Lattice QCD studies of multi-strange baryon–baryon interactions

Kenji Sasaki (CCS, University of Tsukuba)

for HAL QCD collaboration

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

S. Aoki (YITP)
B. Charron (Univ. of Tokyo)
T. Doi (RIKEN)
F. Etminan (Univ. of Tsukuba)
T. Hatsuda (RIKEN)
Y. Ikeda (RIKEN)
T. Inoue (Nihon Univ.)
N. Ishii (Univ. of Tsukuba)
K. Murano (YITP)
H. Nemura (Univ. of Tsukuba)
M. Yamada (Univ. of Tsukuba)
**Introduction**

Baryon-baryon interactions are key to understand nuclear structures and astrophysical phenomena.

Inputs for nuclear structure / reaction, astrophysical phenomenon

**NN interaction**

Properties of BB interactions are not known very well except for NN interaction.

Realistic nucleon-nucleon potential is constructed by fitting large amount of NN scattering data.

**YN / YY interaction**

It is important to know structure of hypernucleus and deep inside of neutron star.

It is not easy to access the multi-strangeness interaction experimentally. Experimental data are insufficient to determine parameters in phenomenological YN and YY interaction model.

Lattice QCD results for YN and YY interactions are highly awaited.
Introduction

Strangeness brought the deeper understanding of BB interaction.

Three flavor (u,d,s) world : SU(3) symmetric limit

\[
\begin{align*}
\Sigma &\quad \Delta &\quad \Sigma^0 &\quad \Sigma^+ &\quad \Sigma^- &\quad \Lambda \\
\Xi &\quad \Xi^0 &\quad \Lambda &\quad \Lambda &\quad \Lambda &\quad \Lambda
\end{align*}
\]

Wide variety of BB interaction

Flavor symmetric

\[1\]

H-dibaryon state is expected

Flavor anti-symmetric

\[8_s\]

Pauli forbidden state

\[8_A\]

Almost forbidden state

Spin singlet

Spin triplet

NN sector
Three flavor (u,d,s) world: broken SU(3) symmetry

In this study, we focus on the S=-4 BB interaction, $\Xi\Xi$ interaction.

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Introduction

Similarity and/or dissimilarity to NN system

Flavor component

\[ p = [u\bar{d}] u = \bar{s}u \]
\[ n = [u\bar{d}] d = \bar{s}d \]

Conjugate flavor structure

Octet baryon

Iso-doublet

\[ \Xi^0 = [su] s = \bar{d}s \]
\[ \Xi = [sd] s = \bar{u}s \]

Iso-triplet state

\[ pp = \bar{s}u\bar{s}u \]
\[ pn + np = \bar{s}u\bar{s}d + \bar{s}d\bar{s}u \]
\[ nn = \bar{s}d\bar{s}d \]

Iso-singlet state

\[ pn - np = \bar{s}u\bar{s}d - \bar{s}d\bar{s}u \]
\[ \Xi^0\Xi^0 = \bar{d}s\bar{d}s \]
\[ \Xi^0\Xi + \Xi\Xi^0 = \bar{d}s\bar{u}s + \bar{u}s\bar{d}s \]
\[ \Xi\Xi = \bar{u}s\bar{u}s \]
\[ \Xi^0\Xi - \Xi\Xi^0 = \bar{d}s\bar{u}s - \bar{u}s\bar{d}s \]
Introduction

Potential

Long range part

Meson exchange contribution is dominant

When meson masses decrease, range of potential becomes longer.

Decreasing ud-quark masses means that the potential range extends

Short range part

Otsuki, Tamagaki, Yasuno PTPS (1965)578
Oka, Shimizu and Yazaki NPA464 (1987)

Quark degrees of freedom is important
Quark Pauli principle
Color magnetic interaction (repulsive for all BB channels except for H dibaryon channel)
Introduction

Meson exchange interaction

Leading contributions are given by $\pi$ and $\eta$ exchange contributions

- Weaker attraction
- $\pi$ exchange contribution in $\Xi\Xi$ is much weaker than NN
- $\eta$ meson mass is much heavier than the pion mass

\[
\pi : g^2 f_\pi(r) \\
\eta : \left[\frac{4\alpha - 1}{\sqrt{3}}\right]^2 g^2 f_\eta(r)
\]

Color magnetic interaction (CMI) and repulsive core

One gluon exchange $\longleftrightarrow$ Dominant contribution at short range region

\[
V_{OGE}^{CMI} \propto \frac{1}{m_{q1} m_{q2}} \langle \lambda_1 \lambda_2 \sigma_1 \cdot \sigma_2 \rangle f\left(r_{ij}\right)
\]

If quark mass decreases, CMI contributions are enhanced.
Short range repulsion could be increased...
Iso-singlet channel

We can access the potential of 10 irreducible representation.
Potential of 10 irrep is expected to be repulsive due to the quark Pauli effect.
It is contrary to NN system where deuteron bound state exist.

Iso-triplet channel

Potential of flavor 27 plet is expected to be strongly attractive

$^1S_0$ in NN system is virtual state

- EFT calculation found that the bound $\Xi\Xi$ state in 1S0 channel.
- Meson exchange model calculations.
  M. Yamaguchi PTP105(2001)627
  Y. Fujiwara PPNP 58(2007)439
- Bound $\Xi\Xi$ state was found by Lattice QCD simulation at m=389MeV
  S.R. Beane PRD85(2012)054511

Search for the bound $\Xi\Xi$ state is interesting to understand more about BB interaction
**QCD to hadronic interactions**

HAL QCD method can derive baryon-baryon potential directly from QCD

**QCD Lagrangian**

\[
L_{QCD} = \bar{q} \left( i \gamma_\mu D^\mu - m \right) q + \frac{1}{4} F^a_{\mu \nu} F^{a \mu \nu}
\]

**Lattice QCD simulation**

1. Measure NBS wave function on the lattice

2. Put NBS wave function in Schroedinger eq

The potential through our method reproduce to the phase shift by QCD

**HAL QCD method**

Nambu-Bethe-Salpeter wave function

**Definition: equal time NBS w.f.**

\[
\Psi_{\nu}(E, t-t_0, \vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(t, \vec{x}+\vec{r}) B_j(t, \vec{x}) | E, \nu, t_0 \rangle
\]

- E: Total energy of system
- \( \nu \): other observables which needs to form the complete set

**Four point correlator**

\[
F_{B_1 B_2}(\vec{r}, t) = \langle 0 | T [ B_1(\vec{r}, t) B_2(0, t) (\bar{B}_2 \bar{B}_1)_{t_0} ] \rangle = \sum_n A_n \Psi_n e^{-E_n t}
\]

Local composite interpolating operators

\[
\begin{align*}
p &= u d u \\
n &= u d d \\
\Xi^0 &= s u s \\
\Xi^- &= s d s \\
\Lambda &= \sqrt{\frac{1}{6}} [ d s u + s u d - 2 u d s ] \\
B &= \epsilon^{abc} (q_a^T C \gamma_5 q_b) q_c \\
\Sigma^+ &= -u s u \\
\Sigma^0 &= -\sqrt{\frac{1}{2}} [ d s u + u s d ] \\
\Sigma^- &= -d s d
\end{align*}
\]

NBS wave function has the same asymptotic form with quantum mechanics.
(NBS wave function is characterized from phase shift)

\[
\Psi(t-t_0, \vec{r}) \approx A \frac{\sin(pr + \delta(E))}{pr}
\]
Define the energy-independent potential in Schrödinger equation (most general form)

\[
\left( \frac{k^2}{2\mu} - H_0 \right) \Psi(\vec{x}) = \int U(\vec{x}, \vec{y}) \Psi(\vec{y}) d^3 y
\]

Recent development: Time dependent method.

We replace \( \psi \) to \( R \) defined below

\[
\partial_t R_\alpha(\vec{x}, E) = \partial_t \left( A \Psi_\alpha(\vec{x}, E) e^{-Et} \right) e^{-m_\alpha t} e^{-m_\beta t} \propto -\frac{p_{\alpha}^2}{2\mu_{\alpha}} R_\alpha(\vec{x}, E)
\]

Performing the derivative expansion for the interaction kernel

\[
\left( -\frac{\partial}{\partial t} - H_0 \right) R(\vec{x}) = \int U(\vec{x}, \vec{y}) R(\vec{y}) d^3 y
\]

Taking the leading order of derivative expansion of non-local potential

\[
U(\vec{x}, \vec{y}) \simeq V_0(\vec{x}) \delta(\vec{x} - \vec{y}) + V_1(\vec{x}, \nabla) \delta(\vec{x} - \vec{y}) \quad \cdots
\]

Finally local potential was obtained as

\[
V(\vec{x}) = -\frac{\partial_t R(\vec{r})}{R(\vec{y})} + \frac{1}{2\mu} \frac{\nabla^2 R(\vec{x})}{R(\vec{x})}
\]
Numerical setup

- 2+1 flavor gauge configurations by PACS-CS collaboration.
  - RG improved gauge action & O(a) improved Wilson-clover quark action
  - $\beta = 1.90$, $a^{-1} = 2.176$ [GeV], $32^3 \times 64$ lattice, $L = 2.902$ [fm].
  - $\kappa_s = 0.13640$ is fixed, $\kappa_{ud} = 0.13700$, 0.13727 and 0.13754 are chosen.

- Flat wall source is considered to produce S-wave B-B state.

- The KEK computer system A & B resources are used.

\[
\begin{array}{|c|c|c|c|}
\hline
 & Esb 1 & Esb 2 & Esb 3 \\
\hline
\text{In unit} & & & \\
\text{of MeV} & & & \\
\hline
\pi & 701 \pm 1 & 570 \pm 2 & 411 \pm 2 \\
\hline
K & 789 \pm 1 & 713 \pm 2 & 635 \pm 2 \\
\hline
m_{\pi}/m_K & 0.89 & 0.80 & 0.65 \\
\hline
N & 1585 \pm 5 & 1411 \pm 12 & 1215 \pm 12 \\
\hline
\Lambda & 1644 \pm 5 & 1504 \pm 10 & 1351 \pm 8 \\
\hline
\Sigma & 1660 \pm 4 & 1531 \pm 11 & 1400 \pm 10 \\
\hline
\Xi & 1710 \pm 5 & 1610 \pm 9 & 1503 \pm 7 \\
\hline
\end{array}
\]

u,d quark masses lighter

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$S=-4$ channels
\[ \Xi \Xi \text{ channel } ^3S_1 \ell=0 \]

Belong to Decuplet (10) in SU(3) limit

*almost forbidden state* \rightarrow \text{Repulsive potential}

Quark mass dependence is small

Esb1 : \( m_{\pi} = 701 \text{ MeV} \)
Esb2 : \( m_{\pi} = 570 \text{ MeV} \)
Esb3 : \( m_{\pi} = 411 \text{ MeV} \)

Flavor decuplet
Belong to 27 plet in SU(3) limit

Qualitative behavior is similar to NN potential

Short range repulsion is increasing,
but no clear difference between potentials measured in each configurations
Phase shift shows an attractive interaction
Attraction becomes weaker as decreasing light quark mass

Fit function: \[ f(r) = A \frac{e^{-mr}}{r} (1 - e^{-Br^2}) + \sum C_i e^{-D_{ir}^2} \]
m is fixed to the measured pion mass

Preliminary
Summary

- We showed preliminary results of $S= -4$ BB potentials with $L=3\text{fm}$.
- Qualitatively, potentials are not so much different from the potential in SU(3) limit reported by Prof. Inoue.
- We can see quark mass dependence of potentials
  - Enhancement of short range core.
  - Potential range is not clearly extended.

Future works

- Increase statistics
- Separation of tensor potential in spin triplet channel
- Try to find whether $^1S_0 \Xi \Xi$ bound state is exist or not.
backups
channel $^3S_1$ I=0

$E_{b1}$: $m_\pi = 701$ MeV
$E_{b2}$: $m_\pi = 570$ MeV
$E_{b3}$: $m_\pi = 411$ MeV

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\[
\Xi\Xi \text{ channel } ^1S_0 \ I=1
\]

\[\begin{align*}
\text{Esb} 1 : m_\pi &= 701 \text{ MeV} \\
\text{Esb} 2 : m_\pi &= 570 \text{ MeV} \\
\text{Esb} 3 : m_\pi &= 411 \text{ MeV}
\end{align*}\]
\( \Xi \Xi \) channel \(^3\)\(S_1 \) \( I=0 \)

\[ \text{Esb1 : } m_\pi = 701 \text{ MeV} \]
\[ \text{Esb2 : } m_\pi = 570 \text{ MeV} \]
\[ \text{Esb3 : } m_\pi = 411 \text{ MeV} \]

Belong to Decuplet in SU(3) limit

almost forbidden state

Repulsive potential

Quark mass dependence is small

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Channel $^1S_0$ $l=1$

Belong to 27plet in SU(3) limit
Qualitative behavior is similar to NN potential

$E_{sb1} : m_\pi = 701$ MeV
$E_{sb2} : m_\pi = 570$ MeV
$E_{sb3} : m_\pi = 411$ MeV