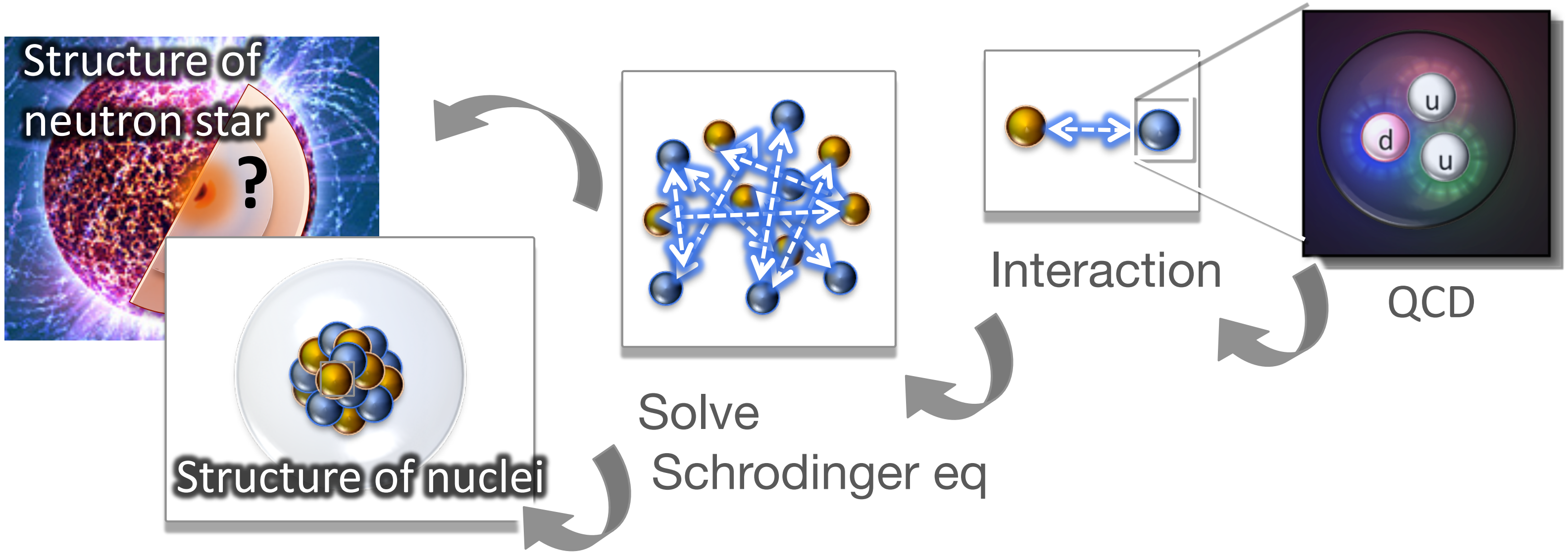
Lattice 2013 the 31st International Symposium on Lattice Field Theory
Germany, Monday, 29 July - Saturday, 3 August.

Quark mass dependence of Spin-Orbit force
in parity odd NN system from Nf=2+1 flavor QCD

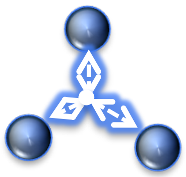
K. Murano for HAL QCD

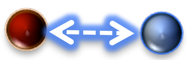
S. Aoki, C. Bruno, T. Doi, T. Hatsuda, Y. Ikeda, T. Inoue,
N. Ishii, H. Nemura, K. Sasaki, M. Yamada,

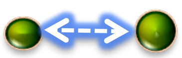
Recently, lattice QCD approach to potentials (HAL's method) has bee proposed :
Ishii, Aoki, Hatsuda Phys. Rev. Lett. 99, 022001 (2007).
Once potential is obtained, we can study nuclear physics based on QCD.



The method has been extended to various systems:


Three body force : T. Doi (Mon, 14:00 1G)


Hyperon system : H. Nemura , K. Sasaki (Fri, 16:50 10C), T. Inoue (Mon, 15:00 1G) , M. Yamada (Fri, 16:30 10C)


meson system: Y. Ikeda (Wed, 12:20 6G)

Theoretical development : beyond inelastic threshold S. Aoki (Tue, 14:40 3G)

Comparison with Luscher's method : C. Bruno (Tue, 15:00 3G) K. Thorsten (Tue, 15:20 3G)

$$\left(\frac{\nabla^2}{m_N}+E\right)\phi(\vec{r};E)=\left[Vc^{(+)}(r)+V_T^{(+)}(r)S_{12}+V_{LS}^{(+)}(r)\vec{L}\cdot\vec{S}\right]P^{+}\phi(\vec{r};E)$$

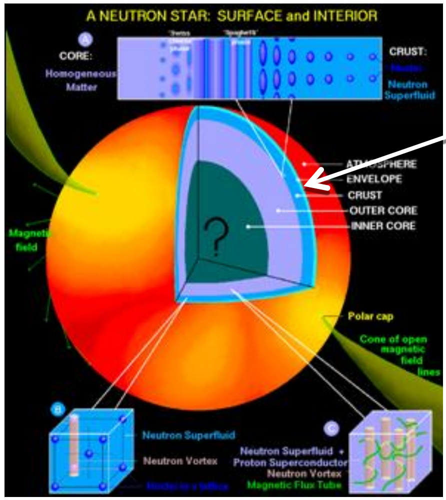
$$+\left[Vc^{(-)}(r)+V_T^{(-)}(r)S_{12}+V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right]P^{-}\phi(\vec{r};E)+\mathcal{O}(\nabla^2)$$

already calculated

Our Work

$V_{LS}^{(+)}$, $V_C^{(-)}$, $V_T^{(-)}$, $V_{LS}^{(-)}$ are needed for the complete determination of NN potentials (up to the next-to- leading order)

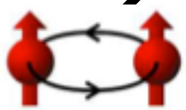
Spin Orbit force is important for the structure of the neutron Star



$^3P_2$ neutron superfluidity in neutron star

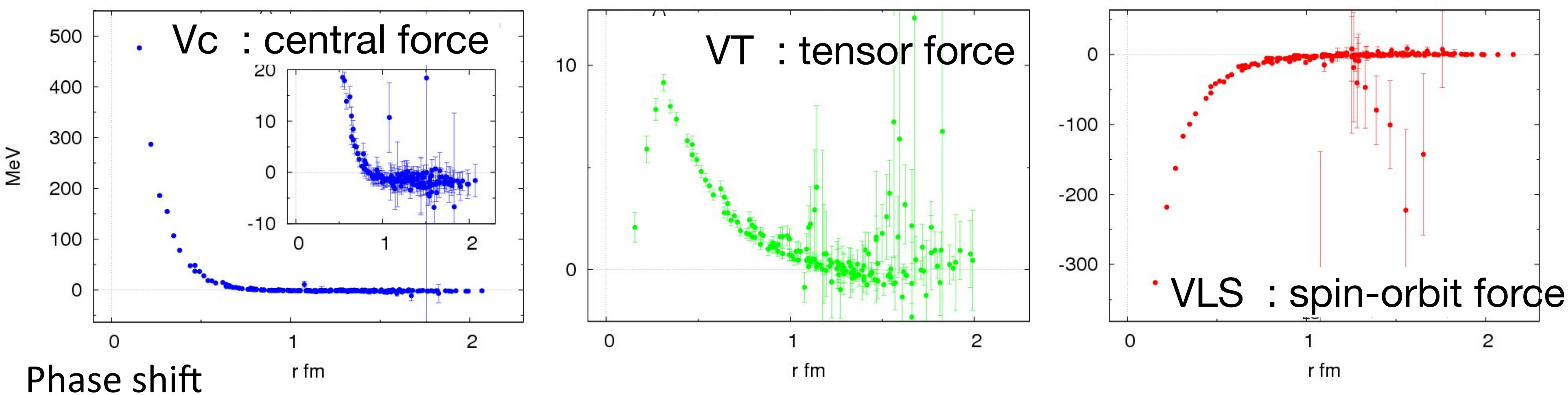
Spin-Orbit force make that
whereas $^3P_0$, $^3P_1$ state is negative,
$^3P_2$ state is atractive in high density.

➔ $^3P_2$ nucleon pairing
(cooper pair) ➔ ($^3P_2$ superfluid)

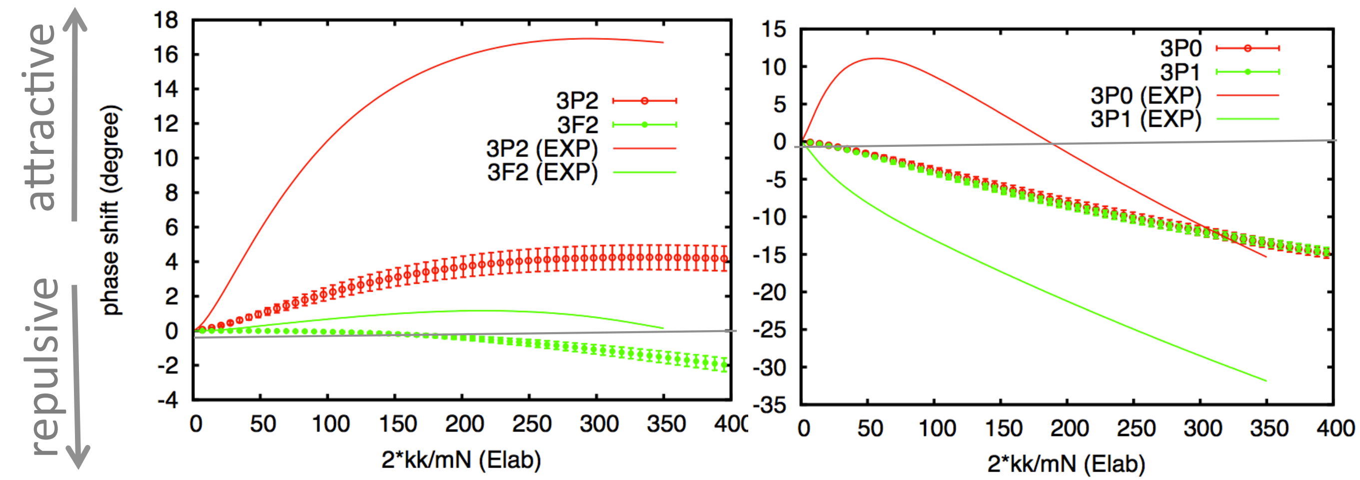


mpi=1.1GeV

In previous conference (Lattice 2012),
We reported a first calculation of Spin-Orbit force:



Phase shift



Although the qualitative behavior of potentials are consistent with phenomenological potentials (such as AV18), their strength are weaker than the experimental one.

These discrepancy is due to heavy pion mass ? ➔ examine a mass dependence

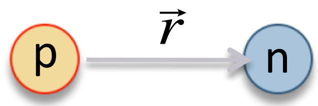
Quark mass dependence of Spin-Orbit force
in parity odd NN system from Nf=2+1 flavor QCD

Outline

- Introduction
- How to calculate Spin-Orbit force from QCD
- On the Lattice
- Set Up
- Preliminary Numerical Results
- Summary and Conclusion

➢How to calculate Spin-Orbit force from QCD

Nambu – Bethe –Salpeter (NBS) wave function



$$\phi(\vec{r};E) \equiv \langle 0 | N(\vec{x}) N(\vec{x}+\vec{r}) | B=2;E\rangle \qquad \hat{N} = \varepsilon_{abc}\left(q^a C\gamma_5 q^b\right) q^c$$

$$q = u, d$$

Potential is defined through :

$$\left(\frac{\nabla^2}{m_N} + E\right)\phi(\vec{r};E) = \int dr' U(\vec{r};\vec{r}')\phi(\vec{r}';E)$$

Derivative expansion

$$= \left[Vc^{(+)}(r) + V_T^{(+)}(r)S_{12} + V_{LS}^{(+)}(r)\vec{L}\cdot\vec{S}\right] P^{+}\phi(\vec{r};E)$$

$$+\left[Vc^{(-)}(r) + V_T^{(-)}(r)S_{12} + V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right] P^{-}\phi(\vec{r};E) + \mathcal{O}(\nabla^2)$$

Projection to odd parity

$$P^{(-)}\left(\frac{\nabla^2}{m_N} + E\right)\phi(\vec{r};E) = \left[Vc^{(-)}(r) + V_T^{(-)}(r)S_{12} + V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right] P^{(-)}\phi(\vec{r};E)$$

In order to solve three unknown parameter, we need three equations.

➔ projecting to three kind of total angular momentum J

$$\left(\frac{\nabla^2}{m_N} + E\right)P^{(J=X)}P^{(-)}\phi(\vec{r};E) = \left[Vc^{(-)}(r) + V_T^{(-)}(r)S_{12} + V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right] P^{(J=X)}P^{(-)}\phi(\vec{r};E)$$

projection operator for total angular momentum J=0, 1, 2

X= 0, 1, 2

➢How to calculate Spin-Orbit force from QCD

$$\left(\frac{\nabla^2}{m_N}+E\right)\psi_X^{(-)}(\vec{r})=\left[Vc^{(-)}(r)+V_T^{(-)}(r)S_{12}+V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right]\psi_X^{(-)}(\vec{r})$$

$$P^{(J=X)}P^{(-)}\phi(\vec{r};E)\equiv\psi_X^{(-)}(\vec{r}),\quad X=0,1,2$$

$J=0$ $$\left(\nabla^2/m_N+E\right)\psi_0^{(-)}(\vec{r})=Vc^{(-)}\psi_0^{(-)}(\vec{r})+V_T^{(-)}S_{12}\psi_0^{(-)}(\vec{r})+V_{LS}^{(-)}\vec{L}\cdot\vec{S}\psi_0^{(-)}(\vec{r})$$

$J=1$ $$\left(\nabla^2/m_N+E\right)\psi_1^{(-)}(\vec{r})=Vc^{(-)}\psi_1^{(-)}(\vec{r})+V_T^{(-)}S_{12}\psi_1^{(-)}(\vec{r})+V_{LS}^{(-)}\vec{L}\cdot\vec{S}\psi_1^{(-)}(\vec{r})$$

$J=2$ $$\left(\nabla^2/m_N+E\right)\psi_2^{(-)}(\vec{r})=Vc^{(-)}\psi_2^{(-)}(\vec{r})+V_T^{(-)}S_{12}\psi_2^{(-)}(\vec{r})+V_{LS}^{(-)}\vec{L}\cdot\vec{S}\psi_2^{(-)}(\vec{r})$$

$$\begin{pmatrix}\left(\nabla^2/m_N+E\right)\psi_0^{(-)}(\vec{r})\\ \left(\nabla^2/m_N+E\right)\psi_1^{(-)}(\vec{r})\\ \left(\nabla^2/m_N+E\right)\psi_2^{(-)}(\vec{r})\end{pmatrix}=\begin{pmatrix}\psi_0^{(-)}(\vec{r}) & S_{12}\psi_0^{(-)}(\vec{r}) & \vec{L}\cdot\vec{S}\psi_0^{(-)}(\vec{r})\\ \psi_1^{(-)}(\vec{r}) & S_{12}\psi_1^{(-)}(\vec{r}) & \vec{L}\cdot\vec{S}\psi_1^{(-)}(\vec{r})\\ \psi_2^{(-)}(\vec{r}) & S_{12}\psi_2^{(-)}(\vec{r}) & \vec{L}\cdot\vec{S}\psi_2^{(-)}(\vec{r})\end{pmatrix}\begin{pmatrix}Vc^{(-)}\\ V_T^{(-)}\\ V_{LS}^{(-)}\end{pmatrix}$$

➢How to calculate Spin-Orbit force from QCD

$$\left(\frac{\nabla^2}{m_N}+E\right)\psi_X^{(-)}(\vec{r}) = \left[Vc^{(-)}(r)+V_T^{(-)}(r)S_{12}+V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right]\psi_X^{(-)}(\vec{r})$$

$$P^{(J=X)}P^{(-)}\phi(\vec{r};E) \equiv \psi_X^{(-)}(\vec{r}), \quad X=0,1,2$$

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$J=1$ $$\left(\nabla^2/m_N+E\right)\psi_1^{(-)}(\vec{r}) = Vc^{(-)}\psi_1^{(-)}(\vec{r}) + V_T^{(-)}S_{12}\psi_1^{(-)}(\vec{r}) + V_{LS}^{(-)}\vec{L}\cdot\vec{S}\psi_1^{(-)}(\vec{r})$$

$J=2$ $$\left(\nabla^2/m_N+E\right)\psi_2^{(-)}(\vec{r}) = Vc^{(-)}\psi_2^{(-)}(\vec{r}) + V_T^{(-)}S_{12}\psi_2^{(-)}(\vec{r}) + V_{LS}^{(-)}\vec{L}\cdot\vec{S}\psi_2^{(-)}(\vec{r})$$

$$\begin{pmatrix}\left(\nabla^2/m_N+E\right)\psi_0^{(-)}(\vec{r})\\ \left(\nabla^2/m_N+E\right)\psi_1^{(-)}(\vec{r})\\ \left(\nabla^2/m_N+E\right)\psi_2^{(-)}(\vec{r})\end{pmatrix} = \underbrace{\begin{pmatrix}\psi_0^{(-)}(\vec{r}) & S_{12}\psi_0^{(-)}(\vec{r}) & \vec{L}\cdot\vec{S}\psi_0^{(-)}(\vec{r})\\ \psi_1^{(-)}(\vec{r}) & S_{12}\psi_1^{(-)}(\vec{r}) & \vec{L}\cdot\vec{S}\psi_1^{(-)}(\vec{r})\\ \psi_2^{(-)}(\vec{r}) & S_{12}\psi_2^{(-)}(\vec{r}) & \vec{L}\cdot\vec{S}\psi_2^{(-)}(\vec{r})\end{pmatrix}}_{M}\begin{pmatrix}Vc^{(-)}\\ V_T^{(-)}\\ V_{LS}^{(-)}\end{pmatrix}$$

After calculating NBS wave functions ψ_0, ψ_1, ψ_2 , Vc, V_T, V_{LS} can be obtained with multiplying the inverse of M from left hand side.

➢How to calculate Spin-Orbit force from QCD

$$\left(\frac{\nabla^2}{m_N}+E\right)\psi_X^{(-)}(\vec{r})=\left[Vc^{(-)}(r)+V_T^{(-)}(r)S_{12}+V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right]\psi_X^{(-)}(\vec{r})$$

$$P^{(J=X)}P^{(-)}\phi(\vec{r};E)\equiv\psi_X^{(-)}(\vec{r}),\quad X=0,1,2$$

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$J=1$ $$\left(\nabla^2/m_N+E\right)\psi_1^{(-)}(\vec{r})=Vc^{(-)}\psi_1^{(-)}(\vec{r})+V_T^{(-)}S_{12}\psi_1^{(-)}(\vec{r})+V_{LS}^{(-)}\vec{L}\cdot\vec{S}\psi_1^{(-)}(\vec{r})$$

$J=2$ $$\left(\nabla^2/m_N+E\right)\psi_2^{(-)}(\vec{r})=Vc^{(-)}\psi_2^{(-)}(\vec{r})+V_T^{(-)}S_{12}\psi_2^{(-)}(\vec{r})+V_{LS}^{(-)}\vec{L}\cdot\vec{S}\psi_2^{(-)}(\vec{r})$$

$$\underbrace{\begin{pmatrix}\psi_0^{(-)}(\vec{r}) & S_{12}\psi_0^{(-)}(\vec{r}) & \vec{L}\cdot\vec{S}\psi_0^{(-)}(\vec{r})\\ \psi_1^{(-)}(\vec{r}) & S_{12}\psi_1^{(-)}(\vec{r}) & \vec{L}\cdot\vec{S}\psi_1^{(-)}(\vec{r})\\ \psi_2^{(-)}(\vec{r}) & S_{12}\psi_2^{(-)}(\vec{r}) & \vec{L}\cdot\vec{S}\psi_2^{(-)}(\vec{r})\end{pmatrix}^{-1}}_{M^{-1}}\begin{pmatrix}\left(\nabla^2/m_N+E\right)\psi_0^{(-)}(\vec{r})\\ \left(\nabla^2/m_N+E\right)\psi_1^{(-)}(\vec{r})\\ \left(\nabla^2/m_N+E\right)\psi_2^{(-)}(\vec{r})\end{pmatrix}=\begin{pmatrix}Vc^{(-)}\\ V_T^{(-)}\\ V_{LS}^{(-)}\end{pmatrix}$$

After calculating NBS wave functions ψ_0,ψ_1,ψ_2 , Vc, V_T, V_{LS} can be obtained with multiplying the inverse of M from left hand side.

➢On the Lattice

NBS wave functions :

$$G^{(i)}(\vec{r}) \equiv \sum_{\vec{x}} \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) \overline{\mathcal{J}}^{(i)} | 0 \rangle$$

$$= \phi(\vec{r};E_0)\ \exp(\text{-E}_0 t) + \phi(\vec{r};E_1)\ \exp(\text{-E}_1 t) + \cdots$$

$$\xrightarrow[t\to\infty]{} \phi(\vec{r};E_0)$$

After time slice saturation, we can obtain grand state of NBS wave function.

Projection of Total Angular momentum on the Lattice :

Instead of angular momentum rep., representation of cubic group is used.

$$P^{J=\Gamma} G^{(i)} = \frac{d^{\Gamma}}{24} \sum_{g\in O} \chi^{*(\Gamma)}(g) S^{-1}(g^{-1}) S^{-1}(g^{-1}) \sum_{j} U(g)_{i,j} G^{(j)}$$

$l=0$	1	2	3	4	...
A_1	T_1	$E+T_2$	$A_2+T_1+T_2$	$A_1+E+T_1+T_2$	...

➢On the Lattice

NBS wave functions :

$$G^{(i)}(\vec{r}) \equiv \sum_{\vec{x}} \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) \overline{\mathcal{J}}^{(i)} | 0 \rangle$$

$$= \phi(\vec{r};E_0)\exp(\text{-E}_0 t) + \phi(\vec{r};E_1)\exp(\text{-E}_1 t) + \cdots$$

$$\xrightarrow[t\to\infty]{} \phi(\vec{r};E_0)$$

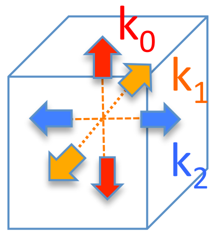
$$\phi^{(0)}(\vec{r};E_0),\ \phi^{(1)}(\vec{r};E_0),\ \phi^{(2)}(\vec{r};E_0)$$

NBS wave functions which corresponding to k0, k1, k2 momentum each other.

$$N(+\vec{k}) = \epsilon_{abc}\left(q_a^t C\gamma_5 q_b\right)\sum_{\vec{x}} q_c(\vec{x})\exp(+\vec{k}\cdot\vec{x})$$

$$\mathcal{J}^{(i)} = P(+\vec{k}_i)N(-\vec{k}_i) - P(-\vec{k}_i)N(+\vec{k}_i)$$

in order to construct NBS parity odd state, momentum wall src is employed.



$$T_1^{(-)} \oplus A_1^{(+)} \oplus E^{(+)}$$

Projection of Total Angular momentum on the Lattice :

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$J=0$	1	2	3	4	...
A_1	T_1	$E+T_2$	$A_2+T_1+T_2$	$A_1+E+T_1+T_2$	...

➢On the Lattice

NBS wave functions :

$$G^{(i)}(\vec r) \equiv \sum_{\vec x} \langle 0 | N(\vec x + \vec r) N(\vec x) \overline{\mathcal{J}}^{(i)} | 0 \rangle$$

$$= \phi(\vec r; E_0)\exp(\text{-}E_0 t) + \phi(\vec r; E_1)\exp(\text{-}E_1 t) + \cdots$$

$$\xrightarrow[t\to\infty]{} \phi(\vec r; E_0)$$

$\phi^{(0)}(\vec r; E_0),\ \phi^{(1)}(\vec r; E_0),\ \phi^{(2)}(\vec r; E_0)$

NBS wave functions which corresponding to
k0, k1, k2 momentum each other.

$$N(+\vec k) = \epsilon_{abc}\left(q_a^t C\gamma_5 q_b\right)\sum_{\vec x} q_c(\vec x)\exp(+\vec k\cdot\vec x)$$

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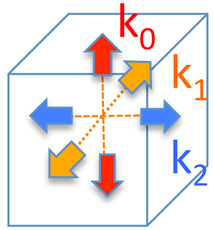


$T_1^{(-)} \oplus A_1^{(+)} \oplus E^{(+)}$

Projection of Total Angular momentum on the Lattice :

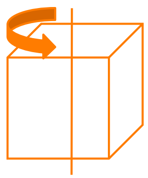
Instead of angular momentum rep., representation of cubic group is used.

representation matrix of three src

$$P^{J=\Gamma}G^{(i)} = \frac{d^\Gamma}{24}\sum_{g\in O}\chi^{*(\Gamma)}(g)S^{-1}(g^{-1})S^{-1}(g^{-1})\sum_j U(g)_{i,j}G^{(j)}$$

ex) rotation around z-axis

90



k0 ➔ k0
k1 ➔ - k2
k2 ➔ k1

$$\begin{pmatrix} 1 & & \\ & & 1 \\ & -1 & \end{pmatrix}$$

representation matrix of quark field

$$\hat R(g)\bar q(x) = \bar q(gx)\ S^{-1}(g^{-1})$$

$J=0$	1	2	3	4	...
A_1	T_1	$E+T_2$	$A_2+T_1+T_2$	$A_1+E+T_1+T_2$	...

➢ Set Up

PACS-CS Configuration: (arXiv:0807.1661)

Japan Lattice Data Grid

management by JLDG + ILDG

Iwasaki + clover Nf=2+1 full QCD

$32^3 \times 64$

$\beta = 1.90$

$a = 0.0907(13)$

$L = 2.9$ fm

Kud	0.13700	0.13727	0.13754
mN (MeV)	1583.2	1411.3	1214.9
mpi (MeV)	701.6	569.9	411.3

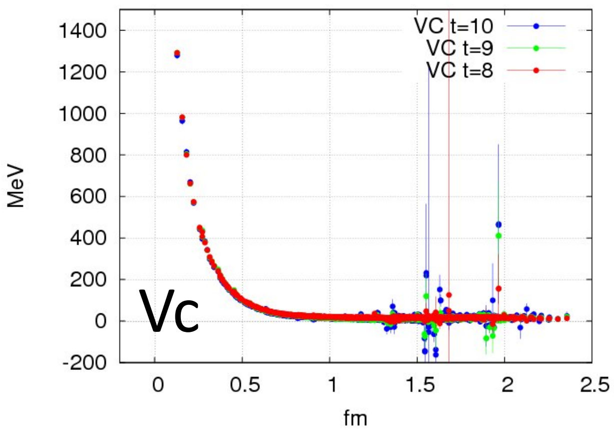
In order to examine mass dependence of potential, we calculate them with three different quark masses.

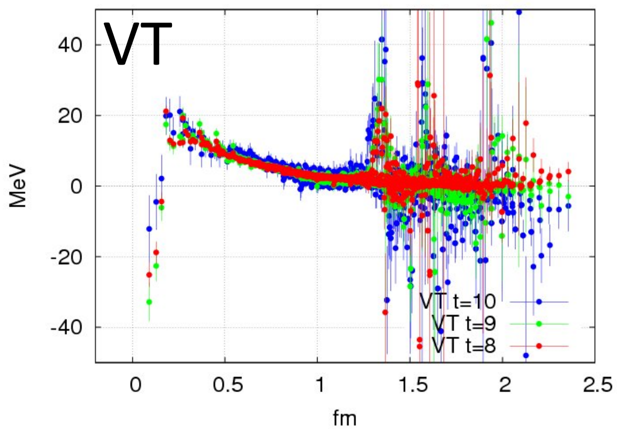
➢ Time-dependent method
(N.Ishii et al., PLB712(2012)437)
Analysis including excited states

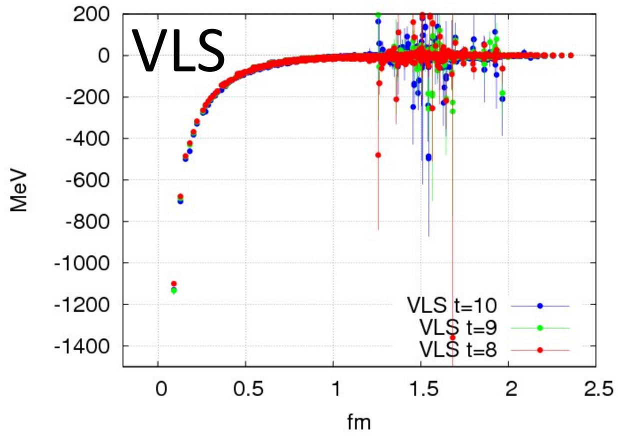
➢Numerical Results : Time slice dependence

t=8 t=9 t=10

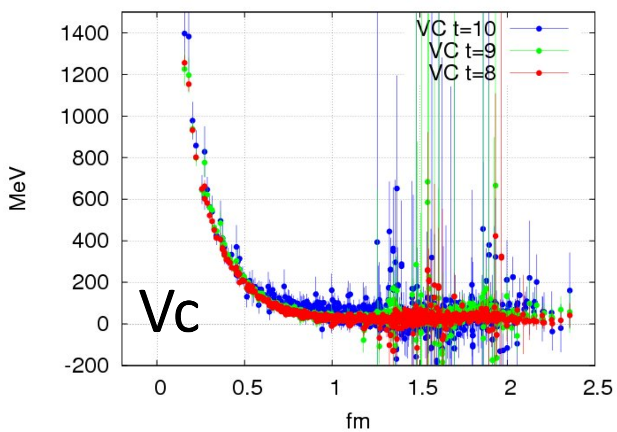
mpi = 702 MeV

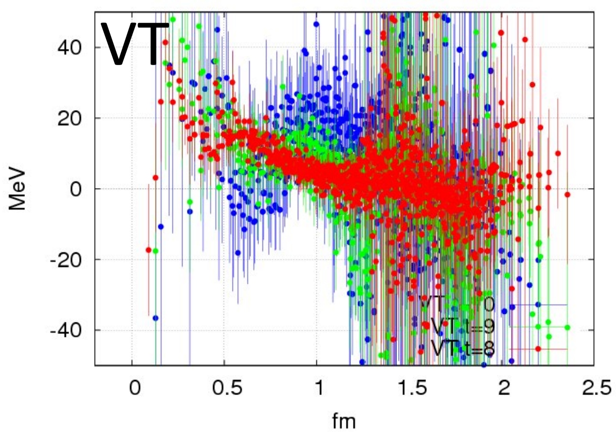


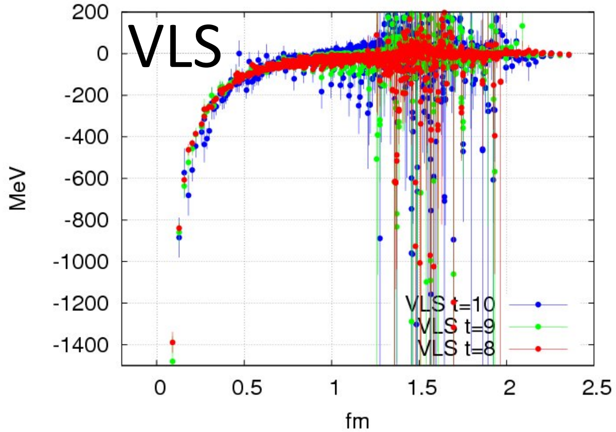




mpi = 411MeV







➢Numerical Results : Time slice dependence

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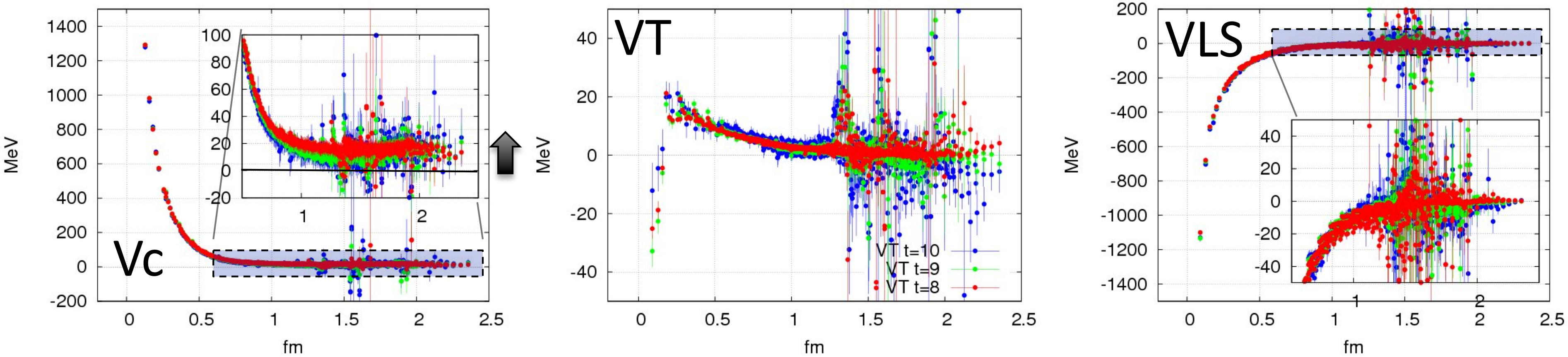
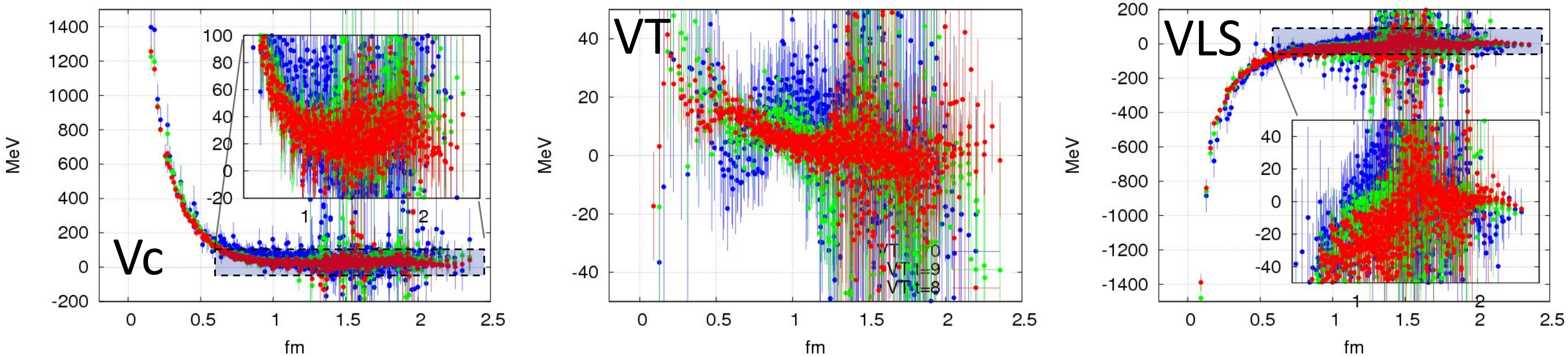


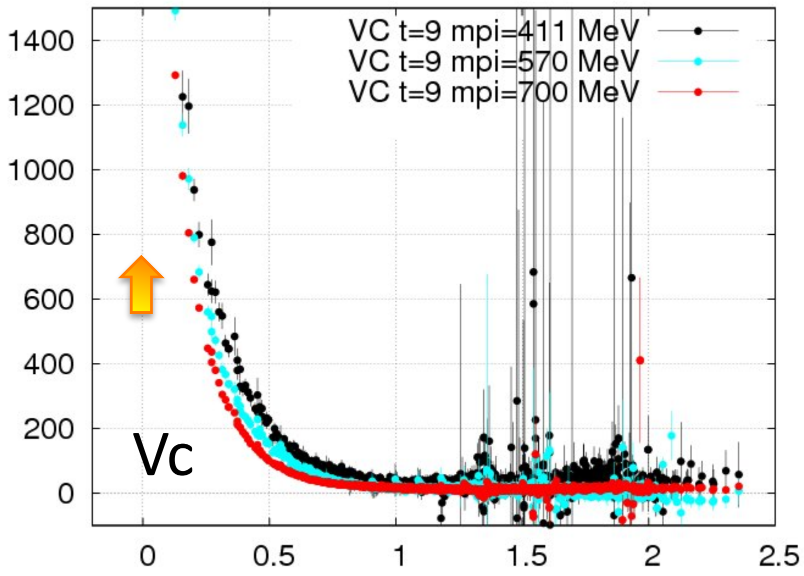
convergence of long range part of Vc is very slow.

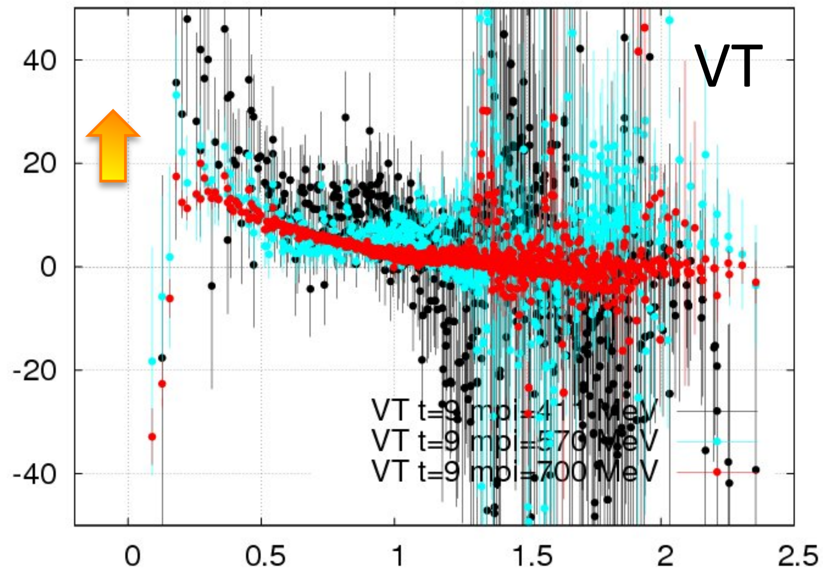
🡐 time slice saturation is not achieved yet.

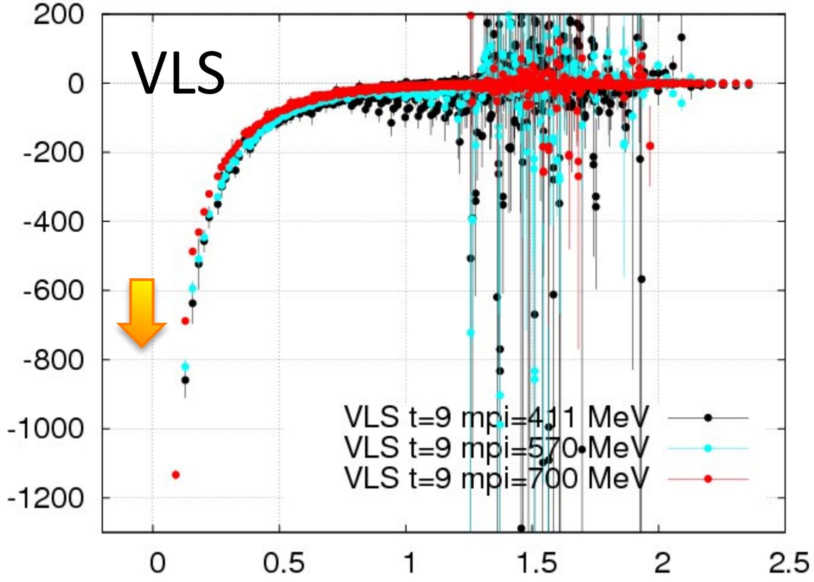
mpi = 411MeV



➢Numerical Results : mass dependence of potentials







mass dependence of Vc, VT and VLS

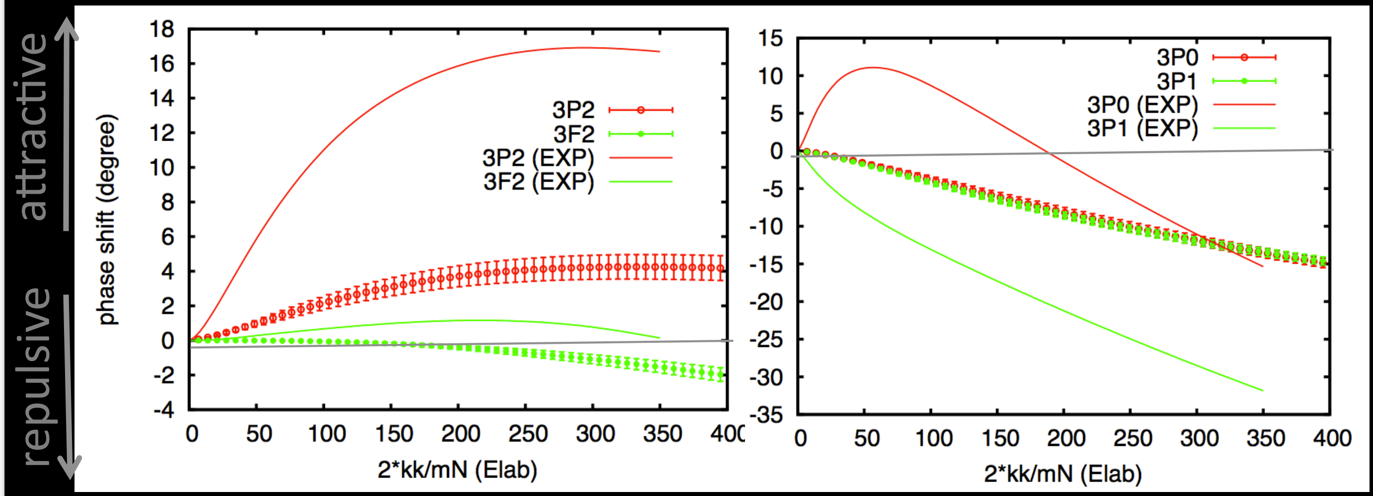
preliminary results

mpi = 411 MeV
570 MeV
700 MeV

* tendency for potentials to glow as quark mass decreases.

➔ consistent with our expectation

➢Numerical Results : mass dependence of potentials



NOW we cannot calculate phase shift by using resultant potentials (since time slice saturation of Vc is not enough), However, we expecting that ..

mass dependence of Vc, VT and VLS

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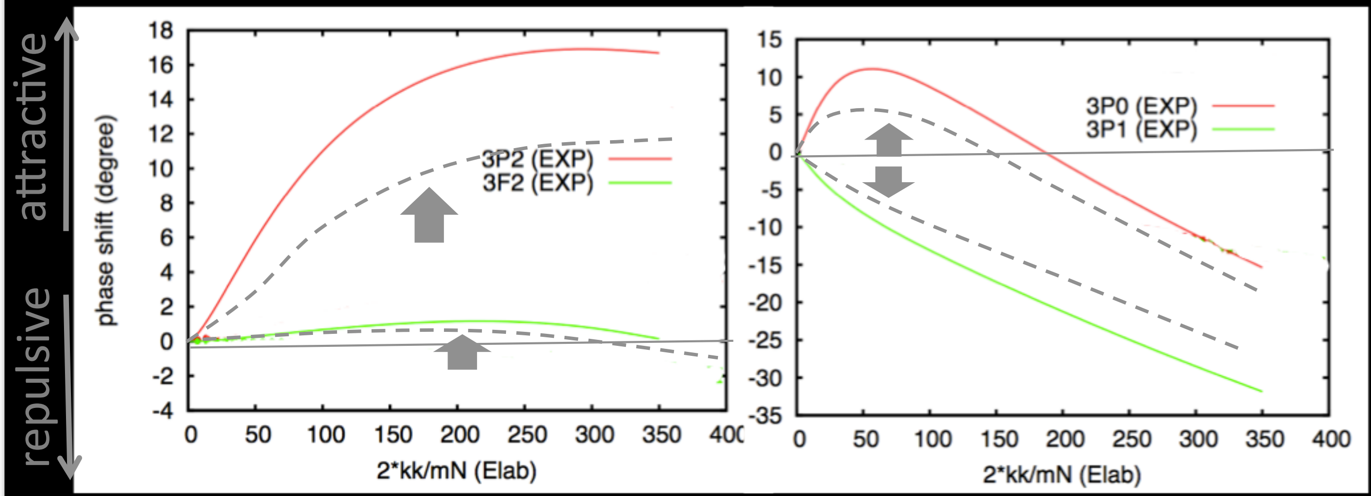

NOW we cannot calculate phase shift by using resultant potentials (since time slice saturation of Vc is not enough), However, we expecting that ..

If (after saturation) phase shift can be calculated, it will approach to EXP.

mass dependence of Vc, VT and VLS

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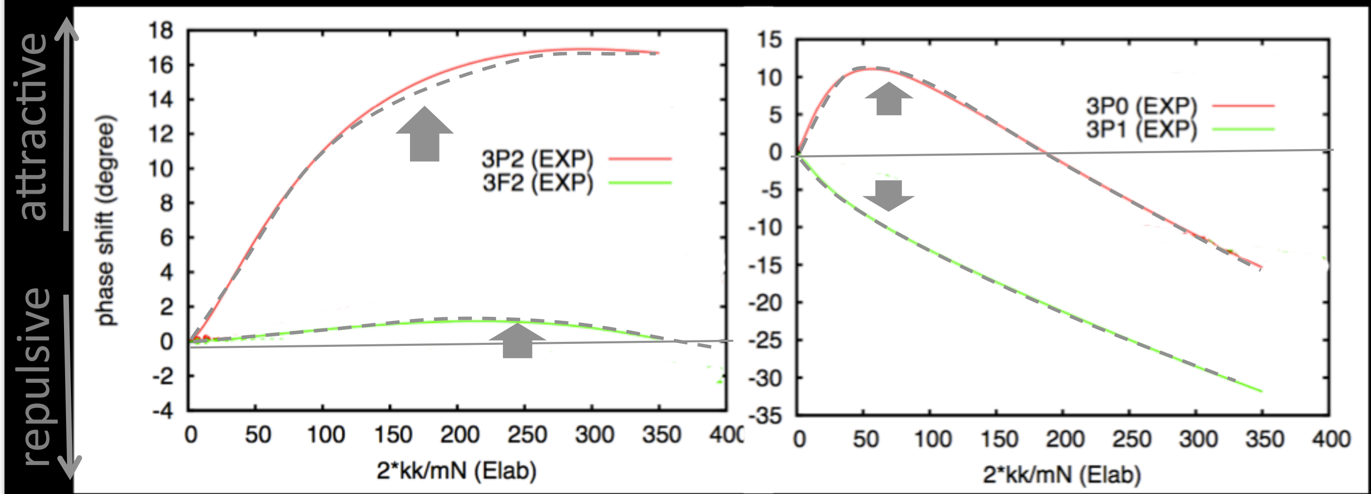
➔ consistent with our expectation

➢Numerical Results : mass dependence of potentials



NOW we cannot calculate phase shift by using resultant potentials (since time slice saturation of Vc is not enough), However, we expecting that ..

If (after saturation) phase shift can be calculated, it will approach to EXP.

If potentials are calculated at **physical point,** the phase shift has to be consistent with EXP

mass dependence of Vc, VT and VLS

preliminary results

mpi = 411 MeV
570 MeV
700 MeV

* tendency for potentials to glow as quark mass decreases.

➔ consistent with our expectation

➢Summary and Conclusion

Recently, parity odd potentials including Spin-Orbit force are successfully calculated.

Although qualitative behavior of resultant potentials are consistent with phenomenological one, their strength are weaker than the expectation from phase shift analysis.

In this talk :

- ➢We have calculated parity odd potentials including Spin-Orbit force by using 2+1 flavor gauge configuration generated by PACS-CS at mpi=700, 570, 411 MeV.
- ➢We have found that the central, tensor and spin-orbit potentials tend to become stronger as the quark mass decreases.
- ➢ Todo
  We will calculate the phase shifts after the time-slice saturation of the central potential is achieved.
  (The convergence of long range part of the central potential is very slow.)

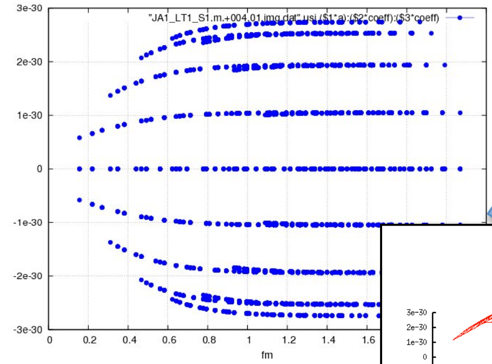
Related Topic :

N. Ishii (10C) "The anti-symmetric LS potential in flavor SU(3) limit from Lattice QCD"

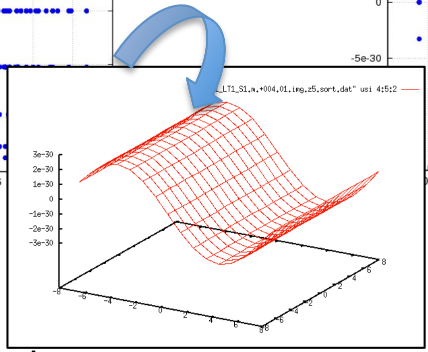
Back Up Slides

3P0

J=1 (A1) L=1 (T1) (imaginary part)

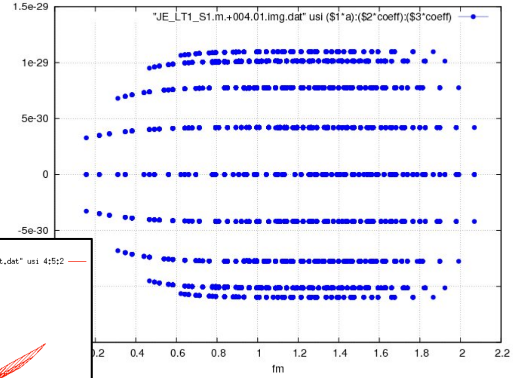


3D-plot



3P2

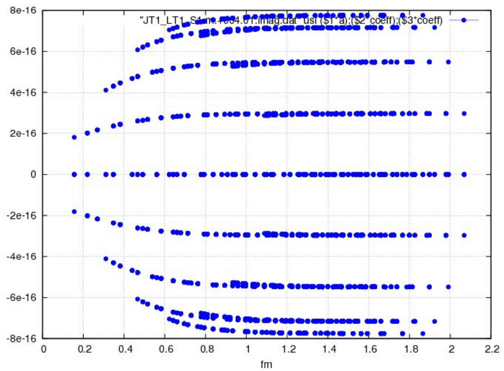
J=2 (E) L=1 (T1)   (imaginary part)

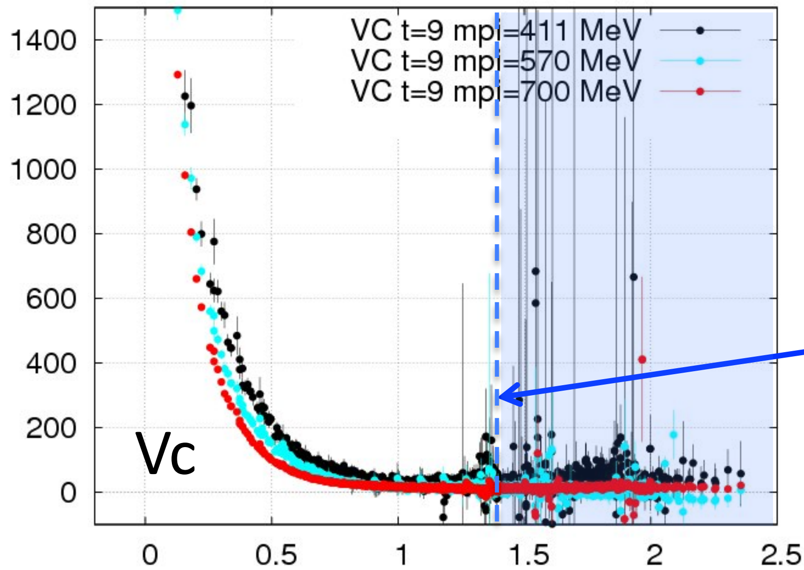


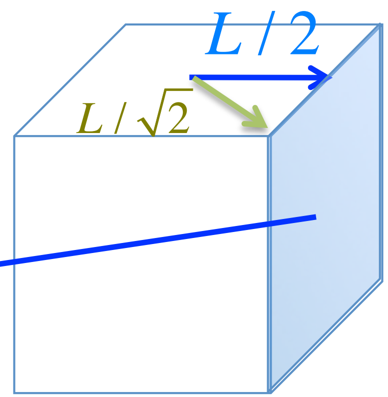
$$\psi(\vec{r}) = R(r)Y_{1m}(\theta,\phi)$$

3P1

J=1 (T1) L=1 (T1)







NBS wave-> potential

$$\left(\frac{\nabla^2}{m_N}+E\right)\phi(\vec{r};E)=K(E;\vec{r})$$

more strict discussion is shown in:

S. Aoki, T. Hatsuda, N. Ishii,
arXiv:0909.5585 [hep-lat]

construct a orthogonal complete set
from NBS wave functions obtained.
(for simplify, we don't take in account degeneracy)

$$\int d^3r\,\tilde{\phi}(\vec{r};E')\phi(\vec{r};E)=\delta_{E,E'}$$

$$U(r,r')\equiv\int dE' K(E',\vec{r})\tilde{\phi}(\vec{r}';E')$$

$$K(E;\vec{r})=\int dr'\,U(\vec{r};\vec{r}')\,\phi(\vec{r}';E)$$

$$\left(\frac{\nabla^2}{m_N}+E\right)\phi(\vec{r};E)=\int dr' U(\vec{r};\vec{r}')\phi(\vec{r}';E)$$

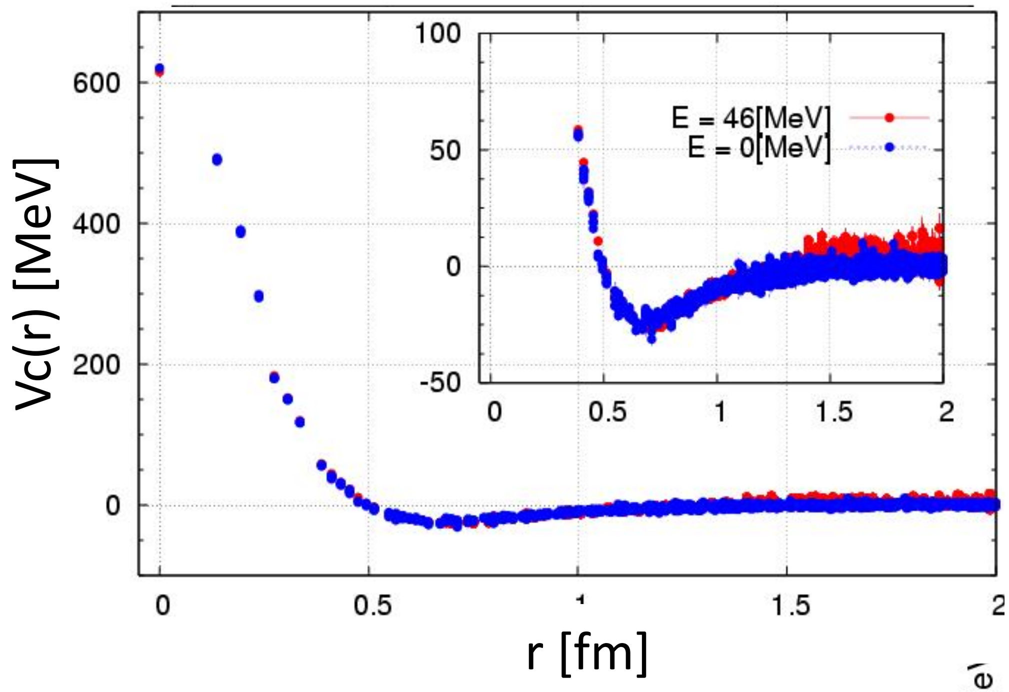
$$k^2=m_N E$$

non-local
Energy independent !

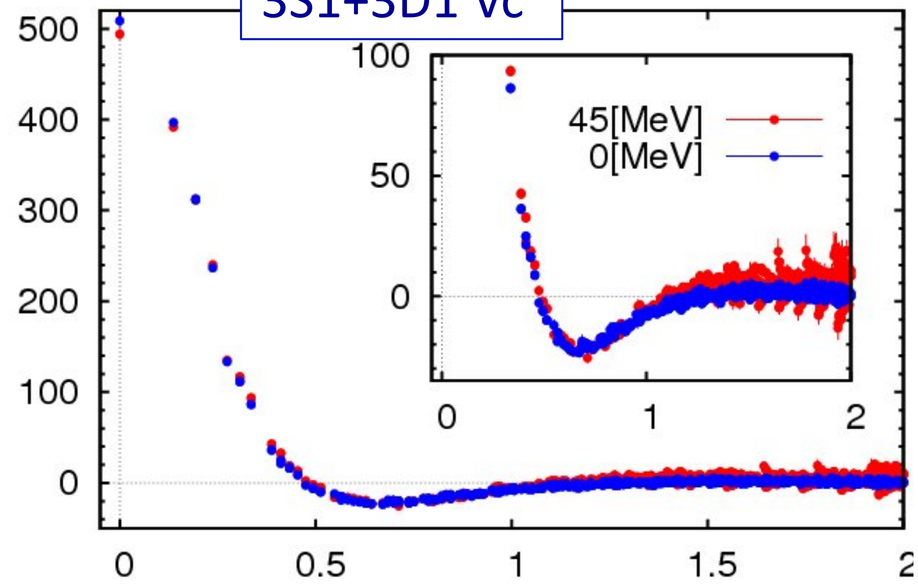
U is Energy independent
by definition !!

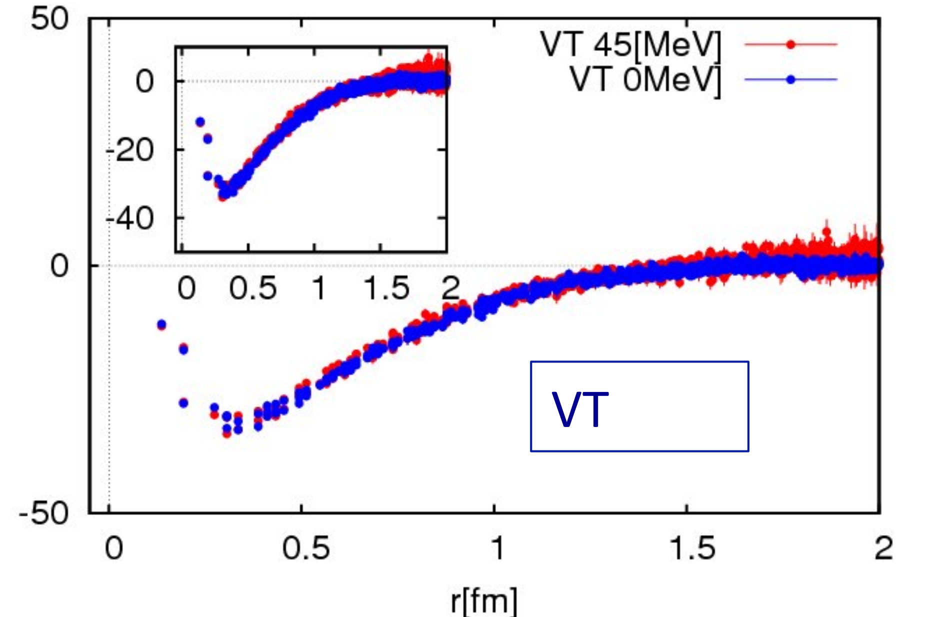
comparison of potentials : 0 MeV and 45 MeV

$^1S_0$ Vc (+-5)



3S1+3D1 Vc





VT

45MeV and 0MeV are consistent

Energy dependence is weak.

Projection operator

$$G_{\alpha,\beta;\alpha'\beta'}(\vec{x},\vec{y},t) \equiv \langle 0 | T[N_\alpha(\vec{x},t) N_\beta(\vec{y},t) \bar{N}_{\alpha'} \bar{N}_{\beta'}] | 0 \rangle \xrightarrow{t\to\infty} \psi_{\alpha,\beta;\alpha',\beta'}(\vec{r})$$

$$\phi^{J=1,S=0,L=\Gamma}(\vec{r}) = P^{S=0}_{\alpha',\beta'} P^{L=\Gamma} P^{J=1} \ \psi_{\alpha,\beta;\alpha',\beta'}(\vec{r})$$

projection of L = Γ

$$P^{(L=\Gamma)} = \frac{d_\Gamma}{24} \sum_{i=1}^{24} \chi^{(\Gamma)}(R_i)^* \ \widehat{R}_i$$

$\exp(-i\omega L)$

projection of J = Γ

$$P^{(J=\Gamma)} = \frac{d_\Gamma}{24} \sum_{i=1}^{24} \chi^{(\Gamma)}(R_i)^* \ \widehat{R}_i$$

$\exp(-i\omega J)$

Projection of total angular momentum J

projection of J (J=T1)

$\exp(-i\omega J)$

$$P^{(J=T_1)} = \frac{3}{24}\sum_{i=1}^{24} \chi^{(T_1)}(R_i)^* \,\widehat{R}_i$$

Projection operator

$$P^{(\Gamma)} = \frac{d_\Gamma}{24}\sum_{i=1}^{24} \chi^{(\Gamma)}(R_i)^* \,\widehat{R}$$

$$P^{J=T_1} G_{\alpha,\beta;\alpha'\beta'}(\vec{x},\vec{y},t) \equiv \langle 0 | T[N_\alpha(\vec{x},t) N_\beta(\vec{y},t) P^{J=T_1} \bar{N}_{\alpha'} \bar{N}_{\beta'}] | 0 \rangle$$

$$= \frac{3}{24}\sum_{i=1}^{24} \chi^{(T_1)}(R_i)^* S_{\alpha',\bar{\alpha}'}(g_i) S_{\beta',\bar{\beta}'}(g_i)\, G_{\alpha,\beta:\bar{\alpha}'\bar{\beta}'}(\vec{x},\vec{y},t)$$

Here, we use..

$$S(g) \equiv \exp(\frac{i}{4}\sigma_{ij}\omega_{ij}), \qquad \sigma_{ij} \equiv -\frac{i}{2}[\gamma_i,\gamma_j], \quad g \in SO(3) \qquad \hat{R}(g)\, q(\vec{x}) = S(g)\, q(g^{-1}\vec{x})$$

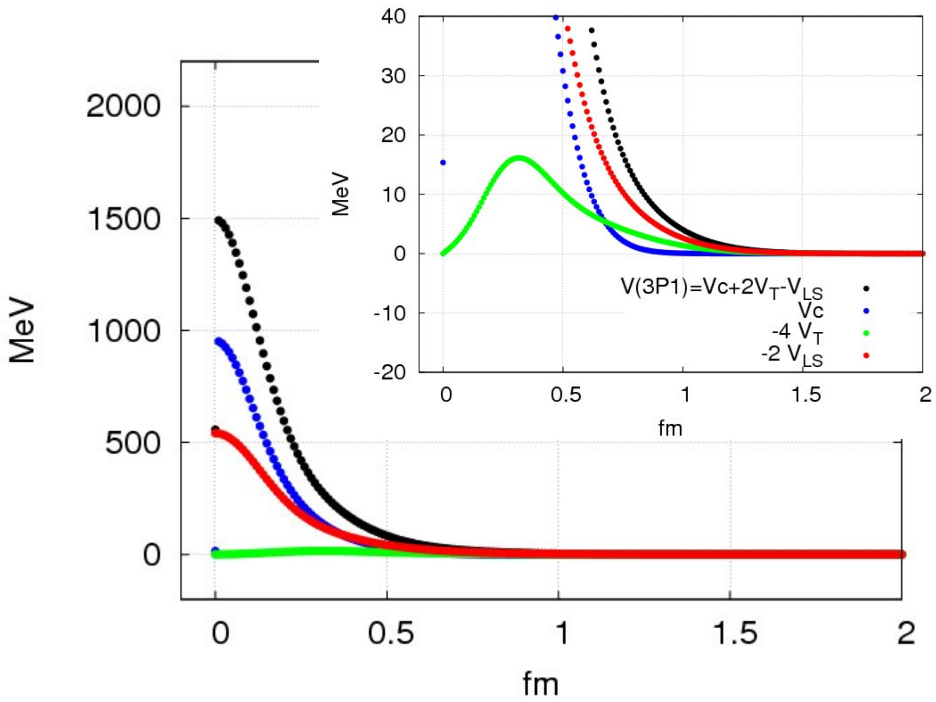
$$\hat{R}(g)\, N_\beta = \sum_{\vec{y}_1,\vec{y}_2,\vec{y}_3} \epsilon_{abc}(q_a^T(g^{-1}\vec{y}_1) S(g)^T C\gamma_5 S(g) q_b(g^{-1}\vec{y}_2)) S(g)\, q_{c,\beta}(g^{-1}\vec{x})$$

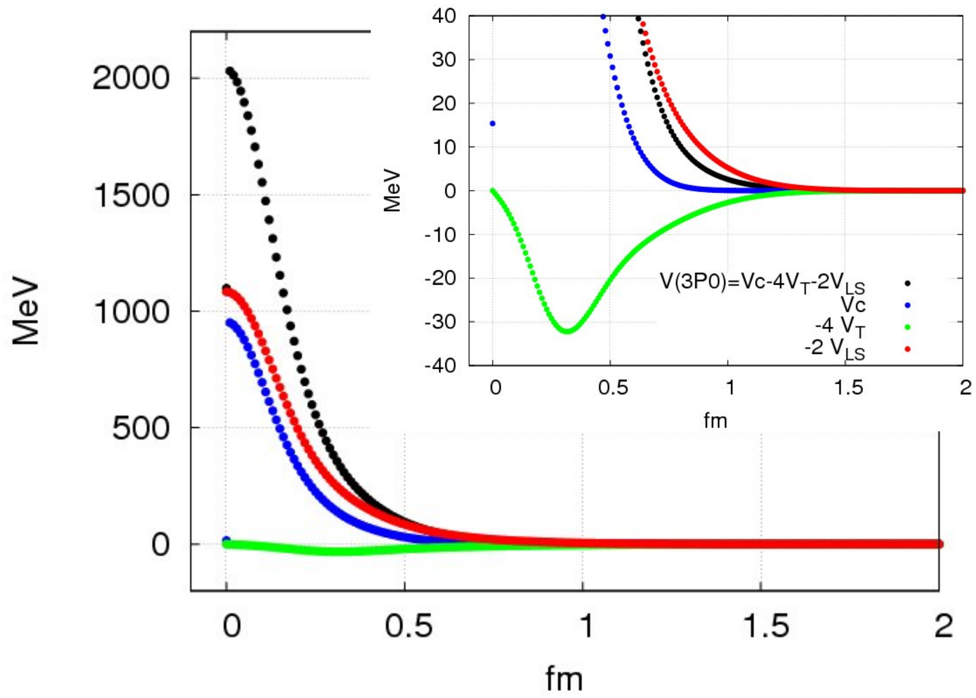
$S(g)^{-1}$

$$= S_{\beta,\bar{\beta}}(g) N_{\bar{\beta}}$$

contribution from potential to each states

$$V(r;{}^3P_1) = Vc(r) + 2V_T(r) - V_{LS}(r) \qquad V(r;{}^3P_0) = Vc(r) - 4V_T(r) - 2V_{LS}(r)$$





- Vc and VT have one-pion exchange, whereas VLS does not have.

- Due to the factor "-4",
  attractive one-pion exchange of VT is expected to dominate long distance
  in the light quark mass region.

$\sim \exp(-m_\pi r)/r$ mpi is heavy (~1GeV) in our lattice set up, p-wave f-wave

	L' \ L	$J-1$	J	$J+1$
p-wave	$J-1$	$V_C - \frac{2(J-1)}{(2J+1)} V_T + (J-1) V_{LS}$	0	$\frac{6\sqrt{J(J+1)}}{(2J+1)} V_T$
	J	0	$V_C + 2V_T - V_{LS}$	0
f-wave	$J+1$	$\frac{6\sqrt{J(J+1)}}{(2J+1)} V_T$	0	$V_C - \frac{2(J+2)}{(2J+1)} V_T - (J+2) V_{LS}$