Pion-Nucleon Scattering in Lattice QCD

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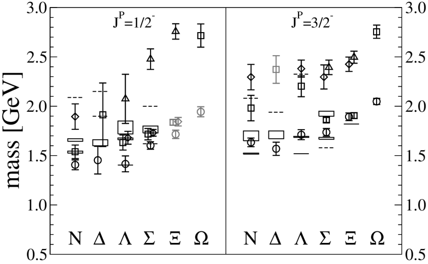
Mainz, Lattice2013

For details see: C.B. Lang, VV Phys. Rev. D 87, 054502 (2013)

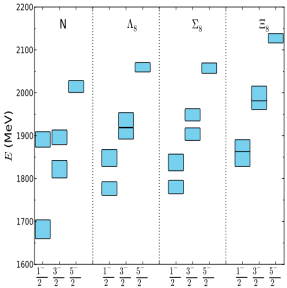
Outline

- **Motivation.** The nucleon spectrum is not well reproduced by lattice calculation and needs a deeper investigation.
- **Pion-Nucleon scattering.** Issues and techniques used to deal with a meson-baryon system on the lattice.
- **Results**. The study of multi-particle systems drastically changes the observed scenario.

Baryon spectroscopy on the lattice



[Engel et al. PRD 87, 074504 (2013)]

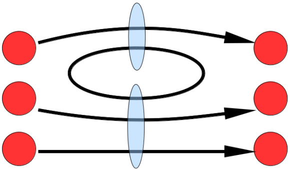


[Edwards et al. PRD 87, 054506 (2013)]

Excited states still represent an outstanding challenge!

N^- spectrum

Nucleon excited states are not stable under strong interactions and their resonant nature has to be taken into account.



$N(1535) \to N\pi$	35-55%
$N(1535) \to N\eta$	32-52%

$N(1650) \to N\pi$	50-90%
$N(1650) \to N\eta$	5-15%
$N(1650) \to \Lambda K$	3-11%

$N\,\pi$ scattering: the issues

- **Many diagrams**

Distillation

- **Backtracking quark lines**

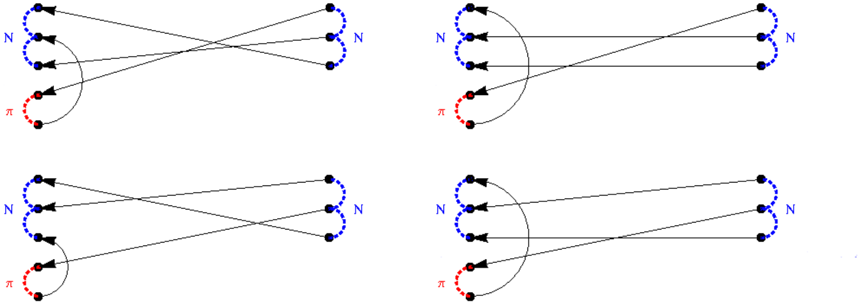
Variational method

- **Many energy levels**

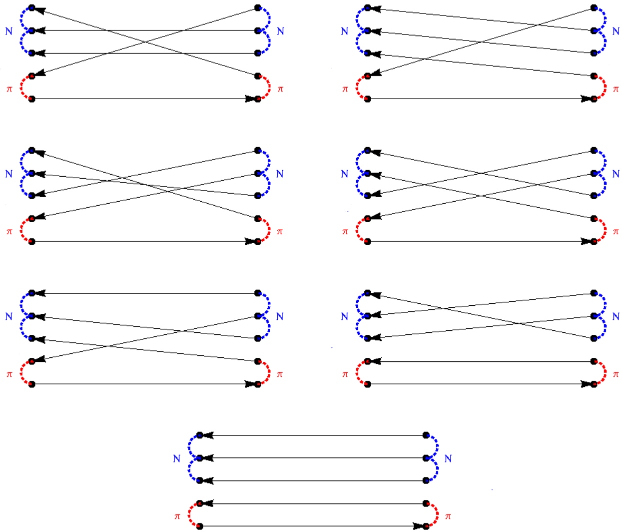
- **Resonances**

Phase shift analysis

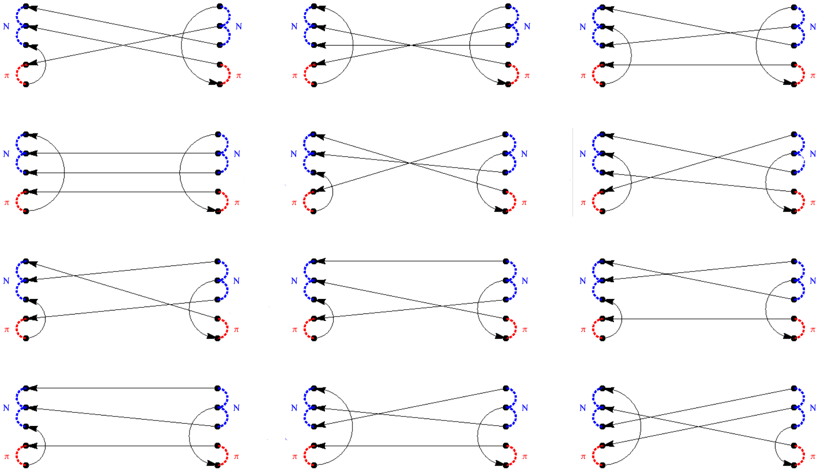
Coupled $N\pi \to N$ system: the diagrams



Coupled $N\pi \to N\pi$ system: the diagrams



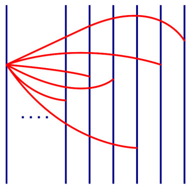
Coupled $N\pi \rightarrow N\pi$ system: the diagrams



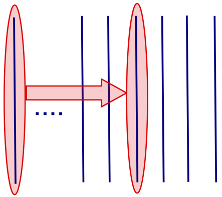
Distillation Method [Peardon et al, Phys. Rev. D 80 (2009) 054506]

The smearing operator is approximated with a truncated expansion written in terms of eigenvectors of the 3D Laplacian

$$q(x) \longmapsto S(x,x')q(x') = \sum_{i=1}^{N_V} v_i(x)v_i^\dagger(x')q(x')$$



vs



$$M^{-1}(x,y)$$

$$\tau_{ij} = v_i^\dagger(x)M^{-1}v_j(y)$$

Extract the excited states: variational method
[Michael, NPB 259 (1985) 58] [Luescher, Wolff. NPB 339 (1990) 222]

- Use several interpolators χ_i to construct a basis with minimum overlap.
- Compute the cross correlations $C_{ij}(t) = \langle \chi_i(t)\chi_j^\dagger(0)\rangle$.
- Solve the generalized eigenvalue problem

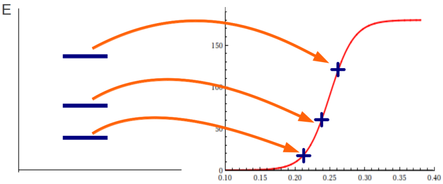
$$C(t)u^{(n)} = \lambda^{(n)}C(t_0)u^{(n)}.$$

- Obtain energy levels from the eigenvalues

$$\lim_{t\to\infty}\lambda^{(n)}(t,t_0) = e^{-E_n(t-t_0)}.$$

Phase shift analysis [Luescher, Commun. Math. Phys. 104, 177 (1986)]

Asymptotically only stable states can be observed and resonances have to be identified by their impact on the finite volume states.



Luescher formula connects the discrete spectrum in finite volume with the elastic scattering phase shift in infinite volume

$$\det[e^{2i\delta}(M(q)-i)-(M(q)+i)]=0$$

Simulation setting

- Wilson Clover action with 2 degenerate flavours.
- Configurations: 280.
- Lattice size: $16^3 \times 32 \ (a = 0.12\,\mathrm{fm})$.
- Pion masses: 266 MeV.

Configurations: [A. Hasenfratz et al., Phys. Rev. D 78 (2008) 054511]
Perambulators: [C.B. Lang et al. Phys. Rev. D 84 (2011) 054503]

Interpolators: single particle

$$N_i = \sum_x P_\pm \epsilon_{abc}\, \Gamma_1^i\, u_a(x)\, [u_b^T(x)\Gamma_2^i\, d_c(x) - d_b^T(x)\Gamma_2^i\, u_c(x)]$$

$$(\Gamma_1^i, \Gamma_2^i) = \{(\mathbf{1},\, C\gamma_5),\, (\gamma_5,\, C),\, (i\mathbf{1},\, C\gamma_4\gamma_5)\}$$

$$P_\pm = (1 \pm \gamma_0)/2$$

$$\pi^+ = \sum_x \bar{d}(x)\gamma_5\, u(x) \qquad \pi^0 = \sum_x \{\bar{d}(x)\gamma_5\, d(x) - \bar{u}(x)\gamma_5\, u(x)\}$$

Interpolators: two particles

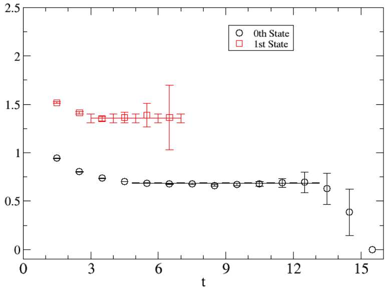
The $N\pi$ (4+1) quark interpolator is built from single particle operators individually projected to zero momentum

$$N\pi(\mathbf{p} = \mathbf{0}) = \gamma_5 \, N(\mathbf{0}) \, \pi(\mathbf{0})$$

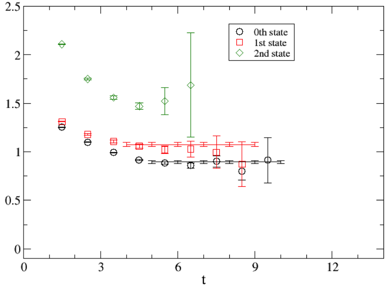
and an additional isospin projection is needed to guarantee the overlap with the nucleon states $1/2^{\pm}$:

$$O_{N\pi} = p\pi_0 + \sqrt{2}\, n\pi_+$$

Single particle sector: N_+ and N_-



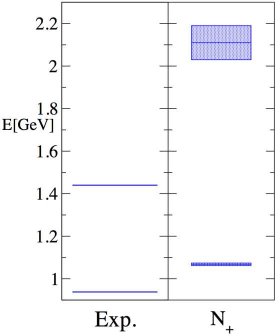
N_+

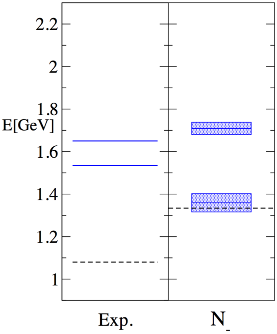


N_-

[C.B. Lang, VV Phys. Rev. D 87, 054502 (2013), arXiv:1212.5055]

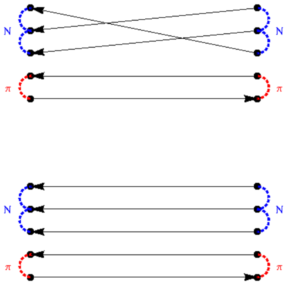
Single particle sector: N_+ and N_-

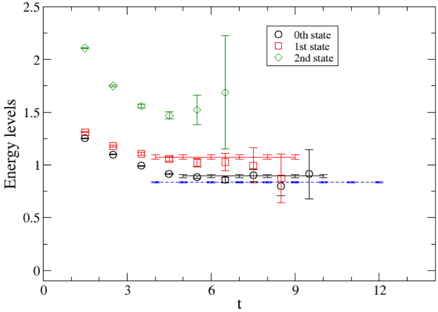




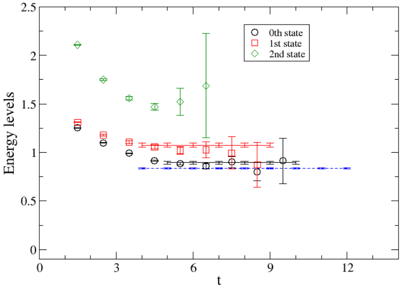
$N\pi$ *s-wave*

Non interacting $N\pi$: the threshold

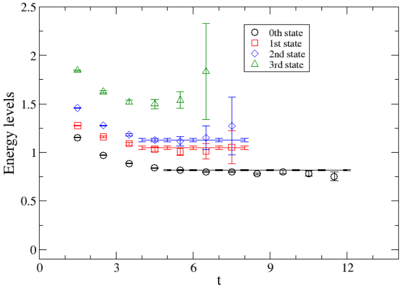




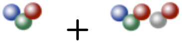
$N\pi$ in s-$wave$



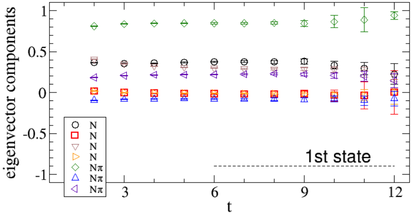
Energy levels
t
0th state
1st state
2nd state
0
0.5
1
1.5
2
2.5
0
3
6
9
12



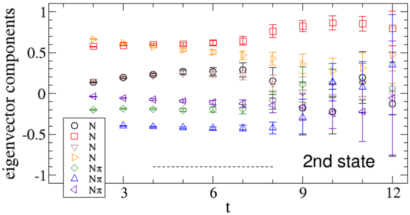
Energy levels
t
0th state
1st state
2nd state
3rd state
0
0.5
1
1.5
2
2.5
0
3
6
9
12



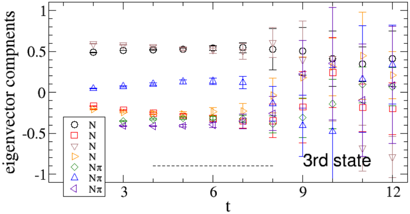
$N\pi$ in *s-wave*



eigenvector components
1
0.5
0
-0.5
-1
3
6
9
12
t
N
N
N
N
Nπ
Nπ
Nπ
1st state



eigenvector components
1
0.5
0
-0.5
-1
3
6
9
12
t
N
N
N
N
Nπ
Nπ
Nπ
2nd state



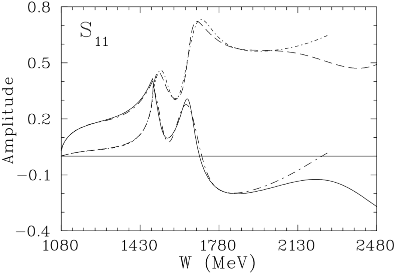
eigenvector compnents
1
0.5
0
-0.5
-1
3
6
9
12
t
N
N
N
N
Nπ
Nπ
Nπ
3rd state

Energy levels: interpretation

For a system of interacting $N\pi$, the energy levels can be computed inverting the Luescher relation

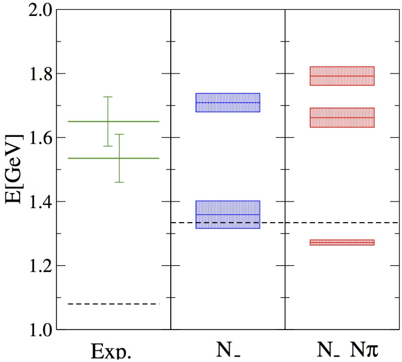
$$\tan\delta(q) = \frac{\pi^{3/2} q}{Z_{00}(1, q^2)} \qquad \textit{for} \qquad \mathbf{P} = \mathbf{0}$$

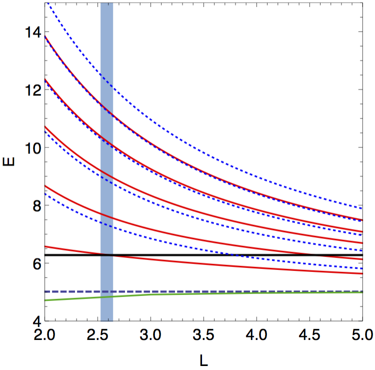
but a phase shift parametrization has to be assumed.



[Arndt et al., Phys.Rev.C74, 045205(2006)]

Energy levels: interpretation



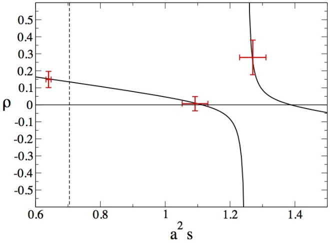


Resonace parameters

The resonance parameters are extracted fitting the phase shift. Assuming two elastic resonances with identical coupling we obtain

$$\rho(s) = \sqrt{s}\,\Gamma(s)\cot\delta(s)$$

$$m_R = 1.678 \text{ GeV}$$
$$m_R = 1.873 \text{ GeV}$$



[For a review on different approaches: M. Doering - Sat 3 Aug 09:45.]

Summary

- Excited states still represent an outstanding challenge for lattice QCD.
- Two-particle system and phase shift analysis provide new information on the resonances of the QCD spectrum.
- A lot more has to be done!