$B_s \to D_s \ell\bar{\nu}_\ell$ near zero recoil from twisted mass LQCD

Form factors from SM and beyond

Mariam Atoui

based on work with D. Bećirević V. Morénas F. Sanfilippo

The 31st International Symposium on Lattice Field Theory
Mainz - Germany
Weak Decays and Matrix Elements session

Laboratoire de Physique Corpusculaire LPC
Université Blaise Pascal
Clermont Fd-France

July 29 - August 03 2013

Outline

1. Exclusive B to D decays
2. Lattice calculation of form factors
3. Simulation and numerical results
4. Conclusions

1. Exclusive B to D decays
2. Lattice calculation of form factors
3. Simulation and numerical results
4. Conclusions

Exclusive $\bar{B} \to D\,\ell\,\bar{\nu}$ *decay*

Motivation :

$B \to D^{(*)}\ (\mu, e)\ (\bar{\nu}_\mu, \bar{\nu}_e)$

- Allows to determine the CKM matrix element V_{cb}
- Theoretical uncertainty on $|V_{cb}|$ comes from form factors

$\Longrightarrow$ Improve lattice precision

τ channel : $B \to D\ \tau\ \bar{\nu}_\tau$

$$\boxed{R(D) \;=\; \frac{\mathscr{B}\,(B \to D\ \tau\ \bar{\nu}_\tau)}{\mathscr{B}\,(B \ \to \ D\ \ell\ \bar{\nu}_\ell)}} : \begin{cases} \text{Babar Collaboration} & \rightsquigarrow\ 0.440 \pm 0.058 \pm 0.042 \\ \text{SM} & \rightsquigarrow\ 0.31 \pm 0.02 \end{cases}$$

about 2.0 σ tension between measurements and SM predictions.

[Becirevic *et al.* arXiv :1301.4037]

New Physics in $b \to c$?

- Consider additional tensor and scalar operators in the effective Hamiltonian

$$\boxed{\mathscr{H}_{\text{eff}} = \mathscr{H}_{\text{eff}}^{\text{SM}} + \mathscr{H}_{\text{eff}}^{\text{NP}}}$$

- Revisit the SM prediction

Proposal : Lattice calculations of $\bar{B}_s \to D_s \, \ell \, \bar{\nu}$ channels in and beyond SM.

New Physics in $b \to c$?

- Consider additional tensor and scalar operators in the effective Hamiltonian

$$\boxed{\mathscr{H}_{\text{eff}} = \mathscr{H}_{\text{eff}}^{\text{SM}} + \mathscr{H}_{\text{eff}}^{\text{NP}}}$$

- Revisit the SM prediction

Proposal : Lattice calculations of $\bar{B}_s \to D_s\, \ell\, \bar{\nu}$ channels in and beyond SM.

Why $B_s \to D_s$?

- Could be done at LHCb and especially at Super-Belle
- No averaging between neutral and charged modes
- Soft photon pollution smaller

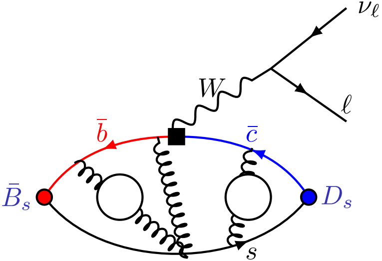
$$\frac{\mathscr{B}(B_s \to D_s \gamma_{\text{soft}}\, \ell\, \nu)}{\mathscr{B}(B_s \to D_s \ell \nu)} < \frac{\mathscr{B}(B \to D^0 \gamma_{\text{soft}} \ell \nu)}{\mathscr{B}(B \to D^0 \gamma\, \ell\, \nu)}$$

- No extrapolation in the valence quark mass needed

$b \to c\ \ell\bar{\nu}_\ell$ effective Hamiltonian

In Standard Model (SM) :

$$\mathscr{H}_{\text{eff}}^{\text{SM}} \propto \bar{c}\,\gamma_\mu\,(1-\gamma_5)\,b\;\bar{\ell}\,\gamma^\mu\,(1-\gamma_5)\,\bar{\nu}_\ell$$



Hadronic matrix elements

$$\left\langle D_s(p_{D_s})\,\middle|\,V_\mu\,\middle|\,\bar{B}_s(p_{B_s})\right\rangle = F_+(q^2)\,(p_{B_s}+p_{D_s})_\mu + (p_{B_s}-p_{D_s})_\mu\;\left[F_0(q^2)-F_+(q^2)\right]\left(\frac{m_{B_s}^2-m_{D_s}^2}{q^2}\right) \qquad \left[0<q^2\leq(m_B-m_D)^2\right]$$

$$\left\langle D_s(p_{D_s})\,\middle|\,A_\mu\,\middle|\,\bar{B}_s(p_{B_s})\right\rangle = 0$$

New Physics scenario

$$\mathscr{H}_{\text{eff}}^{\text{NP}} \propto g_T\ \bar{c}\ (\sigma_{\mu\nu} b)\ \left(\bar{\ell}\,\sigma^{\mu\nu}\bar{\ell}_R \nu_L\right) + g_s\ (\bar{c}\,b)\ \left(\bar{\ell}_R\,\nu_L\right)$$

g_T : Coupling of the new tensor term **g_s : Coupling of the scalar term**

$$\left\langle D_s(p_{D_s}) \,\middle|\, \bar{c}\,\sigma_{\mu\nu}\ b \,\middle|\, B_s(p_{B_s}) \right\rangle =$$

$$-\,i\ \left(p_{B_s\,\mu}\,p_{D_s\,\nu}\ -\ p_{D_s\,\mu}\,p_{B_s\,\nu}\right)\ \frac{2\ F_T(q^2)}{m_{B_s} + m_{D_s}}$$

$$\Longrightarrow \quad \text{3 form factors :} \quad \underbrace{\underbrace{F_+,\ F_0}_{\text{In SM}} \quad \text{and} \quad F_T}_{\text{NP}}$$

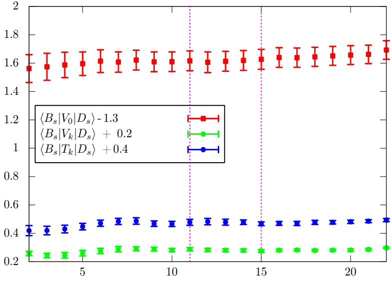
Hadronic matrix element

$$\boxed{\mathscr{R}(t)} = \frac{\mathscr{C}^{(3)}(t,t_i,t_f,\vec{p}_i,\vec{p}_f)}{\mathscr{C}^{(2)}_{(B_s)}(t-t_i,\vec{p}_f)\cdot\mathscr{C}^{(2)}_{(D_s)}(t_f-t,\vec{p}_i)}\cdot\sqrt{\mathscr{Z}_{B_s}}\cdot\sqrt{\mathscr{Z}_{D_s}}$$

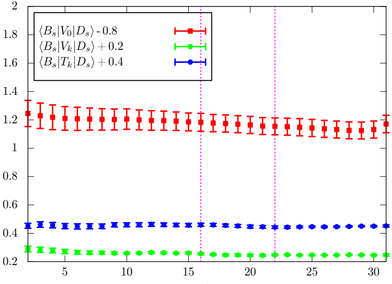
$$\xrightarrow[t-t_i\to\infty]{t_f-t\to\infty} \langle D_s(\vec{p}_f)|(V_\mu,\,T_\mu)|B_s(\vec{p}_i)\rangle$$

$\mathscr{Z}_M = ||\langle 0|\mathscr{O}_M|M\rangle||^2$ obtained from the fit with $\mathscr{C}^{(2)}_{(M)}$

$$\boxed{\mathscr{C}^{(2)}(t) = \frac{\mathscr{Z}_{B_s,D_s}}{2\,m_{B_s,D_s}}\cosh\left(m_{B_s,D_s}\left[\frac{T}{2}-t\right]\right)\,e^{-m_{B_s,D_s}\frac{T}{2}}}$$



$\beta = 3.90,\ a\mu_h = 0.4114,\ a\mu_l = 0.0085$



$\beta = 4.20,\ a\mu_h = 0.3525,\ a\mu_l = 0.0065$

Hadronic matrix element

$$\boxed{\mathscr{R}(t)} = \frac{\mathscr{C}^{(3)}(t, t_i, t_f, \vec{p}_i, \vec{p}_f)}{\mathscr{C}^{(2)}_{(B_s)}(t - t_i, \vec{p}_f) \cdot \mathscr{C}^{(2)}_{(D_s)}(t_f - t, \vec{p}_i)} \cdot \sqrt{\mathscr{Z}_{B_s}} \cdot \sqrt{\mathscr{Z}_{D_s}}$$

$$\xrightarrow[t-t_i \to \infty]{t_f - t \to \infty} \langle D_s(\vec{p}_f) | (V_\mu,\, T_\mu) | B_s(\vec{p}_i) \rangle$$

$\mathscr{Z}_M = ||\langle 0|\mathscr{O}_M|M\rangle||^2$ obtained from the fit with $\mathscr{C}^{(2)}_{(M)}$

$$\boxed{\mathscr{C}^{(2)}(t) \;=\; \frac{\mathscr{Z}_{B_s,D_s}}{2\, m_{B_s,D_s}} \cosh\left(m_{B_s,D_s} \left[\frac{T}{2} - t \right] \right) \; e^{-m_{B_s,D_s} \frac{T}{2}}}$$

- Full propagating heavy quarks
- Vector and tensor operators are renormalized with renormalization constants computed non-perturbatively in the RI-MOM scheme

C. Alexandrou, *et al.* [Phys.Rev., D86 :014505, 2012.]

Simulation setup

- Wilson twisted mass Dirac operator with two degenerate flavors ($N_f = 2$)

$$Q^{(x)} = \gamma_\mu D_\mu + m + i\mu\gamma_5 + \frac{a}{2}\Box \quad , \quad m + 4 = \frac{1}{2\kappa}$$

- "tree-level Symanzik" improved Gauge-action

β	3.8	3.9	3.9	4.05	4.2	4.2
$L^3 \times T$	$24^3 \times 48$	$24^3 \times 48$	$32^3 \times 64$	$32^3 \times 64$	$32^3 \times 64$	$48^3 \times 96$
μ_{sea1}	0.0080	0.0040	0.0030	0.0030	0.0065	0.0020
μ_{sea2}	0.0110	0.0064	0.0040	0.0060		
μ_{sea3}		0.0085		0.0080		
μ_{sea4}		0.0100				
a [fm]	0.098(3)	0.085(3)	0.085(3)	0.067(2)	0.054(1)	0.054(1)
μ_c	0.2331(82)	0.2150(75)	0.2150(75)	0.1849(65)	0.1566(55)	0.1566(55)

- $m_\pi \in [280, 500]$ **MeV**

Kinematics

- B_s rest frame $\quad p_{B_s} = (m_{B_s}, \vec{0})$
- $p^{\mu}_{D_s} = (E_{D_s}, p, p, p)$ with $p = \frac{\pi\theta_0}{L}$: twisted boundary conditions

  Chosen $\theta_0 \Rightarrow w \in \{1, 1.004, 1.02, 1.04, 1.06\}$ (near zero recoil) $\quad w_{\max} = 1.546$

  D_s obey the free-boson lattice dispersion relation :

$$\boxed{4\,\sinh^2\left[\frac{E_{D_s}}{2}\right] = 4\left(3\,\sin^2\left[\frac{\theta_0\,\pi}{2L}\right]\right) + 4\,\sinh^2\left[\frac{m_{D_s}}{2}\right]}$$

- Different recoils $w = v_{D_s} \cdot v_{B_s} = \sqrt{1 + \frac{3(\theta_0\pi)^2}{(m_{D_s}\,L)^2}}$: do not depend on m_{B_s}

  $\Longrightarrow$ Extrapolate to physical $B_s \to D_s$ at fixed values of w!

HQET decomposition

$$\frac{1}{\sqrt{m_{B_s} m_{D_s}}} \left\langle D_s(p_{D_s}) \,\middle|\, V_\mu \,\middle|\, \bar{B}_s(p_{B_s}) \right\rangle = h_+(w)\,(v_{B_s} + v_{D_s})_\mu + (v_{B_s} - v_{D_s})_\mu\, h_-(w)$$

$$v = p/m, \qquad w = \frac{m_{B_s}^2 + m_{D_s}^2 - q^2}{2\, m_{B_s}\, m_{D_s}}$$

- The decay rate depends on $G(w)^{B_s \to D_s}$ (in the limit of vanishing lepton masses)

$$\boxed{G(w) \;=\; h_+(w) \;-\; \frac{m_{D_s} - m_{B_s}}{m_{D_s} + m_{B_s}}\; h_-(w)}$$

needed to extract V_{cb}

  - Experimentalists : shape of $G(w)|V_{cb}|$
  - Lattice QCD provides normalization : the zero recoil point (Isgur-Wise point) $G(1)$

$G_s(w)$ at zero recoil ($w = 1$)

$$G_s(w) = h_+(w)\left[1 - \left(\frac{m_{B_s} - m_{D_s}}{m_{B_s} + m_{D_s}}\right) H(w)\right] \qquad H(w) = \frac{h_-(w)}{h_+(w)}$$

$$\Rightarrow \boxed{G_s(1) = h_+(1)\left[1 - \left(\frac{m_{B_s} - m_{D_s}}{m_{B_s} + m_{D_s}}\right) H(1)\right]}$$

- $h_+(1)$ can be extracted from $F_0(q^2_{\mathrm{max}})$ $\quad q^2_{\mathrm{max}}=(m_{B_s}-m_{D_s})^2$
- $H(1)$ is not directly accessible from the lattice at zero recoil

$G_s(w)$ at zero recoil ($w = 1$)

$$G_s(w) \; = h_+(w)\left[1 - \left(\frac{m_{B_s} - m_{D_s}}{m_{B_s} + m_{D_s}}\right) H(w)\right] \qquad H(w) = \frac{h_-(w)}{h_+(w)}$$

$$\Rightarrow \boxed{G_s(1) = h_+(1)\left[1 - \left(\frac{m_{B_s} - m_{D_s}}{m_{B_s} + m_{D_s}}\right) H(1)\right]}$$

- $h_+(1)$ can be extracted from $F_0(q^2_{\mathrm{max}})$ $\quad q^2_{\mathrm{max}}=(m_{B_s}-m_{D_s})^2$
- $H(1)$ is not directly accessible from the lattice at zero recoil

Strategy

- Compute $F_{+,0}$ at few small recoil momenta $\Rightarrow H(w)$
- Extrapolate to $w = 1 \Rightarrow H(1)$

$G_s(1)$ *in the continuum*

Determine $G(1)_s^{\text{latt}}\ (\ \mu_h^{(i)}\ ,\ \ a^2,\ \mu_l^{\text{sea}}\ )$ for each :

- gauge ensemble
- light quark mass

⇒ Extrapolate to the continuum

$$G_s(1)\ = \left[A_s + B_s\,\mu_l^{\text{sea}}\ + C_s\frac{a^2}{a_{3.9}^2}\right]$$

- heavy quark mass $\quad \mu_h^i \quad i\in\{1,\cdots,9\}$

$$\mu_c\ \leq \mu_h^i \leq\ \mu_b \qquad \frac{\mu_h^{i+1}}{\mu_h^i}\ = \lambda\ =\ 1.17$$

⇒ Allow extrapolation to the physical b quark by means of the *"ratio method"*

Extrapolation procedure

At successive values of $\mu_h^{(i)} = \lambda \mu_h^{(i-1)}$

1. Determine the ratio

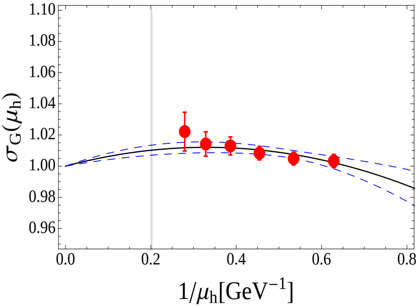
$$\sigma_i(a^2, \mu_l) = \frac{G_s(1,\, \mu_h^{(i)})}{G_s(1,\, \mu_h^{(i-1)})}$$

2. Extrapolate σ_i to the continuum

$$\sigma_i = \left[A_s + B_s\, \mu_l^{\text{sea}} + C_s \frac{a^2}{a_{3.9}^2} \right]$$

No significant dependence on μ_l^{sea} for $B_s \to D_s$ FFs especially for their σ_i

values of σ_i in the continuum :



σ_2	1.003(4)	σ_5	1.013(5)
σ_3	1.005(4)	σ_6	1.014(7)
σ_4	1.008(4)	σ_7	1.022(12)

Errors at $\mu_h = \mu_b/\lambda^2$ are large but do not make any impact on the extrapolation to the infinite quark mass limit where $\sigma = 1$.

value of σ at the physical b quark :

fit with : $$\boxed{\sigma_{(\mu_h)} \; = 1 + \frac{a}{\mu_h} + \frac{b}{\mu_h^2}}$$

Determination of $G_s(1)$

$\sigma \rightsquigarrow \; G_s^{\text{phys.}}(1) = 1.077(39)$

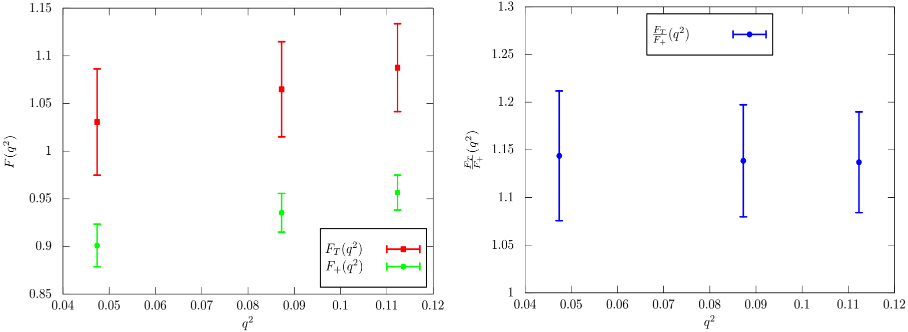
Compatible with $B \to D$ form factor

For comparaison	G(1)
Quenched LQCD G.M.dD *et al.* (2007)	1.026(17)
Unquenched LQCD Okamato *et al.*(2005)	1.074(24)

$F_T/F_+(w, a^2, \mu_l, \mu_h)$

$\frac{F_T}{F_+}$ does not depend on q^2 $\Rightarrow$ $\frac{F_T}{F_+}(w) \simeq$ constant

$\beta = 3.9,\ L = 24,\ a\mu_l = 0.0085,\ a\mu_c = 0.215,\ a\mu_b = 0.4114$



- F_T has larger error bars than F_+ and F_0
- F_T/F_+ shows a slight variation with q^2

near zero recoil

The same strategy of extrapolation is applied to $\frac{F_T}{F_+}$ and $\frac{F_0}{F_+}$

- $w = 1.004 \rightsquigarrow \; q^2_{(B_s \to D_s)} = 11.5 \text{ GeV}^2$
  - $$\boxed{\frac{F_T(w)}{F_+(w)} = 1.222(63) \quad \frac{F_0(w)}{F_+(w)} = 0.827(17)}$$
- $w = 1.02 \rightsquigarrow \; q^2_{(B_s \to D_s)} = 11.2 \text{ GeV}^2$
  - $$\boxed{\frac{F_T(w)}{F_+(w)} = 1.238(65) \quad \frac{F_0(w)}{F_+(w)} = 0.821(21)}$$

1. Exclusive B to D decays

2. Lattice calculation of form factors

3. Simulation and numerical results

4. Conclusions

Conclusions

- Dynamical lattice computation of the $\bar{B}_s \to D_s \ell \bar{\nu}$ form factors

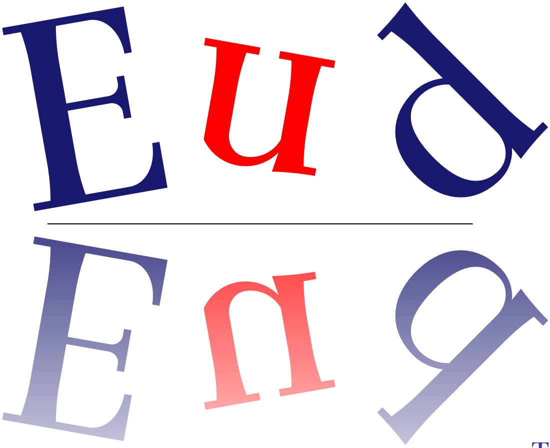
  (unquenched QCD + non perturbative renormalization)

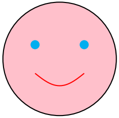
  Results are still preliminary :

  - Computation of $\boxed{G_s(1) : 1.077(39)}$
  - Study of $\boxed{F_0/F_+}$ and $\boxed{F_T/F_+}$ near the zero recoil limit :
    - First LQCD determination of F_T/F_+
    - Better constrained study of NP effects
  - Extrapolation to the bottom quark region using ratios of physical quantities

- Prospective

  - Higher statitics requirement
  - Same strategy can be applied to study $B \to D\ell\bar{\nu}$





Thank you for your attention

Gaussian smearing techniques

- Smearing of the quark fields

$$\psi_q^n = \left(\frac{1+\kappa_g\, H}{1+6\kappa_g}\right)^n \psi_q$$

$$H_{ij} = \sum_{\mu=1}^{3}\left(U_{i,\mu}^{n_\alpha}\delta_{i+\mu,j} + U_{i-\mu,\mu}^{n_\alpha\,\dagger}\delta_{i-\mu,j}\right)$$

  - $n = 30 \quad \kappa_g = 4$
- APE smearing for the gauge fields in H

$$U_{i,\mu}^{n_\alpha} = \mathrm{Proj}_{SU(3)}\left[(1-\alpha)U_{i,\mu}^{n_\alpha -1} + \frac{\alpha}{6}V_{i,\mu}^{n_\alpha}\right]$$

  - $n_\alpha = 20 \quad \alpha = 0.5$