Determination of Karsch Coefficients for 2-Colour QCD

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Outline

QC_2D

- QC_2D is a QCD-like theory which doesn't have a sign problem for even number of flavours.
- It has a hadronic phase and deconfinement.
- Include a diquark source term to counter effects of IR fluctuations. Physical limit J to zero must be taken.

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The Action.

- We use the ordinary Wilson plaquette action for the gauge sector.

$$S_G(\beta,\gamma_g) = -\frac{\beta}{N_c}\left[\frac{1}{\gamma_g}\sum Re\ Tr\ U_{ij}(x) + \gamma_g \sum Re\ Tr\ U_{i0}(x)\right]$$

- For the fermion sector, we use an unimproved Wilson fermion action with hopping parameter κ.

$$S_Q(m,\gamma_q) = \sum\left[\bar{\psi}^{\alpha}(x)\psi^{\alpha}(x) + \gamma_q\kappa\bar{\psi}^{\alpha}(x)(D_0\psi)^{\alpha}(x)\right] + \kappa\sum\left[\bar{\psi}^{\alpha}(x)(D_i\psi)^{\alpha}(x)\right]$$

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The Parameters

- Vary parameters around a central set, in this case β =1.9, κ=0.1680 on a 12^3x24 lattice which gives lattice spacing $a = 0.178(6)$fm and a pion mass $m_\pi = 717(25)$MeV.
- Define:

$$\beta_s = \frac{\beta}{\gamma_g}, \qquad \beta_t = \gamma_g, \qquad \kappa_t = \gamma_q \kappa, \qquad \kappa_s = \kappa$$

$$\xi_+ = \frac{1}{2}\{\xi_g + \xi_q\} \qquad \xi_- = \frac{1}{2}\{\xi_g - \xi_q\}$$

What are they?

Use the Derivative Method to calculate energy density

$$\varepsilon(T) = -\frac{\xi}{N_s^3 a_s^3 N_t a_t} \left\langle \left. \frac{\partial S(\beta, \kappa, \gamma_g, \gamma_q)}{\partial \xi} \right|_{a_s} \right\rangle + \mu n_q$$

Need $\frac{\partial \beta}{\partial \xi}, \frac{\partial \gamma_g}{\partial \xi}, \frac{\partial \gamma_q}{\partial \xi}, \frac{\partial \kappa}{\partial \xi}$

$$\begin{aligned}
\xi_+ - 1 &= a_1 \Delta\gamma_g + b_1 \Delta\gamma_q + c_1 \Delta\beta + d_1 \Delta\kappa \\
\frac{a - a_0}{a_0} &= a_2 \Delta\gamma_g + b_2 \Delta\gamma_q + c_2 \Delta\beta + d_2 \Delta\kappa \\
\frac{M - M_0}{M_0} &= a_3 \Delta\gamma_g + b_3 \Delta\gamma_q + c_3 \Delta\beta + d_3 \Delta\kappa \\
\xi_- &= a_4 \Delta\gamma_g + b_4 \Delta\gamma_q + c_4 \Delta\beta + d_4 \Delta\kappa
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Also get the β functions $\frac{\partial \beta}{\partial a_s}, \frac{\partial \kappa}{\partial a_s}$

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Static Quark Potential.

- This is used to calculate the lattice spacing a_s.
- This is done by measuring the potential between a quark anti-quark pair. Then fit the static quark potential to the Cornell potential.

$$V(r) = C + \frac{\alpha}{r} + \sigma r$$

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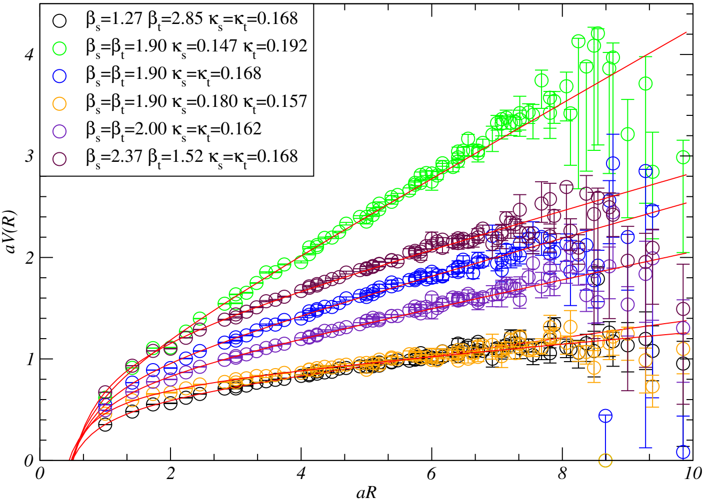
Results.



β_s=1.27 β_t=2.85 κ_s=κ_t=0.168
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β_s=β_t=2.00 κ_s=κ_t=0.162
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aV(R)
aR
0
1
2
3
4
0
2
4
6
8
10

Sideways Potential.

- This is used to calculate the gauge anisotropy ξ_g.
- Compare wilson loop ratios.

$$R_{ss}(x,y) \equiv \frac{W_{ss}(x,y)}{W_{ss}(x+1,y)}, \quad R_{st}(x,t) \equiv \frac{W_{st}(x,t)}{W_{st}(x+1,t)}$$

- The gauge anisotropy is then

$$\xi_g = \frac{V_{xt}(R_2) - V_{xt}(R_1)}{V_{xy}(R_2) - V_{xy}(R_1)}$$

- Currently only considers four sided objects (squares,rectangles).

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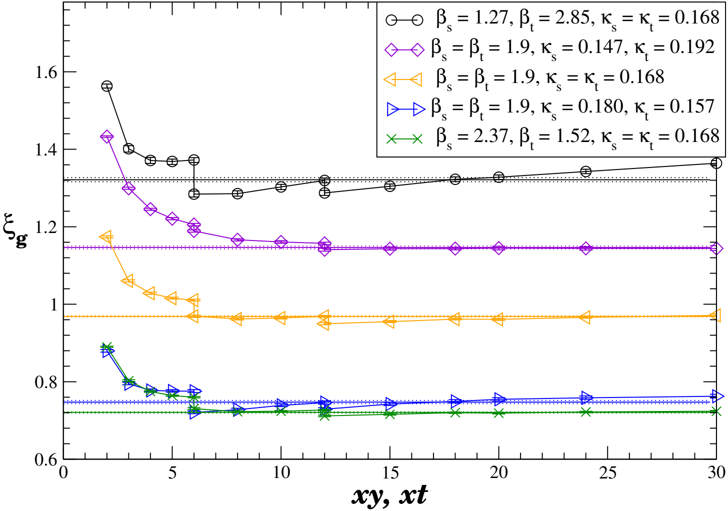
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ξ_g
1.6
1.4
1.2
1
0.8
0.6
0
5
10
15
20
25
30
xy, xt

Meson Dispersion.

- This gives the mass term M and the fermion anisotropy ξ_q.
- Measure the mass ratio of the pion and rho mesons.
- Give them momenta by using FFT and measure the dispersion relation.

$$a_\tau^2 E^2 = a_\tau^2 m_\pi^2 + \frac{a_s^2 p^2}{\xi_q^2}$$

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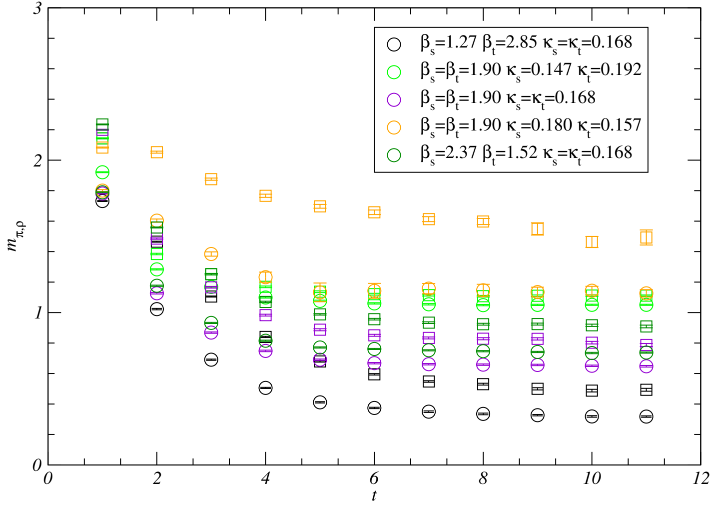
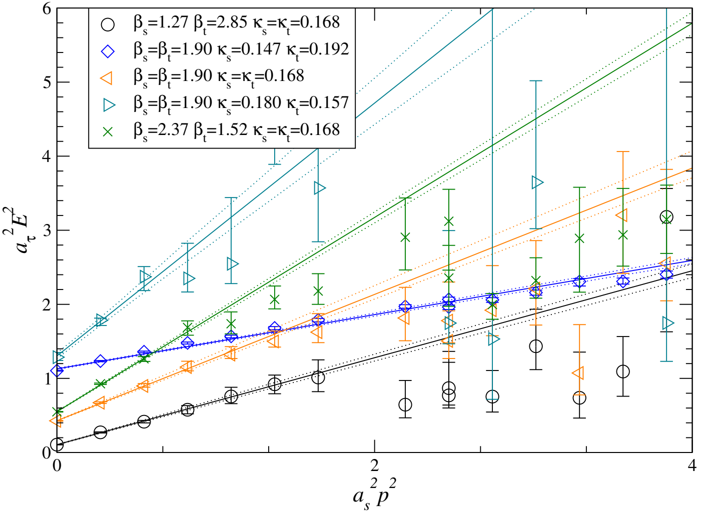
Mass Results.



Dispersion Results.



The Fit.

i	$\gamma_g; a_i$	$\gamma_q; b_i$	$\beta; c_i$	$\kappa; d_i$	χ^2/N_{df}
ξ_+	0.761^{+30}_{-29}	$-1.66^{+0.88}_{-0.49}$	$-2.58^{+0.70}_{-0.39}$	-39^{+12}_{-7}	19.4
a	-0.503^{+48}_{-55}	$-5.14^{+0.79}_{-1.42}$	$-5.94^{+0.74}_{-1.01}$	-88^{+10}_{-17}	2.8
M	-0.531^{+29}_{-46}	$0.15^{+1.1}_{-0.57}$	$-0.59^{+0.96}_{-0.61}$	-15^{+16}_{-8}	22.6
ξ_-	0.096^{+28}_{-29}	$-0.84^{+0.52}_{-0.85}$	$-0.46^{+0.38}_{-0.73}$	-6^{+7}_{-12}	1.9

c_i	$\frac{\partial c_i}{\partial \xi_+}$	$a\frac{\partial c_i}{\partial a}$	$M\frac{\partial c_i}{\partial M}$	$\frac{\partial c_i}{\partial \xi_-}$
γ_g	0.90^{+4}_{-14}	-0.51^{+19}_{-15}	0.13^{+32}_{-58}	$1.4^{+1.2}_{-1.6}$
γ_q	0.13^{+40}_{-5}	0.22^{+12}_{-70}	$-0.55^{+2.11}_{-0.29}$	$-2.9^{+5.7}_{-0.6}$
β	$0.59^{+0.24}_{-1.37}$	$-1.4^{+2.3}_{-0.5}$	$3.7^{+1.9}_{-7.0}$	8^{+8}_{-19}
κ	-0.052^{+69}_{-15}	0.075^{+24}_{-15}	-0.22^{+35}_{-8}	-0.39^{+88}_{-23}

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Beta Function

c_i	$a\frac{\partial c_i}{\partial a}$	$M\frac{\partial c_i}{\partial M}$
β	-1.02^{+17}_{-29}	0.73^{+26}_{-13}
κ	0.057^{+15}_{-9}	-0.047^{+8}_{-16}

using isotropic data

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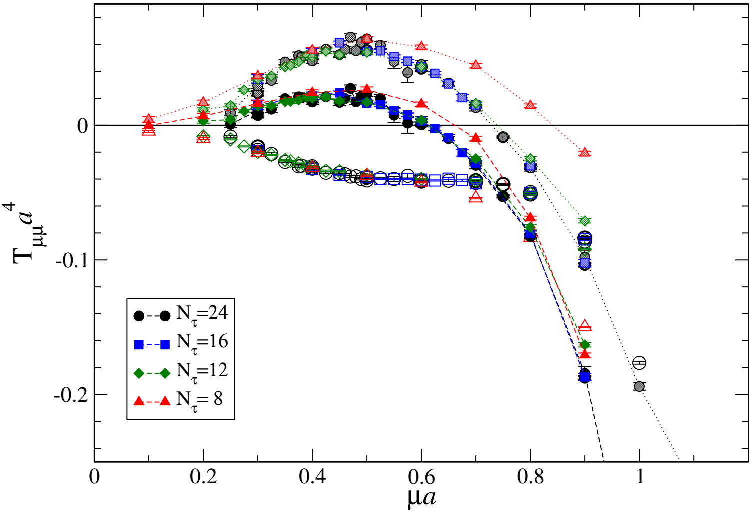
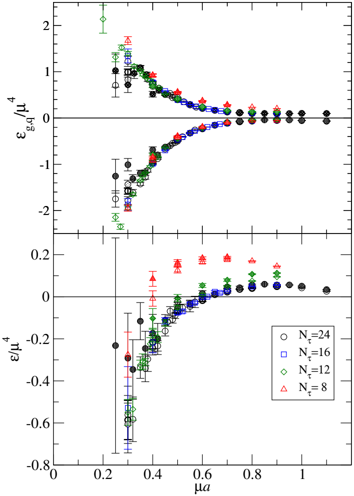
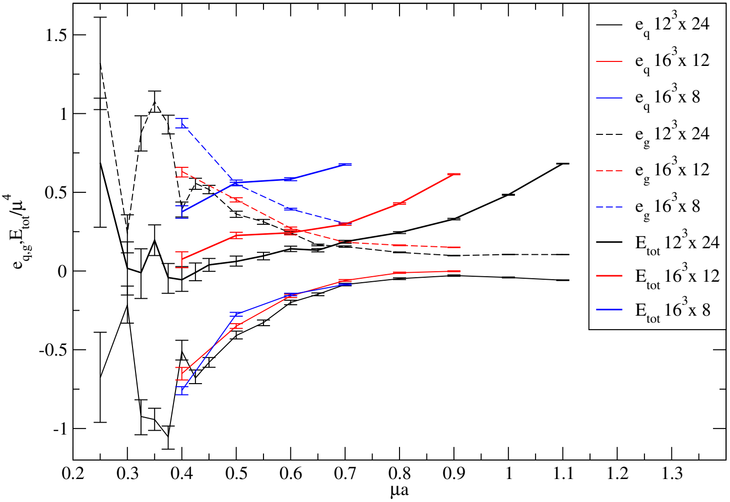
Trace Anomaly



Combined Energy Density I.



Combined Energy Density II.



Summary

- Vary parameters to allow for measurement of observables central set.
- Calculate observables, (combination of meson dispersion, static potential, sideways potential and wilson flow).
- Perform four dimensional fit and invert.
- What's next?
  - Incorporate wilson flow code in Karsch Coeffiecent code suite.
  - Look at writing a more efficient static potential code.
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Further Reading

- S. Hands, S. Kim and J. Skullerud. *A Quarkyonic Phase in Dense Two Color Matter?* Phys. Rev. D81, 091502 (2010), [1001.1682]
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- S. Cotter, P. Giudice, S. Hands, J. Skullerud. *Towards the phase diagram of dense two-colour matter.* Phys. Rev. D87, 034507, (2013), [arXiv:1210.4496].
- T. Boz, S. Cotter, L. Fister, D. Mehta, J. Skullerud *Phase transitions and gluodynamics in 2-colour matter at high*