Temporal mesonic correlators at NLO for any quark mass

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Talk based on
[YB, M. Laine, in preparation]

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Introduction

**Massive vector current correlator in thermal QCD at NLO**
- Vector current correlators as observable
- NLO Calculation
- Results and comparison to the lattice

**Massive scalar current correlator in thermal QCD at NLO**
- (Re-)normalization issues
- Mass dependence and comparison with lattice
- Fermionic contribution to bulk viscosity

Conclusion
1. Introduction

Some temporal mesonic correlators

Vector (spatial) current correlator

\[ G_{V,ii}(\tau) = \int_x \langle (\bar{\Psi} \gamma_{\mu,i} \Psi)(\tau,x) (\bar{\Psi} \gamma^\mu_i \Psi)(0,0) \rangle_T \]

Physics:

> Transport peak, heavy quark diffusion (large \( \tau \))
> Quarkonium physics, bound state (intermediate \( \tau \))
> High energy scattering (small \( \tau \))
1. Introduction

Some temporal mesonic correlators

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> Transport peak, heavy quark diffusion (large \(\tau\))
> Quarkonium physics, bound state (intermediate \(\tau\))
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Scalar current correlator

\[ G_S(\tau) = \int_x \langle (\bar{\Psi} \Psi)(\tau, x) (\bar{\Psi} \Psi)(0, 0) \rangle_T \]

Physics:

> Part of the energy momentum tensor
> Fermionic contribution to the bulk viscosity (small \(\tau\))
How to extract physics from Euclidean correlators?

Physical quantities we are interested in are defined in Minkowski space. They can be fitted from the correlator’s spectrum

\[ \rho(\omega) = \text{Disc}[C_E(-i\omega)], \quad \text{or} \quad C_E(\tau) = \int d\omega K(\omega, \tau)\rho(\omega). \]

**Perturbation theory:**

> Handle Euclidean and Minkowski
> Not accurate for large coupling
> Hard to catch infrared physics

**Lattice:**

> Contains all the physics
> Bound to Euclidean
> Need analytic continuation to Minkowski space...

Can our understanding from perturbation theory help?
2. Vector current correlators as observable

Leading order perturbation theory

> $G_{00}(\tau)$ is constant $\Rightarrow$ one defines the susceptibility $\chi = \beta G_{00}$

   — At LO, $G_{00} = -4N_c \int_p T n'_F(E_p)$

> $G_V$ and $G_{ii}$ have both a constant and a $\tau$-dependent part:

> For $G_{ii}$, at $\tau = \beta/2$ the constant part = time dependent part.
NLO Calculation: Diagrams and renormalization

We calculated the thermal NLO vector correlator for any $M, T$:


$> \text{The “genuine” 2-loop graphs for } G_V(\omega) \text{ are}$

$\begin{array}{c}
\begin{tikzpicture}
    \node (obs) at (0,0) [obs] {};
    \node (q1) at (-2,0) [quark] {};
    \node (q2) at (2,0) [quark] {};
    \node (k1) at (-1,1) [propagator] {};
    \node (k2) at (1,1) [propagator] {};
    \draw (q1) -- (k1) -- (k2) -- (q2);
\end{tikzpicture}
\end{array} + \begin{array}{c}
\begin{tikzpicture}
    \node (obs) at (0,0) [obs] {};
    \node (q1) at (-2,0) [quark] {};
    \node (q2) at (2,0) [quark] {};
    \node (k1) at (-1,1) [propagator] {};
    \node (k2) at (1,1) [propagator] {};
    \node (k3) at (2,2) [propagator] {};
    \draw (q1) -- (k1) -- (k2) -- (k3) -- (q2);
\end{tikzpicture}
\end{array}$

$> \text{Mass counterterm: } \delta M^2$

$\begin{array}{c}
\begin{tikzpicture}
\end{tikzpicture}
\end{array}$

$> \text{Pole mass scheme } \delta M^2 = -\frac{6g^2C_F M^2}{(4\pi)^2} \left( \frac{1}{\varepsilon} + \ln \frac{\mu^2}{M^2} + \frac{4}{3} \right)$

$\Rightarrow G_{00}, G_V \text{ UV and IR finite after mass renormalization}$
Results and comparison to the lattice

To compare the curves, scale:

\[ G_{ii} \] and \( \chi \) to the free, \( M=0 \) result:

\[ G_{ii}^{\text{free}} = 2N_c T^3 \left( \frac{1}{6} + \frac{2 \cos(2\pi \tau T)}{\sin^2(2\pi \tau T)} + \pi (1 - 2\tau T) \frac{1 + \cos^2(2\pi \tau T)}{\sin^3(2\pi \tau T)} \right) \]

\[ \chi^{\text{free}} = \frac{N_c T^2}{3} \]

\[ T = 1.45 T_c, \quad T_c = 1.25 \Lambda_{\text{MS}} \]

\[ M/T = 1 \quad M/T = 5 \]

[Ding, Francis, Kaczmarek, Karsch, Satz, Soeldner 1204.4945; Ding, Francis, Kaczmarek, Karsch, Laermann, Soeldner 1012.4963]
Results and comparison to the lattice

To compare the curves, scale:

> \( G_{ii} \) to the reconstructed correlator

- **NLO:**
  \[
  G_{ii}^{\text{NLO},\text{rec}}(\tau, T) = \int \frac{d\omega}{2\pi} (-\rho_{V}^{\text{NLO,vac}}) K(\tau, \omega)
  \]

- **Lattice:**
  \[
  G_{ii}^{\text{lattice,rec}}(\tau, T, T') = \sum_{\tau'} G(\tau', T')
  \]

\( T = 1.45 T_c, T_c = 1.25 \Lambda_{\overline{\text{MS}}} \)

M/T = 1

M/T = 5
Addition of a transport peak

The spectral function $\rho_{ii}/\omega$ has a $\delta(\omega)$ transport peak (LO, NLO)

> In the full result one expects a Lorentzian shape.

> Replace in $G_{ii}$ the constant part by a $\tau$-dependent part from

$$\rho_{ii}^{(L)}(\omega \sim 0) \approx 3D\chi \frac{\omega\eta^2}{\omega^2 + \eta^2}$$

> We vary $D$ tuning $\eta$ to keep the area under $\rho(\omega)/\omega$ constant
Addition of a bound state peak

In the temperature range of interest → at most one peak.

> Slightly to the left from the free quark-antiquark threshold
> Model this by a skewed Breit-Wigner shape added to NLO

\[
\rho_{ii}^{(\text{BW})}(\omega \sim 2M) \approx \frac{A \omega^2 \gamma^2}{(\omega - 2M + \Delta M)^2 + \gamma^2}
\]

> Set \(\Delta M \equiv 2\gamma\), add to the thermal NLO result
> Add to vacuum result with \(A \rightarrow 5A\), \(\gamma \rightarrow \gamma/5\)
3. Scalar channel

Renormalization

The scalar correlator is defined as

\[ G_S(\tau) = \int_x \langle (\bar{\Psi}\Psi)(\tau, x) (\bar{\Psi}\Psi)(0, 0) \rangle_T \]

> Not renormalizable through a redefinition of the mass!

However

\[ M_b^2 G_S(\tau) \]

> Is finite if we express \( M_b \) as function of \( M_{pole} \):

\[
M_b^2 = M^2 - \frac{6g^2 C_F M^2}{(4\pi)^2} \left( \frac{1}{\varepsilon} + \ln \frac{\bar{\mu}^2}{M^2} + \frac{4}{3} \right)
\]

> \( M_b^2 G_S(\tau) \) is part of the energy momentum tensor

In the following we will calculate \( M_b^2 G_S(\tau) \)
(Re-)normalization issues

At NLO in the pole mass scheme → negative contribution to the spectral function that dominates at $\omega \gg M$
⇒ Euclidean correlator decreases at small $\tau$.

[YB, Laine, Vepsäläinen, JHEP 02 (2009) 008]
Unphysical behavior in the pole mass scheme ⇒ use $\overline{MS}$ scheme
Mass dependence and comparison with lattice

\[ MS \text{ scheme } \rightarrow \delta M^2 = -\frac{6g^2C_F M^2}{(4\pi)^2} \left( \frac{1}{\varepsilon} \right) \]

> Now \( M \rightarrow m(\mu) \) and we refer to \( M = m(\mu = 2\text{GeV}) \).

> Renormalization scale \( \mu \propto \omega, 1/\tau \)

⇒ Plot the results in the \( MS \) scheme:

At small \( \tau \), \( M_b^2 G_{S}^{\text{NLO}}(\tau) \)

> Does not match \( M_b^2 G_{\text{free}}(\tau) \)

⇒ Matches \( m(\mu(\tau))^2 G_{\text{free}}(\tau) \)

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Fermionic contribution to bulk viscosity

Even if this should be extracted for the spectral function, we can compare $G_S(\tau)$ to the gluon part of the trace anomaly $G_\theta(\tau)$.

⇒ Charm quark might give a large contribution to bulk viscosity!
4. Conclusion

On renormalization:

> The Vector (time and spatial comp) and Axial Vector channels are renormalizable through a redefinition of the mass.

> The $M_b^2 \times \text{Scalar}$ and $M_b^2 \times \text{Pseudo Scalar}$ are renormalizable through a redefinition of the mass.

   — For the latter, use the $\overline{MS}$ scheme.

   — They scale to $m(\mu(\tau))^2 G_{\text{free}}(\tau)$ at small $\tau$.

On physical properties:

> The determination of transport and bound state properties requires very precise lattice data.

> Continuum extrapolated data would help as the UV part can be subtracted or used as prior for the analytical continuation.

> The charm quark could be important for the bulk viscosity.