite volume effects may be very large: this is not surprising, we are putting photons in a box! to settle this point we should be able to significantly reduce the other uncertainties
- on the other hand, thanks to the efforts of the different collaborations, we are now calculating not just *guessing* isospin breaking effects! a large uncertainty on IB effects is a small and reliable uncertainty on the given observable

$$1\% \times 30\% \quad = \quad 0.3\%$$

- lattice calculation of QCD+QED IB corrections to hadronic matrix elements are much more complicated: further theoretical work is needed!
- however, once a well defined prescription to separate QED from QCD IB effects has been implemented, the QCD corrections to quantities such as the $K\ell 2$ decay rate can be (and have been) obtained

BACKUP

collaboration	quark action	n_f^{QCD}	n_f^{QED}	M_π (MeV)	N_a, [(fm)]	N_L, [(fm)]	method
PACS-CS	npSW	$2+1$	$1+1+1$	135	$1,\ [0.09, 0.09]$	$1,\ [2.9, 2.9]$	$e^{ieA}U$
RBC-UKQCD	DW	$2+1$	$1+1+1$	250	$1,\ [0.11, 0.11]$	$2,\ [1.8, 2.7]$	$e^{ieA}U$
MILC	Asqtad	$2+1$	0	233	$3,\ [0.06, 0.12]$	$5,\ [2.4, 3.6]$	$e^{ieA}U$
BMW	2-HEX tlSW	$2+1$	0	120	$5,\ [0.05, 0.12]$	$17,\ [1.9, 6]$	$e^{ieA}U$
RM123	TM	2	0	270	$4,\ [0.05, 0.10]$	$6,\ [1.6, 2.6]$	$(1+ieA)U$
NPLQCD	Asqtad/DW	$2+1$	0	290	$1,\ [0.12, 0.12]$	$1,\ [2.5, 2.5]$	U
UK-QCD-SF	npSW	$2+1$	0	290	$2,\ [0.06, 0.075]$	$2,\ [1.8, 2.4]$	U

- at present several collaborations are providing lattice results including the effects of isospin breaking

  see also **A.Portelli, arXiv:1307.6056**

- pure QCD projects, U, obtain results with $\hat{m}_d \neq \hat{m}_u$ but neglecting electromagnetic interactions
- QED+QCD projects use different methods: $(1+i\hat{e}A)U$ means that isospin breaking effects are treated at first order with respect to $\hat{\alpha}_{em}$ and $(\hat{m}_d - \hat{m}_u)/\Lambda_{QCD}$
- first results beyond the electroquenched approximation, $n_f^{QED} \neq 0$, have recently been obtained