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Temporal mesonic correlators at NLO for any quark mass

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Talk based on

[YB, M. Laine, JHEP 1211 (2012) 086]

[YB, M. Laine, in preparation]

Lattice 2013, Johannes Gutenberg University Mainz

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Outline

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Introduction

Massive vector current correlator in thermal QCD at NLO

- Vector current correlators as observable
- NLO Calculation
- Results and comparison to the lattice

Massive scalar current correlator in thermal QCD at NLO

- (Re-)normalization issues
- Mass dependence and comparison with lattice
- Fermionic contribution to bulk viscosity

Conclusion

Some temporal mesonic correlators

Vector (spatial) current correlator

$$G_{V,ii}(\tau) = \int_x \langle \left(\bar{\Psi}\gamma_{\mu,i}\Psi\right)(\tau,x)\left(\bar{\Psi}\gamma^{\mu,}_{\;\;i}\Psi\right)(0,0)\rangle_T$$

Physics:

- Transport peak, heavy quark diffusion (large τ)
- Quarkonium physics, bound state (intermediate τ)
- High energy scattering (small τ)

Some temporal mesonic correlators

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Vector (spatial) current correlator

$$G_{V,\,ii}(\tau) = \int_x \langle \left(\bar{\Psi}\gamma_{\mu,\,i}\Psi\right)(\tau,x)\left(\bar{\Psi}\gamma^{\mu,}_{\;\;i}\,\Psi\right)(0,0)\rangle_T$$

Physics:

- Transport peak, heavy quark diffusion (large τ)
- Quarkonium physics, bound state (intermediate τ)
- High energy scattering (small τ)

Scalar current correlator

$$G_S(\tau) = \int_x \langle \left(\bar{\Psi}\Psi\right)(\tau,x)\left(\bar{\Psi}\Psi\right)(0,0)\rangle_T$$

Physics:

- Part of the energy momentum tensor
- Fermionic contribution to the bulk viscosity (small τ)

How to extract physics from Euclidean correlators?

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Physical quantities we are interested in are defined in Minkowski space.
They can be fitted from the correlator's spectrum

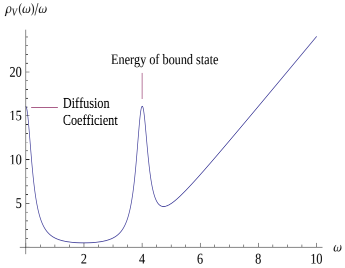
$$\rho(\omega) = \mathrm{Disc}[C_E(-i\omega)], \quad \text{or} \quad C_E(\tau) = \int d\omega K(\omega,\tau)\rho(\omega).$$

Perturbation theory:

- > Handle Euclidean and Minkowski
- > Not accurate for large coupling
- > Hard to catch infrared physics

Lattice:

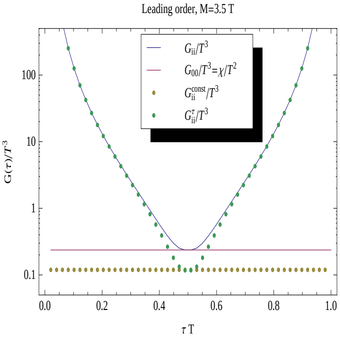
- > Contains all the physics
- > Bound to Euclidean
- > Need analytic continuation to Minkowski space...

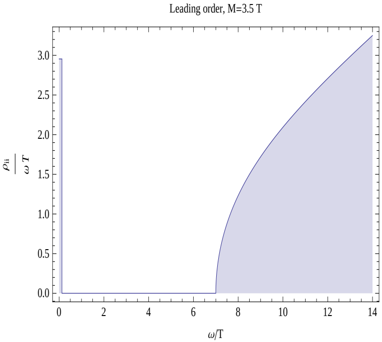


Can our understanding from perturbation theory help?

Leading order perturbation theory

- $G_{00}(\tau)$ is constant $\Rightarrow$ one defines the susceptibility $\chi = \beta G_{00}$
  - At LO, $G_{00} = -4N_c \int_p T n'_{\mathrm{F}}(E_p)$
- G_V and G_{ii} have both a constant and a τ-dependent part:





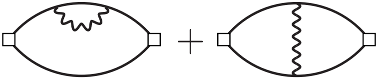
- For G_{ii}, at $\tau = \beta/2$ the constant part = time dependent part.

NLO Calculation: Diagrams and renormalization

We calculated the thermal NLO vector correlator for any M, T:

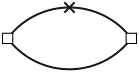
[YB, M.Laine, JHEP 1211 (2012) 086]

> The "genuine" 2-loop graphs for $G_V(\omega)$ are



> Mass counterterm:

δM^2



> Pole mass scheme $\delta M^2 = -\frac{6g^2 C_{\mathrm{F}} M^2}{(4\pi)^2}\left(\frac{1}{\varepsilon} + \ln\frac{\bar{\mu}^2}{M^2} + \frac{4}{3}\right)$

$\Rightarrow$ G_{00}, G_V UV and IR finite after mass renormalization

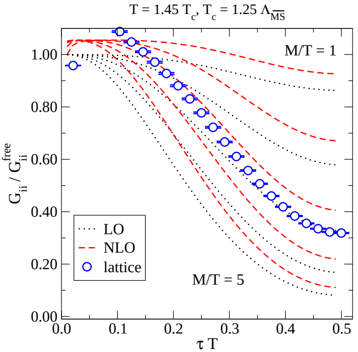
Results and comparison to the lattice

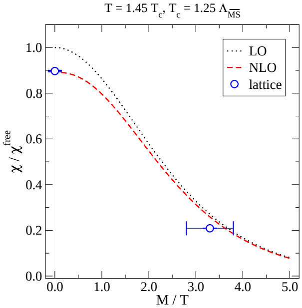
To compare the curves, scale:

> G_{ii} and χ to the free, M=0 result:

— $G_{ii}^{free} = 2N_cT^3\left(\frac{1}{6} + \frac{2\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} + \pi(1-2\tau T)\frac{1+\cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)}\right)$

— $\chi^{free} = \frac{N_cT^2}{3}$





[Ding, Francis, Kaczmarek, Karsch, Satz, Soeldner 1204.4945;

Ding, Francis, Kaczmarek, Karsch, Laermann, Soeldner 1012.4963]

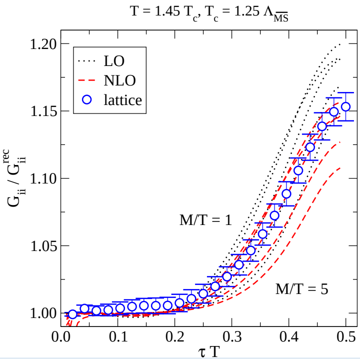
Results and comparison to the lattice

To compare the curves, scale:

> G_{ii} to the reconstructed correlator

— NLO: $G_{ii}^{\mathrm{NLO},rec}(\tau,T) = \int \frac{d\omega}{2\pi}(-\rho_V^{\mathrm{NLO},vac})K(\tau,\omega)$

— Lattice: $G_{ii}^{\mathrm{lattice},rec}(\tau,T,T') = \sum_{\tau'(T,T')} G(\tau',T')$



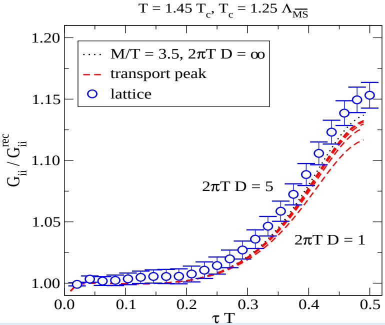
Addition of a transport peak

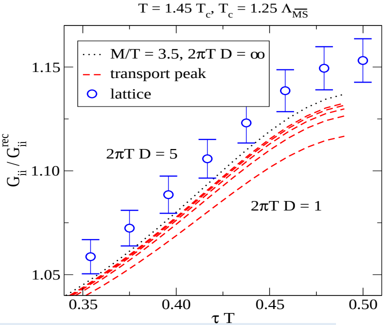
The spectral function ρ_{ii}/ω has a $\delta(\omega)$ transport peak (LO, NLO)

- In the full result one expects a Lorentzian shape.
- Replace in G_{ii} the constant part by a τ-dependent part from

$$\rho_{ii}^{(\mathrm{L})}(\omega \sim 0) \approx 3D\chi \frac{\omega\eta^2}{\omega^2+\eta^2}$$

- We vary D tuning η to keep the area under $\rho(\omega)/\omega$ constant





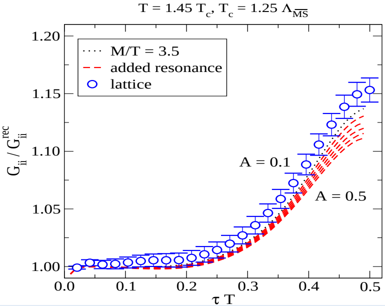
Addition of a bound state peak

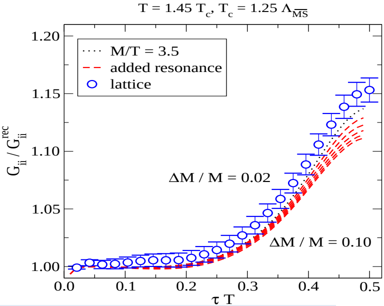
In the temperature range of interest → at most one peak.

- Slightly to the left from the free quark-antiquark threshold
- Model this by a skewed Breit-Wigner shape added to NLO

$$\rho_{ii}^{(\mathrm{BW})}(\omega \sim 2M) \;\approx\; \frac{A\,\omega^2\gamma^2}{(\omega - 2M + \Delta M)^2 + \gamma^2}$$

- Set $\Delta M \equiv 2\gamma$, add to the thermal NLO result
- Add to vacuum result with $A \to 5A$, $\gamma \to \gamma/5$





Renormalization

The scalar correlator is defined as

$$G_S(\tau) = \int_x \langle \left(\bar{\Psi}\Psi\right)(\tau, x) \left(\bar{\Psi}\Psi\right)(0,0)\rangle_T$$

> Not renormalizable through a redefinition of the mass!

However

$$M_b^2 G_S(\tau)$$

> Is finite if we express M_b as fuction of M_{pole}:

$$M_b^2 = M^2 - \frac{6g^2 C_{\mathrm{F}} M^2}{(4\pi)^2}\left(\frac{1}{\varepsilon} + \ln\frac{\bar{\mu}^2}{M^2} + \frac{4}{3}\right)$$

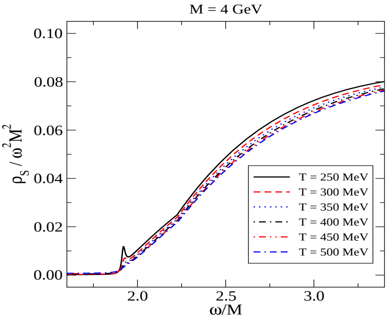
> $M_b^2 G_S(\tau)$ is part of the energy momentum tensor

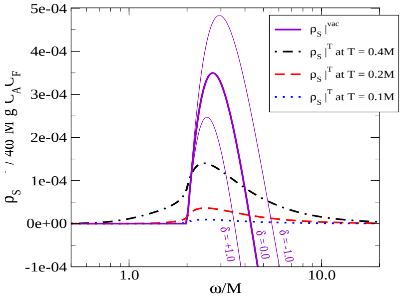
In the following we will calculate $M_b^2 G_S(\tau)$

(Re-)normalization issues

At NLO in the pole mass scheme $\rightarrow$ negative contribution to the spectral function that dominates at $\omega \gg M$

$\Rightarrow$ Euclidean correlator decreases at small τ.





[YB, Laine, Vepsäläinen, JHEP 02 (2009) 008]

Unphysical behavior in the pole mass scheme $\Rightarrow$ use $\overline{MS}$ scheme

Mass dependence and comparison with lattice

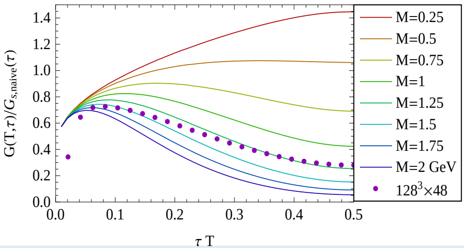
$\overline{MS}$ scheme $\rightarrow \delta M^2 = -\frac{6g^2 C_F M^2}{(4\pi)^2}\left(\frac{1}{\varepsilon}\right)$

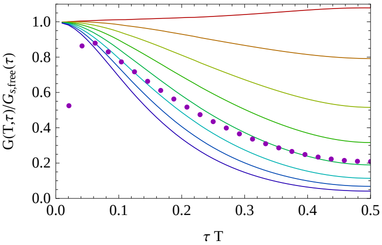
- Now $M \rightarrow m(\mu)$ and we refer to $M = m(\mu = 2GeV)$.
- Renormalization scale $\mu \propto \omega, 1/\tau$

$\Rightarrow$ Plot the results in the $\overline{MS}$ scheme:

At small τ, $M_b^2 G_S^{\mathrm{NLO}}(\tau)$

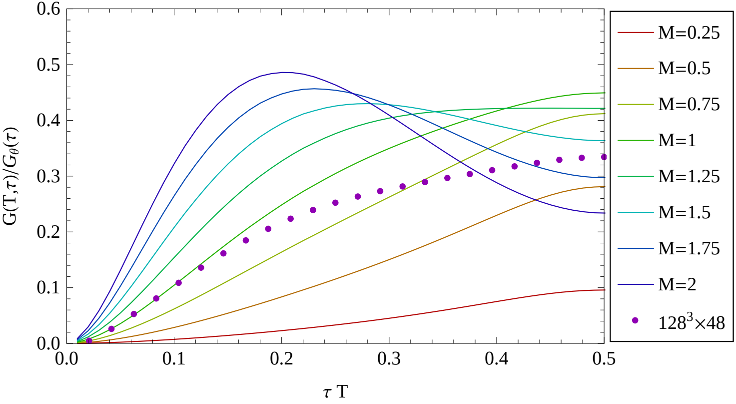
- Does not match $M_b^2 G_{\mathrm{free}}(\tau)$
- Matches $m(\mu(\tau))^2 G_{\mathrm{free}}(\tau)$





Fermionic contribution to bulk viscosity

Even if this should be extracted for the spectral function, we can compare $G_S(\tau)$ to the gluon part of the trace anomaly $G_\theta(\tau)$.



$\Rightarrow$ Charm quark might give a large contribution to bulk viscosity!

4. Conclusion

On renormalization:

- The Vector (time and spatial comp) and Axial Vector channels are renormalizable through a redefinition of the mass.
- The $M_b^2\times$Scalar and $M_b^2\times$Pseudo Scalar are are renormalizable through a redefinition of the mass.
  - For the latter, use the $\overline{MS}$ scheme.
  - They scale to $m(\mu(\tau))^2 G_{\mathrm{free}}(\tau)$ at small τ.

On physical properties:

- The determination of transport and bound state properties requires very precise lattice data.
- Continuum extrapolated data would help as the UV part can be subtracted or used as prior for the analytical continuation.
- The charm quark could be important for the bulk viscosity.