Omega-Omega Interaction on the Lattice

M.Yamada for HAL QCD Collaboration
University of Tsukuba

S. Aoki, C. Bruno, T. Doi, F. Etminan, T. Hatsuda, Y. Ikeda,

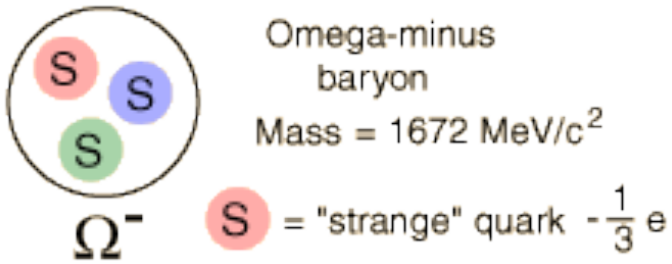
T. Inoue, N. Ishii, K. Murano, H. Nemura, K. Sasaki

Lattice 2013 August 2, 2013, Mainz, Germany

Introduction

My target

Ω-Ω interaction



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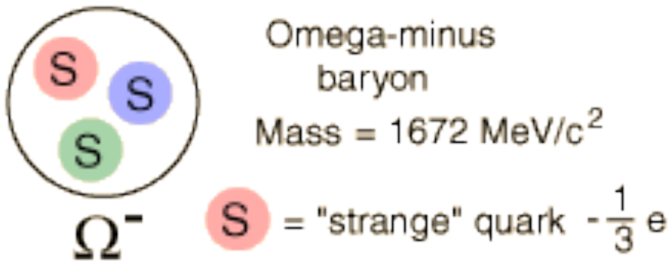

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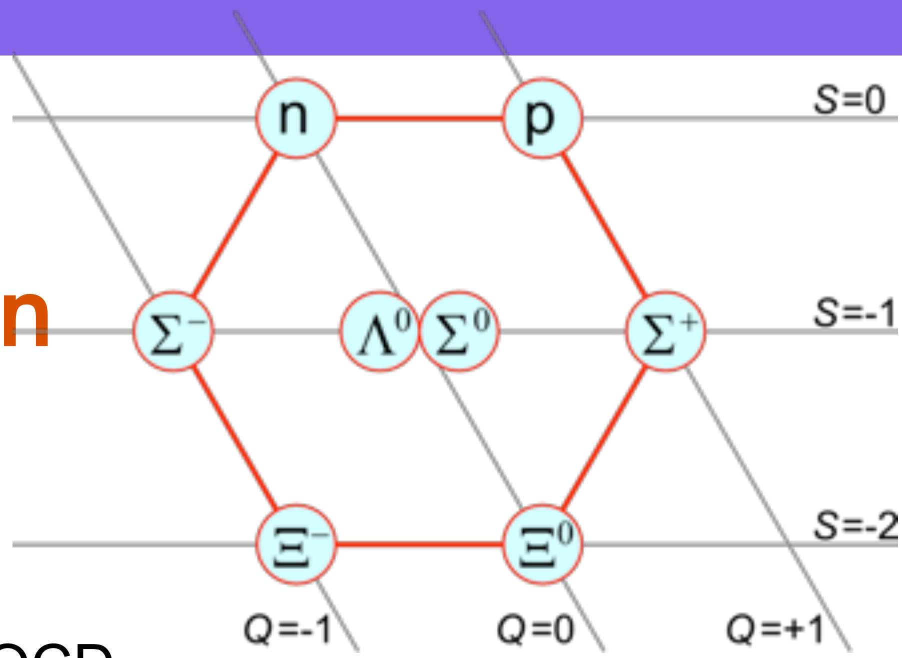

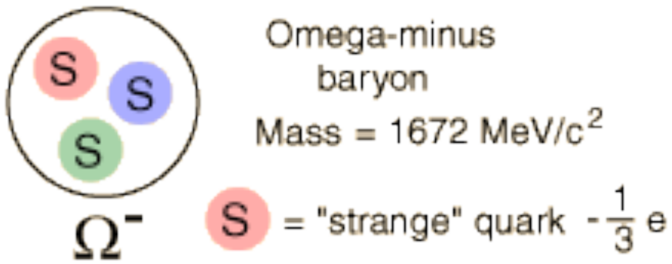

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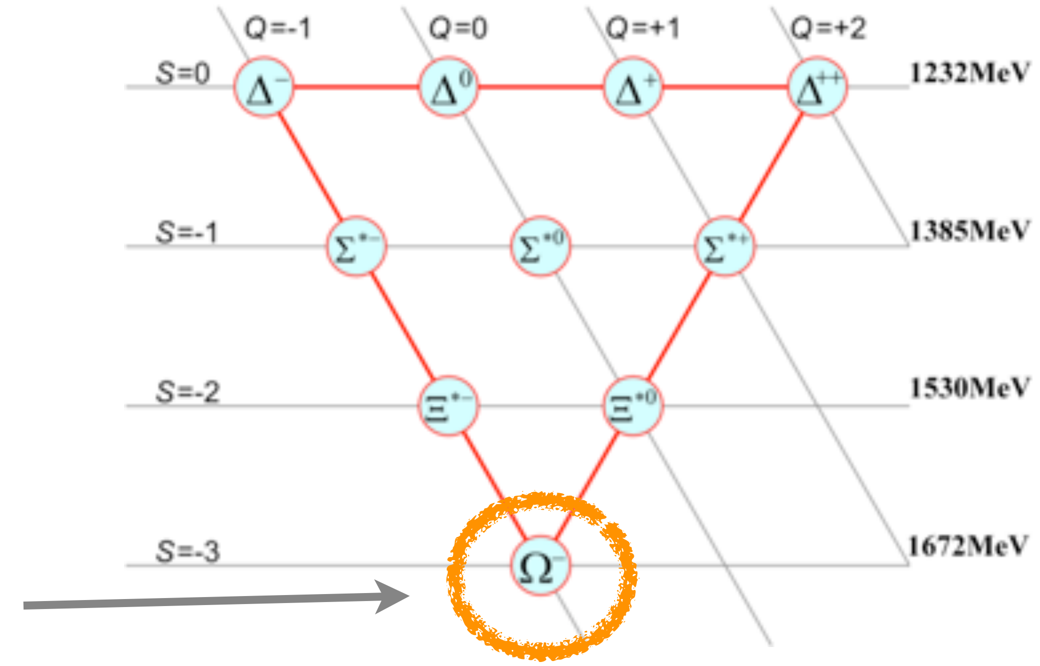

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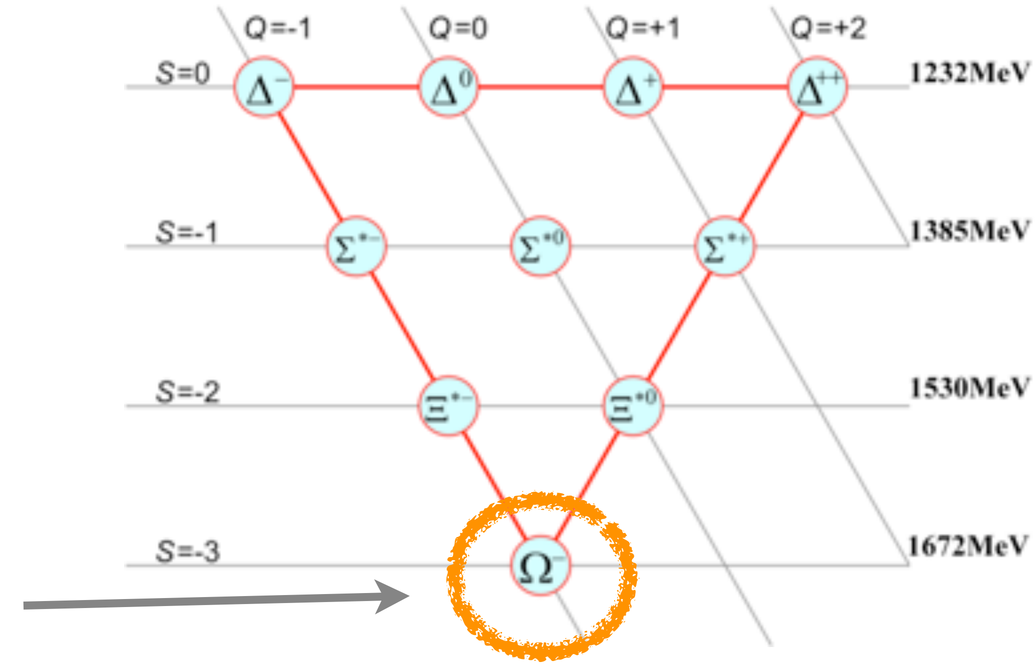

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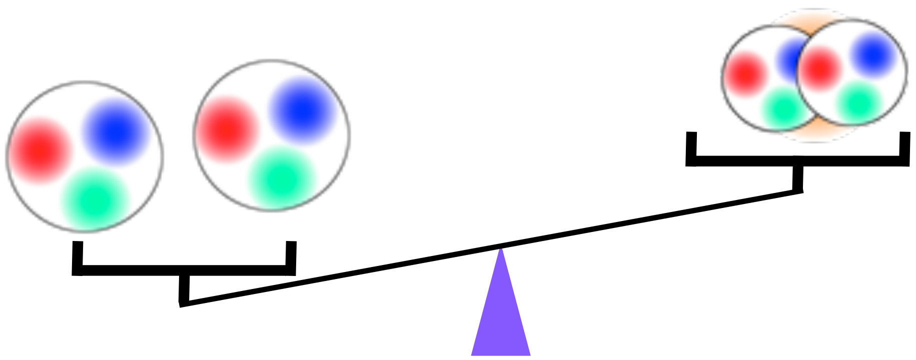

or

interaction energy

$$\Delta M_{\Omega\Omega} = E_{\Omega\Omega} - 2M_{\Omega} = -166\mathrm{MeV}$$

(SU(3) Chiral Quark Model)

$$E_{\Omega\Omega} \equiv 2\sqrt{k^2 + M_{\Omega}^2}$$

[Z.Y.Zhang et al. Phys.Rev.C .61, 065204]
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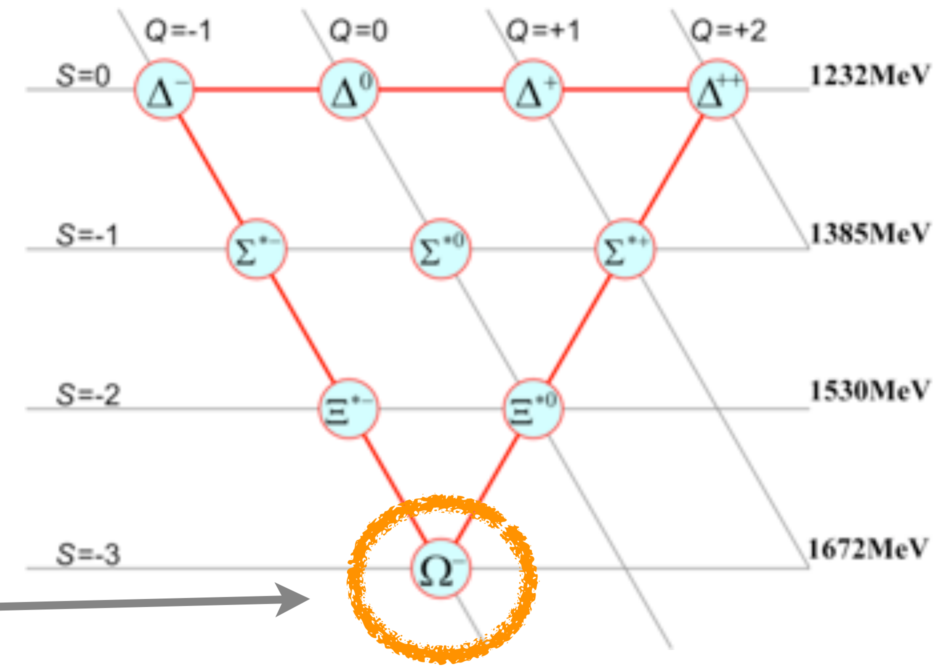

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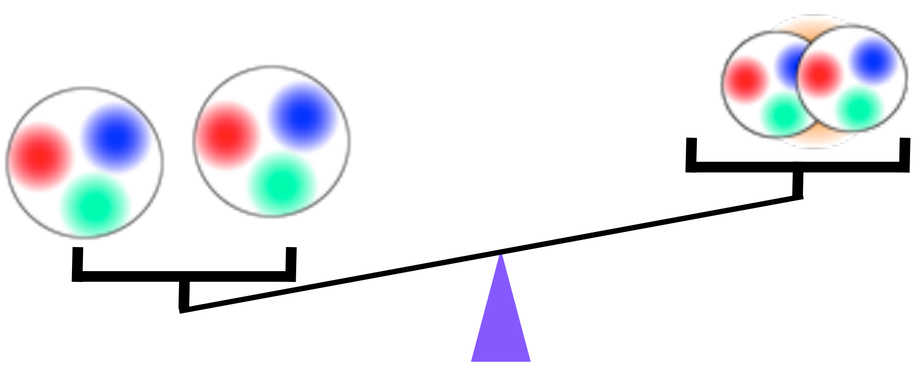

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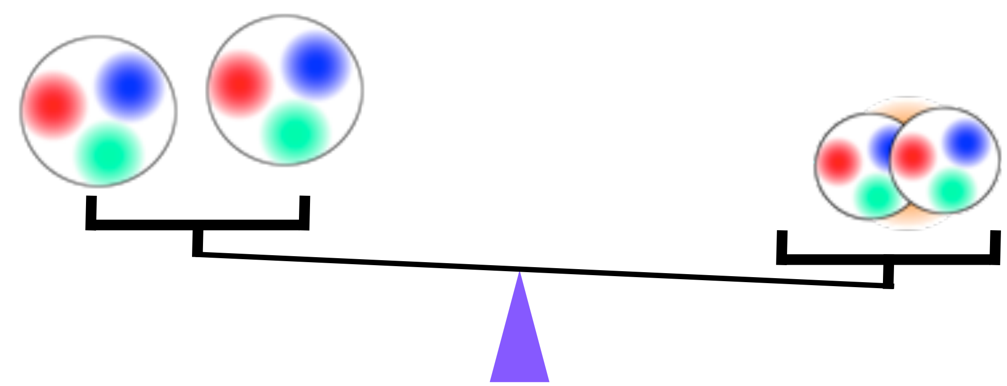

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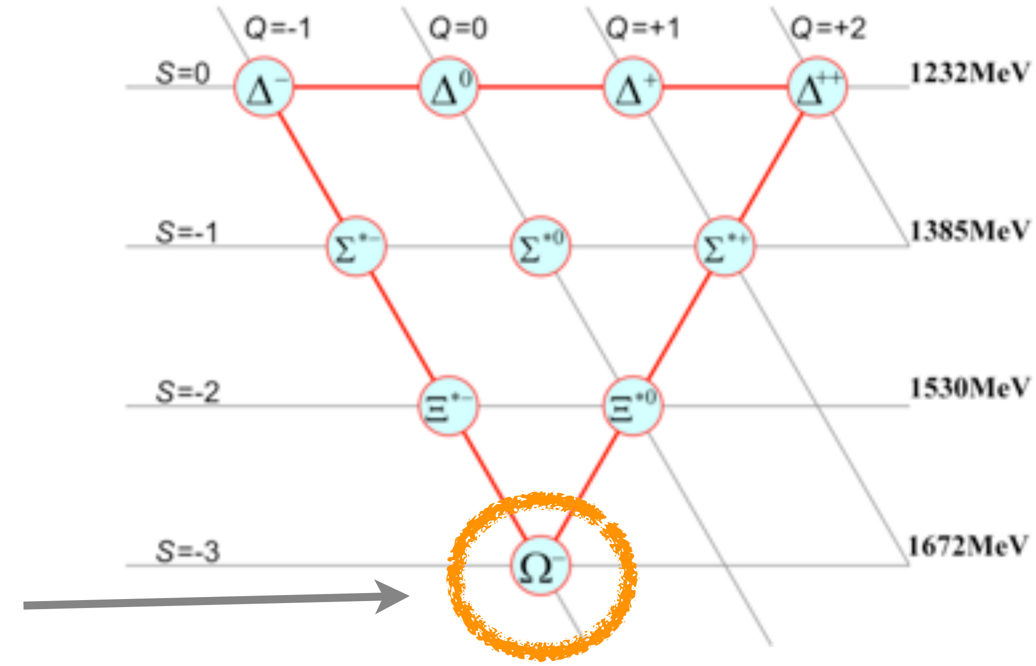

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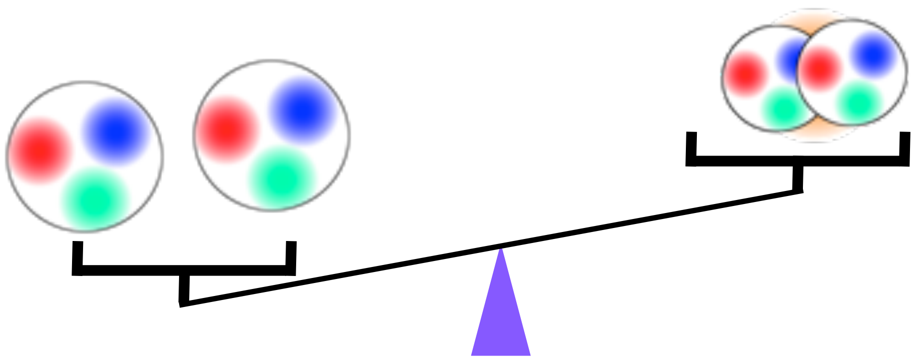

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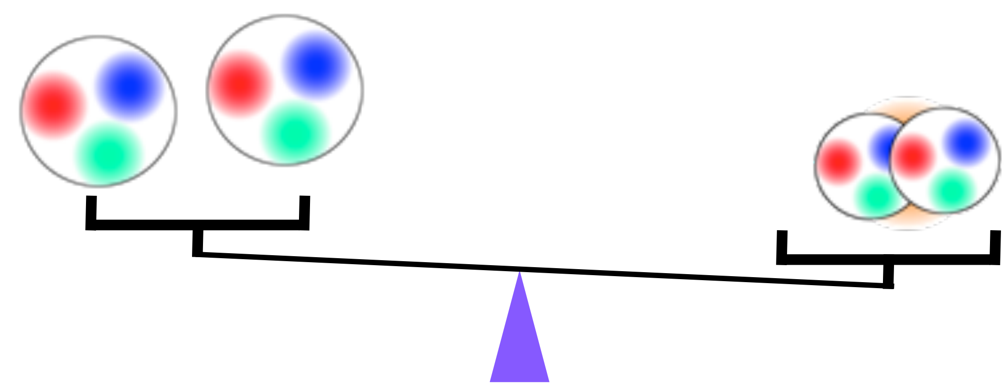

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Report from another group (Lattice QCD simulation)

Lüscher's method [Lüscher CMP105(86)153, NPB354(91)531]

Buchoff et al. : L=3fm Ω=1628[MeV]

J=0 : weak repulsion a= - 0.16 ± 0.22 fm [arXiv:1201.3596]

J=2 : strong repulsion

J.Wasem @Lattice2012

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no definite conclusion, attraction or repulsion

determine a nature of J=0 Omega-Omega interaction, attractive or repulsive

Out line

- Formulation

1. Construction of the potential [HAL QCD method]
2. time dependent method
3. Symmetry of Omega-Omega system

- Lattice QCD Simulation results

4. Potential
5. phase shift

- Conclusion & Future work

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Construction of the potential

[S. Aoki,etal. Prog. Theor. Phys., 123:89]

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Basic Idea

Schrödinger eq

Commonly used method

in Quantum mechanics

Potential(Given) → wave function(result)

via Schrödinger eq

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Potential(Result) ← via Schrödinger eq ← wave function

We can get

Given by Lattice QCD calculation

Construction of the potential

Q. What is the wave function in QCD ?

Construction of the potential

Q.What is the wave function in QCD ?

A. Nambu-Bethe-Salpeter(NBS) wave function

Ω interpolating filed

$$\psi_{\boldsymbol{k}}(\boldsymbol{r}) \equiv \langle 0 | \, \boldsymbol{\Omega}(\boldsymbol{r}) \boldsymbol{\Omega}(\boldsymbol{0}) \, | \overline{\boldsymbol{\Omega}}(\boldsymbol{k}) \overline{\boldsymbol{\Omega}}(-\boldsymbol{k}); \boldsymbol{in} \rangle$$

same quntum number Ω-Ω

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Because

NBS wave has the same asymptotic form of the scattering wave in quantum mechanics.

Wave function↔phase shift↔S-matrix

$$\psi_{k}(r) \simeq e^{i\delta(k)} \frac{\sin(kr - \frac{l\pi}{2} + \delta(k))}{kr}$$

[C.-J.D Lin et al., NPB619(2001)467.]

Energy independent potential U(r,r') is defined from NBS wave function.

$$(\frac{k^2}{m} + \frac{1}{m}\nabla^2)\psi_{k}(r) = \int d^3r' U(r,r')\psi_{k}(x')$$

because of

This potential reproduces the phase shift faithfully

we can extract an interaction kernel (potential) which is defined through the NBS wave function which gives the correct scattering phase shift at asymptotic state.

Construction of the potential

Extraction of the NBS wave from Lattice QCD

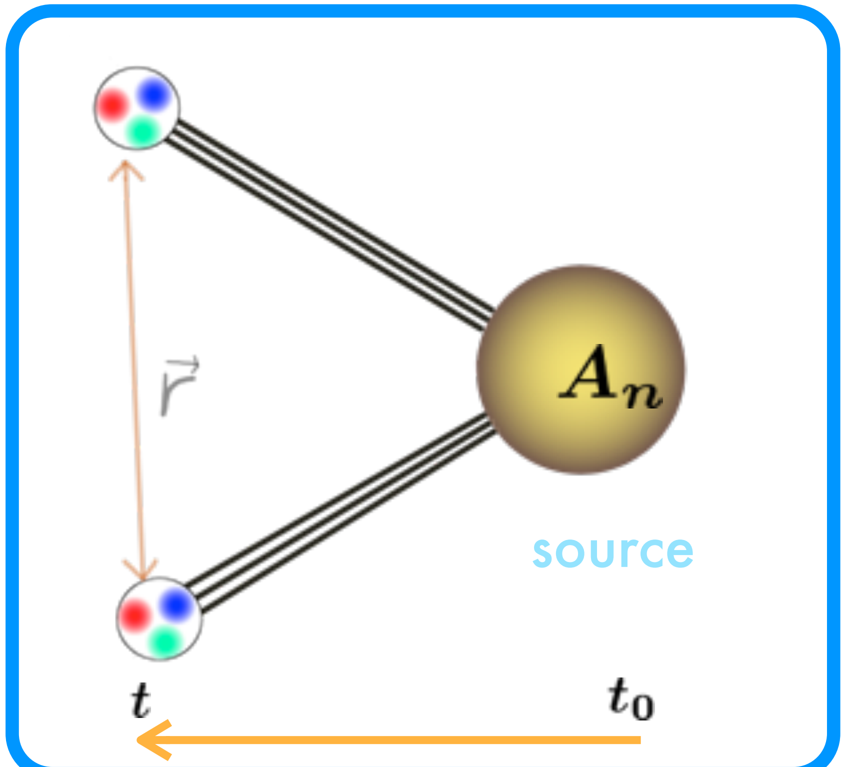
$$C_{\Omega\Omega}(\vec{x},\vec{y},t,t_0) \equiv \langle 0|\, \Omega(\vec{x},t)\Omega(\vec{y},t)\overline{\Omega}(0,t_0)\overline{\Omega}(0,t_0)\, |0\rangle$$

$$= \sum_n \langle 0|\, \Omega(\vec{x},t)\Omega(\vec{y},t)\, |n\rangle\, e^{-E_n(t-t_0)}\, \langle n|\, \overline{\Omega}(0,t_0)\overline{\Omega}(0,t_0)\, |0\rangle$$

$\psi_{k_n}(\vec{x}-\vec{y},n)$ $\qquad$ A_n

$$= \sum_n A_n \psi_{k_n}(x-y,n) e^{-E_n(t-t_0)} + \cdots$$

Image



inelastic contributions

Excited states are suppressed exponentially at large $t - t_0$
We can get the NBS wave at ground state

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$$1 = \sum_n |n\rangle\langle n|$$

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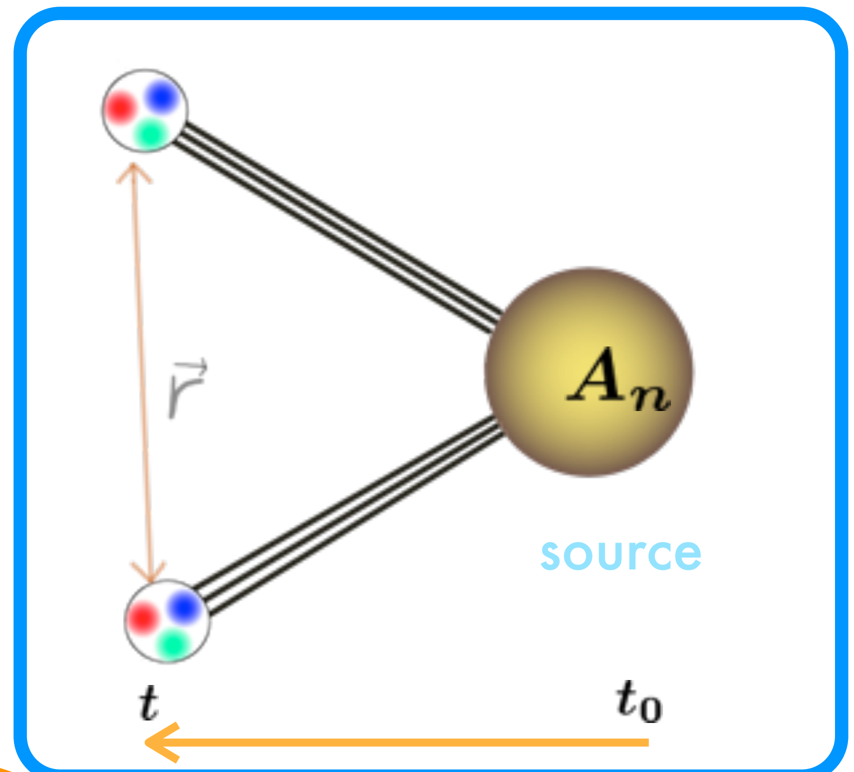

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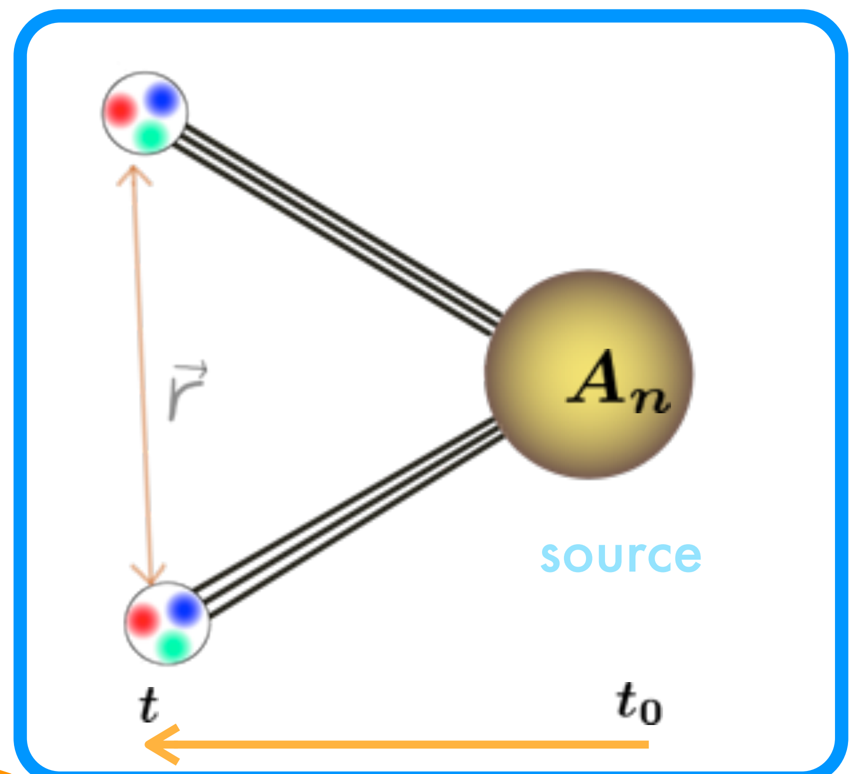

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An origin of large statistical errors

HAL QCD method

[N.Ishii et al.,PLB712(2012)437.]

Time dependent Schrodinger-type equation

$$\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2} + \frac{1}{m}\nabla^2 - \frac{\partial}{\partial t}\right)R = \int dr' U(r,r') R$$

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Time dependent Schrodinger-type equation

R-correlator is defined as

$$W_n \equiv 2\sqrt{m^2 + \overrightarrow{k}_n^2} - 2m$$

$$R \equiv \frac{\Psi(r,t)}{e^{-2mt}} = \sum_n \phi_n(r) e^{-W_n t}$$

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①

$$-\frac{\partial}{\partial t}R = \sum_n W_n \phi_n(r) e^{-W_n t} = \sum_n \left(\frac{\overrightarrow{k}_n^2}{m} - \frac{W^2}{4m}\right)\phi_n(r) e^{-W_n t}$$

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① From identity

$$W_n = \frac{\vec{k}_n^{\,2}}{m} - \frac{W_n^2}{4m}$$

$$\frac{W_n^2}{4m} = \frac{1}{4m}\left(4m^2 + 4\vec{k}_n^{\,2} + 4m^2 - 8m\sqrt{m^2 + \vec{k}_n^{\,2}}\right)$$

$$= 2m + \frac{\vec{k}_n^{\,2}}{m} - 2\sqrt{m^2 + \vec{k}_n^{\,2}}$$

$$= \frac{\vec{k}_n^{\,2}}{m} - W_n$$

$$\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2} + \frac{1}{m}\nabla^2 - \frac{\partial}{\partial t}\right) R = \int dr' U(r,r') R$$

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$$= \sum_n \left(\frac{\vec{k}_n^{\,2}}{m} - \frac{1}{4m}\frac{\partial^2}{\partial t^2}\right) \phi_n(r) e^{-W_n t}$$

②

$$= \left(-\frac{1}{m}\nabla^2 - \frac{1}{4m}\frac{\partial^2}{\partial t^2}\right) R + \int dr' U(r,r') R$$

$$\left(\frac{1}{4m}\frac{\partial^2}{\partial t^2} + \frac{1}{m}\nabla^2 - \frac{\partial}{\partial t}\right) R = \int dr' U(r,r') R$$

② NBS wave function satisfies Schorodinger eq.

$$\left(\frac{k^2}{m} + \frac{1}{m}\nabla^2\right)\psi_k(r) = \int d^3r' U(r,r') \psi_k(x')$$

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[N.Ishii et al.,PLB712(2012)437.]

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We can calculate energy independent non-local potential
without relying on the ground state saturation!

Symmetry of Ω-Ω

Ω operator is defined as

$$\Omega_{\alpha,k} \equiv \varepsilon^{abc} s^a (C\gamma_k) s^b s^c_\alpha$$

blue is $\mathbf{spin1}$ index, red is $\mathbf{spin\frac{1}{2}}$ index

We treat spin 3/2 made from spin 1 and spin 1/2 linear combination by using highest weight construction

- one Ω case($\mathbf{spin\frac{3}{2}}$)

$$\mathbf{spin\frac{1}{2}} \otimes \mathbf{spin1} = \mathbf{spin\frac{3}{2}} \oplus \mathbf{spin\frac{1}{2}}$$

- consider two Ω case (Ω-Ω interaction)

$$\mathbf{spin\frac{3}{2}} \otimes \mathbf{spin\frac{3}{2}} = \mathbf{spin3} \oplus \mathbf{spin2} \oplus \mathbf{spin1} \oplus \mathbf{spin0}$$

Symmetry of Ω-Ω

Conserved quantity J, J_z, P

- parity $P = (-1)^L$
- quantum spin $(-1)^{S+1}$

fermionic condition

$$(-1)^L \times (-1)^{S+1} = -1 \quad \Leftarrow \quad \psi_1\psi_2 = -\psi_2\psi_1$$

Which L ,S is allowed at J^P

	P=+	P=-
J=0	S=0 L=0 , S=2 L=2	S=1 L=1 , S=3 L=3
J=1	S=2 L=2	S=1 L=1 , S=3 L=3
J=2	S=2 L=0 , S=0 L=2 , S=2 L=2 , S=2 L=4	S=1 L=1 , S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
J=3	S=2 L=2 , S=2 L=4	S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
J=4	S=2 L=2 , S=0 L=4 , S=2 L=4 , S=2 L=6	S=3 L=1 , S=1 L=3 , S=3 L=3 , S=1 L=5 , S=3 L=5 , S=3 L=7

	P=+	P=-
J=0	S=0 L=0 , S=2 L=2	S=1 L=1 , S=3 L=3
J=1	S=2 L=2	S=1 L=1 , S=3 L=3
J=2	S=2 L=0 , S=0 L=2 , S=2 L=2 , S=2 L=4	S=1 L=1 , S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
J=3	S=2 L=2 , S=2 L=4	S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
J=4	S=2 L=2 , S=0 L=4 , S=2 L=4 , S=2 L=6	S=3 L=1 , S=1 L=3 , S=3 L=3 , S=1 L=5 , S=3 L=5 , S=3 L=7

at source

at sink

	P=+	P=-
J=0	S=0 L=0 , S=2 L=2	S=1 L=1 , S=3 L=3
J=1	S=2 L=2	S=1 L=1 , S=3 L=3
J=2	S=2 L=0 , S=0 L=2 , S=2 L=2 , S=2 L=4	S=1 L=1 , S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
J=3	S=2 L=2 , S=2 L=4	S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
J=4	S=2 L=2 , S=0 L=4 , S=2 L=4 , S=2 L=6	S=3 L=1 , S=1 L=3 , S=3 L=3 , S=1 L=5 , S=3 L=5 , S=3 L=7

at source

L=0 $\Leftarrow$ We use wall source

at sink

	P=+	P=-
J=0	S=0 L=0 , S=2 L=2	S=1 L=1 , S=3 L=3
J=1	S=2 L=2	S=1 L=1 , S=3 L=3
J=2	S=2 L=0 , S=0 L=2 , S=2 L=2 , S=2 L=4	S=1 L=1 , S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
J=3	S=2 L=2 , S=2 L=4	S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
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at source

L=0 ⇐ We use wall source

J=0 ⇐ S=0

at sink

	P=+	P=-
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J=1	S=2 L=2	S=1 L=1 , S=3 L=3
J=2	S=2 L=0 , S=0 L=2 , S=2 L=2 , S=2 L=4	S=1 L=1 , S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
J=3	S=2 L=2 , S=2 L=4	S=3 L=1 , S=1 L=3 , S=3 L=3 , S=3 L=5
J=4	S=2 L=2 , S=0 L=4 , S=2 L=4 , S=2 L=6	S=3 L=1 , S=1 L=3 , S=3 L=3 , S=1 L=5 , S=3 L=5 , S=3 L=7

at source

L=0 ⇐ We use wall source

J=0 ⇐ S=0

at sink

We can extract S=0 L=0 ,S=2 L=2

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1. How to construct the potential [HAL QCD method]
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Lattice set up

Japan Lattice Data Grid

[T. Ishikawa et al., Phys. Rev. D78 (2008)011502(R)]

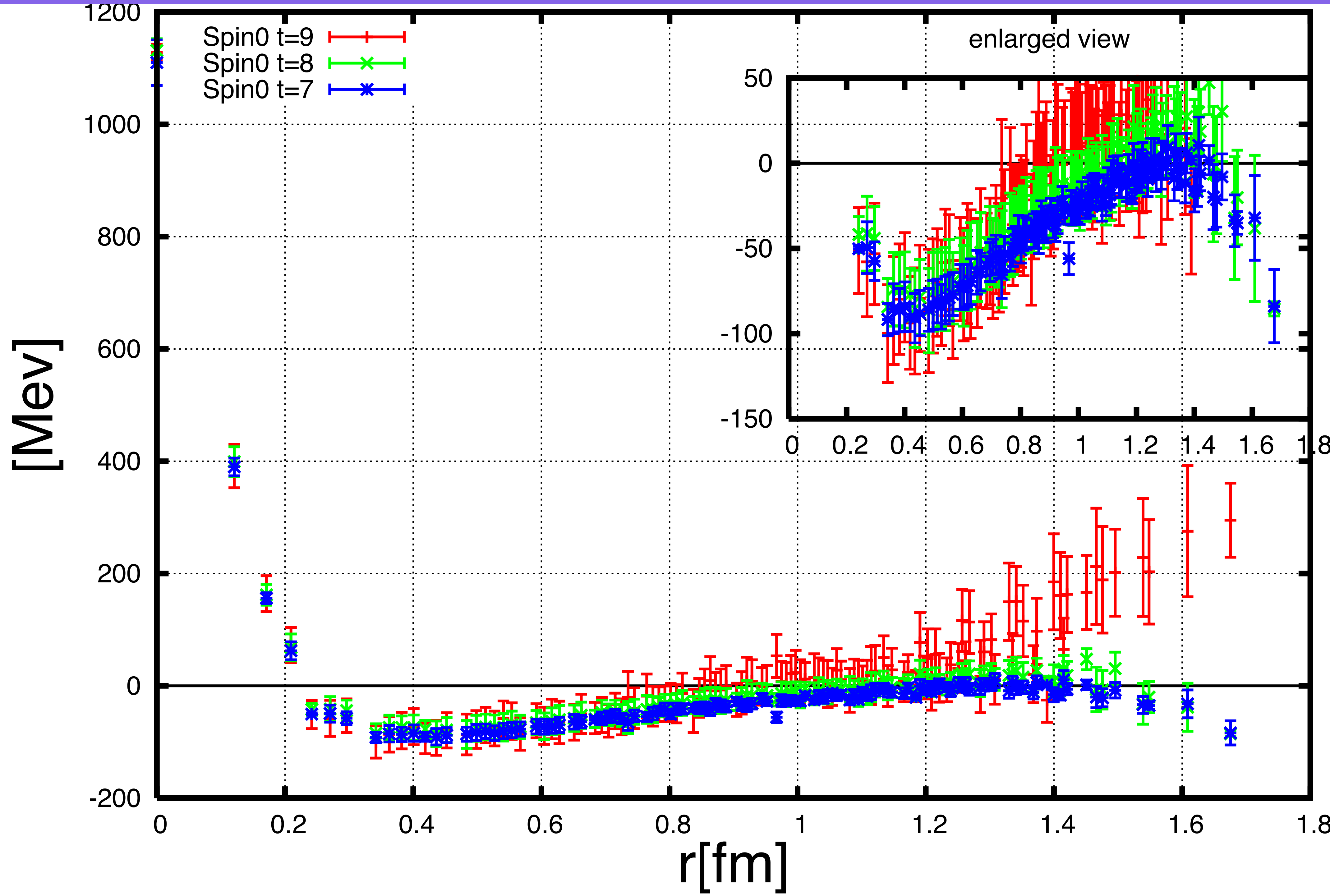
- 2+1 flavor full QCD gauge configurations generated by CP-PACS/JLQCD collaboration
- RG improved gauge action & O(a) improved Wilson quark action
- β=1.83
- lattice spacing a=0.1219(19) fm
- lattice volume $16^3 \times 32$ L~1.9 fm
- hopping parameters $K_s = 0.13710$ $K_{ud} = 0.13760$ giving $M_\Omega = 2108\,\mathrm{MeV}$ $M_\pi = 875\,\mathrm{MeV}$
- flat wall source(P=0)

Experiment value of Ω mass is 1672 MeV

BG/Q @KEK

Potential Ω-Ω

Ω-Ω potential



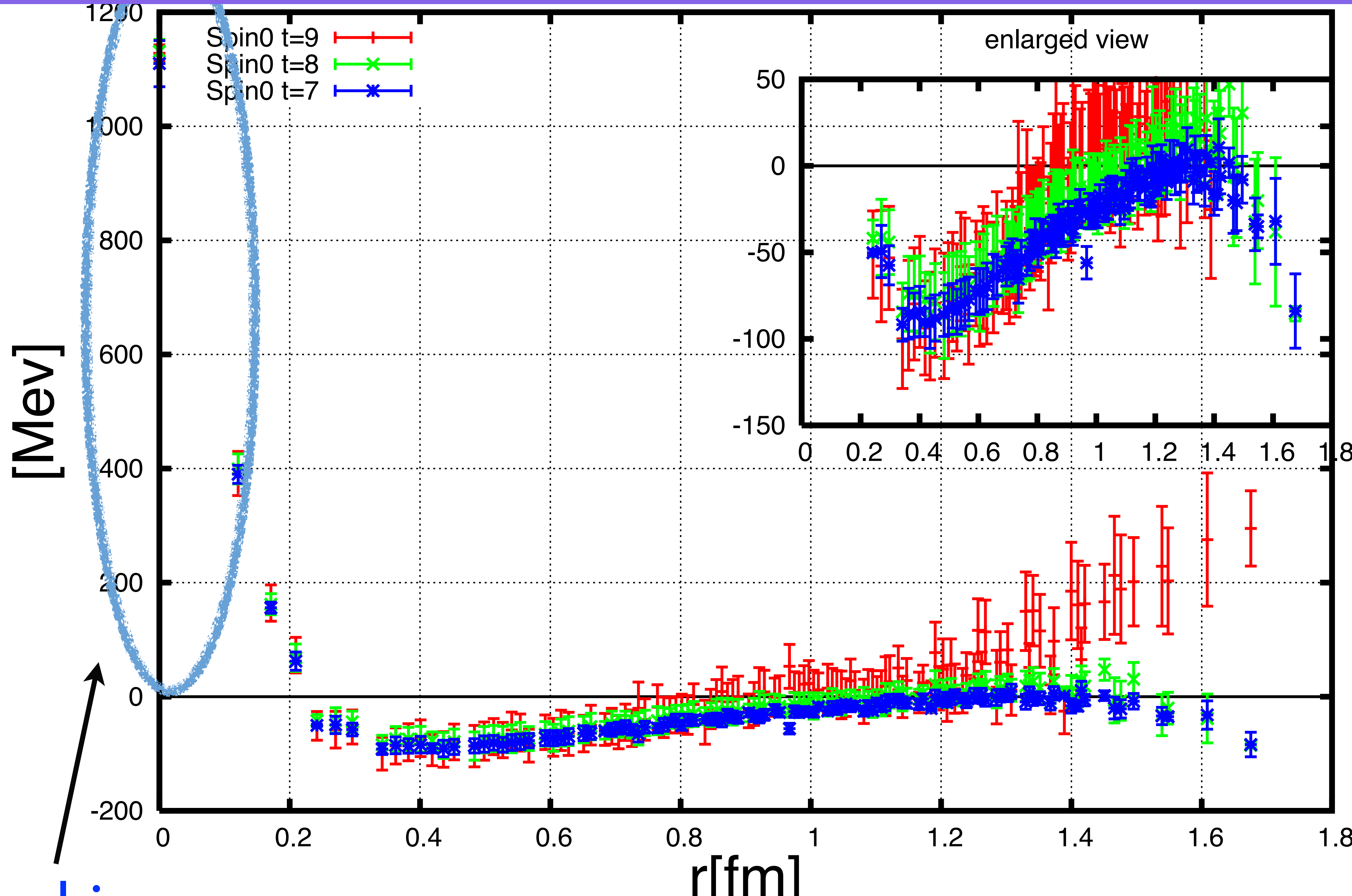
a=0.1209 [fm]

t is relative time between source and sink

$$t \equiv t_1 - t_0$$

$$C_{\Omega\Omega}(\vec{x}, \vec{y}, t_1, t_0) \equiv \langle 0 | \, \Omega(\vec{x}, t_1)\Omega(\vec{y}, t_1)\overline{\Omega}(0, t_0)\overline{\Omega}(0, t_0) \, |0\rangle$$

Ω-Ω potential



a=0.1209 [fm]

repulsive core

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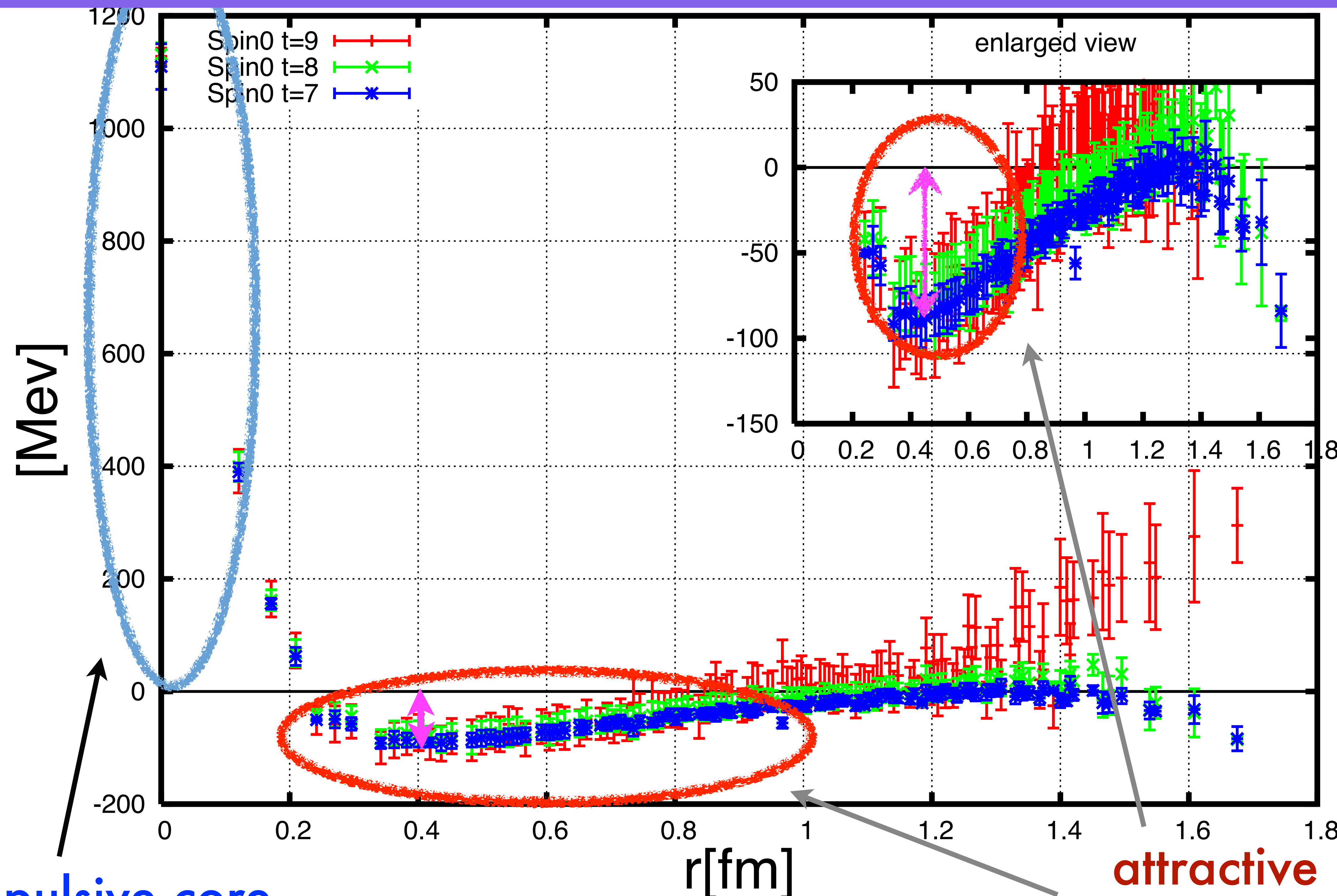

a=0.1209 [fm]

repulsive core

attractive pocket
depth 80 MeV

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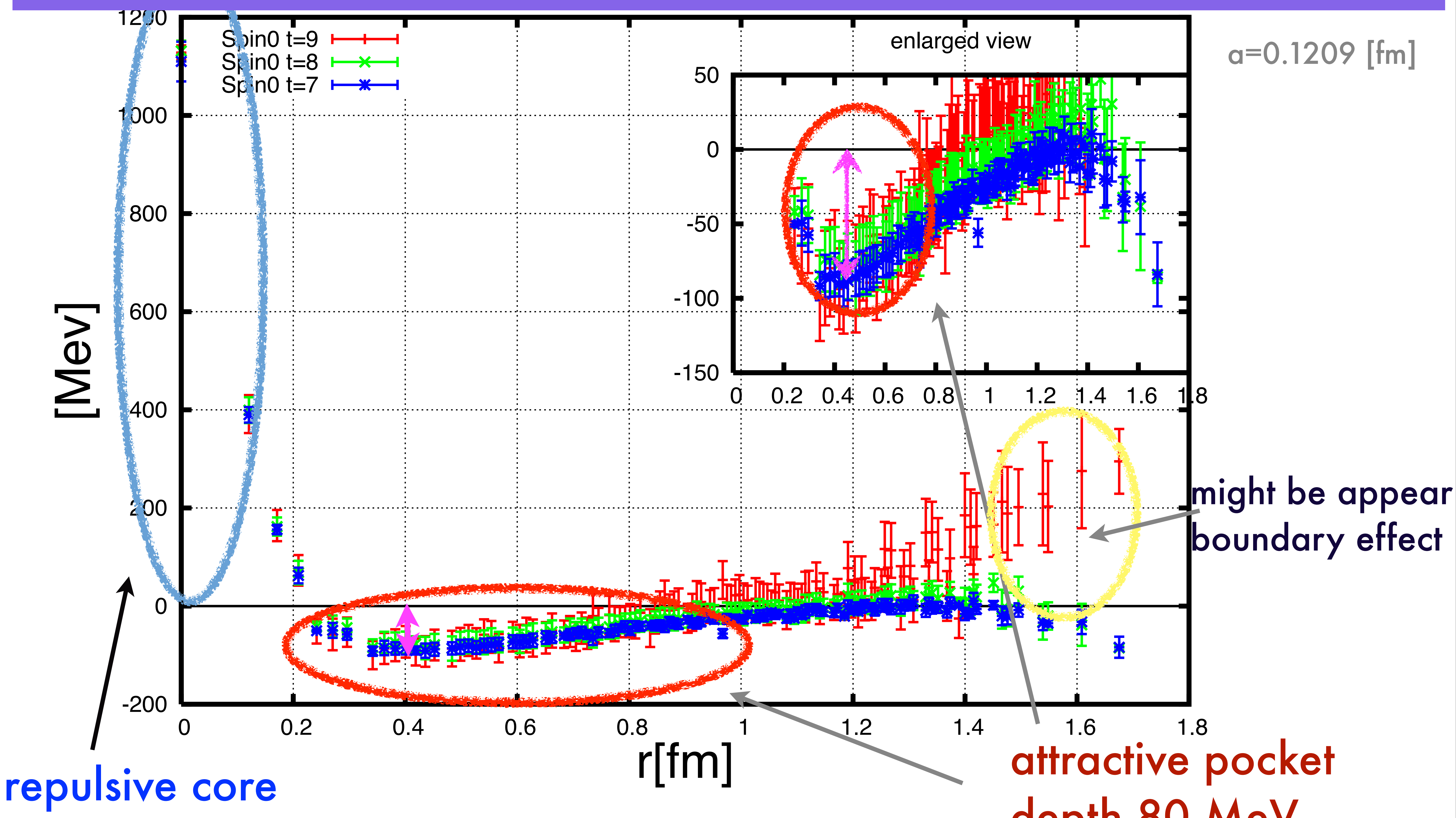

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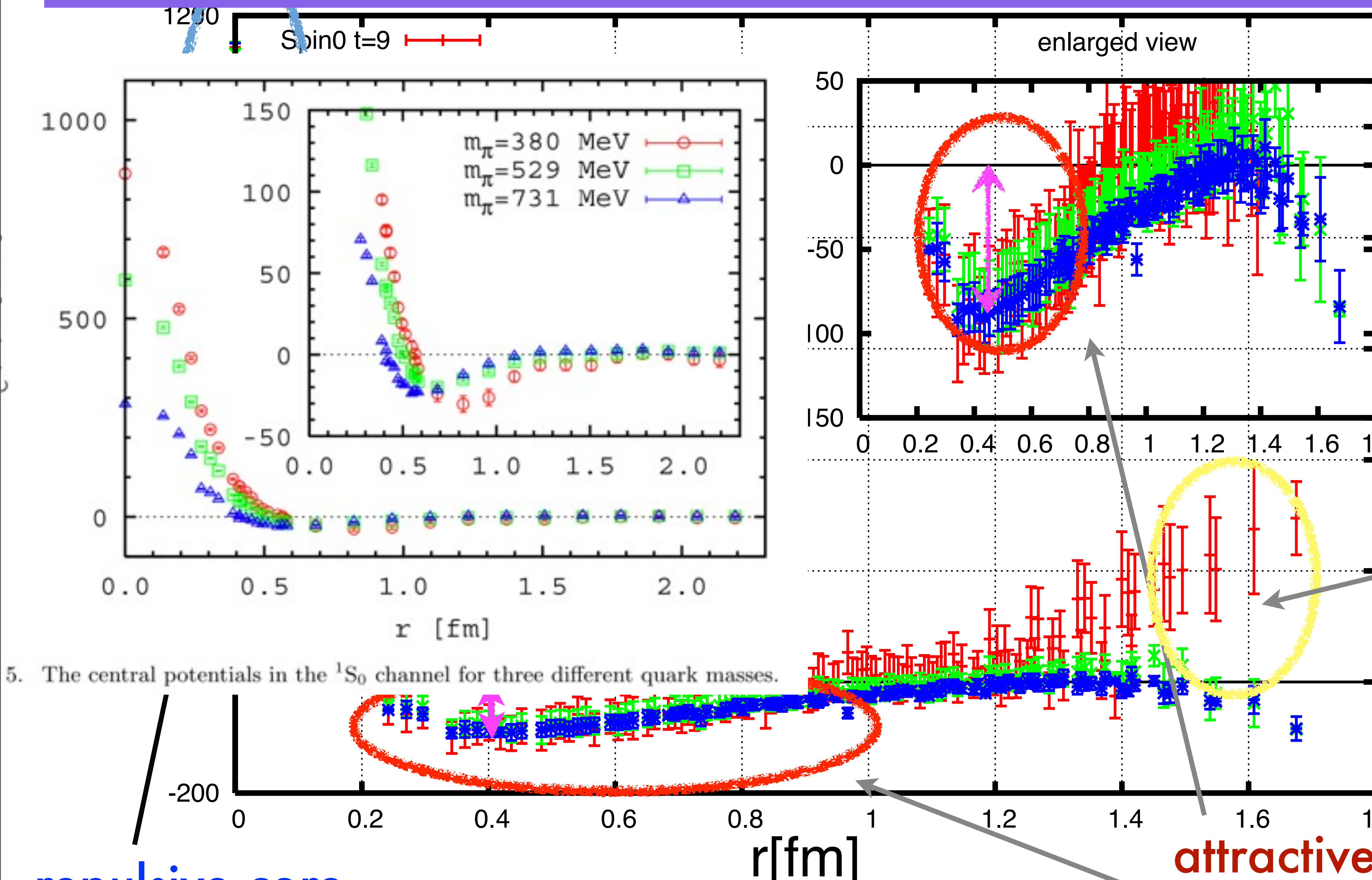
Ω-Ω potential

a=0.1209 [fm]



might be appear boundary effect

attractive pocket
depth 80 MeV

repulsive core

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$$t \equiv t_1 - t_0$$

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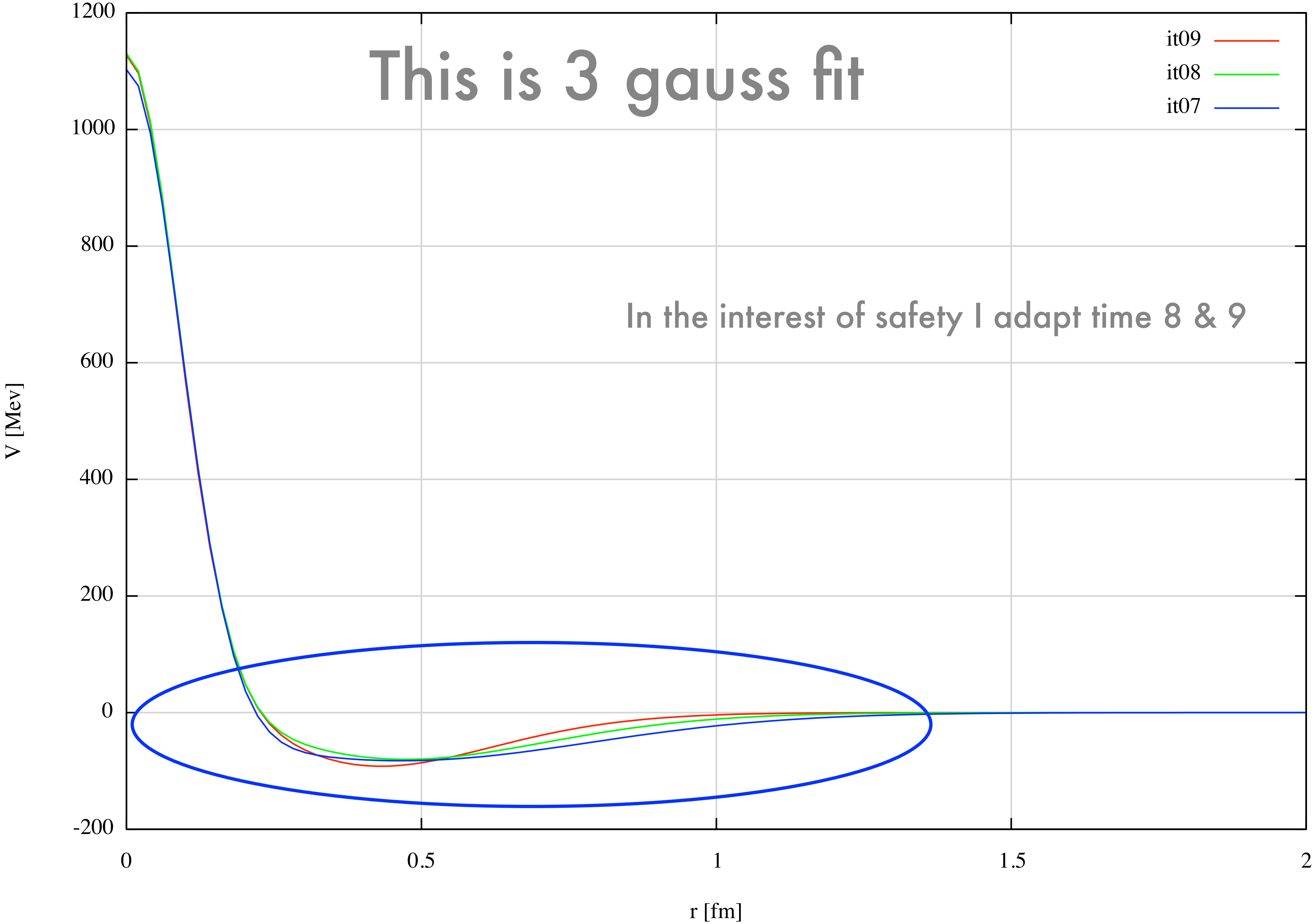
Ω-Ω potential



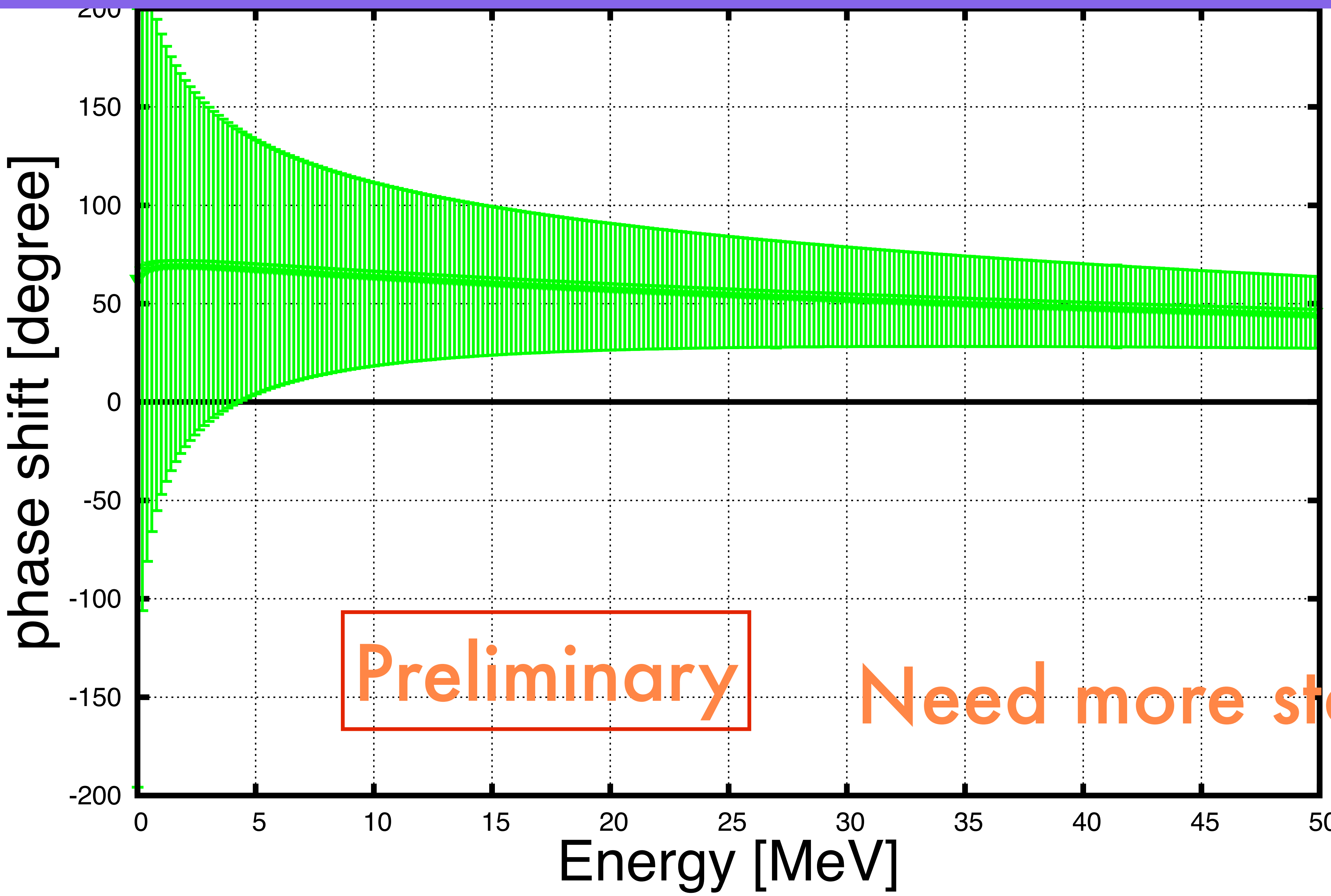
fit function form is little change but, not so change at each time slice

Phase shift

- 3gauss fit
- solve Schrödinger equation

no bound sate

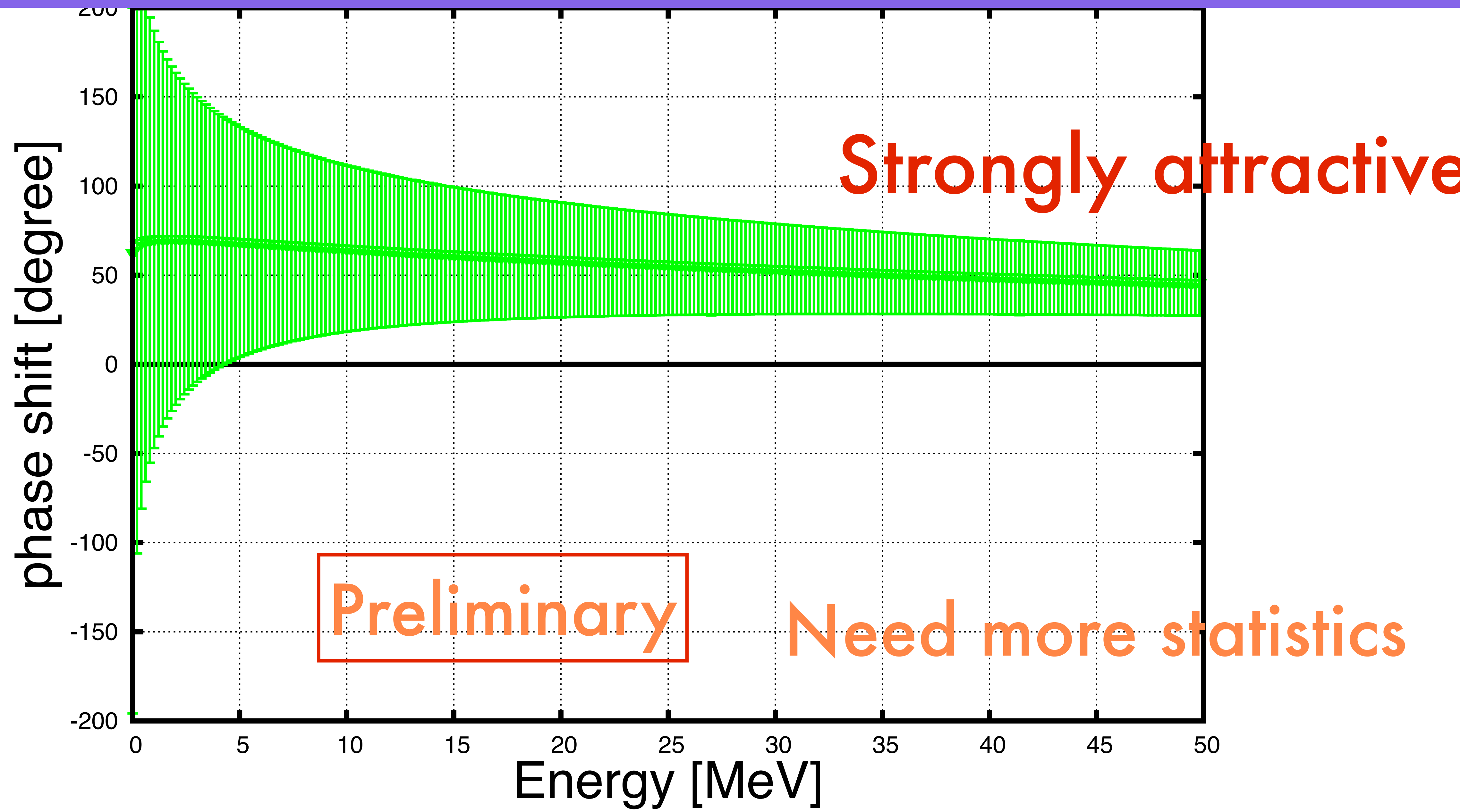
Phase shift t=8



Preliminary

Need more statistics

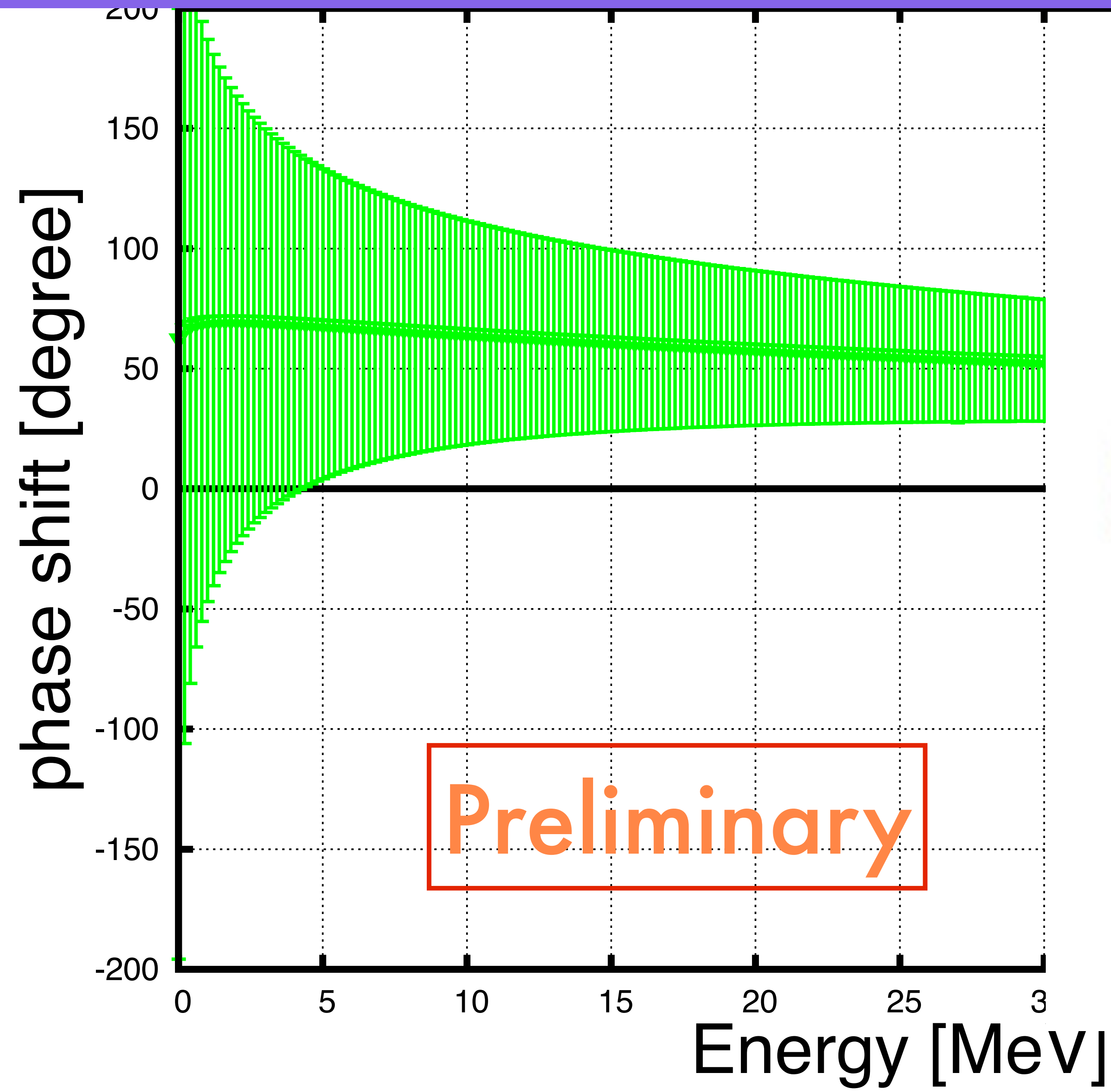
Phase shift t=8

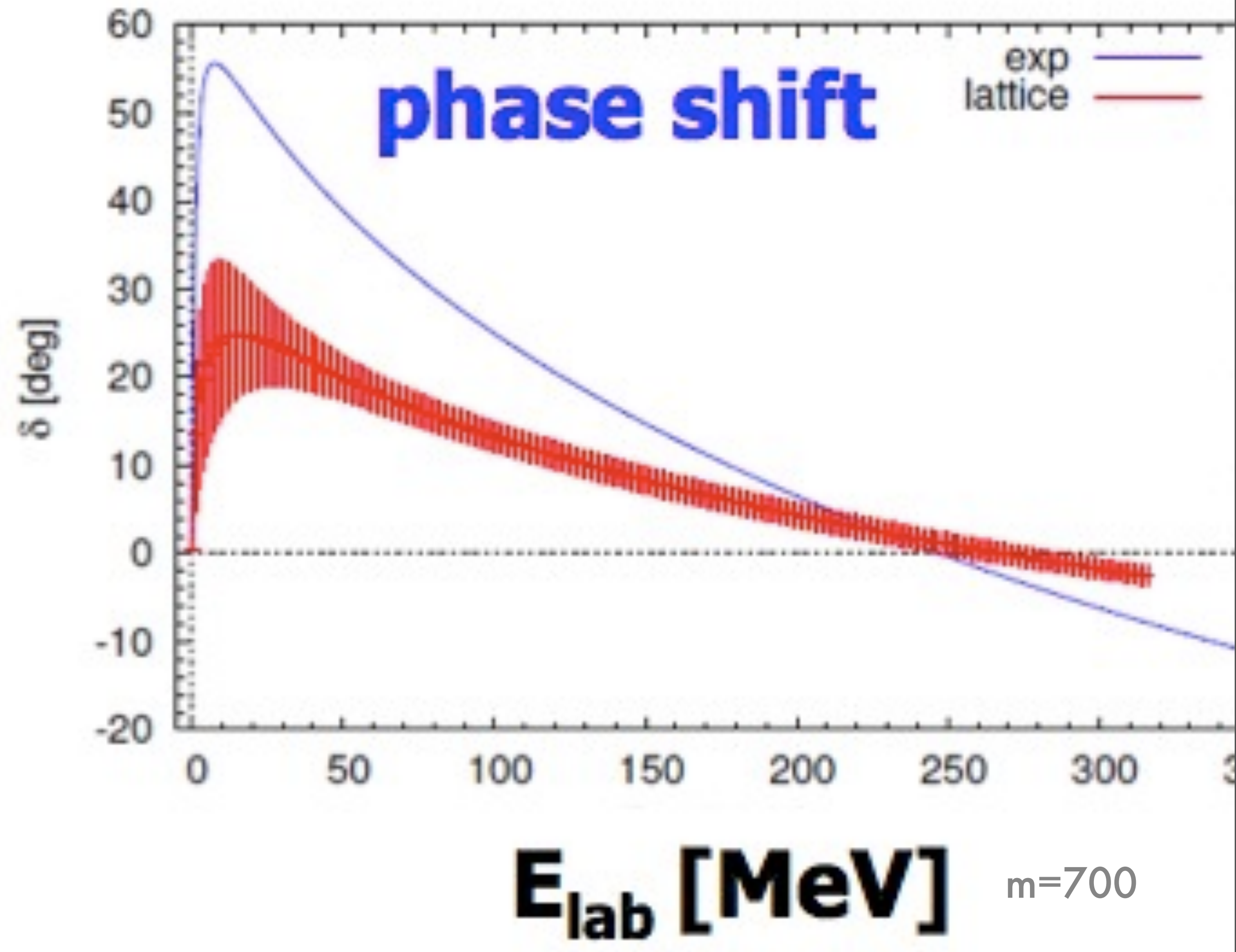


phase shift peak is much as large as 70degree

It suggests strongly attractive

Phase shift t=8





phase shift peak is much as large as 70degree

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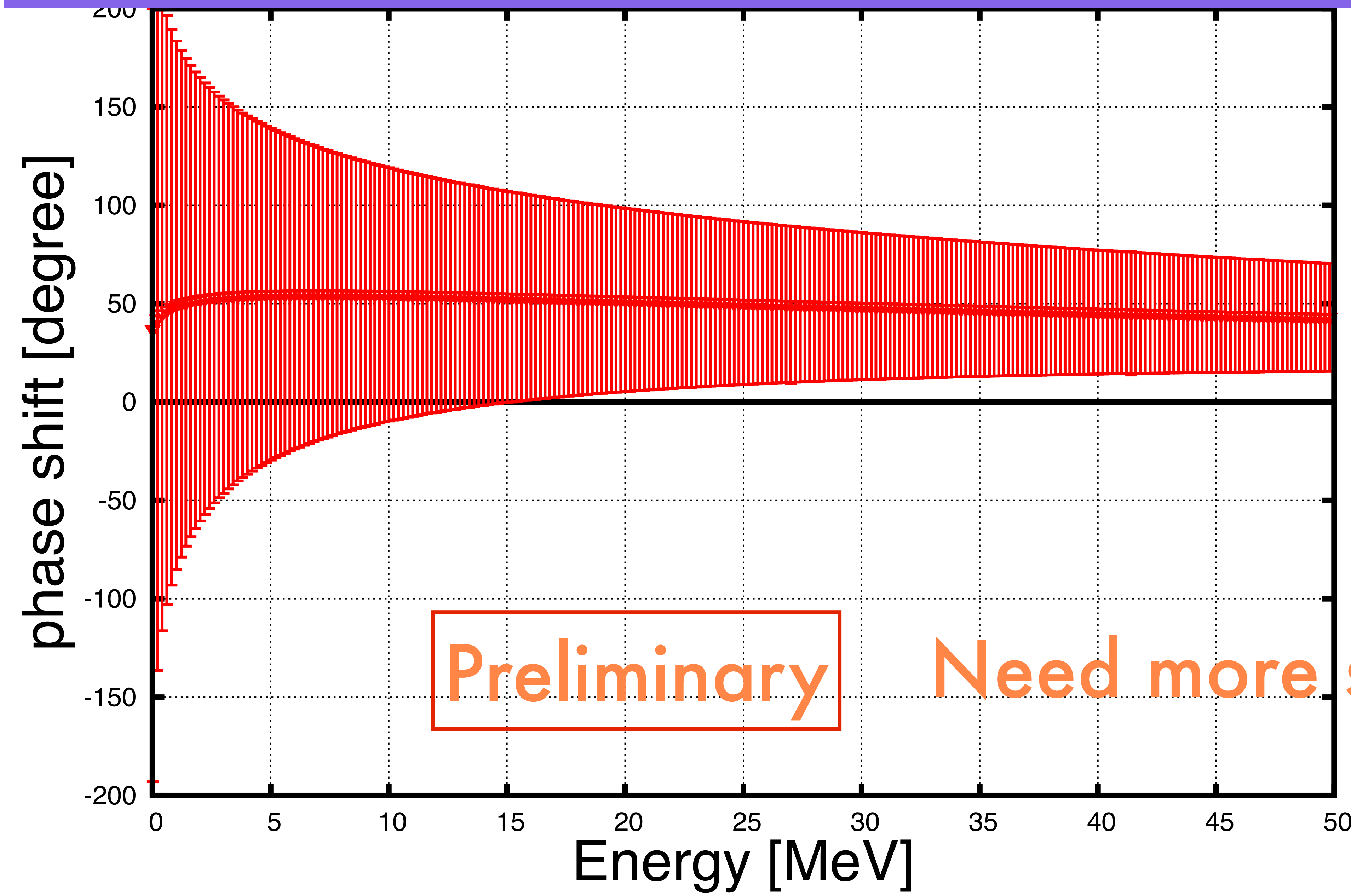
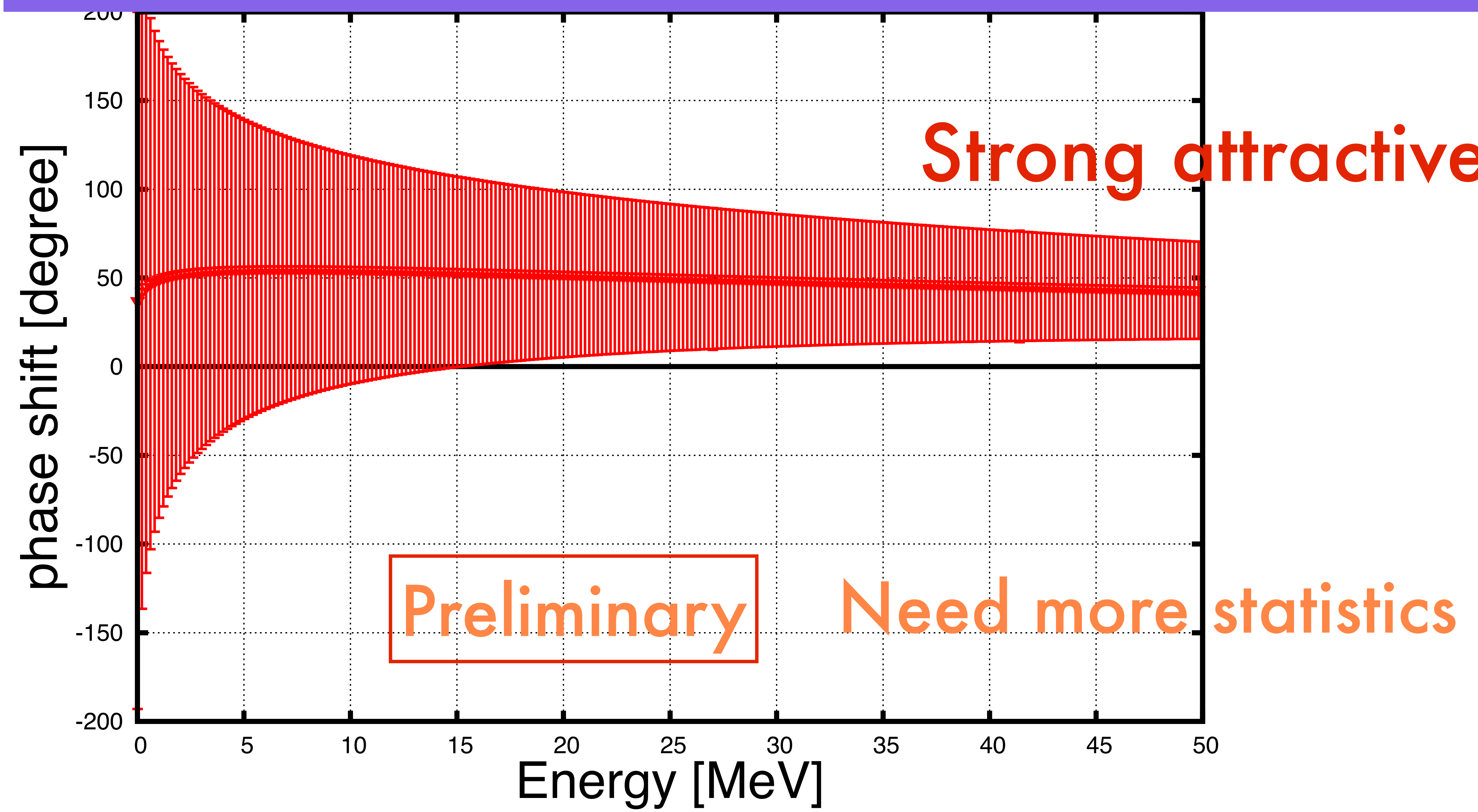
Phase shift t=9

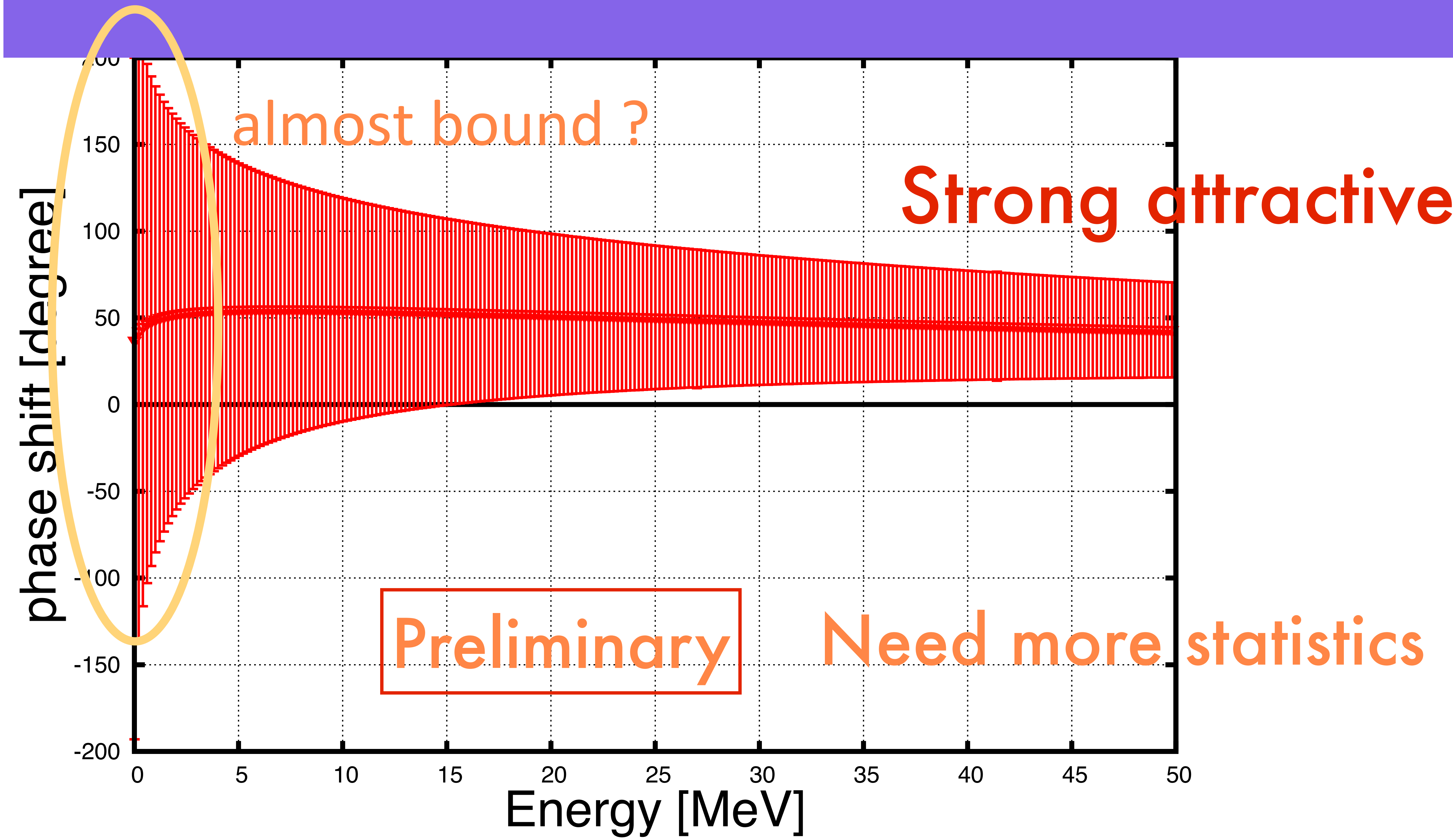


Need more statistics

Phase shift t=9



Phase shift t=9



Conclusion &Future work

- We extended HAL method to decouplet-decouplet system.
- J=0 Omega-Omega interaction is strongly attractive but we can not decide whether the bound state exists or not due to large errors.
- Future studies: larger volumes and lighter quark masses. bound state exist or not ?

Contact
E-mail: sinyamada@het.ph.tsukuba.ac.jp
web: http://www-het.ph.tsukuba.ac.jp/~sinyamada/index.html

Thank you!

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E-mail: sinyamada@het.ph.tsukuba.ac.jp
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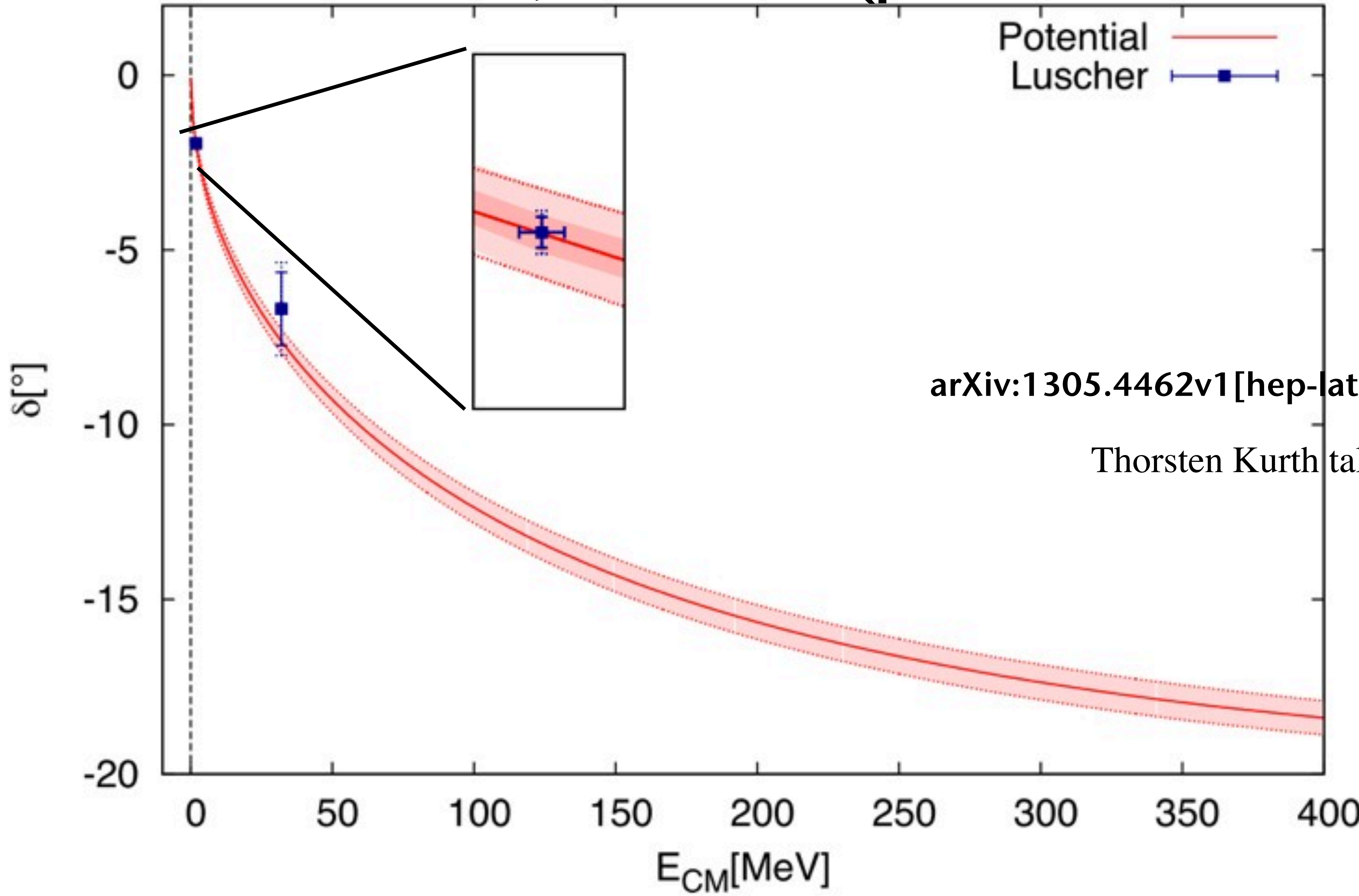
Back up slide

comparison Luscher method, HAL method(phase shift π-πchannel)



arXiv:1305.4462v1[hep-lat]

Thorsten Kurth talked at 7/30

The result of phase shift have been found to agree well between the two methods!

It's difficult to compare these methods without calculating finite volume method at large t and more statics!

method	potential (our work)	finite volume (Buchoff et al.)
fermion mass	heavy(π=875)	light(π=390)
Lattice volume	1.9[fm]	3.9[fm]
ground sate saturation	not need	need
results	strongly attractive	weakly repulsive

I think it's important to check different two methods.

Back up slide

Mass dependence (N-N interaction)

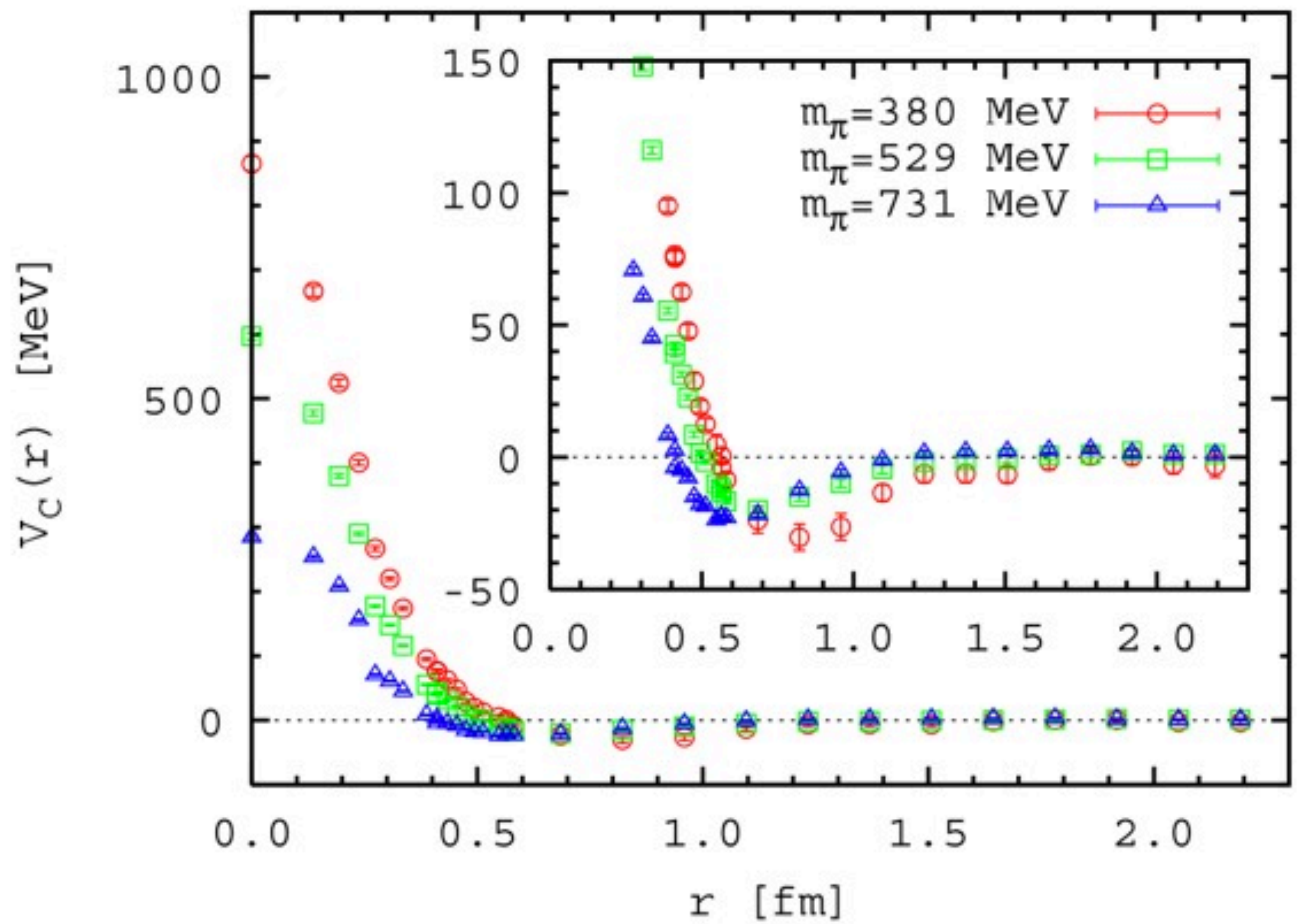


Fig. 5. The central potentials in the $^1S_0$ channel for three different quark masses.

light u,d

We expect Ω-Ω is similar to N-N case

[Sinya Aoki et al. Prog. Theor. Phys. 123 ,89]

S=0 ⇐ a special circumstance in Ω-Ω system

- flavor is completely symmetry
- wall source

source operator

$$\overline{\Omega} = \varepsilon^{abc}(\gamma_k C)_{\beta\gamma}\overline{s}^a_\alpha \overline{s}^b_\beta \overline{s}^c_\gamma$$

a,b,c: color index
α,β,γ: spin index

highest state in Ω-Ω (spin3)

$$\overline{s}^a_{\frac{1}{2}}(x)\overline{s}^b_{\frac{1}{2}}(x)\overline{s}^c_{\frac{1}{2}}(x)\overline{s}^a_{\frac{1}{2}}(y)\overline{s}^b_{\frac{1}{2}}(y)\overline{s}^c_{\frac{1}{2}}(y)$$

For simply neglect ε , γC

We can make all state using lowering operator

spin3⇒spin2⇒spin1⇒spin0

For example one term of spin2 state

$$\overline{s}^a_{\frac{1}{2}}(x)\overline{s}^b_{\frac{1}{2}}(x)\overline{s}^c_{\frac{1}{2}}(x)\overline{s}^a_{\frac{1}{2}}(y)\overline{s}^b_{\frac{1}{2}}(y)\overline{s}^c_{-\frac{1}{2}}(y)$$

spin2 term is written by linear combination of these terms.

$$\overline{s}^a_{\frac{1}{2}}(x)\overline{s}^b_{\frac{1}{2}}(x)\overline{s}^c_{\frac{1}{2}}(x)\overline{s}^a_{\frac{1}{2}}(y)\overline{s}^b_{\frac{1}{2}}(y)\overline{s}^c_{-\frac{1}{2}}(y)$$

wall source⇒ $$\left(\sum_x \overline{s}^a_{\frac{1}{2}}(x)\right)\left(\sum_{x'} \overline{s}^b_{\frac{1}{2}}(x')\right)\left(\sum_{x"} \overline{s}^c_{\frac{1}{2}}(x")\right)\left(\sum_y \overline{s}^a_{\frac{1}{2}}(y)\right)\left(\sum_{y'} \overline{s}^b_{\frac{1}{2}}(y')\right)\left(\sum_{y"} \overline{s}^c_{-\frac{1}{2}}(y")\right)$$

$$= -\left(\sum_y \overline{s}^a_{\frac{1}{2}}(y)\right)\left(\sum_{x'} \overline{s}^b_{\frac{1}{2}}(x')\right)\left(\sum_{x"} \overline{s}^c_{\frac{1}{2}}(x")\right)\left(\sum_x \overline{s}^a_{\frac{1}{2}}(x)\right)\left(\sum_{y'} \overline{s}^b_{\frac{1}{2}}(y')\right)\left(\sum_{y"} \overline{s}^c_{-\frac{1}{2}}(y")\right)$$

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$$= \mathbf{0}$$ Spin2 state should be 0

$$\bar{s}^{a}_{\frac{1}{2}}(x)\bar{s}^{b}_{\frac{1}{2}}(x)\bar{s}^{c}_{\frac{1}{2}}(x)\bar{s}^{a}_{\frac{1}{2}}(y)\bar{s}^{b}_{\frac{1}{2}}(y)\bar{s}^{c}_{-\frac{1}{2}}(y)$$

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fermionic

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x,y are inner arguments

$$= -\left(\sum_x \overline{s}^a_{\frac{1}{2}}(x)\right)\left(\sum_{x'} \overline{s}^b_{\frac{1}{2}}(x')\right)\left(\sum_{x"} \overline{s}^c_{\frac{1}{2}}(x")\right)\left(\sum_y \overline{s}^a_{\frac{1}{2}}(y)\right)\left(\sum_{y'} \overline{s}^b_{\frac{1}{2}}(y')\right)\left(\sum_{y"} \overline{s}^c_{-\frac{1}{2}}(y")\right)$$

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$$= 0$$

Spin2 state should be 0

Spin0 remain

$$\left(\sum_x \overline{s}^a_{\frac{1}{2}}(x)\right)\left(\sum_{x'} \overline{s}^b_{\frac{1}{2}}(x')\right)\left(\sum_{x"} \overline{s}^c_{\frac{1}{2}}(x")\right)\left(\sum_y \overline{s}^a_{-\frac{1}{2}}(y)\right)\left(\sum_{y'} \overline{s}^b_{-\frac{1}{2}}(y')\right)\left(\sum_{y"} \overline{s}^c_{-\frac{1}{2}}(y")\right)$$

Existence of energy independent nonlocal potential

We assume linear independence of NBS wave function

There is a dual bases

$$\int d^3r \tilde{\psi}_{k'}(r)\psi_k(r) = (2\pi)^3\delta^3(k'-k)$$

We define K

$$K_k(r) \equiv (\nabla^2 + k^2)\psi_k(r)$$

$$= \int \frac{d^3k'}{(2\pi)^3} K_{k'}(r) \int d^3r' \tilde{\psi}_{k'}(r')\psi_k(r')$$

$$= \int d^3r' \left\{ \int \frac{d^3k'}{(2\pi)^3} K_{k'}(r)\tilde{\psi}_{k'}(r') \right\} \psi_k(r')$$

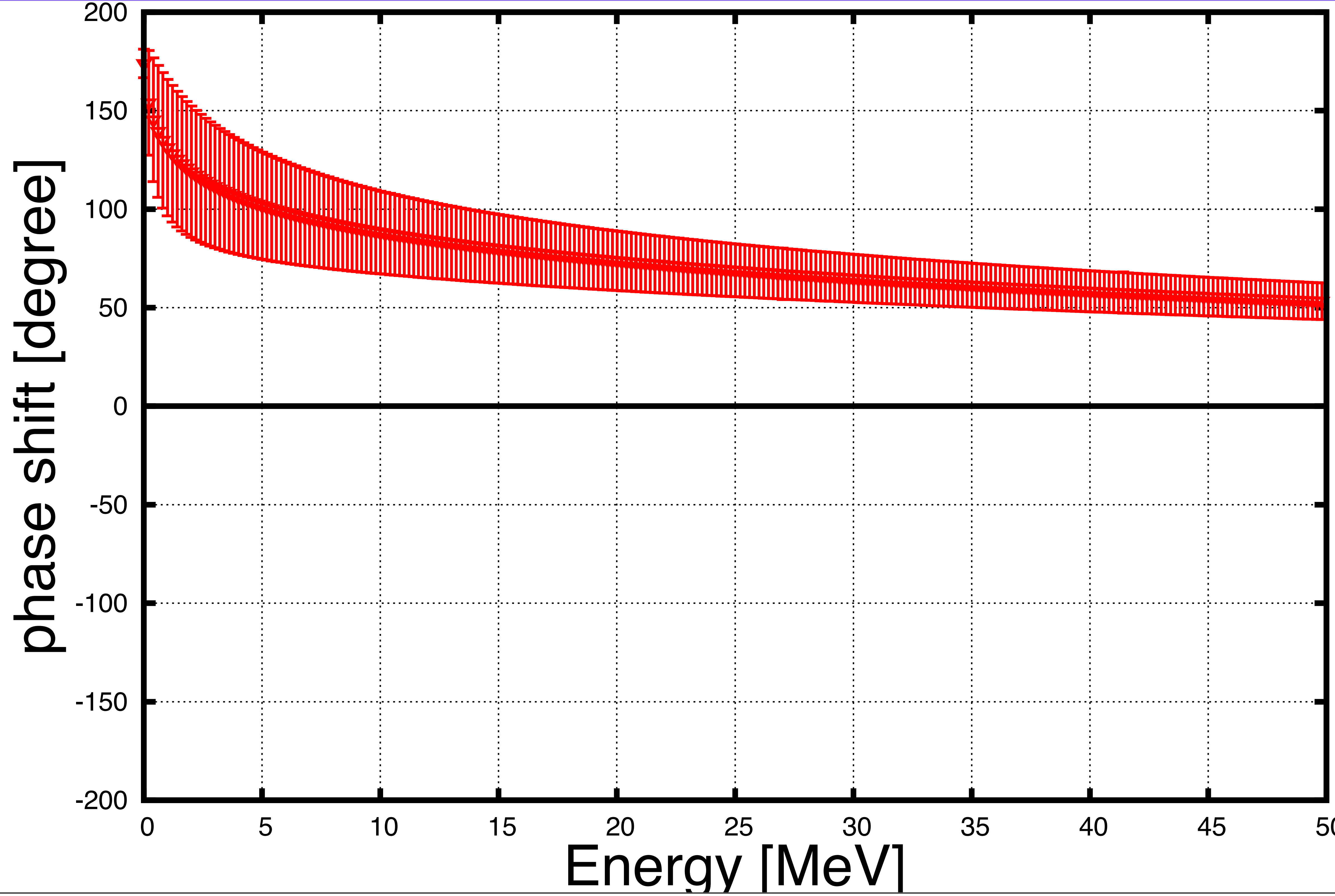
If we define

$$U(r,r') \equiv \frac{1}{m}\int \frac{d^3k'}{(2\pi)^3} K_{k'}(r)\tilde{\psi}_{k'}(r')$$

Then we have

$$(\frac{k^2}{m} + \frac{1}{m}\nabla^2)\psi_k(r) = \int d^3r' U(r,r')\psi_k(r')$$

Phase shift t=7



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