Ab Initio Calculation of Finite Temperature Charmonium Potentials

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Lattice XXXI Mainz

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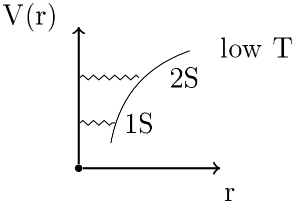
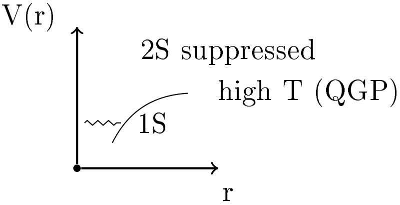
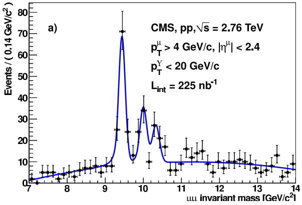
J/ψ Suppression

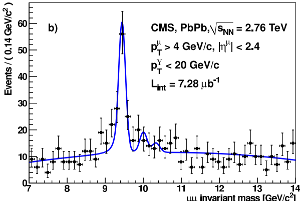
At large color charge density, inter-quark potential is modified.

$$V(r) = -\frac{k}{r} + \sigma r \rightarrow -\frac{\alpha}{r} e^{-r/r_D(T)}$$ *Matsui and Satz 1986*









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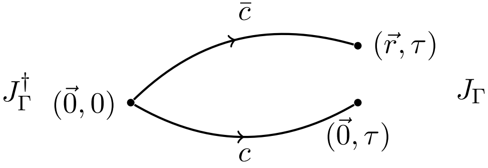
Outline

Observable

Derivation of formula for potential in terms of observable

Simulation and results

Local-Extended Charmonium Correlators



Define general charmonium interpolator,

$$J_\Gamma(x;\mathbf{r}) = \bar{c}(x)\Gamma U(x, x+\mathbf{r})c(x+\mathbf{r})$$

then correlation functions can be written,

$$\begin{aligned} C_\Gamma(\mathbf{r},\tau) &= \sum_{\mathbf{x}} \langle J_\Gamma(\mathbf{x},\tau;\mathbf{r}) J_\Gamma^\dagger(0;\mathbf{0})\rangle \\ &= \sum_j \frac{\psi_j^*(\mathbf{0})\psi_j(\mathbf{r})}{2E_j}\left(e^{-E_j\tau} + e^{-E_j(N_\tau-\tau)}\right) \end{aligned}$$

Convolving Propagators (in Coulomb Gauge)

$$
\begin{aligned}
C_\Gamma(\mathbf{r},\tau) &= \sum_{\mathbf{x}} \langle 0|\bar{c}(\mathbf{x},\tau)\Gamma c(\mathbf{x}+\mathbf{r},\tau)\bar{c}(\mathbf{0},0)\Gamma c(\mathbf{0},t)|0\rangle \\
&= -\sum_{\mathbf{x}} S^\dagger_{\bar{c}}(\mathbf{x}+\mathbf{r},\tau:\mathbf{0},0) S_c(\mathbf{0},0:\mathbf{x},\tau)
\end{aligned}
$$

$$
S(\mathbf{y}) = \frac{1}{V}\sum_{\mathbf{q}} \tilde{S}(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{y}},
$$

$$
\begin{aligned}
C_\Gamma(\mathbf{r},\tau) &= -\sum_{\mathbf{x}} \frac{1}{V^2}\sum_{\mathbf{p},\mathbf{q}} \tilde{S}^\dagger_{\bar{c}}(\mathbf{p})\tilde{S}_c(\mathbf{q}) e^{i\mathbf{p}\cdot(\mathbf{x}+\mathbf{r})} e^{i\mathbf{q}\cdot(\mathbf{x})} \\
&= -\frac{1}{V}\sum_{\mathbf{p},\mathbf{q}} \tilde{S}^\dagger_{\bar{c}}(\mathbf{p})\tilde{S}_c(\mathbf{q})\delta_{\mathbf{p}+\mathbf{q},\mathbf{0}} e^{i\mathbf{p}\cdot(\mathbf{r})} \\
&= -\frac{1}{V}\sum_{\mathbf{p}} \tilde{S}^\dagger_{\bar{c}}(\mathbf{p})\tilde{S}_c(-\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{r}}
\end{aligned}
$$

priv. comm. S. Aoki

HAL QCD Approach

HAL QCD Collaboration 2012

Consider only forward moving contribution of $C_\Gamma(\mathbf{r},\tau)$,

$$\begin{aligned} C_\Gamma(\mathbf{r},\tau) &= \sum_j \frac{\psi_j^*(\mathbf{0})\psi_j(\mathbf{r})}{2E_j} e^{-E_j\tau} \\ &= \sum_j \Psi_j(\mathbf{r}) e^{-E_j\tau} \end{aligned}$$

Differentiate w.r.t. τ,

$$\frac{\partial}{\partial\tau} C_\Gamma(\mathbf{r},\tau) = -\sum_j E_j \Psi_j(\mathbf{r}) e^{-E_j\tau}$$

Now consider Schrödinger equation for $\Psi_j(\mathbf{r})$,

$$\left(-\frac{\nabla^2}{2\mu} + V_\Gamma(\mathbf{r})\right) \Psi_j(\mathbf{r}) = E_j \Psi_j(\mathbf{r})$$

$$
\begin{aligned}
\frac{\partial}{\partial \tau} C_\Gamma(\mathbf{r},\tau) &= \sum_j \left(\frac{\nabla^2}{2\mu} - V_\Gamma(\mathbf{r}) \right) \Psi_j(\mathbf{r}) e^{-E_j \tau} \\
&= \left(\frac{\nabla^2}{2\mu} - V_\Gamma(\mathbf{r}) \right) \sum_j \Psi_j(\mathbf{r}) e^{-E_j \tau} \\
&= \left(\frac{\nabla^2}{2\mu} - V_\Gamma(\mathbf{r}) \right) C(\mathbf{r},\tau)
\end{aligned}
$$

$$
\Longrightarrow \qquad V_\Gamma(\mathbf{r}) = \left(\frac{\nabla^2 C_\Gamma(\mathbf{r},\tau)}{2\mu} - \frac{\partial C_\Gamma(\mathbf{r},\tau)}{\partial \tau} \right) \frac{1}{C_\Gamma(\mathbf{r},\tau)}
$$

S -Waves

The S-wave potential can be expressed as:

$$V_\Gamma(\mathbf{r}) = V_C(\mathbf{r}) + s_1 \cdot s_2 V_S(\mathbf{r}).$$

$s_1 \cdot s_2 = -3/4, 1/4$ for the pseudoscalar and vector respectively.

Hence,

$$V_C(\mathbf{r}) = \frac{1}{4} V_{PS}(\mathbf{r}) + \frac{3}{4} V_V(\mathbf{r})$$

and

$$V_S(\mathbf{r}) = V_V(\mathbf{r}) - V_{PS}(\mathbf{r})$$

Simulation Details

N_s	N_τ	T(MeV)	T/T_c	N_{cfg}
24	40	140	0.76	500
24	36	156	0.84	500
24	32	175	0.95	1000
24	28	201	1.08	1000
24	24	232	1.26	1000

Ensembles

Two-Plaquette Symanzik gauge action, $N_f = 2 + 1$ dynamical sea quarks,
Anisotropic Clover fermion action with stout-link smearing,
Anistropy: $a_s/a_\tau = 3.5$.

Measurement

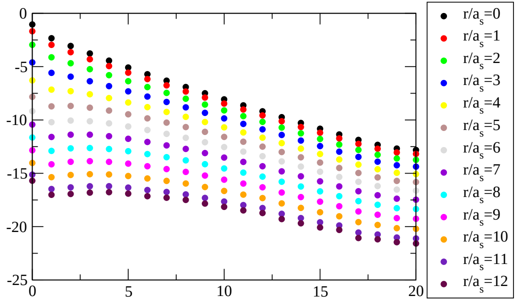
Anisotropic Clover fermion action for stout-link smearing,
Pseudoscalar effective mass tuned to experimental η_c mass,
Gaussian-smeared sources.

QDP++/Chroma: Edwards and Joó

Results

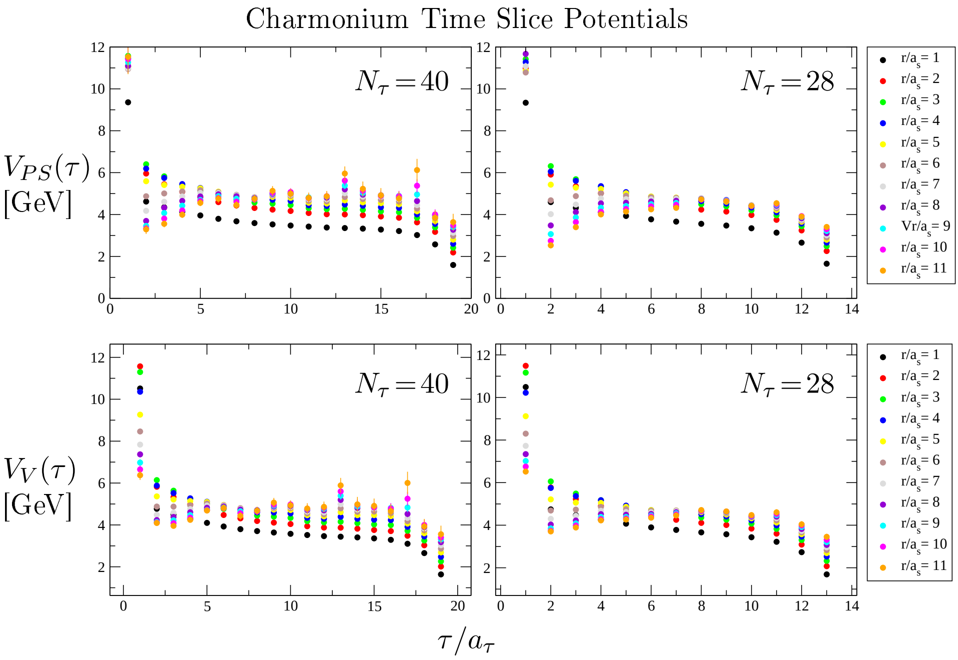
Charmonium Time Slice Correlators
$(N_s = 24, N_\tau = 40,\ \text{Pseudoscalar})$

$\log C(r,\tau)$

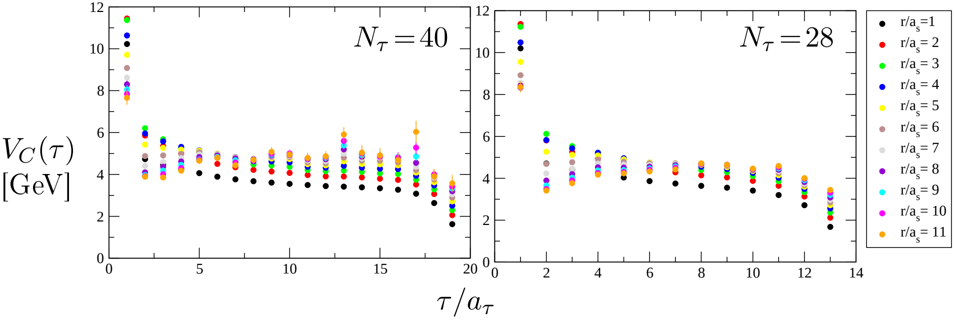


τ/a_τ

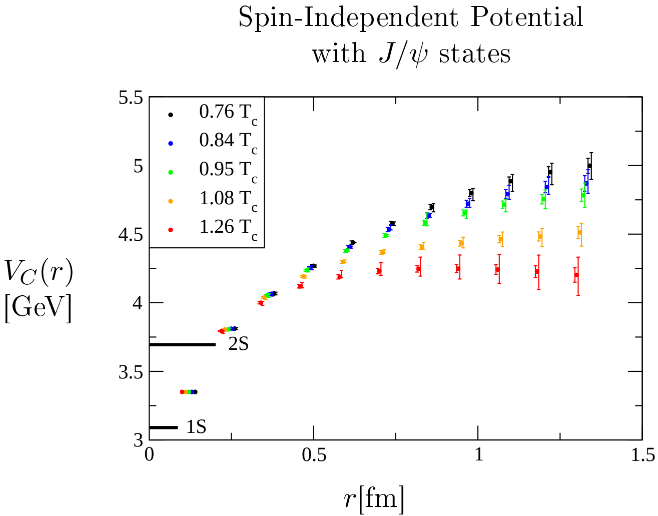
$$V_\Gamma(\mathbf{r}) = \left(\frac{\nabla^2 C_\Gamma(\mathbf{r},\tau)}{2\mu} - \frac{\partial C_\Gamma(\mathbf{r},\tau)}{\partial\tau}\right)\frac{1}{C_\Gamma(\mathbf{r},\tau)}$$


Charmonium Time Slice Potentials
$N_\tau = 40$
$N_\tau = 28$
$V_{PS}(\tau)$ [GeV]
$V_V(\tau)$ [GeV]
τ/a_τ
r/a_s = 1
r/a_s = 2
r/a_s = 3
r/a_s = 4
r/a_s = 5
r/a_s = 6
r/a_s = 7
r/a_s = 8
Vr/a_s = 9
r/a_s = 9
r/a_s = 10
r/a_s = 11

Spin-Independent Charmonium Time Slice Potentials

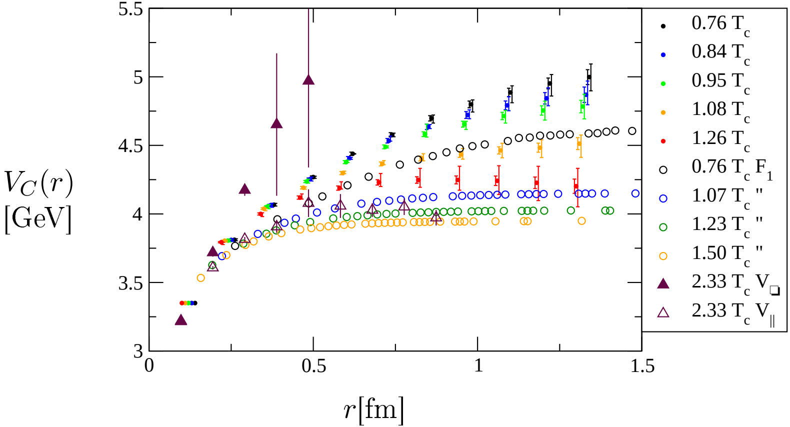


$$V_C(\mathbf{r}) = \frac{1}{4}V_{PS}(\mathbf{r}) + \frac{3}{4}V_V(\mathbf{r})$$

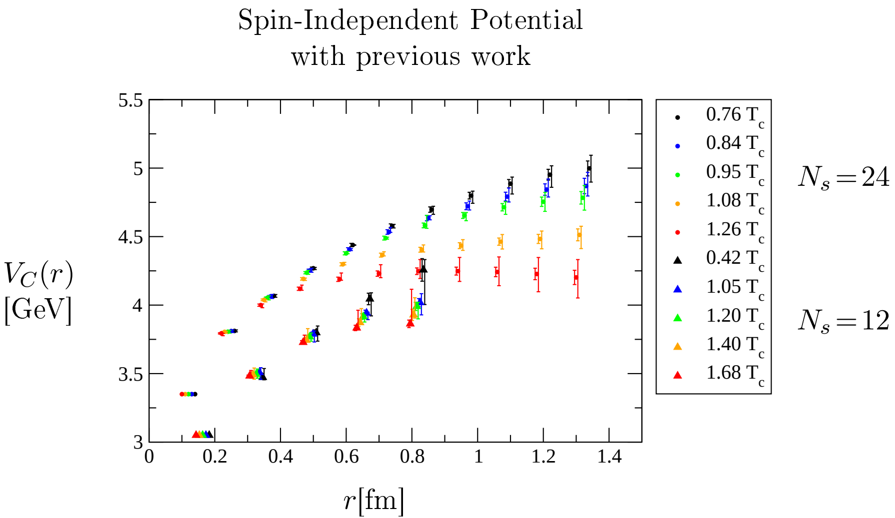


Spin-Independent Potential
with J/ψ states
$V_C(r)$ [GeV]
r[fm]
0.76 T_c
0.84 T_c
0.95 T_c
1.08 T_c
1.26 T_c
2S
1S
3
3.5
4
4.5
5
5.5
0
0.5
1
1.5

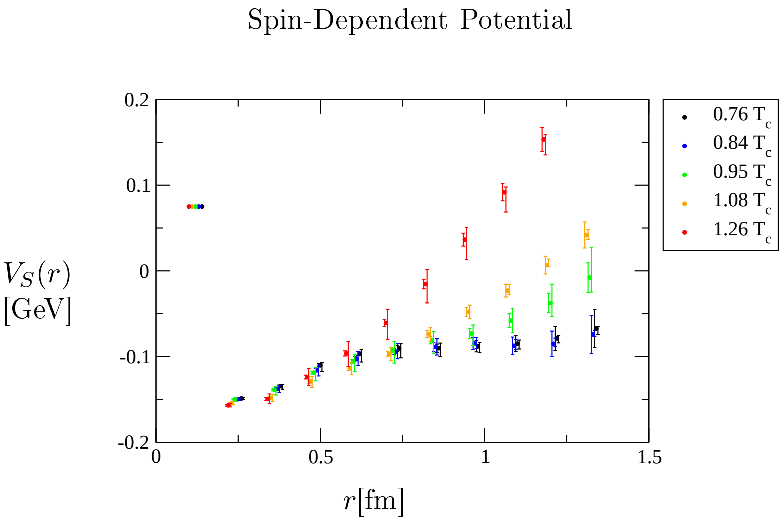
Spin-Independent Potential
with results of other groups



O. Kaczmarek, F. Zantow, Phys. Rev. D71, 114510 (2005)
Y. Burnier, A. Rothkopf, Phys. Rev. D86, 051503 (2012)



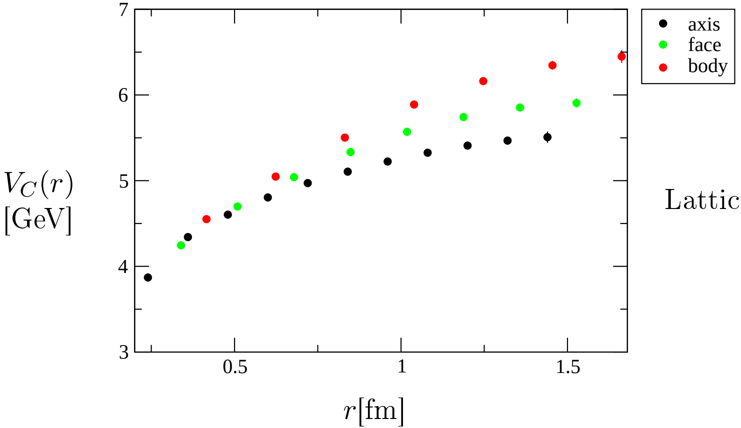
Spin-Independent Potential
with previous work
$V_C(r)$
[GeV]
r[fm]
0.76 T_c
0.84 T_c
0.95 T_c
1.08 T_c
1.26 T_c
0.42 T_c
1.05 T_c
1.20 T_c
1.40 T_c
1.68 T_c
$N_s = 24$
$N_s = 12$



Spin-Dependent Potential
0.2
0.1
0
-0.1
-0.2
0
0.5
1
1.5
$V_S(r)$
[GeV]
r[fm]
0.76 T_c
0.84 T_c
0.95 T_c
1.08 T_c
1.26 T_c

Spin-Independent Potential

$N_\tau = 40$ axis, face, and body separations



Lattice artefact?

Conclusions

A temperature dependence consistent with deconfinement is observed with the HAL QCD approach

To investigate the highest temperatures require configurations with many points in temporal dimension

Next step is to calculate Sommer's r_I to obtain potential at more values of r