G_2-QCD: Spectroscopy and the phase diagram at zero temperature and finite density

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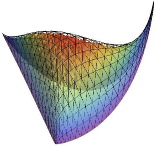

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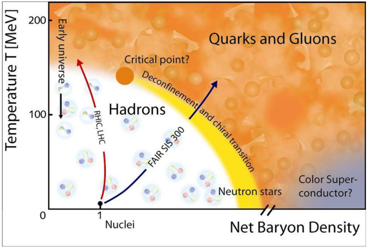
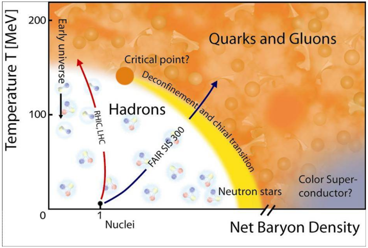


$$\mathcal{Z}_{\mathrm{GC}}(T,\mu) = \int \mathcal{D}\mathcal{U}\,\mathcal{D}\,\Psi\mathcal{D}\bar{\Psi}\; e^{-S_E[\mathcal{U},\Psi,\bar{\Psi}]-\mu N[\Psi,\bar{\Psi}]}$$

$\det D[\mathcal{U},\mu] \in \mathbb{C} \overset{\mathcal{U}\in SU(3)}{\Longrightarrow}$ Sign problem

Standard lattice Monte-Carlo techniques are not efficient

$\Longrightarrow$ QCD-like theories without sign problem: G_2-*QCD*



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$$= \int \mathcal{D}\mathcal{U}\,\det D[\mathcal{U},\mu]\, e^{-S_E[\mathcal{U}]}$$

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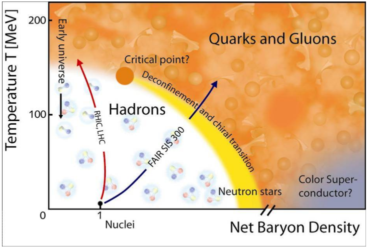

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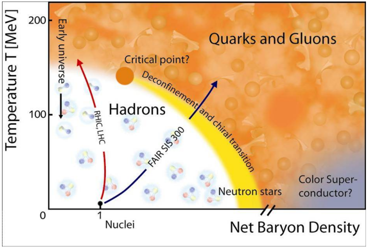

$$\mathcal{Z}_{\mathrm{GC}}(T,\mu) = \int \mathcal{D}\mathcal{U}\,\mathcal{D}\,\Psi\mathcal{D}\bar{\Psi}\; e^{-S_E[\mathcal{U},\Psi,\bar{\Psi}]-\mu N[\Psi,\bar{\Psi}]}$$
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$$\det D[\mathcal{U},\mu] \in \mathbb{C} \quad \overset{\mathcal{U}\in SU(3)}{\Longrightarrow} \quad \text{Sign problem}$$

Standard lattice Monte-Carlo techniques are not efficient

$\Longrightarrow$ QCD-like theories without sign problem: *G_2-QCD*

What is G_2-QCD ?

Replace $SU(3)$ by the exceptional Lie group G_2

$$\det D[\mathcal{U}, \mu] \geq 0$$

Other QCD-like theories without sign problem: SU(2)-QCD (two-color QCD), adjoint QCD . . .

Investigate the full phase diagram of a gauge theory with fermionic baryons and fundamental quarks with Monte-Carlo methods

$SU(3)$ is a subgroup of G_2:
G_2-QCD shares many important features with QCD

Is G_2-QCD really a QCD-like theory ?

What is the contribution of fermionic baryons to the G_2-QCD phase diagram ?

Can we learn something about the QCD sign problem ?

Exceptional confinement

- G_2 is the smallest Lie-group which is simply connected and has a trivial center
- All representations of G_2 are real
- The group has rank 2 and hence possesses two fundamental representations

$$(7) \sim \text{quarks}, \quad (14) \sim \text{gluons}$$

- Similar as in $SU(3)$ two or three quarks can build a colour singlet

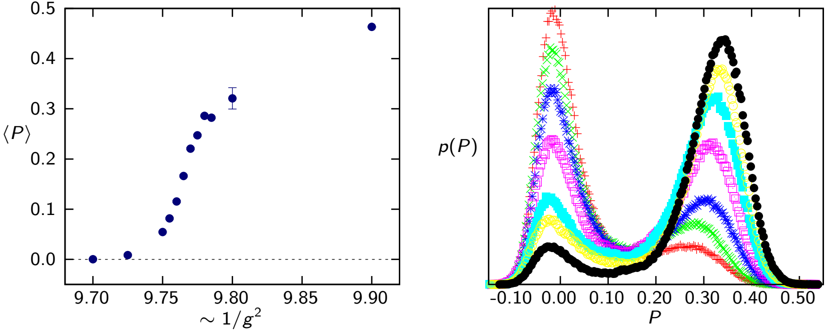
$$(7) \otimes (7) = (1) \oplus \cdots \quad , \quad (7) \otimes (7) \otimes (7) = (1) \oplus \cdots$$

- In contrast gluons can screen the colour charge of a single static quark

$$(7) \otimes (14) \otimes (14) \otimes (14) = (1) \oplus \cdots$$

In G_2 gluodynamics...

- the Polyakov loop is no longer an order parameter for confinement ...



- ...but it serves as an approximate order parameter which changes rapidly at the phase transition and is small (but non-zero) in the confining phase
- First order confinement deconfinement phase transition

K. Holland, P. Minkowski, M. Pepe and U. J. Wiese, Nucl. Phys. **B668** (2003) 207

G_2-QCD in the continuum

Lagrange density for N_f (Dirac) flavour G_2-QCD

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi}\left(\mathrm{i}\,\not{D} - m + \mathrm{i}\,\gamma_0\,\mu\right)\Psi$$

- Decompose the Dirac spinor $\Psi = \chi + \mathrm{i}\,\eta$ into Majorana spinors

$$\mathcal{L}_{\mathrm{matter}} = \bar{\Psi}\left(\mathrm{i}\,\not{D} - m + \mathrm{i}\,\gamma_0\,\mu\right)\Psi = \begin{pmatrix}\bar{\chi}\\ \bar{\eta}\end{pmatrix}\begin{pmatrix}\mathrm{i}\,\not{D} - m & \mathrm{i}\,\gamma_0\,\mu\\ -\mathrm{i}\,\gamma_0\,\mu & \mathrm{i}\,\not{D} - m\end{pmatrix}\begin{pmatrix}\chi\\ \eta\end{pmatrix}$$

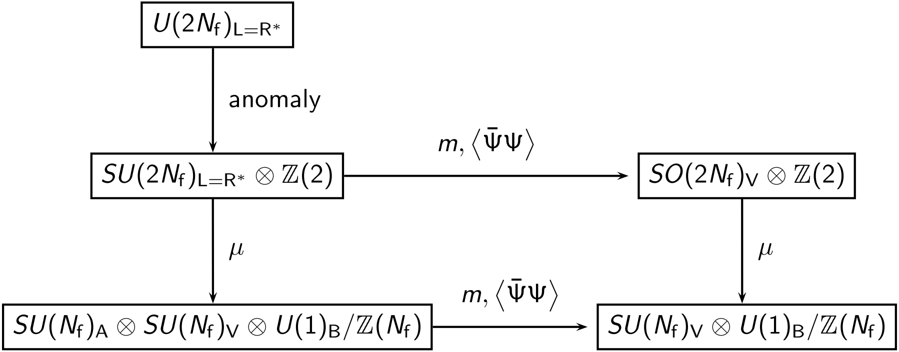
- For vanishing baryon chemical potential $\mu = 0$

$$\mathcal{L}_{\mathrm{matter}} = \bar{\lambda}\left(\mathrm{i}\,\not{D} - m\right)\lambda,$$

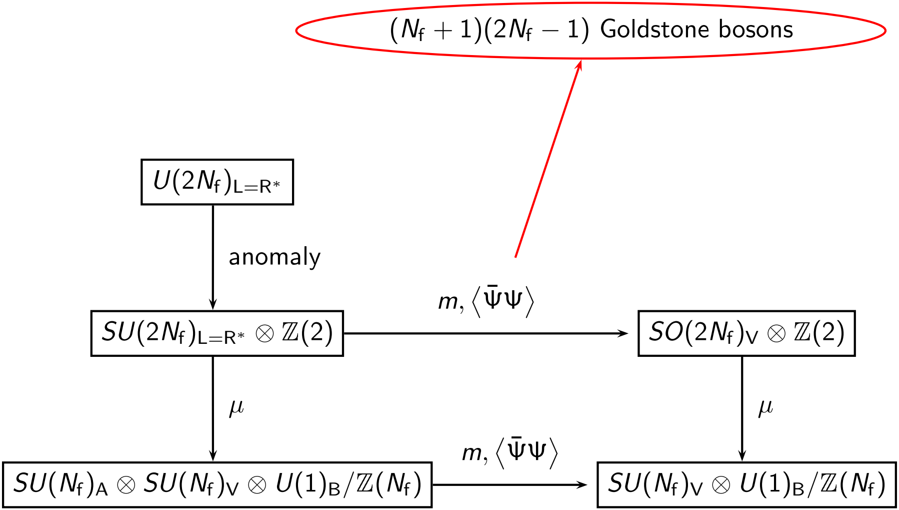
where $\lambda = (\chi, \eta)$ is a $2N_f$ component Majorana spinor.

Chiral symmetry of G_2-QCD

$$U(2N_{\mathrm{f}})_{\mathrm{L=R^*}} = SU(2N_{\mathrm{f}})_{\mathrm{L=R^*}} \otimes U(1)_{\mathrm{A}}/\mathbb{Z}(2N_{\mathrm{f}})$$



$U(2N_f)_{L=R^*}$
anomaly
$SU(2N_f)_{L=R^*} \otimes \mathbb{Z}(2)$
$m, \langle \bar{\Psi}\Psi \rangle$
$SO(2N_f)_V \otimes \mathbb{Z}(2)$
μ
μ
$SU(N_f)_A \otimes SU(N_f)_V \otimes U(1)_B/\mathbb{Z}(N_f)$
$m, \langle \bar{\Psi}\Psi \rangle$
$SU(N_f)_V \otimes U(1)_B/\mathbb{Z}(N_f)$


$(N_f+1)(2N_f-1)$ Goldstone bosons
$U(2N_f)_{L=R^*}$
anomaly
$SU(2N_f)_{L=R^*} \otimes \mathbb{Z}(2)$
$m, \langle\bar{\Psi}\Psi\rangle$
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μ
μ
$SU(N_f)_A \otimes SU(N_f)_V \otimes U(1)_B/\mathbb{Z}(N_f)$
$m, \langle\bar{\Psi}\Psi\rangle$
$SU(N_f)_V \otimes U(1)_B/\mathbb{Z}(N_f)$

$N_f = 1$

- Chiral symmetry $SU(2)_{\mathrm{L=R^*}} \otimes \mathbb{Z}(2) \longrightarrow U(1)_{\mathrm{B}} \otimes \mathbb{Z}(2)$
- 2 (would-be) Goldstone bosons

$$d(0^{+-}) = \bar{\chi}\gamma_5\eta = \bar{\Psi}^C\gamma_5\Psi - \bar{\Psi}\gamma_5\Psi^C$$
$$d(0^{++}) = \frac{1}{\sqrt{2}}(\bar{\chi}\gamma_5\chi - \bar{\eta}\gamma_5\eta) = \bar{\Psi}^C\gamma_5\Psi + \bar{\Psi}\gamma_5\Psi^C$$

- Goldstone bosons carry baryon charge $n_q = 2$, i.e. they couple to baryon chemical potential

In contrast to QCD . . .

. . . the Goldstone bosons of chiral symmetry breaking are scalar baryons (diquarks) instead of pseudoscalar mesons

Mesons $n_q = 0$

Name	Operator	Pos.	Spin	Colour	Flavour	J	P
η	$\bar{u}\gamma_5 u$	S	A	S	S	0	-
f	$\bar{u}u$	S	A	S	S	0	+
ω	$\bar{u}\gamma_\mu u$	S	S	S	A	1	-
h	$\bar{u}\gamma_5\gamma_\mu u$	S	S	S	A	1	+
π	$\bar{u}\gamma_5 d$	S	A	S	S	0	-
a	$\bar{u}d$	S	A	S	S	0	+
ρ	$\bar{u}\gamma_\mu d$	S	S	S	A	1	-
b	$\bar{u}\gamma_5\gamma_\mu d$	S	S	S	A	1	+

$$(7) \otimes (7) = (1) \oplus \ldots$$

Baryons $n_q = 1$

Name	Operator	Pos.	Spin	Col.	Flav.	J	P
Hybrid	$\epsilon_{abcdefg} u^a F^p_{\mu\nu} F^q_{\mu\nu} F^r_{\mu\nu} T^{bc}_p T^{de}_q T^{fg}_r$	S	S	A	S	1/2	$\pm$
$\tilde{\Delta}$	$T^{abc}(\bar{u}_a \gamma_\mu u_b) u_c$	S	S	A	S	3/2	$\pm$
$\tilde{N}$	$T^{abc}(\bar{u}_a \gamma_5 d_b) u_c$	S	A	A	A	1/2	$\pm$

$$(7) \otimes (7) \otimes (7) = (1) \oplus \dots$$
$$(7) \otimes (14) \otimes (14) \otimes (14) = (1) \oplus \dots$$

Baryons $n_q = 2$

Name	Operator	Pos.	Spin	Colour	Flavour	J	P	C
$d(0^{++})$	$\bar{u}^C\gamma_5 u + \bar{u}\gamma_5 u^C$	S	A	S	S	0	+	+
$d(0^{+-})$	$\bar{u}^C\gamma_5 u - \bar{u}\gamma_5 u^C$	S	A	S	S	0	+	-
$d(0^{-+})$	$\bar{u}^C u + \bar{u} u^C$	S	A	S	S	0	-	+
$d(0^{--})$	$\bar{u}^C u - \bar{u} u^C$	S	A	S	S	0	-	-
$d(1^{++})$	$\bar{u}^C\gamma_\mu d + \bar{u}\gamma_\mu d^C$	S	S	S	A	1	+	+
$d(1^{+-})$	$\bar{u}^C\gamma_\mu d - \bar{u}\gamma_\mu d^C$	S	S	S	A	1	+	-
$d(1^{-+})$	$\bar{u}^C\gamma_5\gamma_\mu d + \bar{u}\gamma_5\gamma_\mu d^C$	S	S	S	A	1	-	+
$d(1^{--})$	$\bar{u}^C\gamma_5\gamma_\mu d - \bar{u}\gamma_5\gamma_\mu d^C$	S	S	S	A	1	-	-

$$(7) \otimes (7) = (1) \oplus \dots$$

Baryons $n_q = 3$

Name	Operator	Pos.	Spin	Colour	Flavour	J	P
Δ	$T^{abc}(\bar{u}_a^C \gamma_\mu u_b) u_c$	S	S	A	S	3/2	$\pm$
N	$T^{abc}(\bar{u}_a^C \gamma_5 d_b) u_c$	S	A	A	A	1/2	$\pm$

$$(7) \otimes (7) \otimes (7) = (1) \oplus \dots$$

Correlation functions

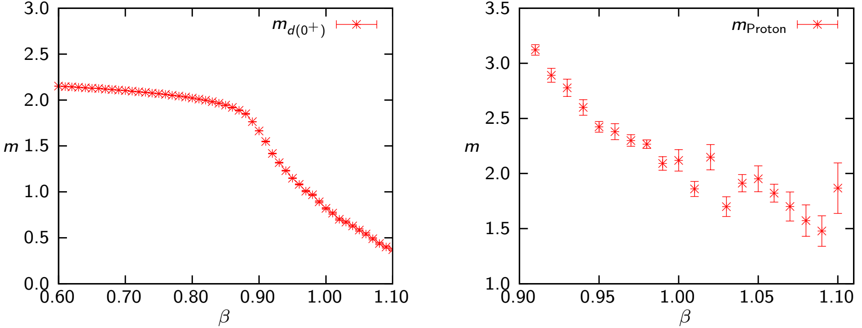
$$C_d(x,y) = \langle d(0^{++})(x)\, d(0^{++})^\dagger(y)\rangle = \langle d(0^{+-})(x)\, d(0^{+-})^\dagger(y)\rangle$$
$$= \left\langle \bar{\chi}(x)\gamma_5\chi(x)\,\bar{\chi}(y)\gamma_5\chi(y)\right\rangle ,$$
$$C_\eta(x,y) = \langle \eta(x)\, \eta^\dagger(y)\rangle$$
$$= 2\left\langle \bar{\chi}(x)\gamma_5\chi(x)\,\bar{\chi}(y)\gamma_5\chi(y)\right\rangle + C_d(x,y)$$

Exact relations between diquark and meson masses

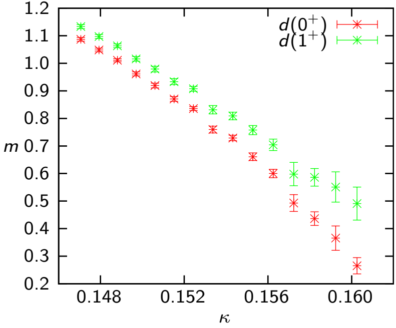
$$m_{d(0^+)} = m_{\pi(0^-)}$$
$$m_{d(0^-)} = m_{a(0^+)}$$
$$m_{d(1^+)} = m_{\rho(1^-)}$$
$$m_{d(1^-)} = m_{b(1^+)}$$

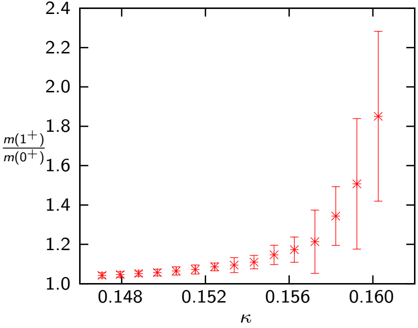
Spectroscopy

Scalar diquark and nucleon mass



Scalar and vector diquark mass



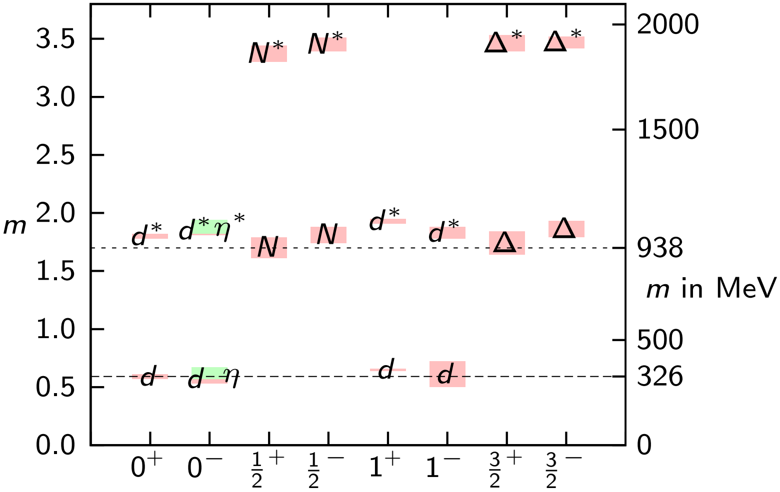


Two different ensembles ...

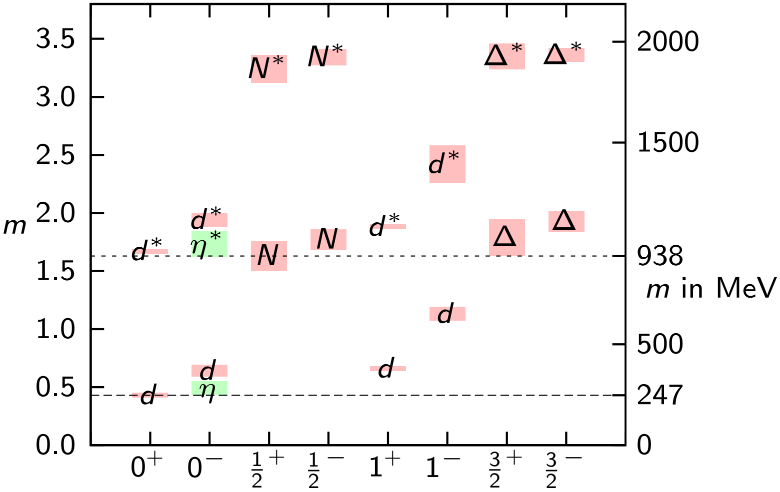
- Heavy ensemble
  - Symanzik improved gauge action, Wilson fermions
  - Volume $V = 8^3 \times 16$, $\beta = 1.05$, $\kappa = 0.147$
  - 35000 MC-Configs (7000 Measurements)
  - Goldstone (diquark) mass $m_{d(0^{+\pm})} = 326$ MeV

- Light ensemble
  - Symanzik improved gauge action, Wilson fermions
  - Volume $V = 8^3 \times 16$, $\beta = 0.96$, $\kappa = 0.15924$
  - 25000 MC-Configs (5000 Measurements)
  - Goldstone (diquark) mass $m_{d(0^{+\pm})} = 247$ MeV

Heavy ensemble

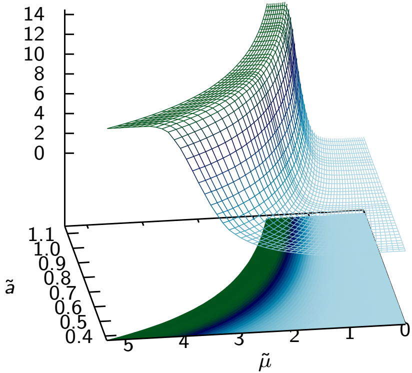


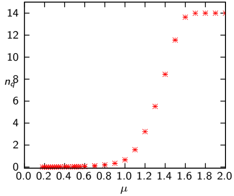
Light ensemble



Finite density

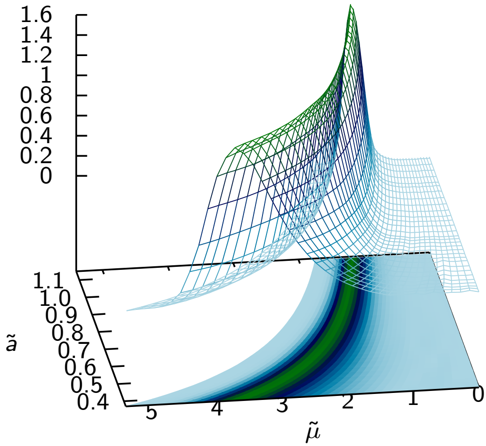
Quark number density $n_q = \frac{1}{V} \frac{\partial \ln Z}{\partial \mu}$





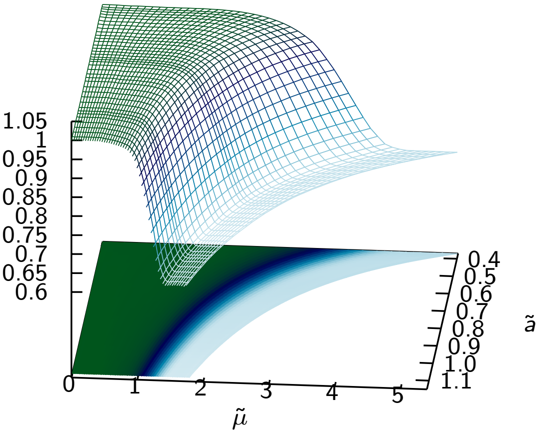
$n_q^{\text{sat}} = 2\, N_{\text{f}}\, N_{\text{c}} = 14$

Polyakov loop P

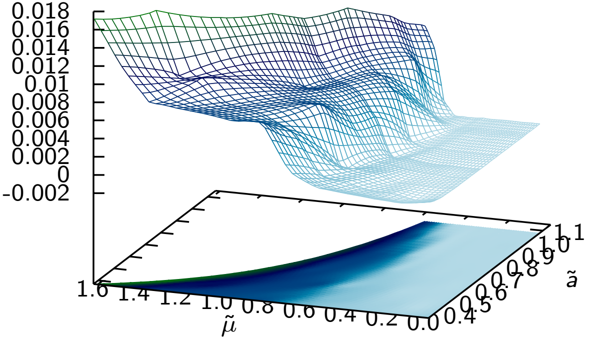


Volume $V = 8^3 \times 16$ $\kappa = 0.147$ $\tilde{a} = m_{d(0^+)}$, $\tilde{\mu} = \mu/m_{d(0^+)}$, $\tilde{T} = T/m_{d(0^+)}$

Chiral condensate $\Sigma = \frac{1}{V}\frac{\partial \ln Z}{\partial m}$

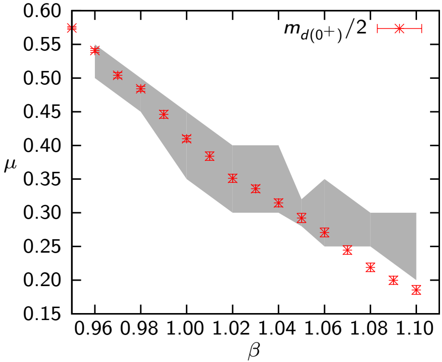


Quark number density $n_q = \frac{1}{V}\frac{\partial \ln Z}{\partial \mu}$



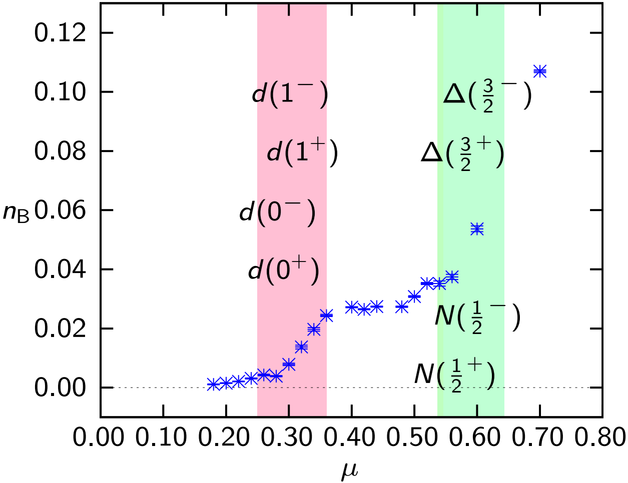
Volume $V = 8^3 \times 16$ $\kappa = 0.147$ $\tilde{a} = m_{d(0^+)}$, $\tilde{\mu} = \mu/m_{d(0^+)}$, $\tilde{T} = T/m_{d(0^+)}$

Onset transition to baryonic matter compared to diquark mass

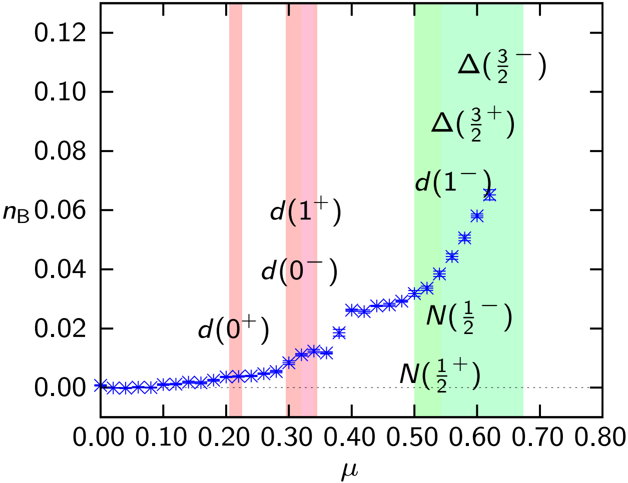


- Onset transition to (bosonic) baryonic matter at $\mu_0 \approx m_{d(0^+)}/2$
- Silver blaze property known from QCD
- Diquarks condensate for $\mu > \mu_0$

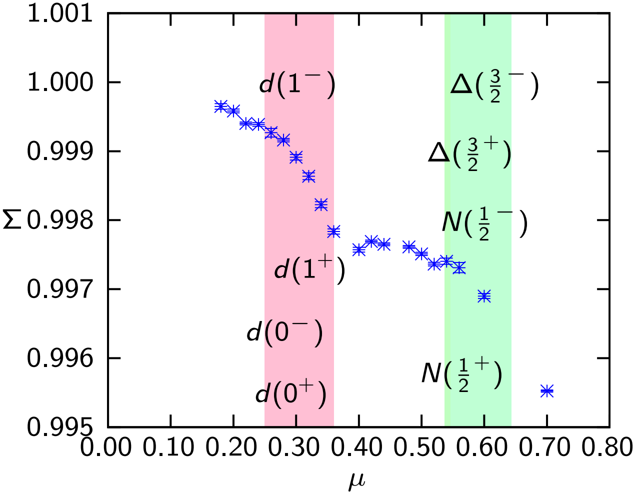
Heavy ensemble: Quark number density



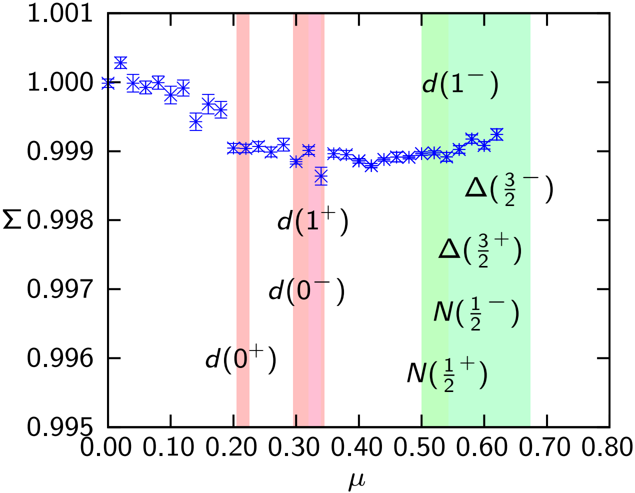
Light ensemble: Quark number density



Heavy ensemble: Chiral condensate



Light ensemble: Chiral condensate



Conclusions

- G_2 gauge theories share many important features with $SU(3)$ gauge theories
- There is no sign problem in G_2-QCD: It is possible to investigate the phase diagram of a theory with fundamental quarks and fermionic baryons even at low temperatures and high densities with lattice simulations
- Mass spectrum on small lattices
- G_2-QCD possesses the silver blaze property
- Various transitions at zero temperature: diquark condensation, onset of nuclear matter and deconfinement/chiral restoration

A. Maas, L. von Smekal, B. H. Wellegehausen and A. Wipf, *The phase diagram of a gauge theory with fermionic baryons*, arXiv:1203.5653 [hep-lat], 2012.