The anti-symmetric LS potential in flavor SU(3) limit from lattice QCD

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for HAL QCD Collaboration

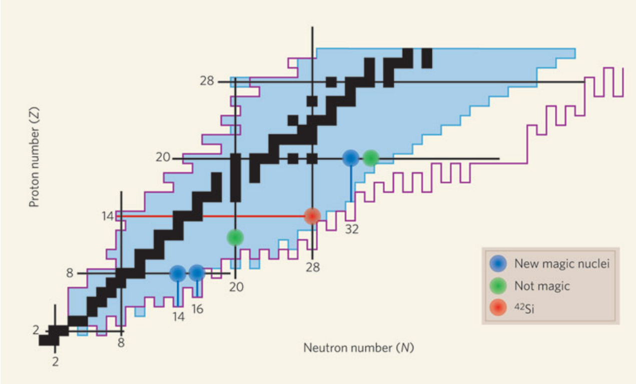
Background

- In NN sector, LS potential is quite strong.

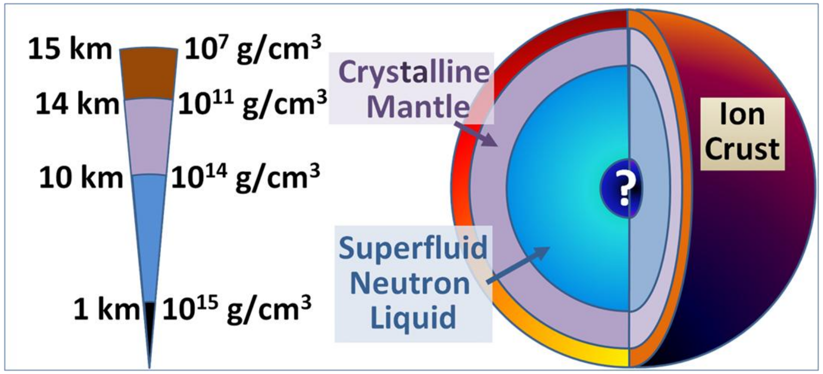
$$V_{NN} = V_0(r) + V_\sigma(r)\vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T(r)\left(3(\hat{r}\cdot\vec{\sigma}_1)(\hat{r}\cdot\vec{\sigma}_2) - \vec{\sigma}_1\cdot\vec{\sigma}_2\right)$$
$$+ V_{SLS}(r)\vec{L}\cdot(\vec{s}_1 + \vec{s}_2) + O(\nabla^2)$$

It has important influence on phenomenology

magic # of atomic nuclei



$^3P_2$ neutron superfluid in neutron star



Next speaker will tell you about this much more !

Background

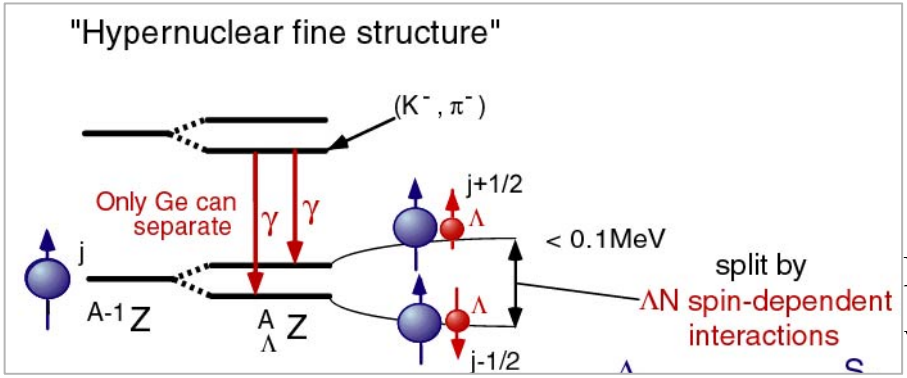
- In Lambda N sector,
  - LS potential splits into two

$$V_{\Lambda N} = V_0(r) + V_\sigma(r)\vec{\sigma}_\Lambda \cdot \vec{\sigma}_N + V_{\mathrm{T}}(r)\left(3(\hat{r}\cdot\vec{\sigma}_\Lambda)(\hat{r}\cdot\vec{\sigma}_N) - \vec{\sigma}_\Lambda \cdot \vec{\sigma}_N\right)$$

$$+V_{\mathrm{LS}}^{(\Lambda)}(r)\vec{L}\cdot\vec{s}_\Lambda + V_{\mathrm{LS}}^{(N)}(r)\vec{L}\cdot\vec{s}_N + O(\nabla^2)$$

(1) Lambda-spin-dependent LS potential (red)
(2) Nucleon-spin-dependent LS potential (blue)

  - **High precision spectroscopy** of p-shell hyper nuclei suggests Strength of **Lambda-spin-dependent LS potential** is weak. [H.Akikawa et al.,PRL88(2002)082501]



Rearrangement

$$V_{\mathrm{LS}}^{(\Lambda)}(r)\vec{L}\cdot\vec{s}_\Lambda + V_{\mathrm{LS}}^{(N)}(r)\vec{L}\cdot\vec{s}_N$$

$$\equiv V_{\mathrm{SLS}}(r)\vec{L}\cdot(\vec{s}_\Lambda + \vec{s}_N) + V_{\mathrm{ALS}}(r)\vec{L}\cdot(\vec{s}_\Lambda - \vec{s}_N)$$

  - Rearranging the two terms, it can be restated that **anti-symmetric LS(ALS) potential is so strong that symmetric LS(SLS) potential is cancelled.**

Quark model → strong cancellation [S.Takeuchi et. al., PTPS137(2000)830]
Meson exch. model → weak cancellation [T.A.Rijken et al., PRC59(1991)21]

Anti-symmetric LS potential

Symmetric LS (SLS)

$$\vec{L}\cdot\vec{S}_{+} = \vec{L}\cdot(\vec{s}_1 + \vec{s}_2)$$

only **diagonal** entry
Total spin S does not change.

SLS	S=0	S=1
S=0	0	0
S=1	0	*

Anti-symmetric LS (ALS)

$$\vec{L}\cdot\vec{S}_{-} = \vec{L}\cdot(\vec{s}_1 - \vec{s}_2)$$

only **off-diagonal** entry
Total spin S has to change.

ALS	S=0	S=1
S=0	0	*
S=1	*	0

- In NN sector, ALS is missing

$$(\mathrm{spin})\otimes(\mathrm{parity})\otimes(\mathrm{isospin}) = -1$$

Change of (spin) cannot be compensated by a change of (pairty)x(isospin)
because (parity) and (isospin) are conserved quantities.

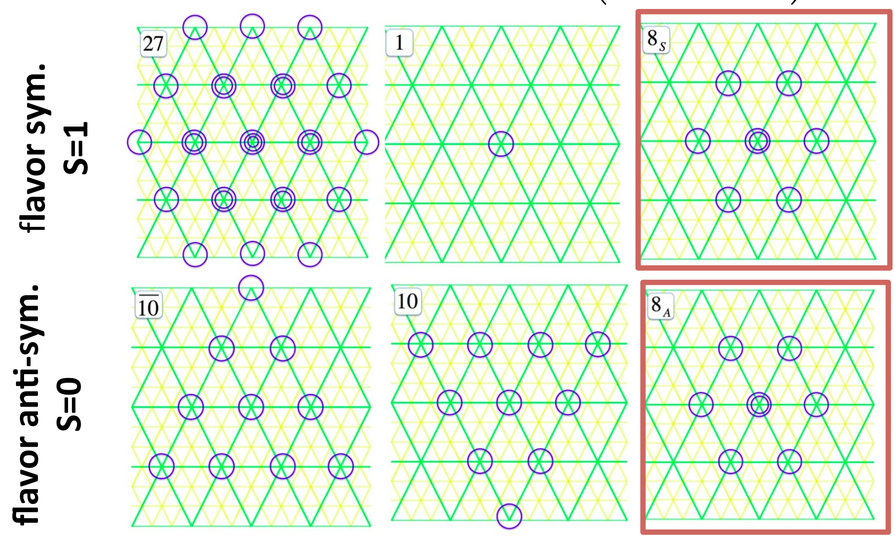
- In two-hyperon sector, ALS exists.

$$(\mathrm{spin})\otimes(\mathrm{parity})\otimes(\mathrm{flavor}) = -1$$

  - Change of (spin) can be compensated by a change of flavor (8s ←→ 8a).
  - After flavor SU(3) is broken, other “representations” can give contributions.

flavor SU(3) rep.’s

$$27\oplus 1\oplus 10^{*}\oplus 10\oplus 8(=8s\oplus 8a)$$

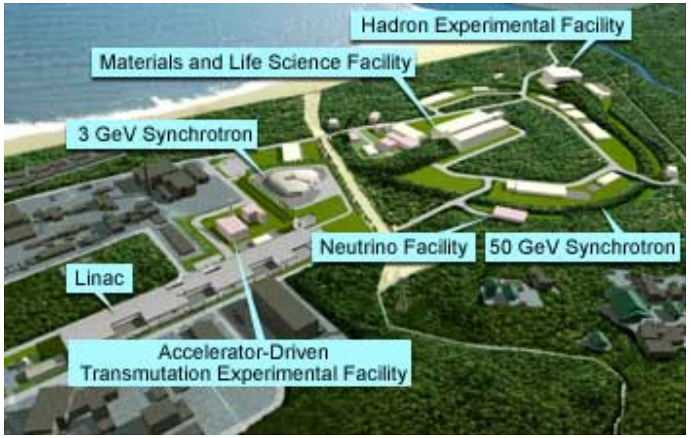


Construction of Hyperon potentials: Exp. v.s. Theory

- **Experimental construction**
  - Due to the short life time of hyperons, direct scattering experiment is difficult.
  - Method employed in the construction of NN interaction cannot be used.
  - Construction of hyperon potentials requires a tremendous efforts.

J-PARC

Exploration of multi-strangeness world



- **Theoretical construction**
  - HAL QCD collaboration recently developed a lattice QCD method to construct hadron potentials from Nambu-Bethe-Salpeter(NBS) wave functions.
  - It is faithful to scattering phases.
  - It has been applied to many systems.
    NN, NY, YY (including coupled channnel),and NNN potentials in the parity-even sector and MM, MB, etc.
  - It has been recently extended to parity-odd sectors and LS potential.
    (K.Murano et al., arXiv:1305.2293)

We apply this extension to the **hyperon sector (parity-odd)** to consider the expected cancellation between the symmetric and the anti-symmetric LS potential between N and Lambda.

HAL QCD method

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi_{\vec{k}}(x,y) \equiv Z_B^{-1} \langle 0 | T[B(x)B(y)] | B(+\vec{k})B(-\vec{k}),\mathrm{in} \rangle$$

- It is related to the S-matrix through the reduction formula

$$\langle B(p_1)B(p_2),\mathrm{out} | B(+\vec{k})B(-\vec{k}),\mathrm{in} \rangle$$
$$= \mathrm{disc.} + \left(iZ_B^{-1/2}\right)^2 \int d^4x_1 d^4x_2 e^{ip_1x_1} \left(\Box_1 + m^2\right) e^{ip_2x_2} \left(\Box_2 + m^2\right) \langle 0 | T[B(x_1)B(x_2)] | B(+\vec{k})B(-\vec{k}),\mathrm{in} \rangle$$

- **Equal-time restriction of NBS wave function** shows **the same asymptotic behavior** as **the non-relativistic scattering wave function** at long distance

$$\psi_{\vec{k}}(\vec{x}-\vec{y}) \equiv \lim_{x_0 \to +0} \psi_{\vec{k}}(\vec{x},x_0;\vec{y},y_0=0)$$

C.-J.D.Lin et al.,NPB619,467(2001).

$$\simeq e^{i\delta(k)} \frac{\sin(kr+\delta(k))}{kr} + \cdots \quad \text{as} \quad r \equiv |\vec{x}-\vec{y}| \to \text{large}$$

- Energy-independent potential is defined by Schrodinger equation:

$$\left(k^2/m - H_0\right)\psi_{\vec{k}}(\vec{r}) = \int d^3r' \, U(\vec{r},\vec{r}')\psi_{\vec{k}}(\vec{r}')$$

Resulting potential U(r,r') reproduces the scattering phase,
because of the asymptotic behavior of the equal-time NBS wave function.

"Time-dependent" method (an efficient way to obtain HAL QCD potentials)

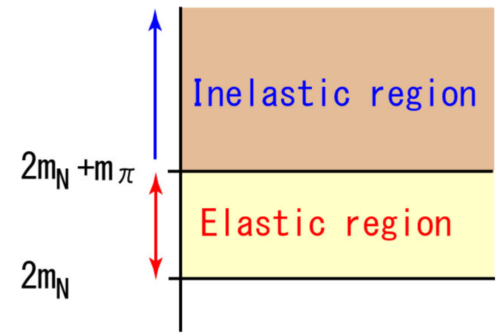
[N.Ishii et al.,PLB712(2012)437.]

- Normalized BB correlator (R-correlator)

$$R(t,\vec{x}-\vec{y}) \equiv e^{2m\cdot t}\left\langle 0 \middle| T\left[B(\vec{x},t)B(\vec{y},t)\cdot \overline{\mathcal{J}}_{BB}(t=0)\right] \middle| 0 \right\rangle$$

$$= \sum_{\vec{k}} a_{\vec{k}} \exp\left(-t\Delta W(\vec{k})\right)\psi_{\vec{k}}(\vec{x}-\vec{y})$$

$$\Delta W(\vec{k}) \equiv 2\sqrt{m_N^2+\vec{k}^2} - 2m_N$$



t has to be sufficiently large to suppress inelastic contribution ($E > 2m_N + m_{pion}$).

- "Time-dependent" Schrodinger-like equation (derivation)

$$\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t}\right)R(t,\vec{x}) = \sum_{\vec{k}} a_{\vec{k}} \frac{\vec{k}^2}{m}\exp\left(-t\Delta W(\vec{k})\right)\psi_{\vec{k}}(\vec{x})$$

$\frac{\vec{k}^2}{m} = \Delta W(\vec{k}) + \frac{\Delta W(\vec{k})^2}{4m}$ is used.

HAL QCD potential U satisfies

$$\left(H_0 + U\right)\psi_{\vec{k}}(\vec{x}) = \frac{\vec{k}^2}{m}\psi_{\vec{k}}(\vec{x})$$

"Time-dependent" Schrodinger-like equation

$$\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right)R(t,\vec{x}) = \int d^3x' U(\vec{x},\vec{x}')R(t,\vec{x}')$$

It enables us to obtain the potential without requiring the ground state saturation.

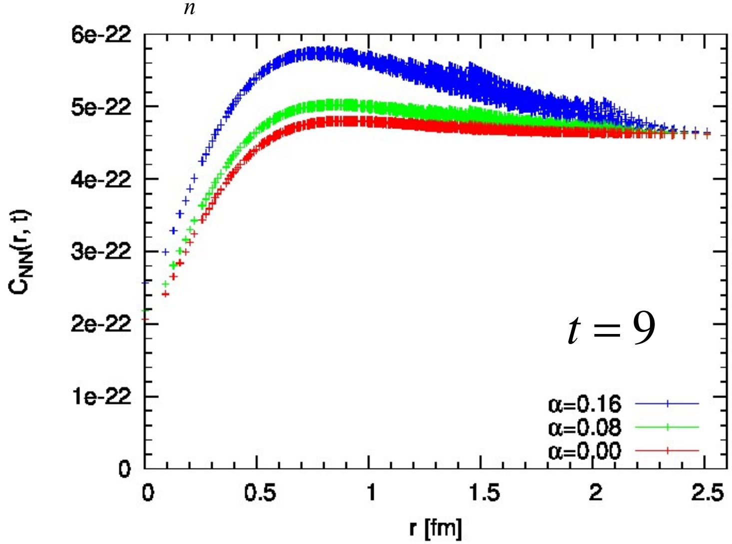
Ground state saturation is not needed. (an example)

- Source functions (with a single real parameter alpha)

$$f(x,y,z) = 1 + \alpha\left(\cos(2\pi x/L) + \cos(2\pi y/L) + \cos(2\pi z/L)\right)$$

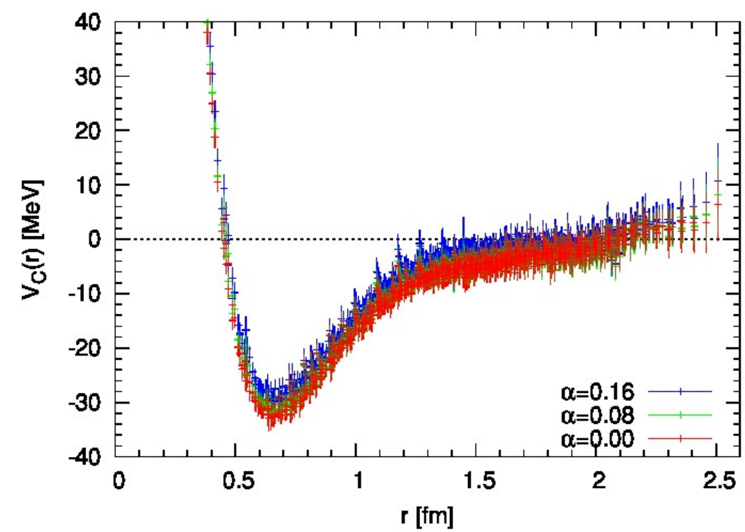
- alpha is used to arrange the mixture of NBS wave functions

$$C_{NN}(\vec{x}-\vec{y},t) \equiv \langle 0 | T[N(\vec{x},t)N(\vec{y},t)\cdot \overline{NN}(t=0;\alpha)] | 0\rangle = \sum_n \psi_n(\vec{x}-\vec{y})\cdot a_n(\alpha)\cdot \exp(-E_n t)$$



Central potential
at the leading order
of derivative expansion

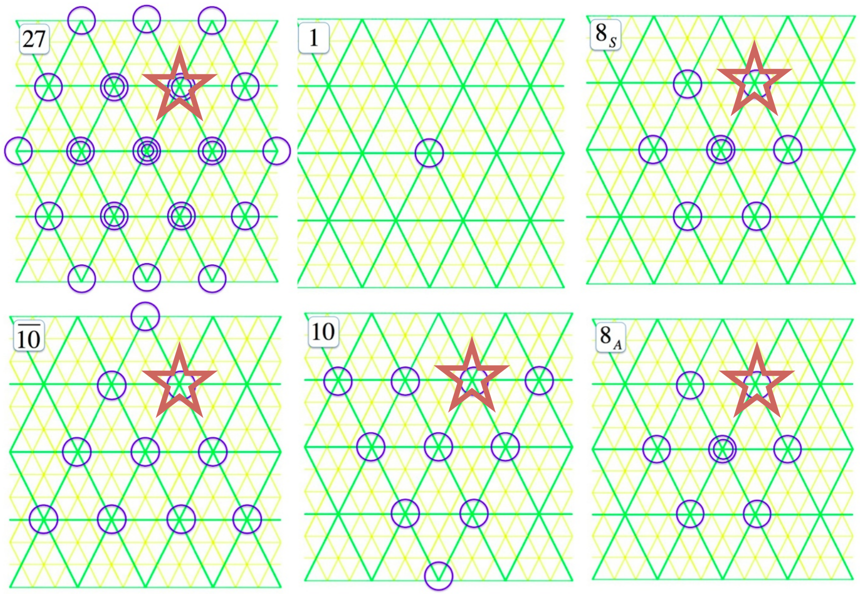
$$V_C(\vec{x}) = -\frac{H_0 R(t,\vec{x})}{R(t,\vec{x})} - \frac{(\partial/\partial t)R(t,\vec{x})}{R(t,\vec{x})} + \frac{1}{4m}\frac{(\partial/\partial t)^2 R(t,\vec{x})}{R(t,\vec{x})}$$



"Time-dependent" Schrodinger-like eq. leads to an alpha-independent result.

Two-hyperon source

- To save the computational cost, we restrict ourselves to S=-1 sector. (At this moment, the code is not efficient)



BGQ@KEK is used.

- We can access flavor rep's of $27 \oplus \overline{10} \oplus 10 \oplus 8_S \oplus 8_A$
- The following 4 operators:

$$\begin{cases} [ud]d \cdot [su]d \\ [ud]u \cdot [ud]s \\ [ud]u \cdot [ds]u \\ [ud]u \cdot [su]d \end{cases}$$

are used to construct 4x4 matrix correlator on the supercomputer.

The results are combined for flavor representations on the workstation afterwards.

Momentum wall source

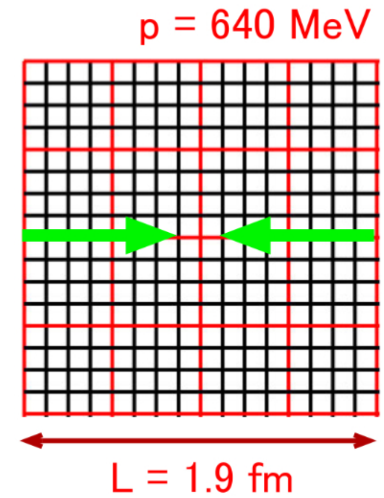
[K.Murano et al, arXiv.1305.2293]

- Two-baryon source with a non-trivial orbital cubic group rep.

$$\overline{\mathcal{J}}_{\alpha\beta}(\vec{p}) \equiv \sum_{\vec{x}_1,\cdots,\vec{x}_6} \overline{B}_\alpha(\vec{x}_1,\vec{x}_2,\vec{x}_3)\overline{B}'_\beta(\vec{x}_4,\vec{x}_5,\vec{x}_6)\cdot \exp(i\vec{p}\cdot(\vec{x}_3-\vec{x}_6))$$

$$B_\alpha(x_1,x_2,x_3) \equiv \epsilon_{abc}\left(q_a^{(1)}(x_1)C\gamma_5 q_b^{(2)}(x_2)\right)q_{c;\alpha}^{(3)}(x_3)$$

$$B_\beta{}'(x_4,x_5,x_6) \equiv \epsilon_{abc}\left(q_a^{(4)}(x_4)C\gamma_5 q_b^{(5)}(x_5)\right)q_{c;\beta}^{(6)}(x_6)$$



- Non-vanishing momentum p is carried by “spectator quark”
- We consider momenta which are parallel (anti-parallel) to the coordinate axes.

Momentum wall source

[K.Murano et al, arXiv.1305.2293]

- Cubic group analysis ➔ "orbital contribution" of source

$$A_1^+(\sim \text{s-wave}) \oplus E^+(\sim \text{d-wave}) \oplus T_1^-(\sim \text{p-wave})$$

➔ It generates NBS wave functions for (parity-odd sector)

$$J^P = 0^-(A_1^-),\ 1^-(T_1^-),\ 2^-(E^- \oplus T_2^-)$$

- Two-hyperon potentials up to NLO

$$V_{BB} = V_{\text{C;S=0}}(r)\mathbb{P}^{(S=0)} + V_{\text{C;S=1}}(r)\mathbb{P}^{(S=1)} + V_{\text{T}}(r)\left(3(\hat{r}\cdot\vec{\sigma}_1)(\hat{r}\cdot\vec{\sigma}_2) - \vec{\sigma}_1\cdot\vec{\sigma}_2\right)$$
$$+V_{\text{SLS}}(r)\vec{L}\cdot\vec{S}_+ + V_{\text{ALS}}(r)\vec{L}\cdot\vec{S}_- + O(\nabla^2)$$

are obtained by solving "t-dep" Schrodinger-like eq

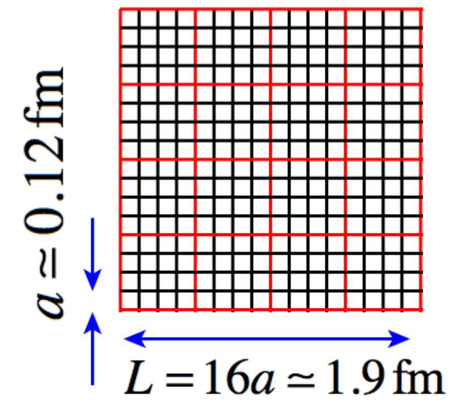
$$\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right)R(t,\vec{x};\bar{\mathcal{J}}) = V_{BB}\cdot R(t,\vec{x};\bar{\mathcal{J}})$$

	S=1	S=0
$J^P=0^-$	$^3P_0$	
$J^P=1^-$	$^3P_1$ ———	$^1P_1$
$J^P=2^-$	$^3P_2 - {}^3F_2$	

by employing sources for 3P0, 3P1, 3P2, 1P1
and flavor representations 27, 10, 10^*, 8.

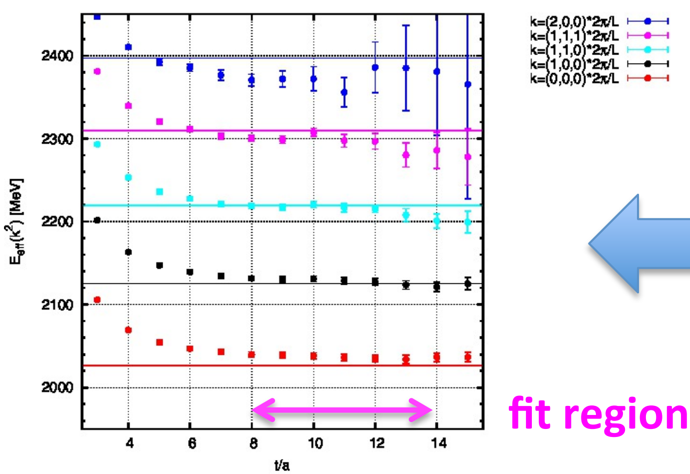
Lattice QCD setup

- 2+1 flavor gauge configuration on 16^3x32 lattice generated by CP-PACS+JLQCD
  - RG improved Iwasaki gauge action at beta=1.83
  - O(a) improved Wilson quark (clover) action with C_{SW}=1.761 at kappa_{uds}=0.1371 (**flavor SU(3) limit**)
    - a=0.121(2) fm; 1/a = 1630.58 MeV; L=32a = 1.93(3) fm
    - m(baryon) = 2051(3) MeV
m(PS) = 1013(1) MeV
  - 700 gauge configurations with 8 source points are used.

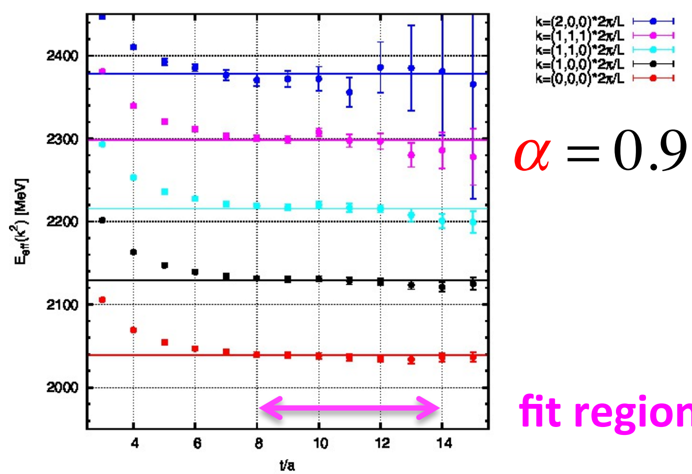


- Relativistic dispersion is violated.

Fit with $E^2(\vec{k}^2) = m^2 + \vec{k}^2$



Fit with $E^2(\vec{k}^2) \simeq m^2 + \alpha\,\vec{k}^2$

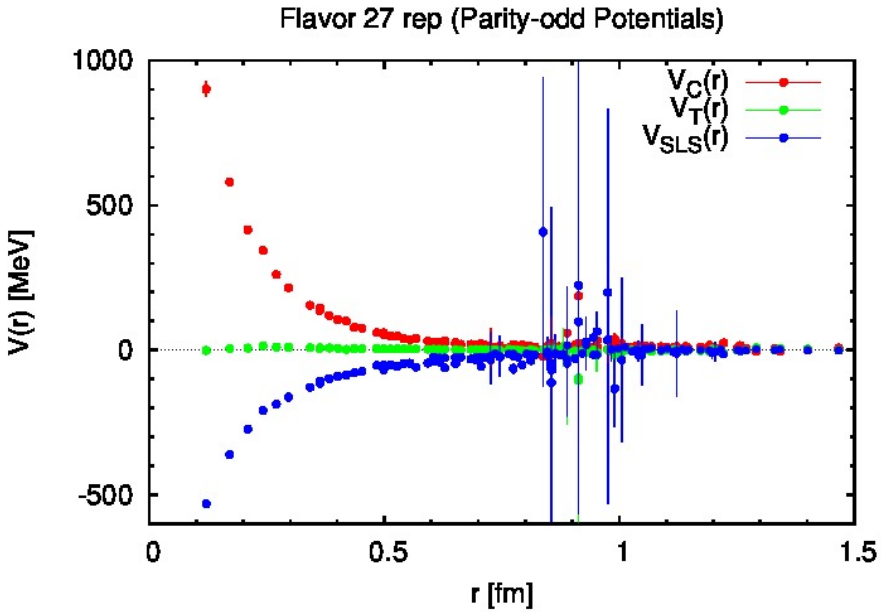


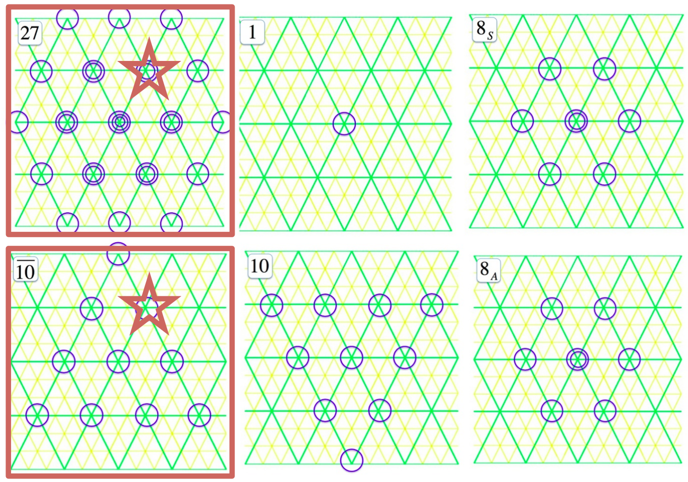
$\alpha = 0.9134(19)$

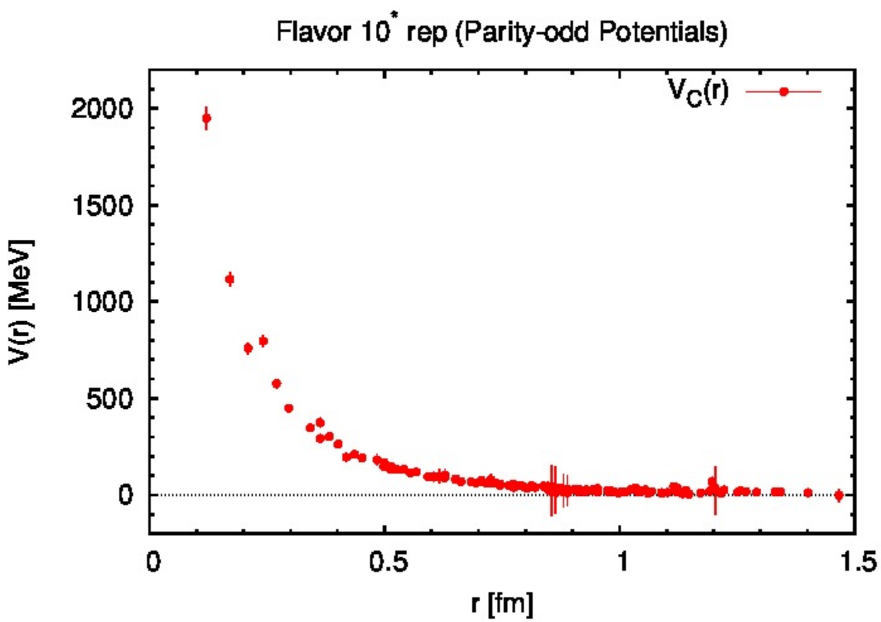
"time-depenent" Schrodinger-like eq. has to be modified as

$$\left(\frac{1}{\alpha}\left\{\frac{1}{4m}\frac{\partial^2}{\partial t^2}-\frac{\partial}{\partial t}\right\}-H_0\right)R(t,\vec{x})=\int d^3x'\,U(\vec{x},\vec{x}')R(t,\vec{x}')$$

Numerical Results(1) 27 & 10^* sector (←→ NN sector)



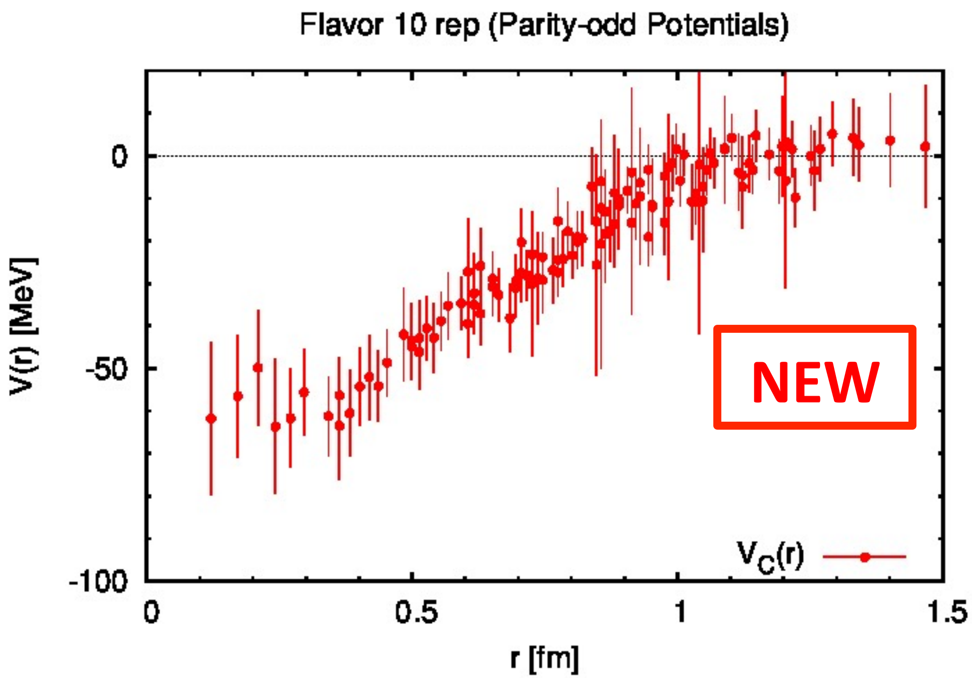


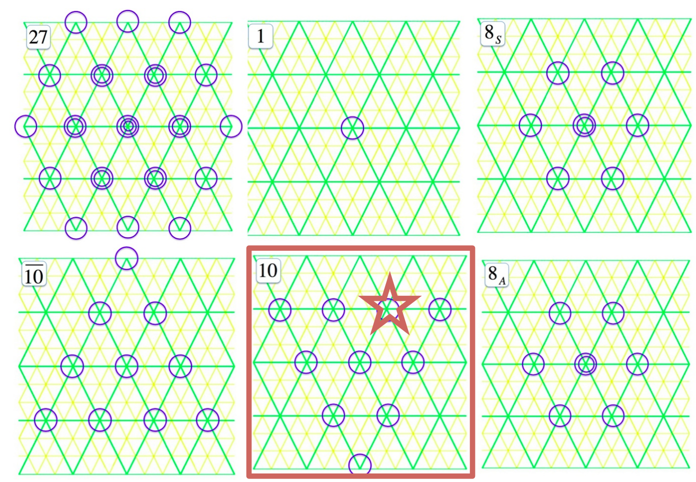


Qualitative behaviors are reproduced.

Next speaker will tell you about this channel much more !

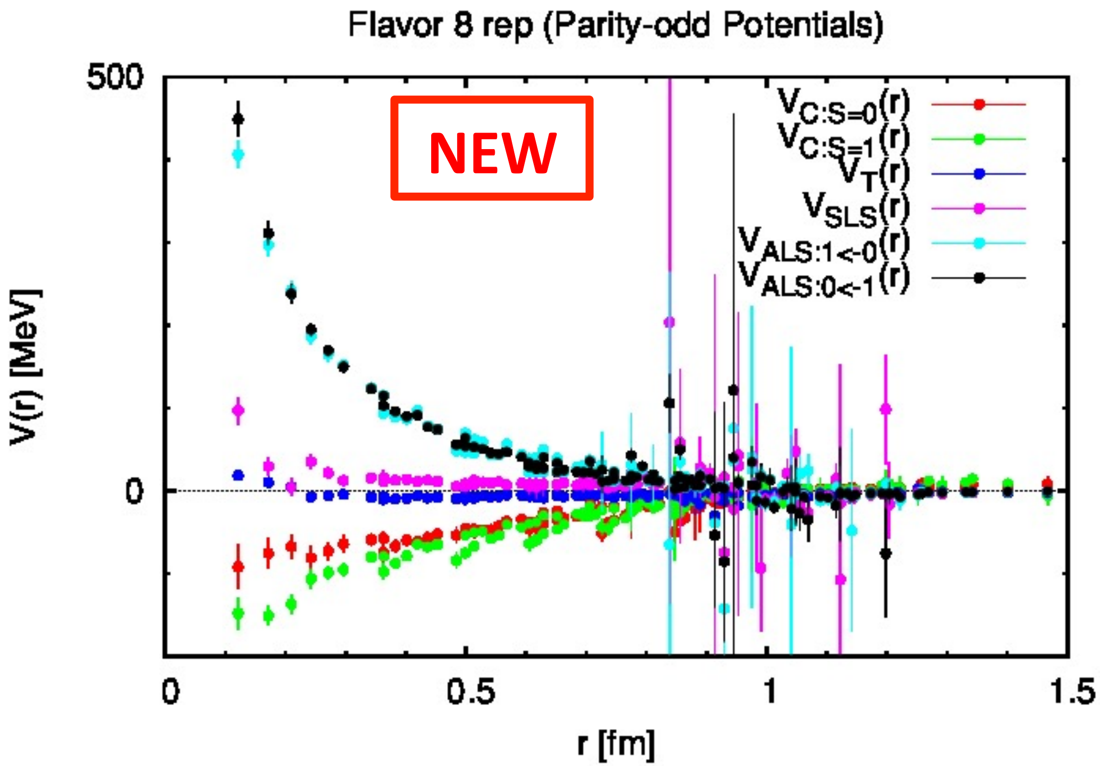
Numerical Results(2): 10 sector

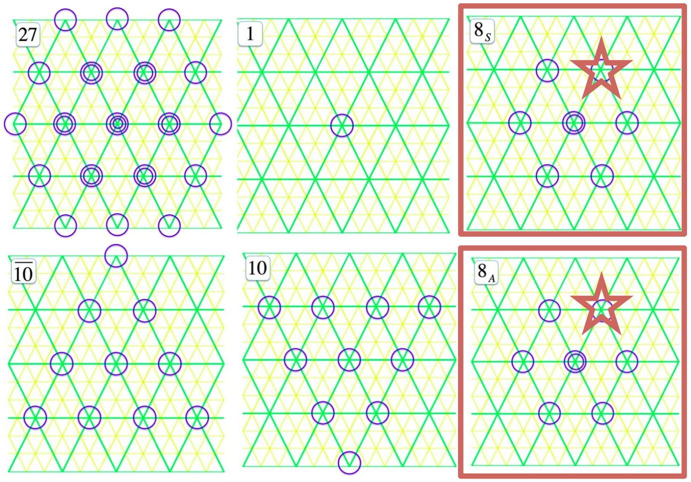




- The central potential in flavor 10 sector (parity-odd) does not have a repulsive core.
  (This is consistent with quark model.)

Numerical Results(3): 8 sector

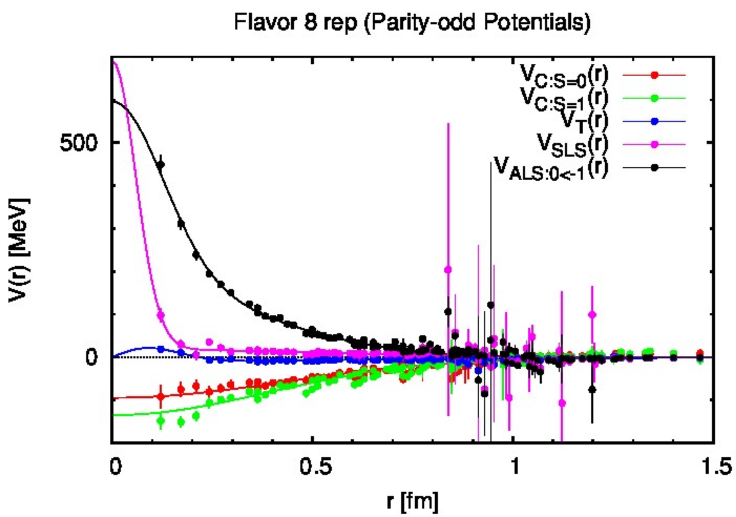


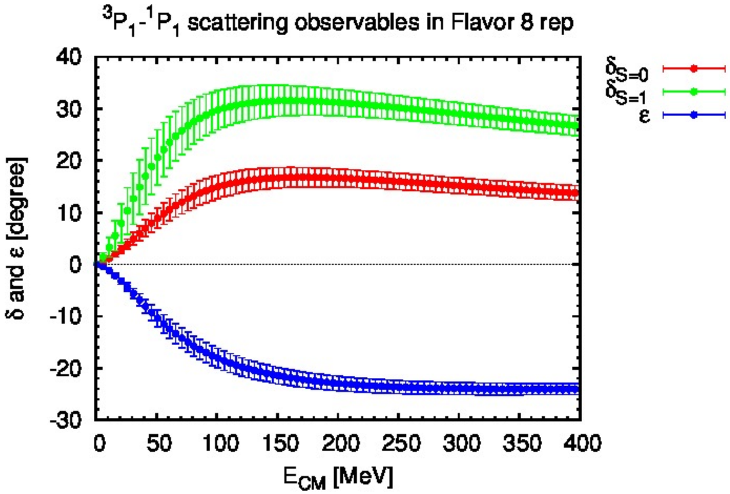


- No repulsive cores in spin-singlet and triplet central potentials in flavor 8 sector (parity-odd).
  [This is consistent with quark model.]
- Large anti-symmetric LS potential is obtained (with good Hermiticity).

Phase shift and mixing parameter (flavor 8 sector)

- Fit with various multi-gaussian functions



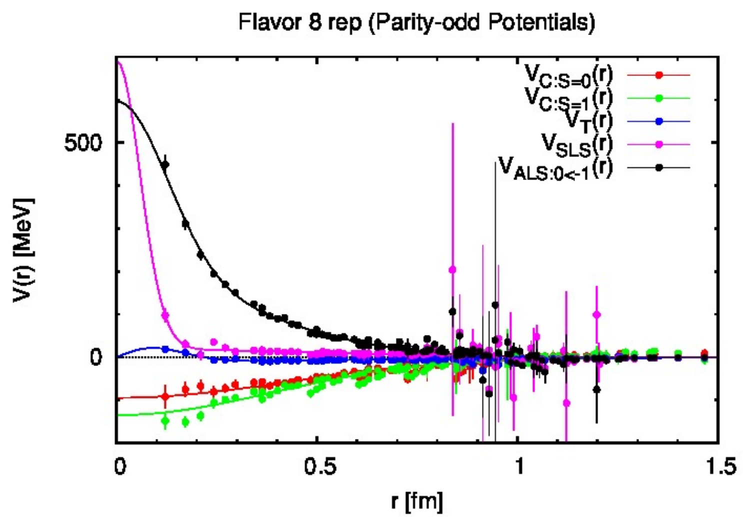


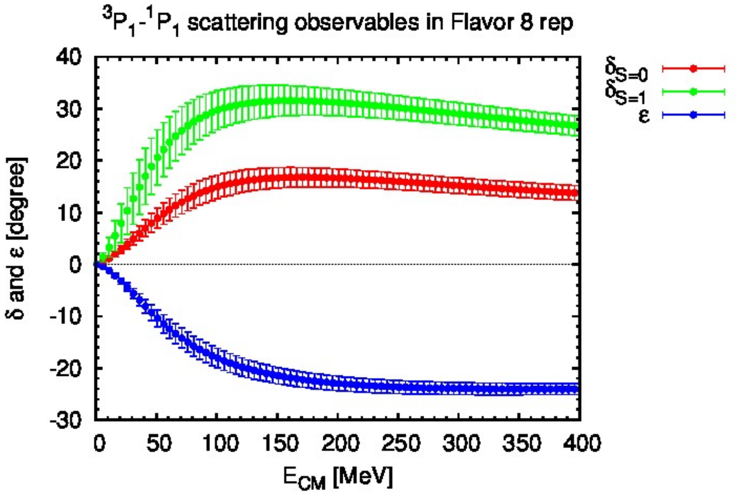
	S=1	S=0
$J^P = 0^-$	$^3P_0$	
$J^P = 1^-$	$^3P_1$ ——	$^1P_1$
$J^P = 2^-$	$^3P_2 - {}^3F_2$	

- Stapp's bar convension is adopted.
- Attractive phase shifts.
- Rather large mixing parameter. (Anti-symmetric LS mixes spin-singlet and spin-triplet sectors)

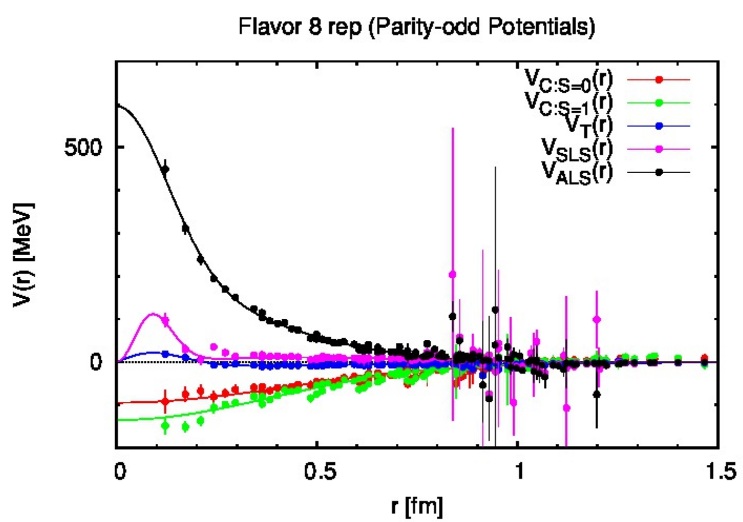
Phase shift and mixing parameter (flavor 8 sector)

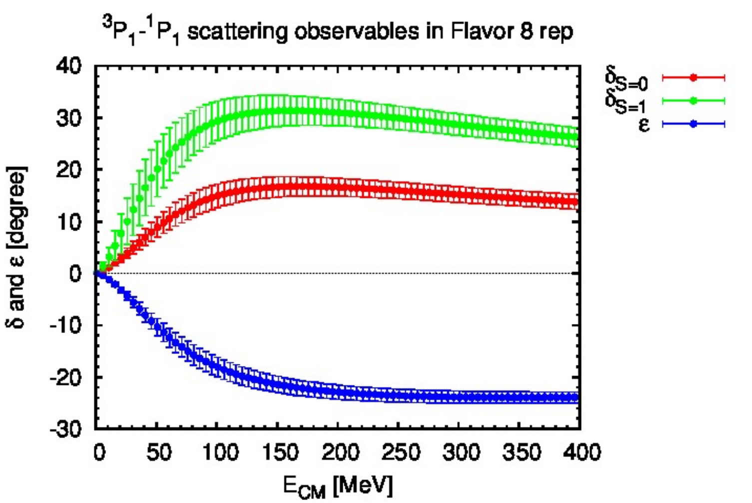
- Smooth parameterizations with various multi-gaussian functions





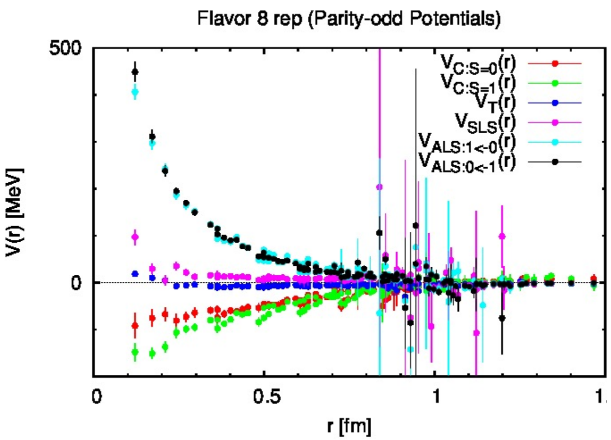
- Another functional form of symmetric LS potential is tried. (Almost nothing changes in the phase shift)

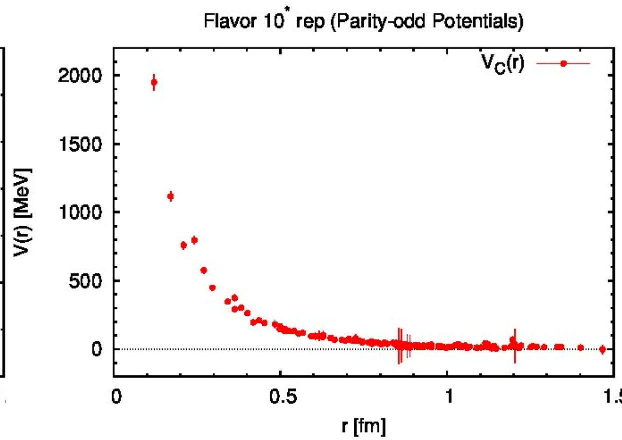


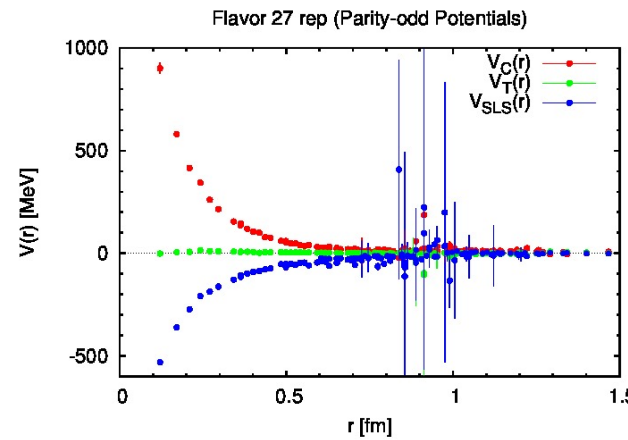


Lambda N potential

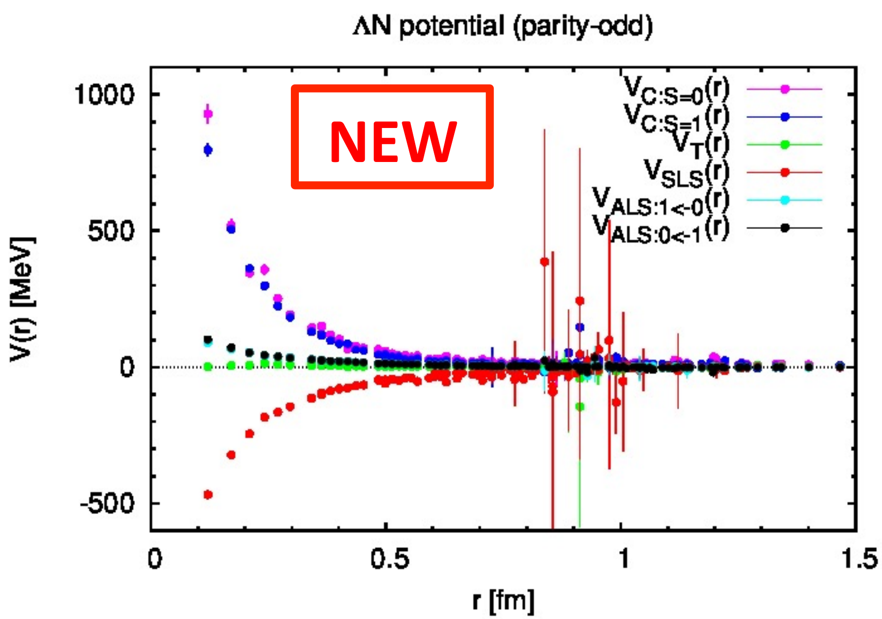
- Lambda N potentials are obtained as linear combinations of 8, 10^* and 27.







$$V_{\Lambda N} = \left(\frac{1}{2}V_{\rm C}^{(\overline{10})} + \frac{1}{2}V_{\rm C;S=0}^{(8)}\right)\mathbb{P}^{(S=0)} + \left(\frac{1}{10}V_{\rm C:S=1}^{(8)} + \frac{9}{10}V_{\rm C}^{(27)}\right)\mathbb{P}^{(S=1)} + \left(\frac{1}{10}V_{\rm T}^{(8)} + \frac{9}{10}V_{\rm T}^{(27)}\right)S_{12}(\hat{r}) + \left(\frac{1}{10}V_{\rm SLS}^{(8)} + \frac{9}{10}V_{\rm SLS}^{(27)}\right)\vec{L}\cdot\vec{S}_+ + \frac{1}{2\sqrt{5}}V_{\rm ALS}^{(8)}\cdot\vec{L}\cdot\vec{S}_-$$



- Weak cancellation
  - Symmetric LS is strong.
    It comes from 27 rep. (90%), i.e., NN LS

$$V_{\rm SLS}^{(\Lambda N)} = \frac{1}{10}V_{\rm SLS}^{(8)} + \frac{9}{10}V_{\rm SLS}^{(27)}$$

  - Anti-symmetric LS is weak.
    It is weakened by a numerical factor 1/(2*sqrt(5))

$$V_{\rm ALS}^{(\Lambda N)} = \frac{1}{2\sqrt{5}}V_{\rm ALS}^{(8)}$$

Summary

◆ We have calculated **parity-odd** two-hyperon potentials in the flavor SU(3) limit for

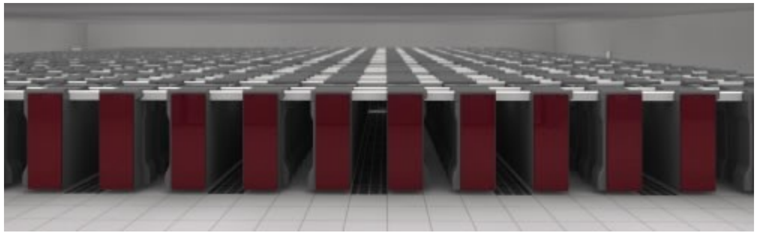
$$8 \oplus 10 \oplus 10^{*} \oplus 27$$

◆ Central potentials for 10^* and 27 have repulsive cores at short distance, whereas central potentials for 8 (spin singlet and triplet) and 10 do not have repulsive core. [This is consistent with quark model]

◆ Rather strong anti-symmetric LS potential is obtained in flavor 8 channel.

◆ 8, 10^* and 27 potentials are combined to give Lambda N potentials(parity-odd)

- ☐ It has a strong symmetric LS potential. (which comes from 27 rep (90%))
- ☐ Anti-symmetric LS potential becomes weakened by a CG factor 1/(2*sqrt(5))
  ➔ weak cancellation !!!
- ☐ The following two possibilities have to be examined
  - ✧ light quark mass effect (m_u == m_d == m_s)
  - ✧ SU(3) breaking effect (m_u == m_d << m_s) ➔ physical quark mass



backup slides

"Time-dependent" method for violated relativistic dispersion

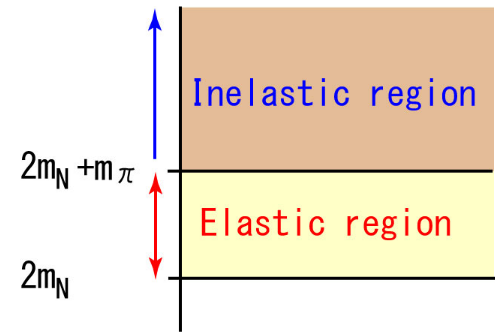
$$E(\vec{k})^2 = m^2 + \alpha \vec{k}^2 + O(k^4)$$

$$\Delta W(\vec{k}) \equiv 2E(\vec{k}) - 2m$$

- Normalized BB correlator (R-correlator)

$$R(t,\vec{x}-\vec{y}) \equiv e^{2m\cdot t}\left\langle 0\left| T\left[B(\vec{x},t)B(\vec{y},t)\cdot \overline{\mathcal{J}}_{BB}(t=0)\right]\right| 0\right\rangle$$

$$= \sum_{\vec{k}} a_{\vec{k}} \exp\left(-t\Delta W(\vec{k})\right)\psi_{\vec{k}}(\vec{x}-\vec{y})$$



t has to be sufficiently large to suppress inelastic contribution ($E > 2m + m_{pion}$).

- "Time-dependent" Schrodinger-like equation (derivation)

$$\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t}\right)R(t,\vec{x}) = \sum_{\vec{k}} a_{\vec{k}}\,\alpha\frac{\vec{k}^2}{m}\exp\left(-t\Delta W(\vec{k})\right)\psi_{\vec{k}}(\vec{x})$$

$\alpha\frac{\vec{k}^2}{m} \simeq \Delta W(\vec{k}) + \frac{\Delta W(\vec{k})^2}{4m}$ is used.

HAL QCD potential U satisfies

$$\left(H_0 + U\right)\psi_{\vec{k}}(\vec{x}) = \frac{\vec{k}^2}{m}\psi_{\vec{k}}(\vec{x})$$

"Time-dependent" Schrodinger-like equation

$$\left(\frac{1}{\alpha}\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t}\right) - H_0\right)R(t,\vec{x}) = \int d^3x'\, U(\vec{x},\vec{x}')R(t,\vec{x}')$$

It enables us to obtain the potential without requiring the ground state saturation.

Existence of energy-independent interaction kernel

- We assume linear independence of NBS wave functions below the pion threshold
  ➔ There exists a dual basis

$$E \equiv 2\sqrt{m_N^2 + \vec{k}^2} < 2m_N + m_\pi$$

$$\int d^3 r\, \widetilde{\psi}_{\vec{k}'}(\vec{r})\psi_{\vec{k}}(\vec{r}) = (2\pi)^3 \delta^3(\vec{k}' - \vec{k})$$

- We have

$$K_{\vec{k}}(\vec{r}) \equiv \left(k^2/m_N - H_0\right)\psi_{\vec{k}}(\vec{r})$$

$$= \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r}) \int d^3r'\, \widetilde{\psi}_{\vec{k}'}(\vec{r})\psi_{\vec{k}}(\vec{r})$$

$$= \int d^3r' \left\{ \int \frac{d^3k}{(2\pi)^3} K_{\vec{k}'}(\vec{r}) \widetilde{\psi}_{\vec{k}'}(\vec{r}') \right\} \psi_{\vec{k}}(\vec{r}')$$

If we define an **energy-independent interaction kernel** by

$$U(\vec{r},\vec{r}') \equiv \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r}) \widetilde{\psi}_{\vec{k}'}(\vec{r})$$

Owing to the integration of k', U(r,r') is energy-independent

then it generates NBS wave functions below the pion threshold

$$\left(k^2/m_N - H_0\right)\psi_{\vec{k}}(\vec{r}) = \int d^3r'\, U(\vec{r},\vec{r}')\psi_{\vec{k}}(\vec{r})$$

for $E \equiv 2\sqrt{m_N^2 + \vec{k}^2} < 2m_N + m_\pi$