The anti-symmetric LS potential in flavor SU(3) limit from lattice QCD

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for HAL QCD Collaboration
In NN sector, LS potential is quite strong.

\[
V_{NN} = V_0(r) + V_\sigma(r) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T(r) \left( 3(\hat{r} \cdot \vec{\sigma}_1)(\hat{r} \cdot \vec{\sigma}_2) - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right) + V_{SLS}(r) \vec{L} \cdot (\vec{s}_1 + \vec{s}_2) + O(\nabla^2)
\]

It has important influence on phenomenology.

Next speaker will tell you about this much more!
Background

◆ In Lambda N sector,

- LS potential splits into two

\[
V_{\Lambda N} = V_0(r) + V_\sigma(r)\vec{\sigma}_\Lambda \cdot \vec{\sigma}_N + V_T(r)(3(\hat{r} \cdot \vec{\sigma}_\Lambda)(\hat{r} \cdot \vec{\sigma}_N) - \vec{\sigma}_\Lambda \cdot \vec{\sigma}_N)
\]

\[+ V_{LS}^{(A)}(r)\vec{L} \cdot \vec{s}_\Lambda + V_{LS}^{(N)}(r)\vec{L} \cdot \vec{s}_N + O(\nabla^2)\]

1. Lambda-spin-dependent LS potential (red)
2. Nucleon-spin-dependent LS potential (blue)


- Rearranging the two terms, it can be restated that anti-symmetric LS(ALS) potential is so strong that symmetric LS(SLS) potential is cancelled.

\[\equiv V_{SLS}(r)\vec{L} \cdot (\vec{s}_\Lambda + \vec{s}_N) + V_{ALS}(r)\vec{L} \cdot (\vec{s}_\Lambda - \vec{s}_N)\]

Quark model $\rightarrow$ strong cancellation [S.Takeuchi et. al., PTPS137(2000)830]
Meson exch. model $\rightarrow$ weak cancellation [T.A.Rijken et al., PRC59(1991)21]
**Anti-symmetric LS potential**

### Symmetric LS (SLS)

\[ \vec{L} \cdot \vec{S}_+ = \vec{L} \cdot (\vec{s}_1 + \vec{s}_2) \]

<table>
<thead>
<tr>
<th>SLS</th>
<th>S=0</th>
<th>S=1</th>
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<tbody>
<tr>
<td>S=0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S=1</td>
<td>0</td>
<td>*</td>
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</tbody>
</table>

*only diagonal entry*

Total spin S does not change.

### Anti-symmetric LS (ALS)

\[ \vec{L} \cdot \vec{S}_- = \vec{L} \cdot (\vec{s}_1 - \vec{s}_2) \]

<table>
<thead>
<tr>
<th>ALS</th>
<th>S=0</th>
<th>S=1</th>
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</thead>
<tbody>
<tr>
<td>S=0</td>
<td>0</td>
<td>*</td>
</tr>
<tr>
<td>S=1</td>
<td>*</td>
<td>0</td>
</tr>
</tbody>
</table>

*only off-diagonal entry*

Total spin S has to change.

**◆ In NN sector, ALS is missing**

\[(\text{spin}) \otimes (\text{parity}) \otimes (\text{isospin}) = -1\]

Change of (spin) cannot be compensated by a change of (parity)x(isospin) because (parity) and (isospin) are conserved quantities.

**◆ In two-hyperon sector, ALS exists.**

\[(\text{spin}) \otimes (\text{parity}) \otimes (\text{flavor}) = -1\]

- Change of (spin) can be compensated by a change of flavor \((8s \leftrightarrow 8a)\).

- After flavor SU(3) is broken, other “representations” can give contributions.

**flavor SU(3) rep.’s**

\[27 \oplus 1 \oplus 10^* \oplus 10 \oplus 8 (= 8s \oplus 8a)\]
Construction of Hyperon potentials: Exp. v.s. Theory

◆ Experimental construction
  □ Due to the short life time of hyperons, direct scattering experiment is difficult.
  □ Method employed in the construction of NN interaction cannot be used.
  □ Construction of hyperon potentials requires a tremendous efforts.

◆ Theoretical construction
  □ HAL QCD collaboration recently developed a lattice QCD method to construct hadron potentials from Nambu-Bethe-Salpeter(NBS) wave functions.
  □ It is faithful to scattering phases.
  □ It has been applied to many systems. NN, NY, YY (including coupled channel), and NNN potentials in the parity-even sector and MM, MB, etc.
  □ It has been recently extended to parity-odd sectors and LS potential. (K.Murano et al., arXiv:1305.2293)

We apply this extension to the hyperon sector (parity-odd) to consider the expected cancellation between the symmetric and the anti-symmetric LS potential between N and Lambda.
HAL QCD method

◆ Nambu-Bethe-Salpeter (NBS) wave function

$$\psi_k(x,y) \equiv Z_B^{-1} \langle 0 \left| T \left[ B(x)B(y) \right] \right| B(+\bar{k})B(-\bar{k}), \text{in} \rangle$$

□ It is related to the S-matrix through the reduction formula

$$\langle B(p_1)B(p_2), \text{out} \left| B(+\bar{k})B(-\bar{k}), \text{in} \rangle$$

$$= \text{disc.} + \left( iZ_B^{-1/2} \right)^2 \int d^4x_1d^4x_2e^{ip_1x_1}(\square_1 + m^2)e^{ip_2x_2}(\square_2 + m^2) \langle 0 \left| T \left[ B(x_1)B(x_2) \right] \right| B(+\bar{k})B(-\bar{k}), \text{in} \rangle$$

□ Equal-time restriction of NBS wave function shows the same asymptotic behavior as the non-relativistic scattering wave function at long distance

$$\psi_k(\bar{x} - \bar{y}) \equiv \lim_{x_0 \to +0} \psi_k(\bar{x}, x_0 ; \bar{y}, y_0 = 0)$$

C.-J.D.Lin et al., NPB619, 467(2001).

$$\approx e^{i\delta(k)} \frac{\sin(kr + \delta(k))}{kr} + \cdots \quad \text{as} \quad r \equiv |\bar{x} - \bar{y}| \to \text{large}$$

◆ Energy-independent potential is defined by Schrodinger equation:

$$\left( k^2 / m - H_0 \right) \psi_k(\bar{r}) = \int d^3r' U(\bar{r}, \bar{r}') \psi_k(\bar{r}')$$

Resulting potential $U(r,r')$ reproduces the scattering phase, because of the asymptotic behavior of the equal-time NBS wave function.
“Time-dependent” method (an efficient way to obtain HAL QCD potentials)

◆ Normalized BB correlator (R-correlator)

\[ R(t, \bar{x} - \bar{y}) \equiv e^{2mt} \langle 0 \left| T \left[ B(\bar{x}, t)B(\bar{y}, t) \cdot \mathcal{J}_{BB}(t = 0) \right] \right| 0 \rangle = \sum_{k} a_{k} \exp \left( -t \Delta W(\bar{k}) \right) \psi_{\bar{k}}(\bar{x} - \bar{y}) \]

\[ \Delta W(\bar{k}) \equiv 2\sqrt{m_{N}^{2} + \bar{k}^{2} - 2m_{N}} \]

\[ \frac{\bar{k}^{2}}{m} = \Delta W(\bar{k}) + \frac{\Delta W(\bar{k})^{2}}{4m} \]

It has to be sufficiently large to suppress inelastic contribution (E > 2m_{N} + m_{\text{pion}}).

◆ “Time-dependent” Schrodinger-like equation (derivation)

\[ \left( \frac{1}{4m} \frac{\partial^{2}}{\partial t^{2}} - \frac{\partial}{\partial t} \right) R(t, \bar{x}) = \sum_{k} a_{k} \frac{\bar{k}^{2}}{m} \exp \left( -t \Delta W(\bar{k}) \right) \psi_{\bar{k}}(\bar{x}) \]

HAL QCD potential U satisfies

\[ (H_{0} + U)\psi_{\bar{k}}(\bar{x}) = \frac{\bar{k}^{2}}{m} \psi_{\bar{k}}(\bar{x}) \]

“Time-dependent” Schrodinger-like equation

\[ \left( \frac{1}{4m} \frac{\partial^{2}}{\partial t^{2}} - \frac{\partial}{\partial t} - H_{0} \right) R(t, \bar{x}) = \int d^{3}x' U(\bar{x}, \bar{x}') R(t, \bar{x}') \]

It enables us to obtain the potential without requiring the ground state saturation.

[N.Ishii et al., PLB712(2012)437.]
Ground state saturation is not needed. (an example)

- Source functions (with a single real parameter $\alpha$)

$$f(x, y, z) = 1 + \alpha \left( \cos\left(\frac{2\pi x}{L}\right) + \cos\left(\frac{2\pi y}{L}\right) + \cos\left(\frac{2\pi z}{L}\right) \right)$$

- $\alpha$ is used to arrange the mixture of NBS wave functions

$$C_{NN}(\bar{x} - \bar{y}, t) \equiv \langle 0 \mid T[N(\bar{x}, t)N(\bar{y}, t) \cdot NN(t = 0; \alpha)] \mid 0 \rangle = \sum_n \psi_n(\bar{x} - \bar{y}) \cdot a_n(\alpha) \cdot \exp(-E_n t)$$

Central potential at the leading order of derivative expansion

$$V_C(\bar{x}) = -\frac{H_0 R(t, \bar{x}) - (\partial / \partial t)R(t, \bar{x})}{R(t, \bar{x})} + \frac{1}{4m} \left(\frac{\partial / \partial t}{R(t, \bar{x})}\right)^2 R(t, \bar{x})$$

“Time-dependent” Schrodinger-like eq. leads to an alpha-independent result.
**Two-hyperon source**

- To save the computational cost, we restrict ourselves to $S=-1$ sector. (At this moment, the code is not efficient)

![Diagram of 27, 10, 8_s, 8_a representations]

- We can access flavor rep’s of $27 \oplus 10 \oplus 10 \oplus 8_s \oplus 8_a$

- The following 4 operators:

\[
\begin{align*}
\end{align*}
\]

are used to construct 4x4 matrix correlator on the supercomputer.

The results are combined for flavor representations on the workstation afterwards.

BGQ@KEK is used.
Momentum wall source

Two-baryon source with a non-trivial orbital cubic group rep.

\[ \mathcal{J}_{\alpha\beta}(\vec{p}) \equiv \sum_{\vec{x}_1,\ldots,\vec{x}_6} B_{\alpha}(\vec{x}_1,\vec{x}_2,\vec{x}_3)B'_{\beta}(\vec{x}_4,\vec{x}_5,\vec{x}_6) \exp(i\vec{p} \cdot (\vec{x}_3 - \vec{x}_6)) \]

\[ B_{\alpha}(x_1,x_2,x_3) \equiv \epsilon_{abc} \left( q_a^{(1)}(x_1) C \gamma_5 q_b^{(2)}(x_2) \right) q_c^{(3)}(x_3) \]

\[ B'_{\beta}(x_4,x_5,x_6) \equiv \epsilon_{abc} \left( q_a^{(4)}(x_4) C \gamma_5 q_b^{(5)}(x_5) \right) q_c^{(6)}(x_6) \]

Non-vanishing momentum \( \vec{p} \) is carried by “spectator quark”

We consider momenta which are parallel (anti-parallel) to the coordinate axes.
Momentum wall source

◆ Cubic group analysis ➔ “orbital contribution” of source

\[ A_1^+ (\sim \text{s-wave}) \oplus E^+ (\sim \text{d-wave}) \oplus T_1^- (\sim \text{p-wave}) \]

➔ It generates NBS wave functions for (parity-odd sector)

\[ J^P = 0^- (A_1^-), \ 1^- (T_1^-), \ 2^- (E^- \oplus T_2^-) \]

◆ Two-hyperon potentials up to NLO

\[ V_{BB} = V_{C;S=0}(r)\mathbb{P}^{(S=0)} + V_{C;S=1}(r)\mathbb{P}^{(S=1)} + V_T(r) \left( 3(\hat{r} \cdot \vec{\sigma}_1)(\hat{r} \cdot \vec{\sigma}_2) - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right) \]

\[ + V_{SLS}(r)\vec{L} \cdot \vec{S}_+ + V_{ALS}(r)\vec{L} \cdot \vec{S}_- + O(\nabla^2) \]

are obtained by solving “t-dep” Schrodinger-like eq

\[
\left( \frac{1}{4m} \frac{d^2}{dt^2} - \frac{\partial}{\partial t} - H_0 \right) R(t, \vec{x}; \vec{J}) = V_{BB} \cdot R(t, \vec{x}; \vec{J})
\]

by employing sources for $3P0$, $3P1$, $3P2$, $1P1$ and flavor representations $27$, $10$, $10^*$, $8$. 

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<tr>
<td>$0^-$</td>
<td>$^3P_0$</td>
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</tr>
<tr>
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<td>$^3P_1$</td>
<td>$^1P_1$</td>
</tr>
<tr>
<td>$2^-$</td>
<td>$^3P_2 - ^3F_2$</td>
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Lattice QCD setup

- 2+1 flavor gauge configuration on 16^3x32 lattice generated by CP-PACS+JLQCD
- RG improved Iwasaki gauge action at beta=1.83
- O(a) improved Wilson quark (clover) action
  with $C_{SW}=1.761$ at kappa_{uds}=0.1371 (flavor SU(3) limit)
  - $a=0.121(2)$ fm; $1/a = 1630.58$ MeV; $L=32a = 1.93(3)$ fm
  - $m($baryon$) = 2051(3)$ MeV
    $m($PS$) = 1013(1)$ MeV
- 700 gauge configurations with 8 source points are used.

- Relativistic dispersion is violated.

Fit with $E^2(\vec{k}^2) = m^2 + \vec{k}^2$

```
        fit region

\begin{equation}
\left( \frac{1}{\alpha} \left\{ \frac{1}{4m} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} \right\} - H_0 \right) R(t,\vec{x}) = \int d^3x' U(\vec{x},\vec{x}')R(t,\vec{x}')
\end{equation}
```

Fit with $E^2(\vec{k}^2) \approx m^2 + \alpha \vec{k}^2$

```
\textcolor{red}{\alpha = 0.9134(19)}
```

"time-dependent" Schrodinger-like eq. has to be modified as
Numerical Results(1) 27 & $10^*$ sector ($\longleftrightarrow$ NN sector)

Qualitative behaviors are reproduced.

Next speaker will tell you about this channel much more!
Numerical Results(2): 10 sector

◆ The central potential in flavor 10 sector (parity-odd) does not have a repulsive core.
(This is consistent with quark model.)
No repulsive cores in spin-singlet and triplet central potentials in flavor 8 sector (parity-odd).

[This is consistent with quark model.]

Large anti-symmetric LS potential is obtained (with good Hermiticity).
Phase shift and mixing parameter (flavor 8 sector)

- Fit with various multi-gaussian functions

- Stapp’s bar conversion is adopted.
- Attractive phase shifts.
- Rather large mixing parameter.
  (Anti-symmetric LS mixes spin-singlet and spin-triplet sectors)

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Phase shift and mixing parameter (flavor 8 sector)

- Smooth parameterizations with various multi-gaussian functions

- Another functional form of symmetric LS potential is tried. (Almost nothing changes in the phase shift)
Lambda N potential

Lambda N potentials are obtained as linear combinations of 8, $10^*$ and 27.

\[
V_{\Lambda N} = \left( \frac{1}{2} V_{C;S=0}^{(10)} + \frac{1}{2} V_{C;S=0}^{(8)} \right) \mathbb{P}^{(S=0)} + \left( \frac{1}{10} V_{C;S=1}^{(8)} + \frac{9}{10} V_{C}^{(27)} \right) \mathbb{P}^{(S=1)} + \left( \frac{1}{10} V_{T}^{(8)} + \frac{9}{10} V_{T}^{(27)} \right) S_{12} (\hat{r}) + \left( \frac{1}{10} V_{SLS}^{(8)} + \frac{9}{10} V_{SLS}^{(27)} \right) \mathbb{L} \cdot \mathbb{S} + \frac{1}{2\sqrt{5}} V_{ALS}^{(8)} \cdot \mathbb{L} \cdot \mathbb{S}
\]

**Weak cancellation**

- Symmetric LS is strong.
  
  It comes from 27 rep. (90%), i.e., NN LS

  \[
  V_{SLS}^{(AN)} = \frac{1}{10} V_{SLS}^{(8)} + \frac{9}{10} V_{SLS}^{(27)}
  \]

- Anti-symmetric LS is weak.
  
  It is weakened by a numerical factor

  \[
  V_{ALS}^{(AN)} = \frac{1}{2\sqrt{5}} V_{ALS}^{(8)}
  \]
Summary

◆ We have calculated parity-odd two-hyperon potentials in the flavor SU(3) limit for 8 ⊕ 10 ⊕ 10* ⊕ 27

◆ Central potentials for 10^* and 27 have repulsive cores at short distance, whereas central potentials for 8 (spin singlet and triplet) and 10 do not have repulsive core. [This is consistent with quark model]

◆ Rather strong anti-symmetric LS potential is obtained in flavor 8 channel.

◆ 8, 10^* and 27 potentials are combined to give Lambda N potentials (parity-odd)
  ◯ It has a strong symmetric LS potential. (which comes from 27 rep (90%))
  ◯ Anti-symmetric LS potential becomes weakened by a CG factor 1/(2*sqrt(5))
    ➔ weak cancellation !!!
  ◯ The following two possibilities have to be examined
    ◦ light quark mass effect (m_u == m_d == m_s)
    ◦ SU(3) breaking effect (m_u == m_d << m_s) ➔ physical quark mass
backup slides
“Time-dependent” method for violated relativistic dispersion

**Normalized BB correlator (R-correlator)**

\[
R(t, \bar{x} - \bar{y}) \equiv e^{2mt} \langle 0 \left| T \left[ B(\bar{x}, t)B(\bar{y}, t) \cdot \overline{J}_{BB}(t = 0) \right] \right| 0 \rangle = \sum_{\bar{k}} a_{\bar{k}} \exp(-t\Delta W(\bar{k})) \psi_{\bar{k}}(\bar{x} - \bar{y})
\]

t has to be sufficiently large to suppress inelastic contribution (E > 2m + m_{\text{pion}}).

**“Time-dependent” Schrodinger-like equation (derivation)**

\[
\left( \frac{1}{4m} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} \right) R(t, \bar{x}) = \sum_{\bar{k}} a_{\bar{k}} \frac{\alpha \bar{k}^2}{m} \exp(-t\Delta W(\bar{k})) \psi_{\bar{k}}(\bar{x})
\]

\[\alpha \frac{\bar{k}^2}{m} \approx \Delta W(\bar{k}) + \frac{\Delta W(\bar{k})^2}{4m}\] is used.

HAL QCD potential U satisfies

\[
(H_0 + U) \psi_{\bar{k}}(\bar{x}) = \frac{\bar{k}^2}{m} \psi_{\bar{k}}(\bar{x})
\]

**“Time-dependent” Schrodinger-like equation**

\[
\left( \frac{1}{\alpha} \left( \frac{1}{4m} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} \right) - H_0 \right) R(t, \bar{x}) = \int d^3 x' U(\bar{x}, \bar{x}') R(t, \bar{x}')
\]

It enables us to obtain the potential without requiring the ground state saturation.

\[
E(\bar{k})^2 = m^2 + \alpha \bar{k}^2 + O(k^4)
\]

\[
\Delta W(\bar{k}) \equiv 2E(\bar{k}) - 2m
\]

\[
2m_N + m_{\pi}
\]

Inelastic region

Elastic region

\[
2m_N
\]

It enables us to obtain the potential without requiring the ground state saturation.
Existence of energy-independent interaction kernel

◆ We assume linear independence of NBS wave functions below the pion threshold

There exists a dual basis

\[ \int d^3r \tilde{\psi}_{\vec{k}'}(\vec{r}) \psi_{\vec{k}}(\vec{r}) = (2\pi)^3 \delta^3(\vec{k}' - \vec{k}) \]

◆ We have

\[ K_{\vec{k}}(\vec{r}) \equiv \left( \frac{k^2}{m_N} - H_0 \right) \psi_{\vec{k}}(\vec{r}) = \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r}) \int d^3r' \tilde{\psi}_{\vec{k}'}(\vec{r}) \psi_{\vec{k}}(\vec{r}) \]

\[ = \int d^3r' \left\{ \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r}) \tilde{\psi}_{\vec{k}'}(\vec{r}') \right\} \psi_{\vec{k}}(\vec{r}') \]

If we define an energy-independent interaction kernel by

\[ U(\vec{r},\vec{r}') \equiv \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r}) \tilde{\psi}_{\vec{k}'}(\vec{r}) \]

Owing to the integration of \( k' \), \( U(r,r') \) is energy-independent

then it generates NBS wave functions below the pion threshold

\[ \left( \frac{k^2}{m_N} - H_0 \right) \psi_{\vec{k}}(\vec{r}) = \int d^3r' U(\vec{r},\vec{r}') \psi_{\vec{k}}(\vec{r}') \]

for \( E \equiv 2\sqrt{m_N^2 + \vec{k}^2} < 2m_N + m_{\pi} \)