$SO(2N)$ and $SU(N)$ gauge theories

Lattice 2013

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Talk Structure

$SO(2N)$ Gauge Theories: Why $SO(2N)$?

- Group equivalence

$$SU(2) \sim SO(3)$$
$$SU(4) \sim SO(6)$$

- Large-N equivalence[1,2]

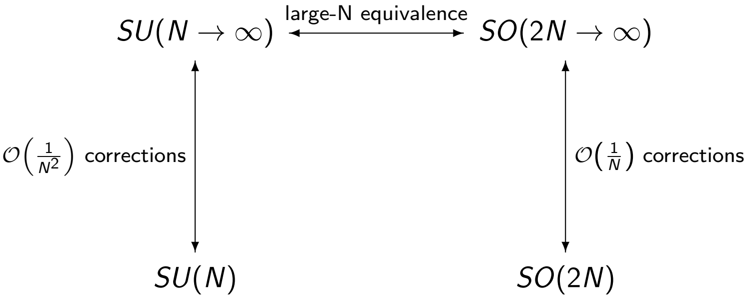
$$SU(N \to \infty) = SO(2N \to \infty)$$
$$g^2\big|_{SU(N\to\infty)} = g^2\big|_{SO(2N\to\infty)}$$

- No sign problem

[1] C. Lovelace, Nucl. Phys. B201 (1982) 333

[2] A. Cherman, M. Hanada, and D. Robles-Llana, Phys. Rev. Lett. 106, 091603 (2011)

Going between $SU(N)$ to $SO(2N)$



Our approach

- Continuum limit at specific $SO(2N)$
- Large-N extrapolation

$SO(2N)$ Gauge Theories: Lattice Setup

- $D = 3 + 1$
  - Bulk transition occurs at very small lattice spacing
  - Very large lattices required to get continuum extrapolation.[3]
- $D = 2 + 1$
  - Bulk transition occurs at larger lattice spacing
- Pure gauge theories

$$S = \beta \sum_{p} \left(1 - \frac{1}{N} \mathrm{Tr} U_p \right) : \beta = \frac{2N}{ag^2}$$

[3] e.g. P. de Forcrand and O. Jahn, Nucl. Phys. B651 (2003) 125

String Tensions: Extracting string tensions

- Polyakov loop operators $l_P(t)$
- Correlators

$$C(t) \equiv \langle l_P(t) l_P(0) \rangle \propto e^{-m_P t}$$

- Nambu-Goto model[4]

$$m_P(l) = \sigma l \left(1 - \frac{\pi}{3\sigma l^2}\right)^{\frac{1}{2}}$$

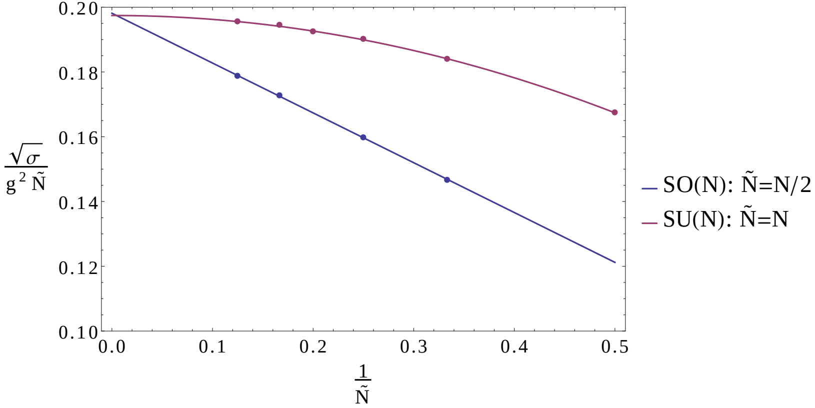
[4] A. Athenodorou, B. Bringoltz, and MT, JHEP 1102 (2011) 030

String Tensions: $SO(2N \to \infty)$ and $SU(N \to \infty)$

Large-N extrapolation of continuum limits

$SO(N)$: $N = 6, 8, 12, 16 \Rightarrow \tilde{N} = 3, 4, 6, 8$

$SU(N)$: $N = 2, 3, 4, 5, 6, 8$



String Tensions: $SO(2N \to \infty)$ and $SU(N \to \infty)$

Large-N extrapolation of $SO(2N)$ and $SU(N)$[5] string tensions

Gauge group	$\left.\frac{\sqrt{\sigma}}{g^2 \tilde{N}}\right\|_{\tilde{N}\to\infty}$
$SO(2N)$	0.1981(6)
$SU(N)$	0.1974(2)

[5] B. Bringoltz and MT, Phys. Lett. B645: 383−388 (2007)

Mass Spectrum: Extracting glueball masses

- Operators for J^P glueballs

$$\phi(t) = \sum_{\vec{x}} \sum_{n} e^{ij\theta_n} \mathrm{Tr}\left\{ U_{R(\theta_n)\mathcal{C}} \pm U_{PR(\theta_n)\mathcal{C}} \right\} : \theta_n = \frac{n\pi}{2}$$

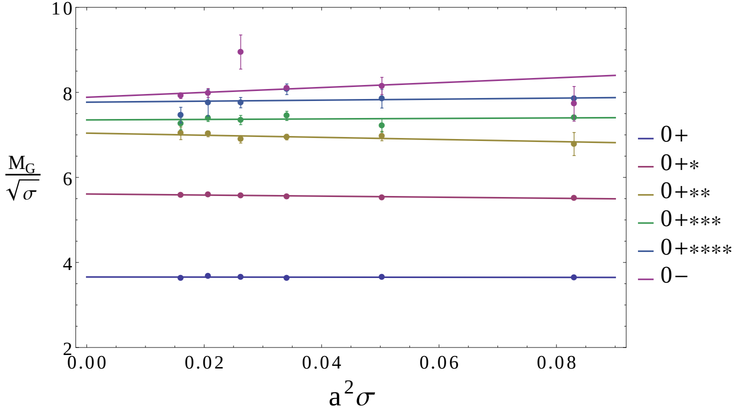
- Variational method[6]
- Correlation functions

$$C(t) \equiv \frac{\langle \phi(t)\phi(0) \rangle}{\langle \phi(0)\phi(0) \rangle} \propto e^{-m_G t}$$

[6] MT, Phys. Rev. D59 (1999) 014512

Mass Spectrum: Continuum extrapolation

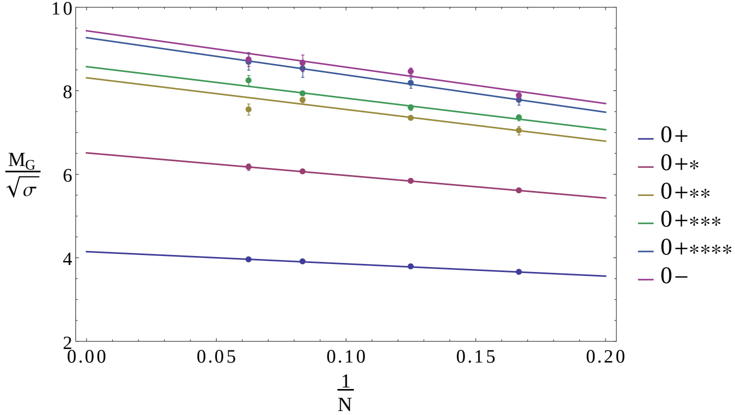
$SO(6)$: $0^{+/-}$ glueball masses



Mass Spectrum: $SO(2N \to \infty)$

Large-N extrapolation: $0^{+/-}$ glueball masses

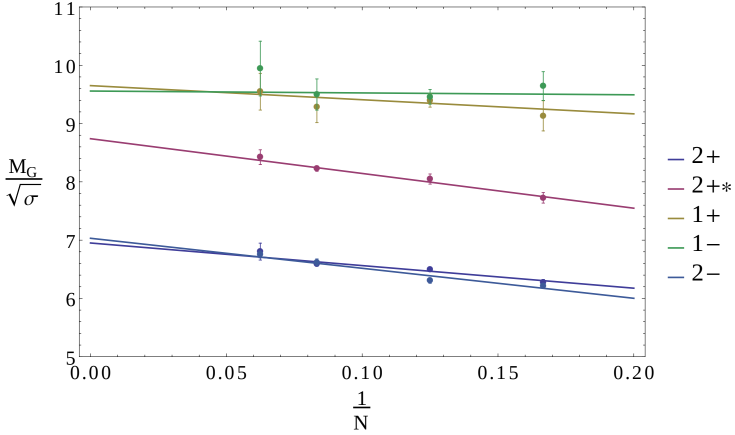
$N = 6, 8, 12, 16$



Mass Spectrum: $SO(2N \to \infty)$

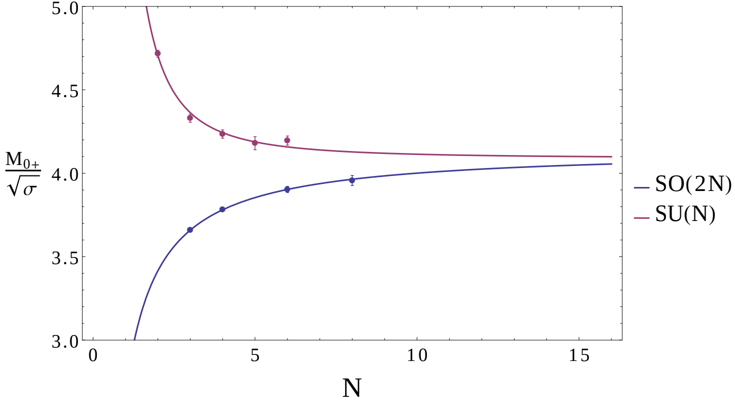
Large-N extrapolation: $1^{+/-}$, $2^{+/-}$ glueball masses

$N = 6, 8, 12, 16$



Mass Spectrum: $SO(2N \to \infty)$ and $SU(N \to \infty)$

Large-N extrapolation of 0^+ glueball mass for $SO(2N)$ and $SU(N)$[7].



[7] B. Lucini and MT, Phys. Rev. D 66, 097502 (2002)

Mass Spectrum: $SO(2N \to \infty)$ and $SU(N \to \infty)$

Large-N extrapolation of $SO(2N)$ and $SU(N)$[8] mass spectra

J^P	$SO(2N \to \infty)$	$SU(N \to \infty)$
0^+	4.14(3)	4.11(2)
0^{+*}	6.51(4)	6.21(5)
2^+	6.95(9)	6.88(6)
2^-	7.03(8)	6.89(21)
0^{+**}	8.31(20)	8.35(20)
0^-	9.44(22)	9.02(30)
2^{+*}	8.74(12)	9.22(32)
1^+	9.65(41)	9.98(25)
1^-	9.56(48)	10.06(40)

[8] B. Lucini and MT, Phys. Rev. D 66, 097502 (2002)

Deconfining Temperature: Finding the deconfining phase transition

- Finite temperature theory

$$T = \frac{1}{a(\beta)L_t}$$

- Deconfining temperature

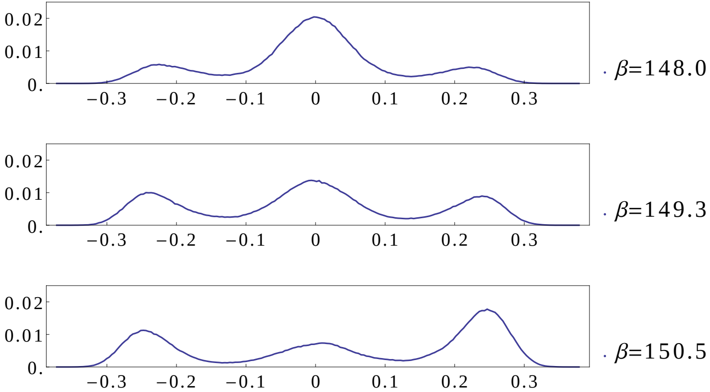
$$T_c = \frac{1}{a(\beta_c)L_t}$$

- Order parameters O

$$\overline{U_t},\ |\overline{l_p}|$$

Deconfining Temperature: Deconfining phase transition

Histograms of $\langle \overline{l_p} \rangle$ in $SO(16)$ on a $8^2 3$ lattice



Deconfining Temperature: Reweighting

- Order parameters O

$$\overline{U_t},\ |\overline{l_p}|$$

- Susceptibility χ_o

$$\chi_o \sim \langle O^2 \rangle - \langle O \rangle^2$$

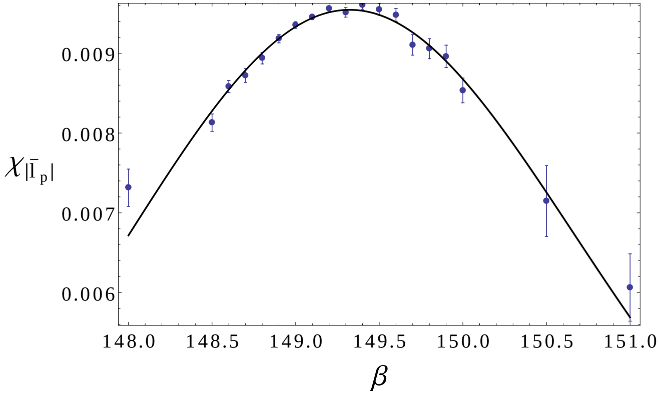
- Reweighting[9]

$$Z(\beta) \equiv \sum_i D(S_i) e^{-\beta S_i}$$

[9] A. Ferrenberg and R. Swendsen, Phys. Rev. Lett. 63, 11951198 (1989)

Deconfining Temperature: Reweighting

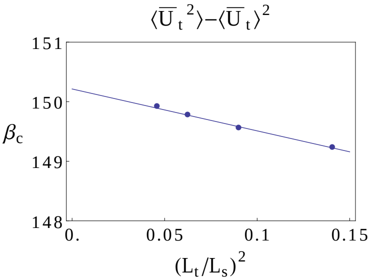
$\chi_{|\overline{l_p}|}$ on $SO(16)$: $8^2 3$ lattice

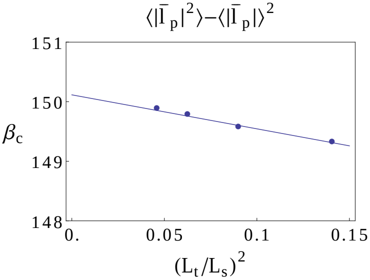


$|\overline{l_p}|$ susceptibility $\Rightarrow \beta_c = 149.32(1)$

Deconfining Temperature: Finite volume extrapolation

$SO(16)$: $\beta_c(V \to \infty)$ at $T_c = \frac{1}{3a}$, $L_s = 8, 10, 12, 14$





$\overline{u_t}$ susceptibility $\Rightarrow \beta_c(V \to \infty) = 150.21(3)$

$|\overline{l_p}|$ susceptibility $\Rightarrow \beta_c(V \to \infty) = 150.16(2)$

Deconfining Temperature: $SO(2N \to \infty)$ and $SU(N \to \infty)$

Large-N extrapolation of $SO(2N)$[10] and $SU(N)$[11] deconfining temperatures

Gauge group	$T_c/\sqrt{\sigma}$
$SO(2N \to \infty)$	0.924(20)
$SU(N \to \infty)$	0.903(23)

[10] F. Bursa, RL, and MT, JHEP 1305:025,2013
[11] J. Liddle and MT, arXiv:0803.2128

Conclusions

- There are large-N equivalences between $SO(2N)$ and $SU(N)$ gauge theories.
- Their pure gauge theories in 2+1 dimensions have matching physical properties at large-N.
  - String tensions
  - Mass spectra
  - Deconfining temperature
- $SO(2N)$ theories may provide a starting point for answering problems with $SU(N)$ QCD theories at finite chemical potential.