Charmonium spectral functions from 2 flavour anisotropic lattice data

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Outline

- Quick overview of lattice parameters and the method used.
- Spectral functions of charmonium at zero momentum.
- Use of the conserved vector current for non-zero momenta.
- Spectral functions of charmonium at non-zero momenta.

Parameters

- The lattices used are ensembles of anisotropic gauge configurations with anisotropy $\xi = a_s/a_\tau = 6$.
- The spatial lattice spacing is $a_s = 0.167$ fm and the number of lattice sites in the spatial direction is $N_s = 12$.
- The fermion action is a fine Hamber-Wu action in the spatial directions and a coarse Wilson action in the temporal direction.
- The number of configurations for each ensemble are thus.

N_τ	T (MeV)	N_{cfgs}
16	459	1000
18	408	700
20	368	1000
24	306	500
28	263	1000
32	230	875

Maximum Entropy Method

The method employed for this research is the Maximum Entropy Method (MEM), as presented by Asakawa, Nakahara and Hatsuda in their 2001 paper [arxiv:hep-lat/0011040v2].

MEM is a well-established way of determining spectral functions on the lattice.

The spectral function $\rho(\omega)$ is related to the imaginary time correlator $G(\tau)$ as follows.

$$G(\tau) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega) \frac{\cosh[\omega(\tau - \frac{\beta}{2})]}{\sinh(\frac{\beta\omega}{2})}.$$

The most probable spectral function is extracted from the Monte Carlo data in this analysis.

Default Models

MEM relies on input information known as a default model, $m(\omega)$. The output should have no dependence on these functions.

The default models used are given below.
m_0 is a free parameter, and m_1 is scaled with the temperature.

Default Models

$$m(\omega) = m_0\omega^2$$

$$m(\omega) = m_0$$

$$m(\omega) = m_0\omega(m_1 + \omega)$$

Conserved vector current

- The spatial component of the conserved vector current is

$$
\begin{aligned}
V_i(\vec{p}, x) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} [&\bar{\psi}(x)(\Gamma_i^{(0)}\psi)(x) \\
&+ \bar{\psi}(x-\hat{i})(\Gamma_i^{(-1)}\psi)(x) \\
&+ \bar{\psi}(x+\hat{i})(\Gamma_i^{(1)}\psi)(x) \\
&+ \bar{\psi}(x+2\hat{i})(\Gamma_i^{(2)}\psi)(x)]
\end{aligned}
$$

- The temporal component is the usual conserved Wilson current.

$$V_t(x) = \frac{1}{2}[\bar{\psi}(x+\hat{t})(r+\gamma_0)U_t^\dagger(x)\psi(x) - \bar{\psi}(x)(r-\gamma_0)U_t(x)\psi(x+\hat{t})]$$

Decomposing the vector current

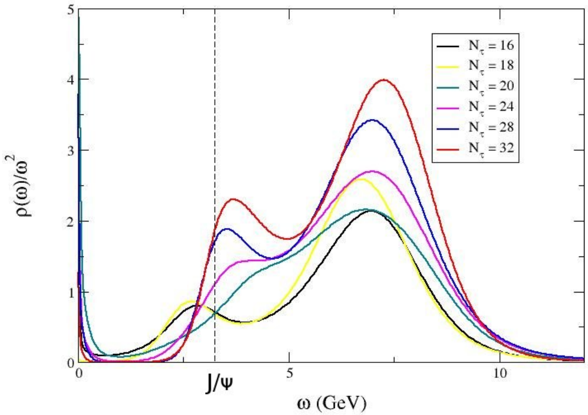
- The conserved vector current is used since it requires **no renormalisation**.
- In order to calculate the heavy quark diffusion, we need the longitudinal polarisation of the conserved vector current.
- The vector meson correlator is decomposed into transverse and longitudinal polarisations as follows.

Decomposition of vector meson correlator into its polarisations

$$V_{ij}(\tau,\vec{p}) = \left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) V_T(\tau,\vec{p}) + \frac{p_i p_j}{p^2} V_L(\tau,\vec{p})$$

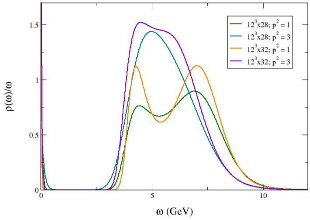
Spectral functions for zero momentum

Spectral functions for zero momentum charmonium using default model $m(\omega) = m_0\omega(m_1 + \omega)$

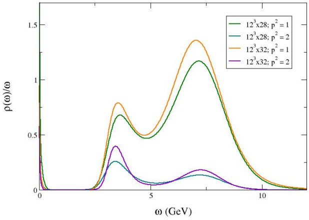


Spectral functions for non-zero momenta

Spectral functions for the **longitudinal** component of non-zero momenta using default model $m(\omega) = m_0\omega(m_1 + \omega)$.

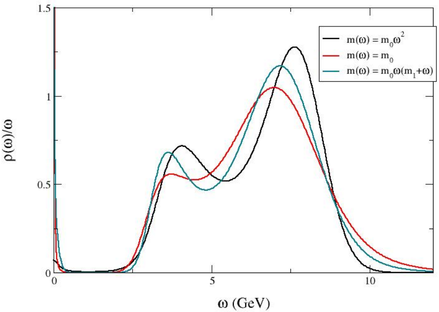


Spectral functions for the **transverse** component of non-zero momenta using default model $m(\omega) = m_0\omega(m_1 + \omega)$.



Default model dependence

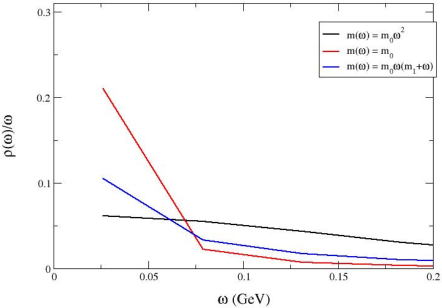
- The shape of the spectral functions should not depend on the input default models.
- This plot shows the default model dependence for the transverse component of momentum $p^2 = 1$ on the $12^3 \times 28$ lattice.



Low frequency zone

Our aim is to calculate the transport coefficients. For this we need to examine the low frequency zone of the spectral functions.

This plot shows the low frequency zones for each default model of the spectral function for zero momentum charmonium from the $12^3 \times 32$ lattice.



Conclusions and Outlook

- We have calculated the spectral functions for lattice sizes $12^3 \times N_\tau$ for $\tau = 16, 18, 20, 24, 28, 32$ with an anisotropy of $\xi = 6$. The calculations were done for both zero momentum and non-zero momentum.
- The spectral functions for non-zero momentum charmonium quickly become unreliable, and only the $12^3 \times 28$ and $12^3 \times 32$ lattices give us significant insight.
- The results obtained do not depend heavily on the default models $m(\omega)$.
- For future work, the low frequency zones will be examined and transport coefficients determined.
- $N_f = 2 + 1$ with finer spatial lattice spacing will also be examined in future.

Hamber-Wu action.

$$
\begin{aligned}
S_{HW} = \frac{1}{a}\Bigg\{ \sum_{x} \bar{\psi}(x)\psi(x) \\
& - \frac{4\kappa}{3} \sum_{x,\mu} \Big[\bar{\psi}(x)(r-\gamma_\mu) U_\mu(x) \psi(x+\hat{\mu}) \\
& - \bar{\psi}(x)(r+\gamma_\mu) U_\mu^\dagger(x-\hat{\mu}) \psi(x-\hat{\mu}) \Big] \\
& + \frac{\kappa}{6U_0} \sum_{x,\mu} \Big[\bar{\psi}(x)(2r-\gamma_\mu) U_\mu(x) U_\mu(x+\hat{\mu}) \psi(x+2\hat{\mu}) \\
& - \bar{\psi}(x)(2r+\gamma_\mu) U_\mu^\dagger(x-\hat{\mu}) U_\mu^\dagger(x-2\hat{\mu}) \psi(x-2\hat{\mu}) \Big] \Bigg\} .
\end{aligned}
$$

Spatial part of conserved vector current.

$$
\begin{aligned}
V_i^A(x) = &- \frac{2}{3}\bar{\psi}(x)(r_A - \gamma_i)U_i(x)\psi(x+\hat{\imath}) \\
&+ \frac{2}{3}\bar{\psi}(x+\hat{\imath})(r_A + \gamma_i)U_i^\dagger(x)\psi(x) \\
&+ \frac{1}{12u_s}\Big[\bar{\psi}(x)(2r_A - \gamma_i)U_i(x)U_i(x+\hat{\imath})\psi(x+2\hat{\imath}) \\
&+ (x \to x - \hat{\imath})\Big] \\
&- \frac{1}{12u_s}\Big[\bar{\psi}(x+\hat{\imath})(2r_A + \gamma_i)U_i^\dagger(x)U_i^\dagger(x-\hat{\imath})\psi(x-\hat{\imath}) \\
&+ (x \to x + \hat{\imath})\Big] .
\end{aligned}
$$