LATTICE 13

JULY 29 – AUGUST 03 2013

MAINZ, GERMANY

$B_{(s)}$ semileptonic decays

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H. Na, and J. Shigemitsu (HPQCD)

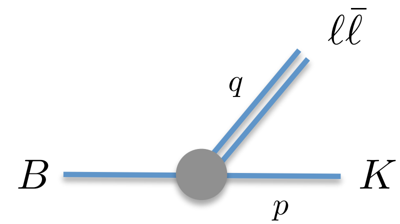
Outline

- $B \to K\ell\bar{\ell}$ calculation [HPQCD, Bouchard et al, 1306.0434; 1306.2384]
  - motivation
  - setup and simulation details
  - correlator fits and matching
  - chiral / continuum and kinematic extrapolations
  - phenomenology
- preliminary results for other decays
  - $B_s \to K\ell\bar{\nu}$ (and $B_s \to \eta_s$)
  - $B_{(s)} \to D_{(s)}\ell\bar{\nu}$
  - $B \to \pi\ell\bar{\nu}$, $\pi\ell\bar{\ell}$
- next steps

$B \to K\ell\bar{\ell}$ motivation

- FCNC $b \to s$ transition probes NP
  - Becirevic et al, 1205.5811
  - Bobeth et al, 1111.2558; JHEP 08 (2012) 030; 1212.2321
  - Altmannshofer and Straub, JHEP 08 (2012) 121
- plenty of experimental results:
  - BaBar, PRD 86, 032012 (2012)
  - Belle, PRL 103, 171801 (2009)
  - CDF, PRL 107, 201802 (2011)
  - LHCb, JHEP 07 (2012) 133; JHEP 1302 (2013) 105
  - promise of improvements at LHCb and BelleII
- No unquenched lattice results. Works in progress:
  - Liu et al, 1101.2726
  - FNAL-MILC, Zhou et al, 1211.1390; A. Kronfeld's talk

$B \to K\ell\bar{\ell}$ setup



- SM $V-A$ leads to hadronic matrix elements $\langle V-A\rangle_{\text{QCD}} = \langle V\rangle_{\text{QCD}}$. In B rest frame

$$\begin{aligned}\langle K|V^0|B\rangle &= \sqrt{2M_B}\; f_{\|}(q^2)\\ \langle K|V^k|B\rangle &= \sqrt{2M_B}\; p^k\; f_{\perp}(q^2)\end{aligned}$$

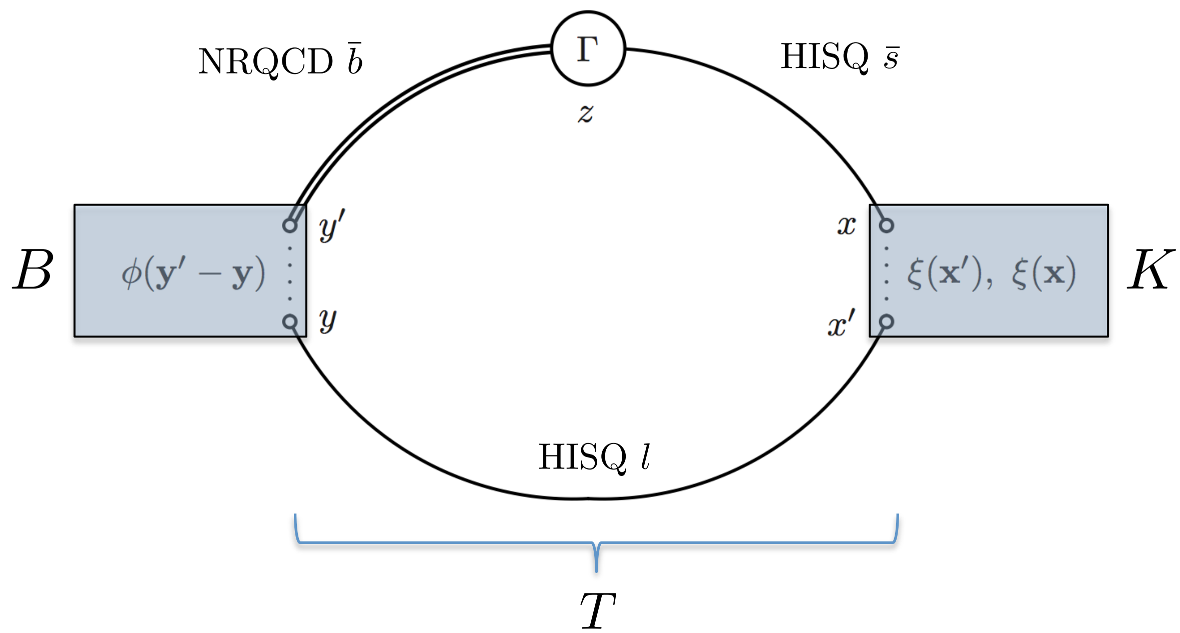
  – phenomenologically relevant form factors

$$\begin{aligned}f_0 &= \frac{\sqrt{2M_B}}{M_B^2 - M_K^2}\left[(M_B - E_K)f_{\|} + \mathbf{p}^2 f_{\perp}\right]\\ f_+ &= \frac{1}{\sqrt{2M_B}}\left[f_{\|} + (M_B - E_K)f_{\perp}\right]\end{aligned}$$

- Generic BSM physics characterized by addition of tensor current

$$\langle K|T^{k0}|B\rangle = \frac{2iM_B p^k}{M_B + M_K}\; f_T(q^2)$$

$B \to K\ell\bar{\ell}$ setup



- Γ: spin-structure of flavor-changing current
- $\phi(\mathbf{y}' - \mathbf{y})$: Gaussian smearing of NRQCD heavy quark
- $\xi(\mathbf{x}')$, $\xi(\mathbf{x})$: $U(1)$ phases for random-wall HISQ sources
- momentum inserted at x

$B \to K\ell\bar{\ell}$ simulation details

- MILC $2+1$ asqtad gauge configurations [MILC, Bazavov et al, RMP 82, 1349 (2010)]

ensemble	$\approx a$ [fm]	$m_l(\text{sea})/m_s(\text{sea})$	N_{conf}	N_{tsrc}	$L^3 \times N_t$
C1	0.12	0.005/0.050	1200	2	$24^3 \times 64$
C2	0.12	0.010/0.050	1200	2	$20^3 \times 64$
C3	0.12	0.020/0.050	600	2	$20^3 \times 64$
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- NRQCD [HPQCD, Lepage et al, PRD 46, 4052 (1992)] b quark, tuned in HPQCD, Na et al, 1202.4914

- HISQ [HPQCD, Follana et al, PRD 75, 054502 (2007)] light, strange valence-quark propagators generated for

  $D \to \pi\ell\bar{\nu}$ [HPQCD, Na et al, PRD 84, 114505 (2011)]
  $D \to K\ell\bar{\nu}$ [HPQCD, Na et al, PRD 82, 14506 (2010)]

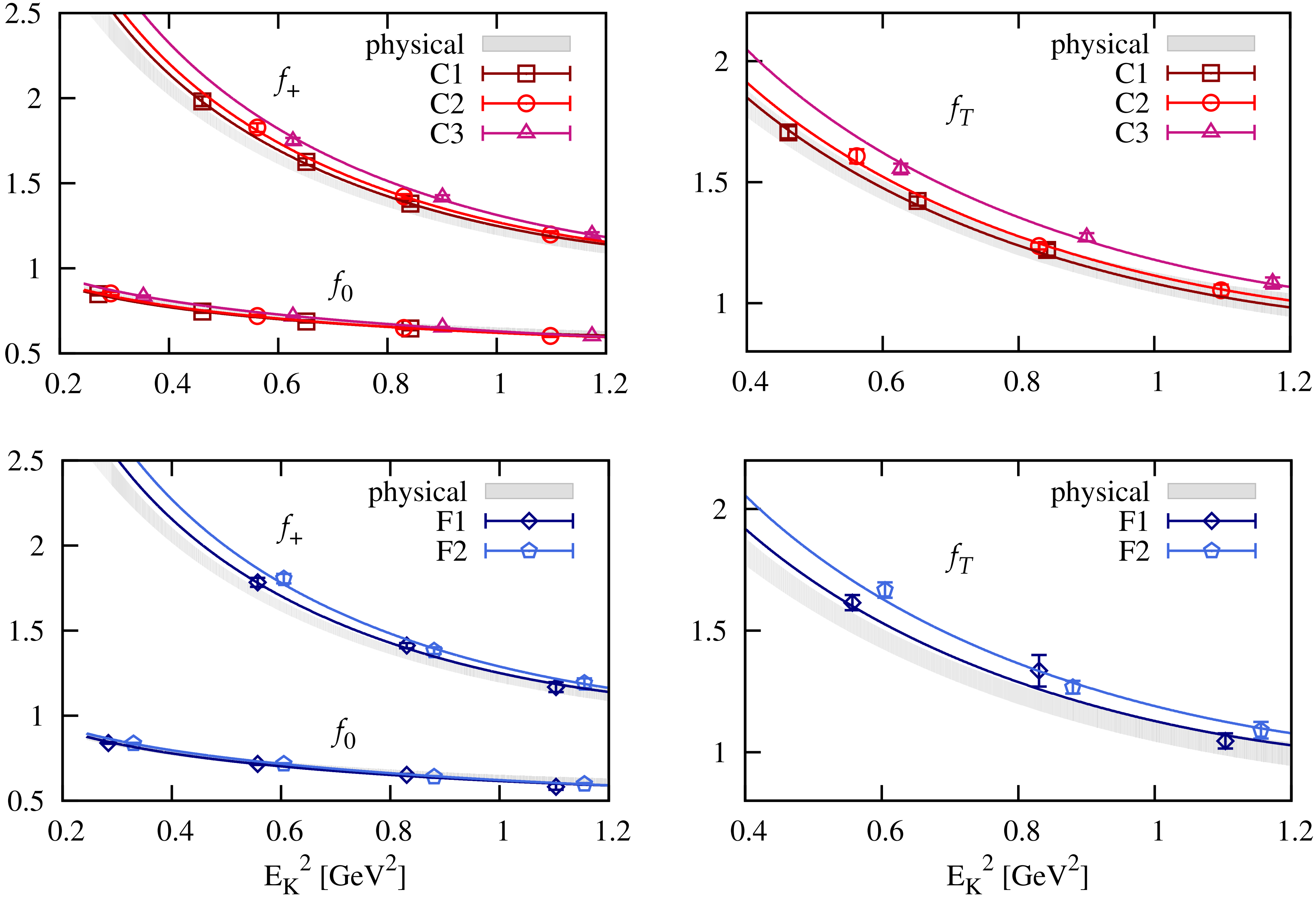
$B \to K\ell\bar{\ell}$ correlator fits and matching

- Bayesian correlator fits
  - 2pts
    - 4 combinations of local, smeared B data
    - 4 K momenta: dispersion relation satisfied to $\mathcal{O}(\alpha_s^2 a^2)$
  - 3pts (simultaneous with 2pts)
    - 3 currents (V_0, V_k, $T_{\mu\nu}$)
    - 4 momenta for V_0, 3 for V_k and $T_{\mu\nu}$
    - multiple T's, both B smearings
    - verify consistent 2pt parameters
- matching [HPQCD, Monahan et al, PRD 87, 034017 (2013)]
  - massless HISQ, 1 loop PT
  - through $\mathcal{O}(\alpha_s, 1/(am_b), \Lambda_{\mathrm{QCD}}/m_b)$

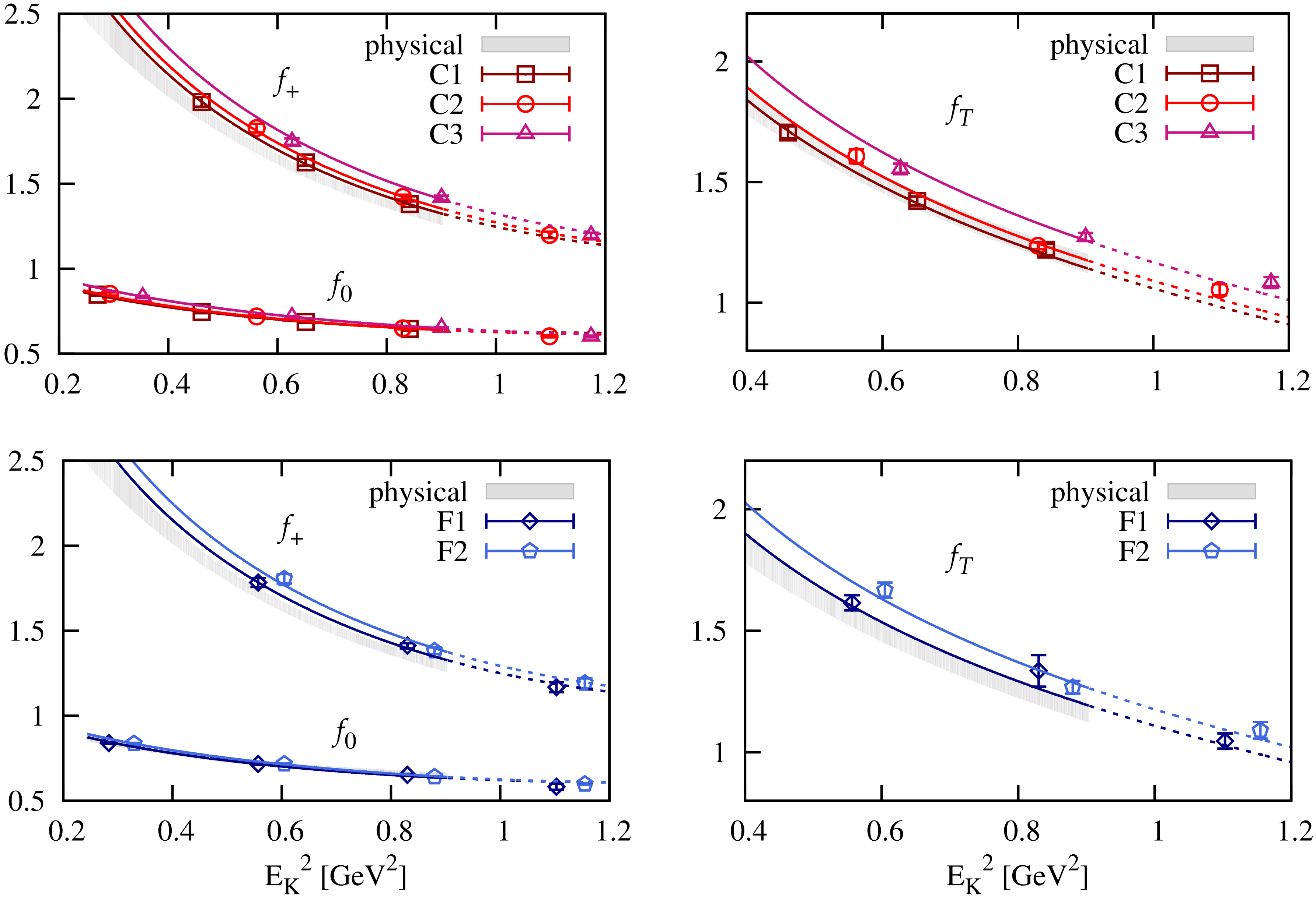
$B \to K\ell\bar{\ell}$ chiral / continuum extrapolation

- NLO chiral logs for $f_{\parallel,\perp}$ from PQsχPT [Aubin & Bernard, PRD 76, 014002 (2007)]
  - swap explicit taste-breaking for generic disc effects
  - fit ansatz for $f_{0,+}$ in terms of $f_{\parallel,\perp}$
- fit ansatz for f_T based on $f_\perp$
  - at LO in $1/m_b$, $f_T \propto f_\perp$
  - same chiral logs
  - same analytic terms but separate coefficients
- inlcude NLO and NNLO chiral analytic terms

- simultaneous extrapolation for f_0, f_+, f_T with $\chi^2/\mathrm{dof} = 35.1/50$



- extrapolation describes data near the chiral scale $\Lambda_\chi \sim 1$ GeV



$B \to K\ell\bar{\ell}$ kinematic extrapolation

- data points, with covariance matrix, from chiral/continuum extrapolation
- model-independent z expansion [Boyd et al, NPB 461, 493 (1996)]

$$q^2 \to z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \qquad \begin{array}{l} t_+ = (M_B + M_K)^2 \\ t_0 \text{ free} \end{array}$$

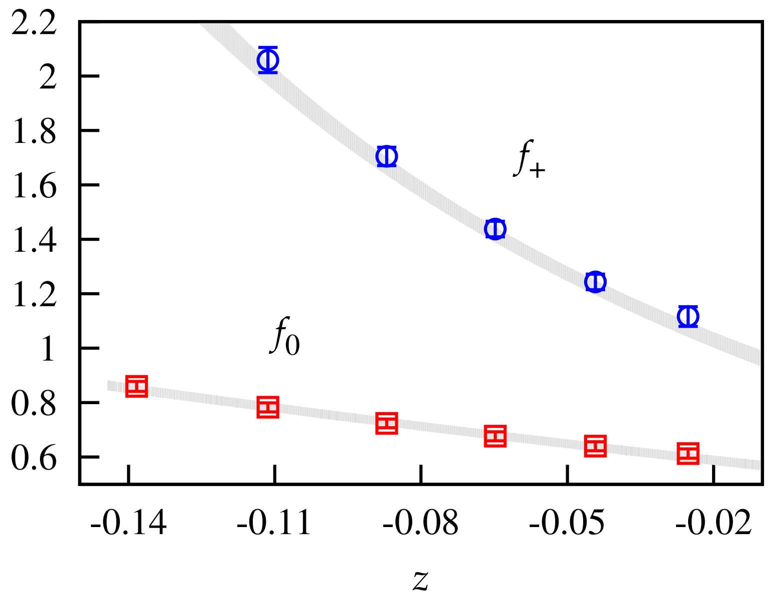
- "BCL" parameterization [Bourrely et al, PRD 79, 013008 (2009); PRD 82, 099902 (2010)]

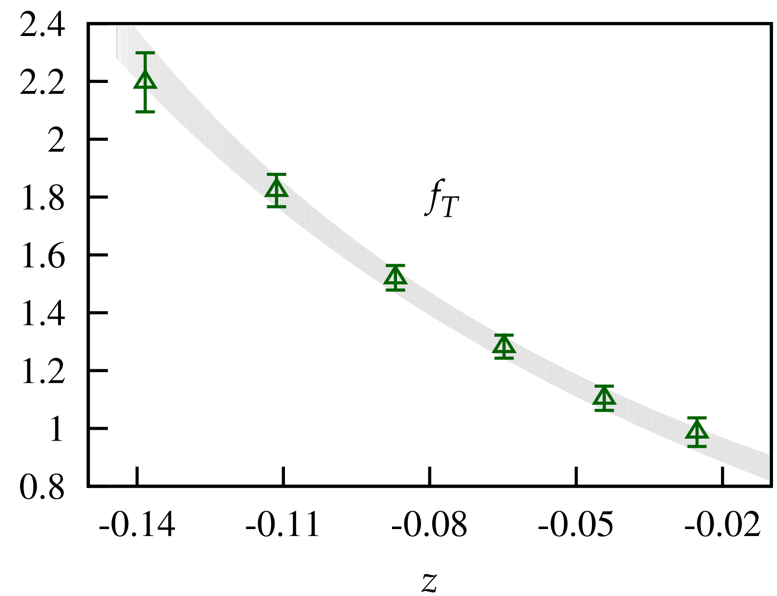
$$\begin{aligned} f_0(q^2) &= \sum_{k=0}^{K} a_k^0 \, z(q^2)^k \\ f_i(q^2) &= \frac{1}{P_i(q^2)} \sum_{k=0}^{K-1} a_k^i \left[z(q^2)^k - (-1)^{k-K} \frac{k}{K} z(q^2)^K \right], \text{ where } i = +, T \end{aligned}$$

- poles removed by "Blaschke" factor, $P_i(q^2) = 1 + q^2/(M_i^{\text{pole}})^2$

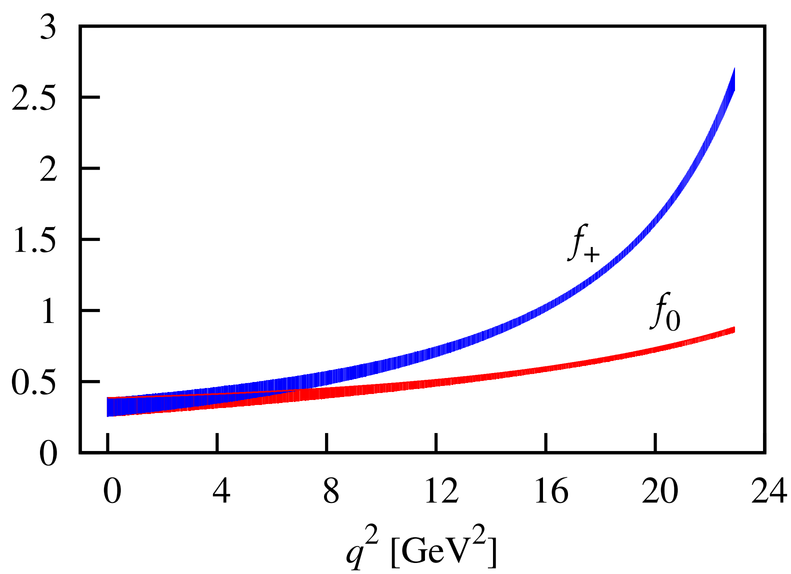
$$M_+^{\text{pole}} = (5325.36 \pm 35) \text{ MeV} \qquad M_T^{\text{pole}} = (5325 \pm 35) \text{ MeV}$$

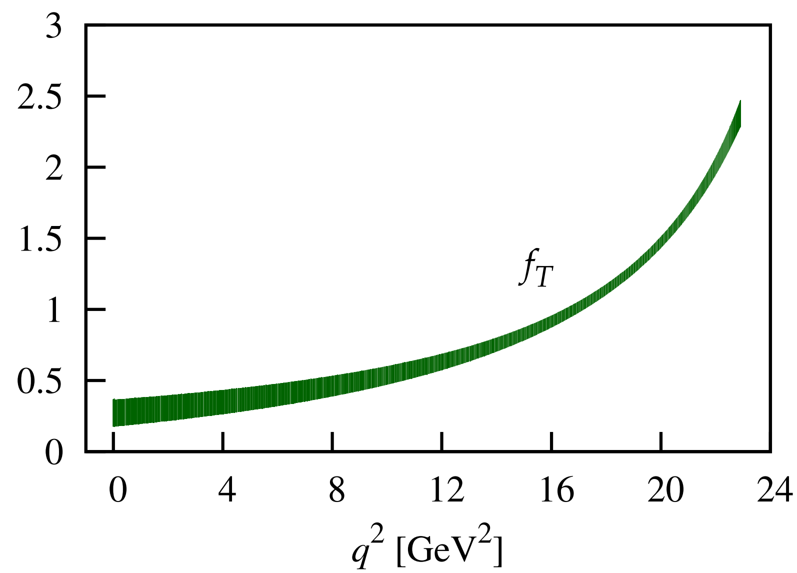
- simultaneous extrapolation for f_0, f_+, f_T with $\chi^2/\mathrm{dof} = 8.58/11$ (svd cut)





- extrapolated curves over full range of q^2



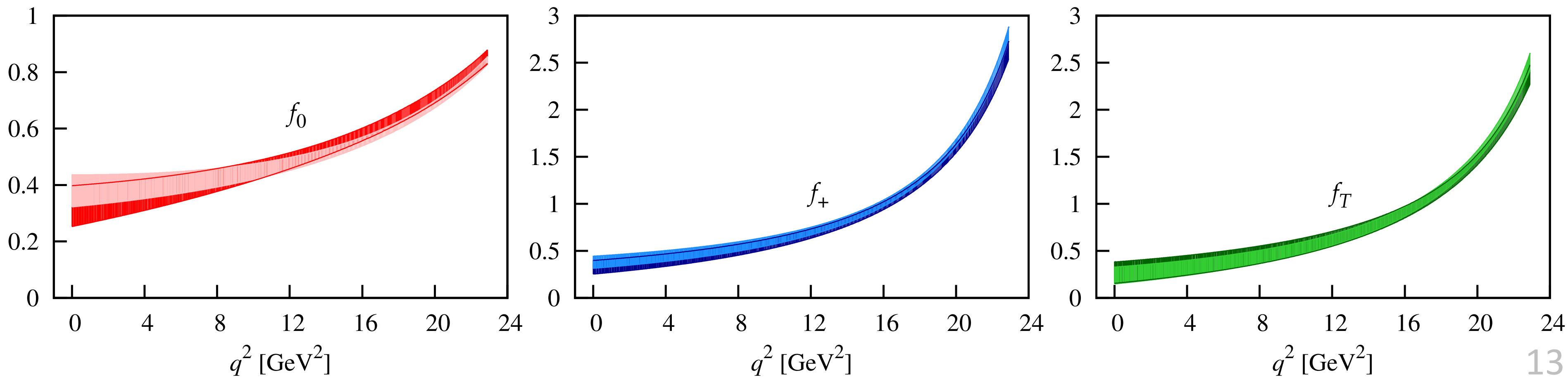


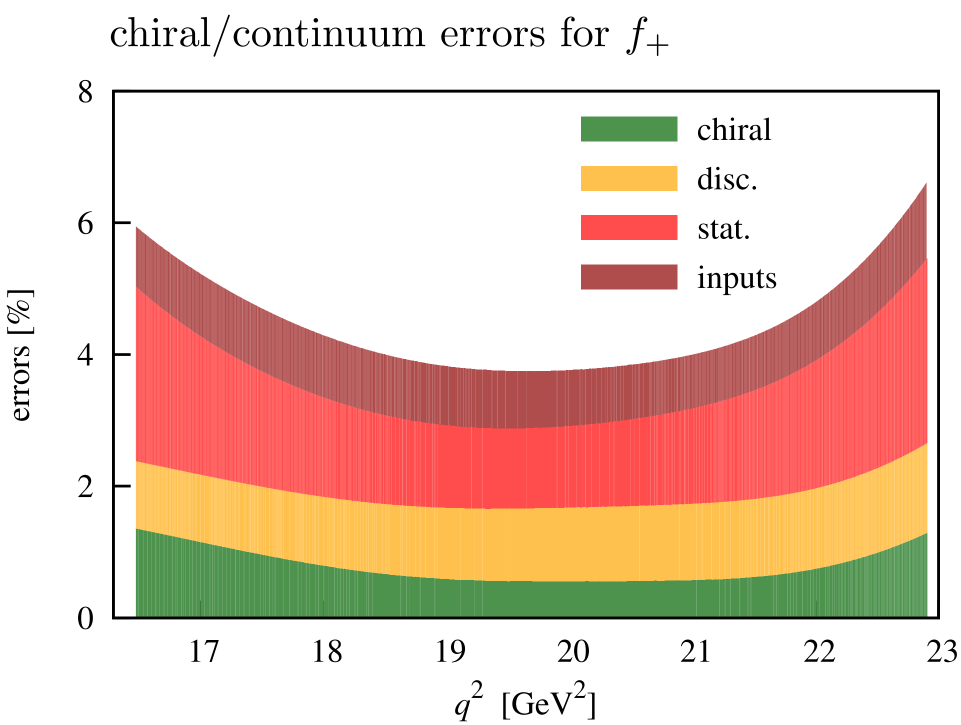
- cross check with "modified" z expansion [HPQCD, Na et al, PRD 82, 114506 (2010); PRD 84, 114505 (2011)]

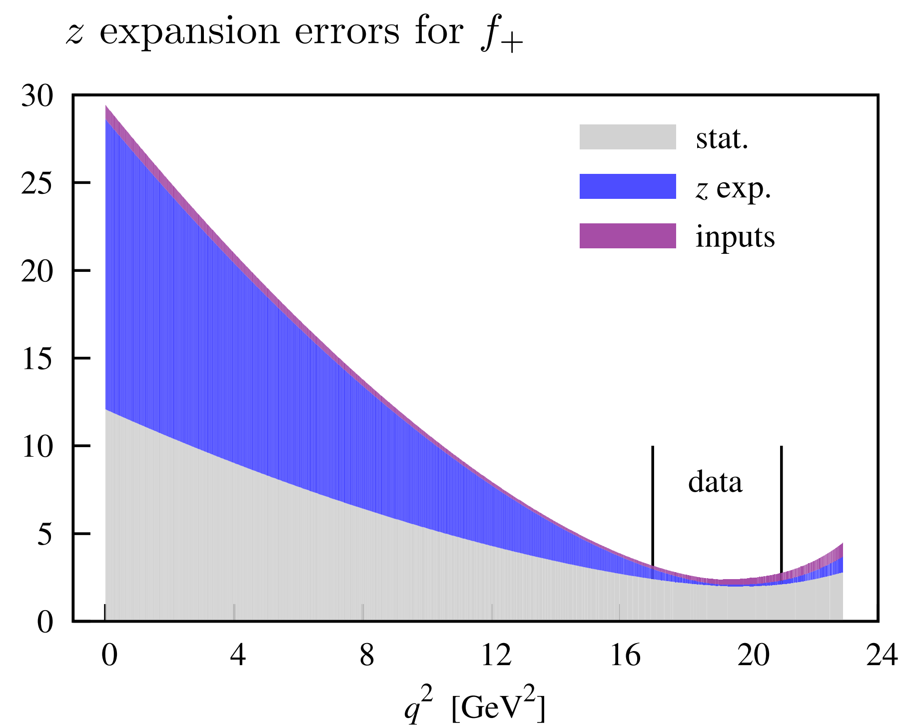
$$
\begin{aligned}
f_0(q^2) &= B_0 \sum_{k=0}^{K} a_k^0 D_k^0 \, z(q^2)^k \\
f_i(q^2) &= \frac{B_i}{P_i(q^2)} \sum_{k=0}^{K-1} a_k^i D_k^i \left[z(q^2)^k - (-1)^{k-K} \frac{k}{K} z(q^2)^K \right]
\end{aligned}
$$

  - calculate q^2, z, P_i separately for each ensemble
  - coefficients of z expansion $a_k \to B a_k D_k$
    - B, D_k are 1 + terms motivated by PQsχPT
    - in chiral and continuum limits $B, D_k \to 1$

two step extrapolation (dark bands) vs. modified z expansion (light bands)

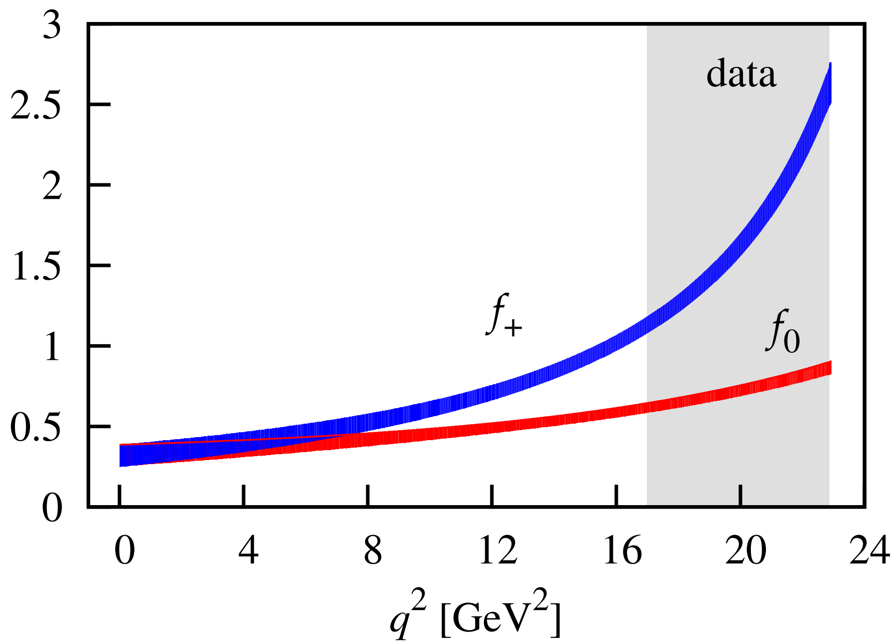


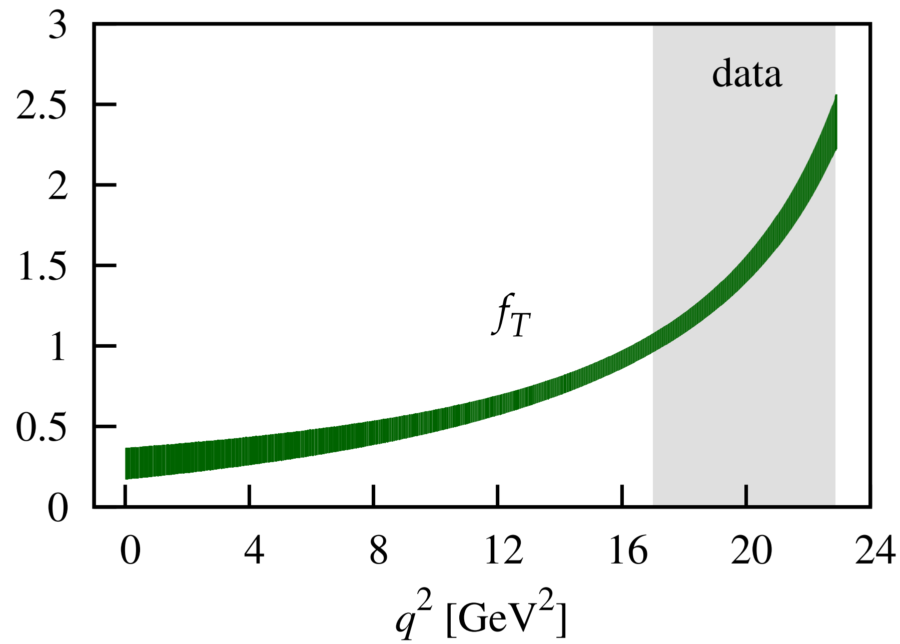




- breakdown of errors [HPQCD, Davies et al, PRD 78, 114507 (2008)]
- truncation (ChPT and z exp), FV, and mass-dep disc errors included
- total fit error from sum in quadrature of z expansion errors
- additional 4% systematic error (**matching**, charm sea, EM/isospin)

$B \to K\ell\bar{\ell}$ form factor results





		$f_0(0) = f_+(0)$	$f_T(0)$	
	this work	0.319 ± 0.066	0.270 ± 0.095	1306.2384
quenched	Bečirević *et al.* [1]	0.33 ± 0.04	0.31 ± 0.04	1205.5811
sum rules	Khodjamirian *et al.* [7]	$0.34^{+0.05}_{-0.02}$	$0.39^{+0.05}_{-0.03}$	JHEP 1009 (2010) 089

TABLE IV: Comparison of form factor results at $q^2 = 0$.

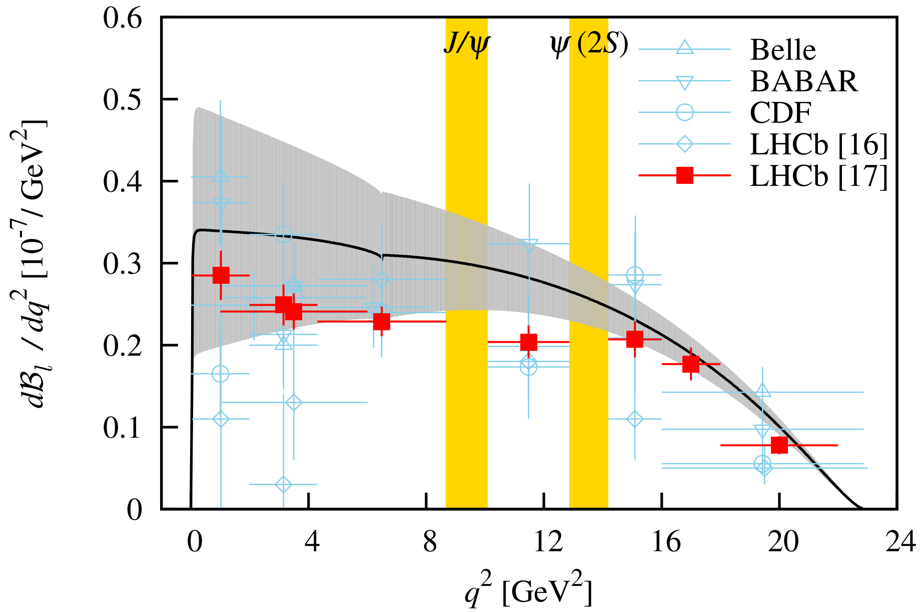
$B \to K\ell\bar{\ell}$ phenomenology

- differential branching fractions

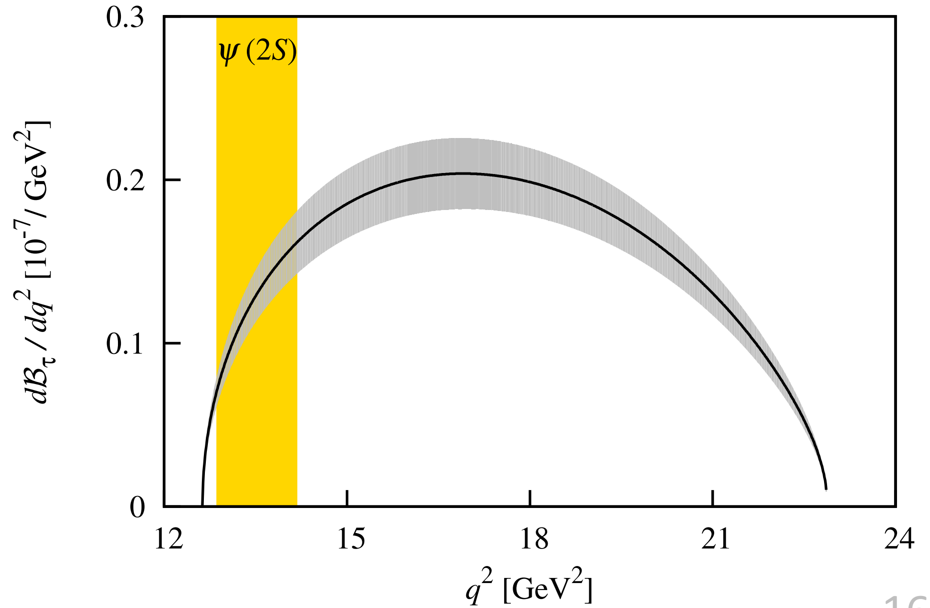
$$\frac{d\mathcal{B}_\ell}{dq^2} = 2\tau_B \left(a_\ell + \frac{1}{3} c_\ell \right)$$

where a_ℓ, c_ℓ are functions of C^i_{Wilson}, f_0, f_+, f_T

SM vs. experiment for light dileptons



SM prediction for ditau



- a new $c\bar{c}$ resonance

  [LHCb, 1307.7595, Monday]

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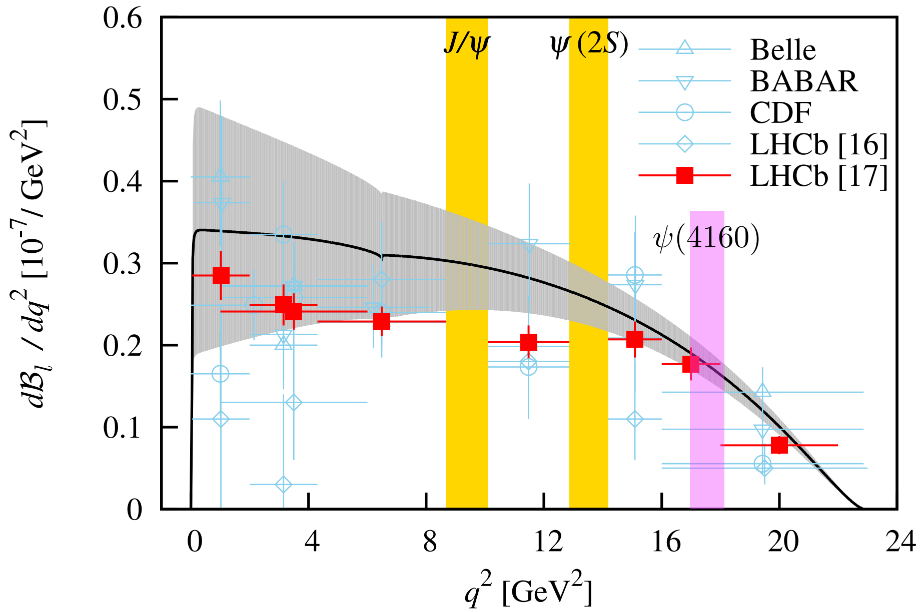

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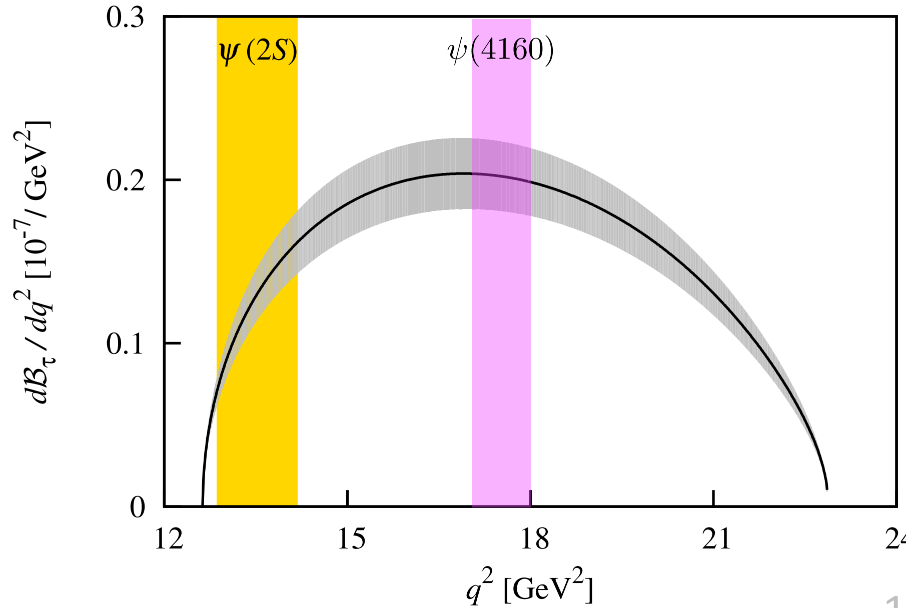

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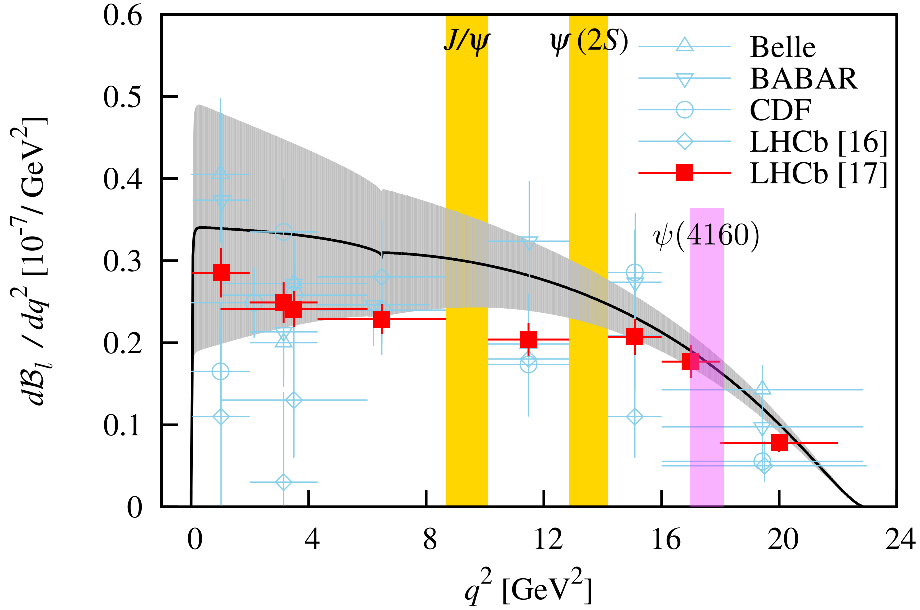

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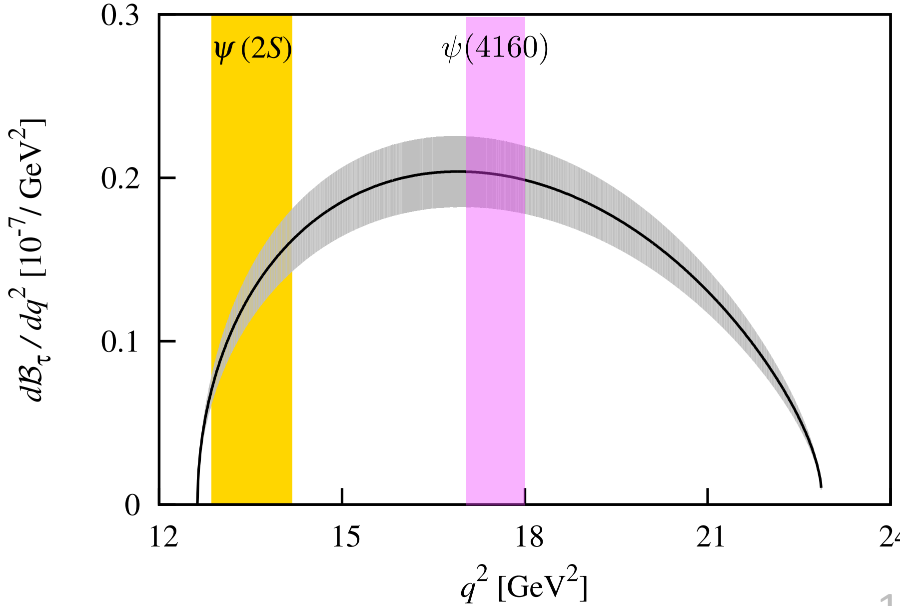
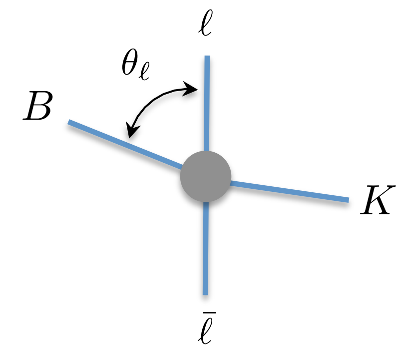


- angular distribution of differential decay rate ($\Gamma_\ell = \mathcal{B}_\ell/\tau_B$)

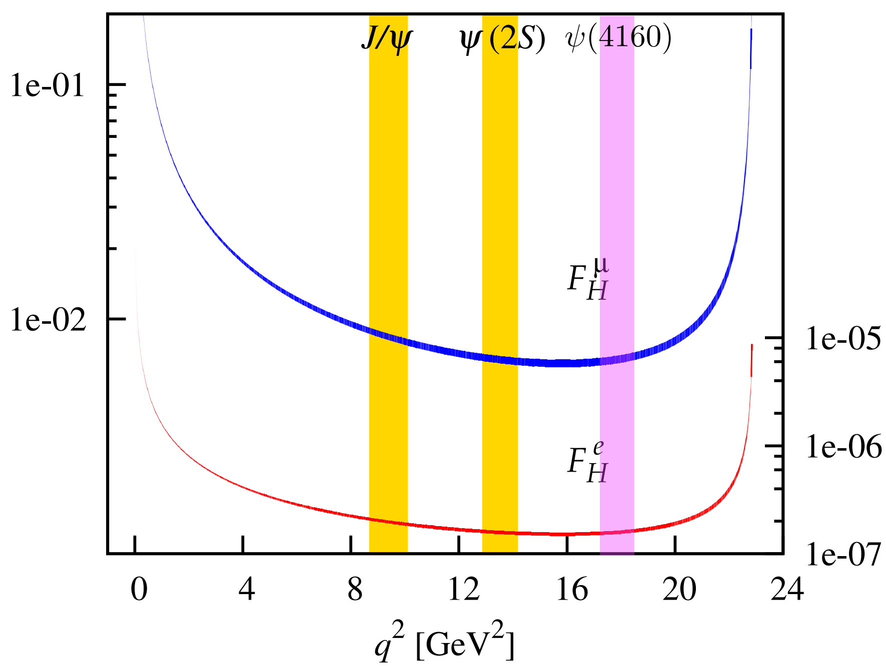
$$\frac{1}{\Gamma_\ell}\frac{d\Gamma_\ell}{d\cos\theta_\ell} = \frac{1}{2}F_H^\ell + A_{FB}^\ell\cos\theta_\ell + \frac{3}{4}(1-F_H^\ell)(1-\cos^2\theta_\ell)$$

"flat term": $F_H^\ell(q^2) = \dfrac{a_\ell + c_\ell}{a_\ell + \frac{1}{3}c_\ell}$

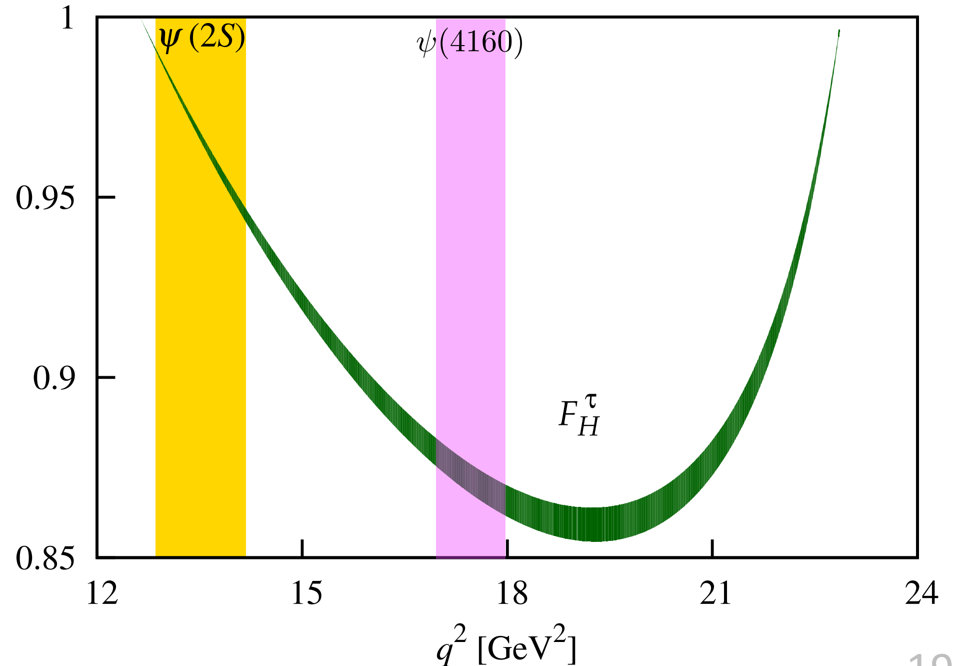


in dilepton rest frame

SM prediction for light dileptons



SM prediction for ditau



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$B_s \to K\ell\bar{\nu}$ (and $B_s \to \eta_s$)

- $B_s \to K\ell\bar{\nu}$
  - should be measured by LHCb and/or BelleII (prediction opportunity)
  - no lattice calculation; FNAL-MILC analysis underway [A. Kronfeld's talk]
  - s spectator makes lattice calculation easier than $B \to \pi\ell\bar{\nu}$
  - alternative exclusive $|V_{ub}|$

- $B_s \to K\ell\bar{\nu}$ and $B_s \to \eta_s$

$$\left.\frac{f_\parallel^{B_s\to K}(q^2_{\max})}{f_\parallel^{B_s\to\eta_s}(q^2_{\max})}\right|_{\text{NRQCD } b} \times \left. f_\parallel^{B_s\to\eta_s}(q^2_{\max})\right|_{\text{HISQ } b}$$

  - $B_s \to K\ell\bar{\nu}$ with essentially no matching error
  - nonperturbative matching factor for $b \to u$ current $(B \to \pi\ell\bar{\nu})$
  - simultaneous fit of $B_s \to K$ and $B_s \to \eta_s$ data requires new methods

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- $B_s \to K\ell\bar{\nu}$ and $B_s \to \eta$

$$\frac{f_{\parallel}^{B_s\to K}(q^2}{f_{\parallel}^{B_s\to\eta_s}(q^2}$$

HISQ b

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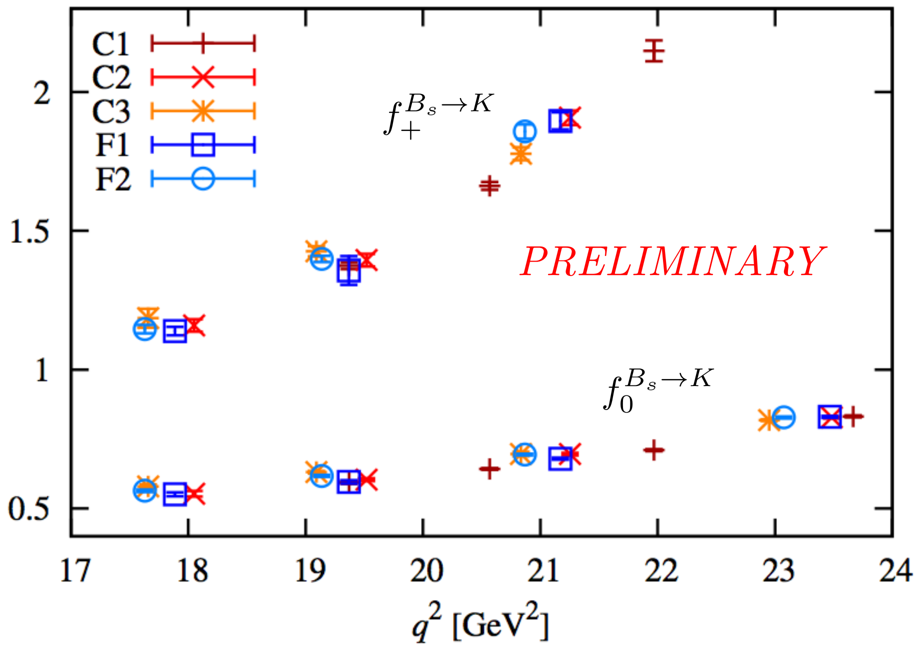
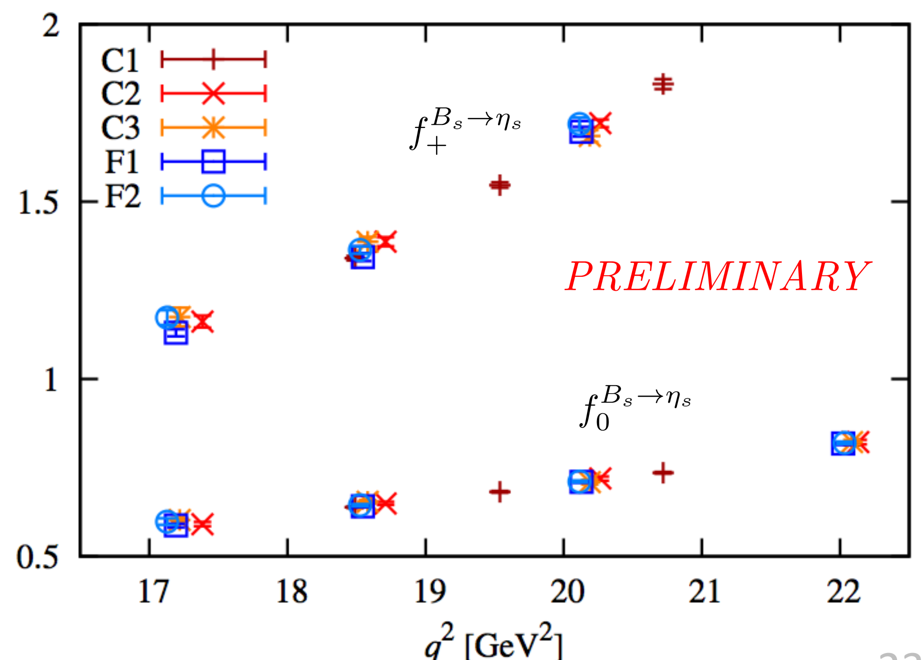
$B_s \to K\ell\bar{\nu}$ (and $B_s \to \eta_s$)





$B_{(s)} \to D_{(s)} \ell \bar{\nu}$

- alternative exclusive $|V_{cb}|$
  - inclusive vs. exclusive
  - other works underway
    * $B \to D\ell\bar{\nu}$ [S. Qiu's talk]
    * $B_s \to D_s\ell\bar{\nu}$ [M. Atoui's talk]
  - $B \to D\ell\bar{\nu}$ projected to be $(2-3)\%$ in 10 yrs [http://belle2.kek.jp/phyics.html]
  - $B \to D^*\ell\bar{\nu}$ is the standard, with $< 1\%$ expt error

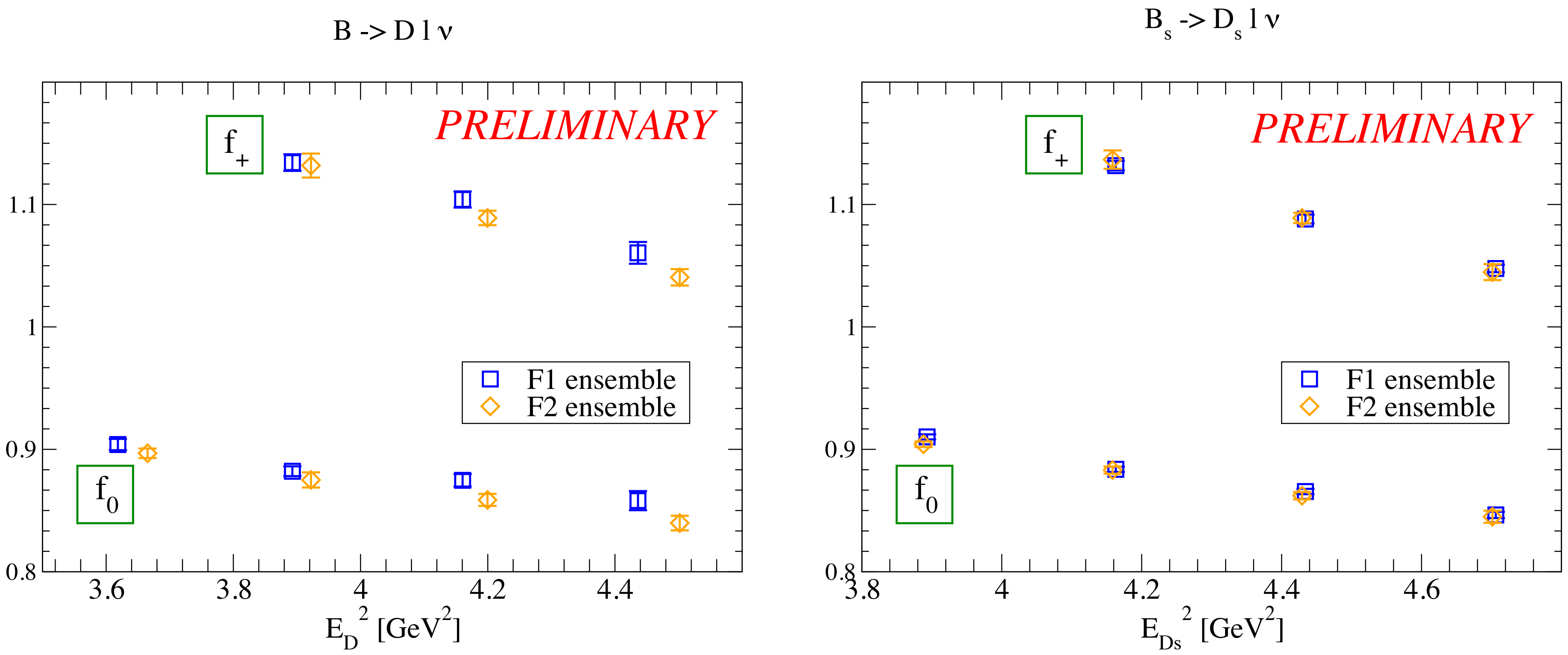
- a useful ratio to constrain NP [FNAL-MILC, Bailey et al, PRL 109, 071802 (2012)]

$$R_D = \frac{\mathcal{B}(B \to \tau\bar{\nu})}{\mathcal{B}(B \to \ell\bar{\nu})}$$

- a useful ratio in $\mathcal{B}(B_s \to \mu\bar{\mu})_{\text{expt}}$ [FNAL-MILC, Bailey et al, PRD 85, 114502 (2012)]

$$\frac{f_0^{B_s \to D_s}(M_\pi^2)}{f_0^{B \to D}(M_K^2)}$$

$$B_{(s)} \to D_{(s)}\ell\bar{\nu}$$



only fine ($a \approx 0.09$ fm) ensemble results shown

$B \to \pi\ell\bar{\nu},\ \pi\ell\bar{\ell}$

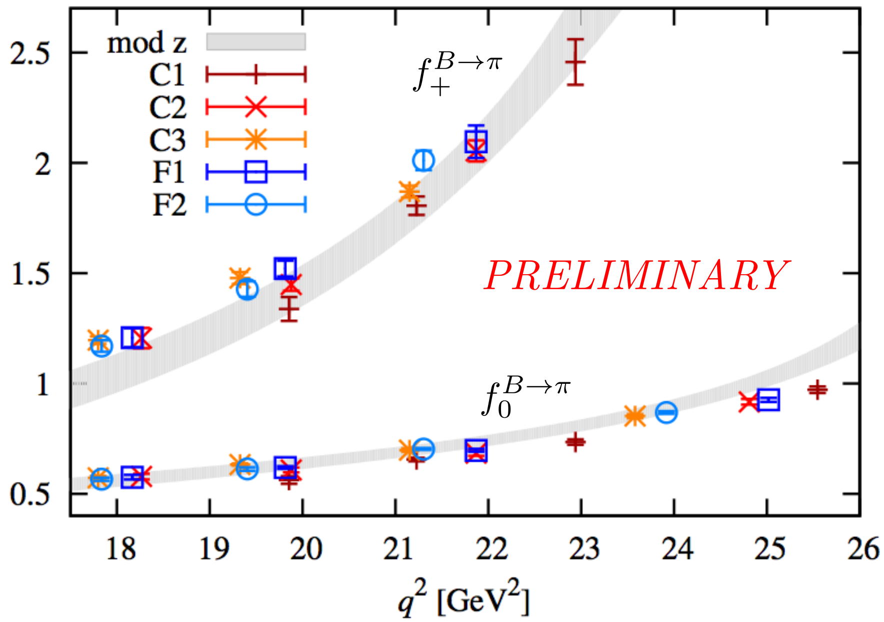
- $B \to \pi\ell\bar{\nu}$
  - "standard" exclusive $|V_{ub}|$
    - inclusive vs. exclusive, $B \to \tau\bar{\nu}$

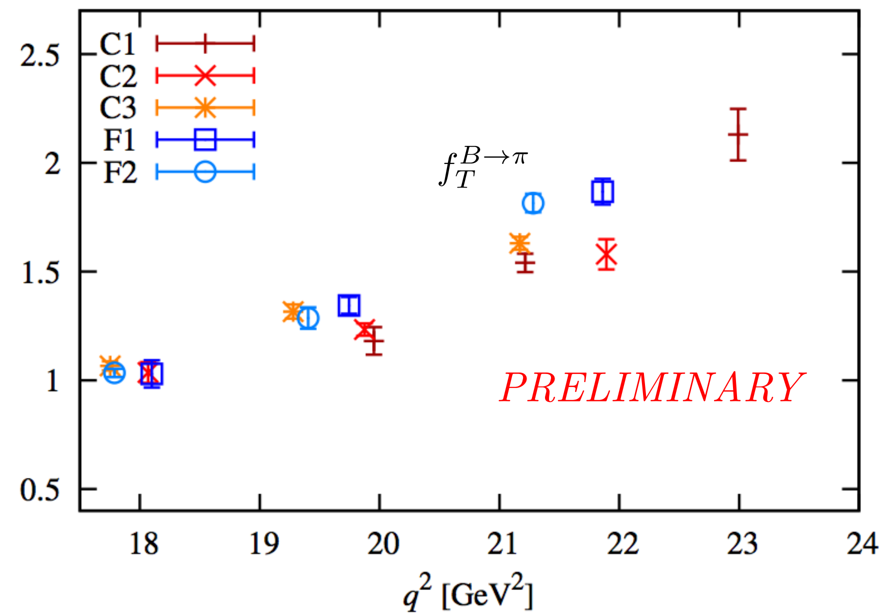
$\delta V_{ub}/V_{ub}$	now	in 5 yrs
expt:	4%	2%
lattice:	8-9%	

  - improve upon HPQCD, PRD 73, 074502 (2006); PRD 75, 119906 (2007)
    - b quark smearing
    - HISQ light valence quarks with random wall sources
    - better scale-determination and quark mass tuning
    - fitting advances (e.g. simultaneous fit multiple separation times)
    - (modified) z expansion

- $B \to \pi\ell\bar{\ell}$
  - $b \to d$ FCNC
  - seen for first time [LHCb, JHEP 12 (2012) 125]

$B \to \pi\ell\bar{\nu},\ \pi\ell\bar{\ell}$





based on preliminary analysis, have since added data...

ensemble	$\approx a$ [fm]	$m_l(\text{sea})/m_s(\text{sea})$	N_{conf}	N_{tsrc}	$L^3 \times N_t$	
C1	0.12	0.005/0.050	1200 (2100)	2 (4)	$24^3 \times 64$	$\longleftarrow\ \mathbf{p}_\pi : \frac{2\pi}{L}(2,0,0)$
C2	0.12	0.010/0.050	1200 (2100)	2	$20^3 \times 64$	
C3	0.12	0.020/0.050	600	2	$20^3 \times 64$	$\longleftarrow\ \mathbf{p}_\pi : \frac{2\pi}{L}(2,0,0)$
F1	0.09	0.0062/0.031	1200 (1800)	4	$28^3 \times 96$	
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next steps

- correlator fitting methods for “very big” fits (e.g. ratios)
- re-evaluate $B \to \pi$ with added data
  - additional improvements needed?
- chiral/continuum and kinematic extrapolations
  - measured tree-level decays: simultaneous with expt (ie. $B \to \pi$)
  - FCNC or tree-level predictions more challenging
    - $*$ modified z expansion?
    - $*$ hard pion ChPT?

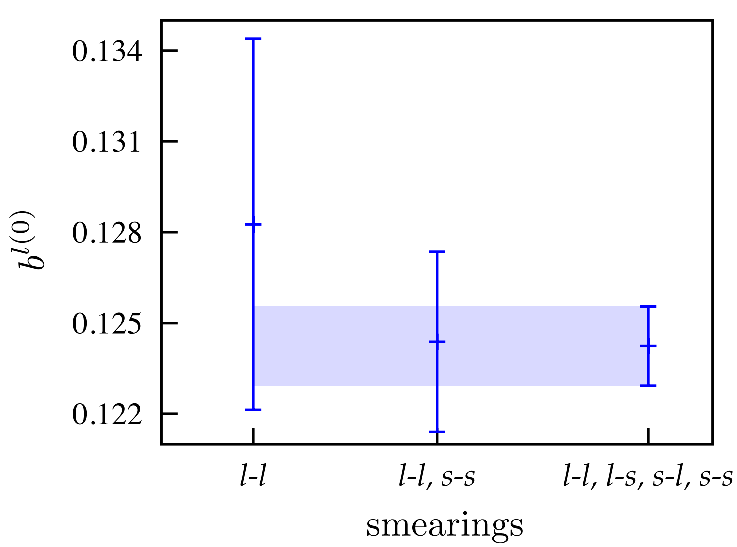
Dankeschön

$B \to K\ell\bar{\ell}$ parent 2pt fits

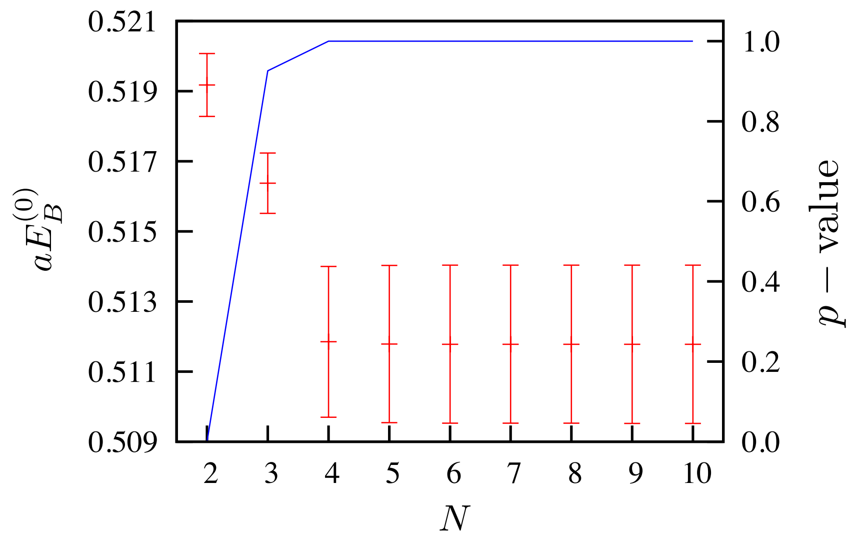
- B rest frame
- Bayesian fit 2×2 matrix of local and Gaussian smeared data

$$C_B^{\alpha\beta}(t) = \sum_{n=0}^{2N-1} b^{\alpha(n)} b^{\beta(n)\dagger} (-1)^{nt} e^{-E_B^{(n)} t} \quad ; \alpha,\ \beta \text{ specify smearing}$$

improvement from smearing



fits are stable vs. N, $t_{\min}$, $t_{\max}$

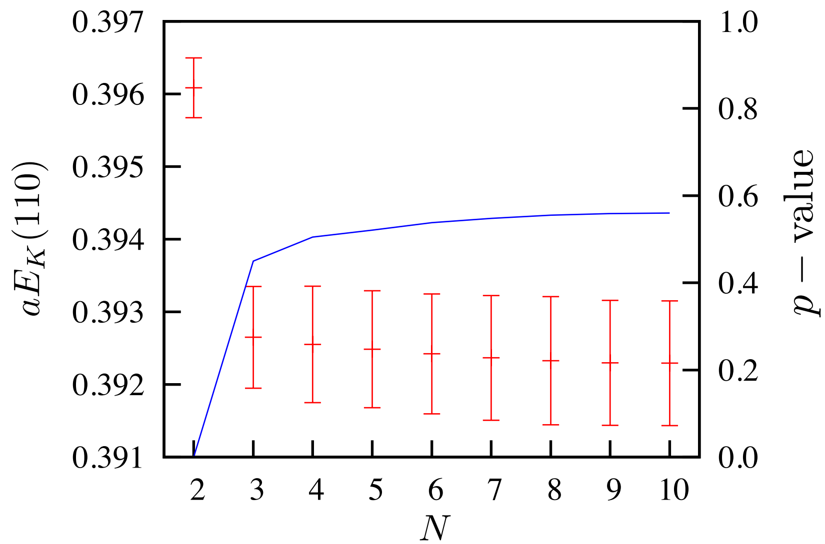


$B \to K\ell\bar{\ell}$ daughter 2pt fits

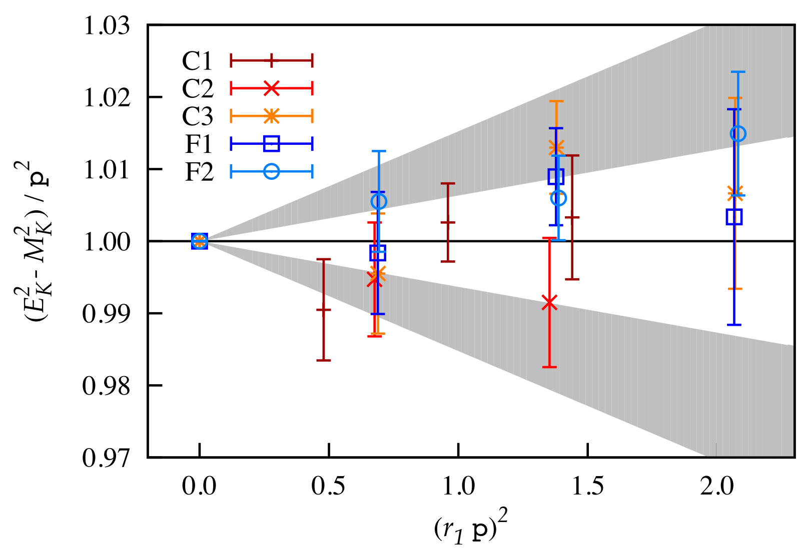
- random-wall sources
- momenta: $\frac{2\pi}{L} \times \{(0,0,0),(1,0,0),(1,1,0),(1,1,1)\}$

$$C_K(t;\mathbf{p}) = \sum_{n=0}^{2N-1} \left|d_{\mathbf{p}}^{(n)}\right|^2 (-1)^{nt} \left(e^{-E_K^{(n)}t} + e^{-E_K^{(n)}(N_t - t)}\right)$$

fits are stable vs. N, $t_{\min}$, $t_{\max}$



dispersion relation satisfied, gray bands correspond to $1 \pm \frac{1}{3}\alpha_s(a\mathbf{p})^2$



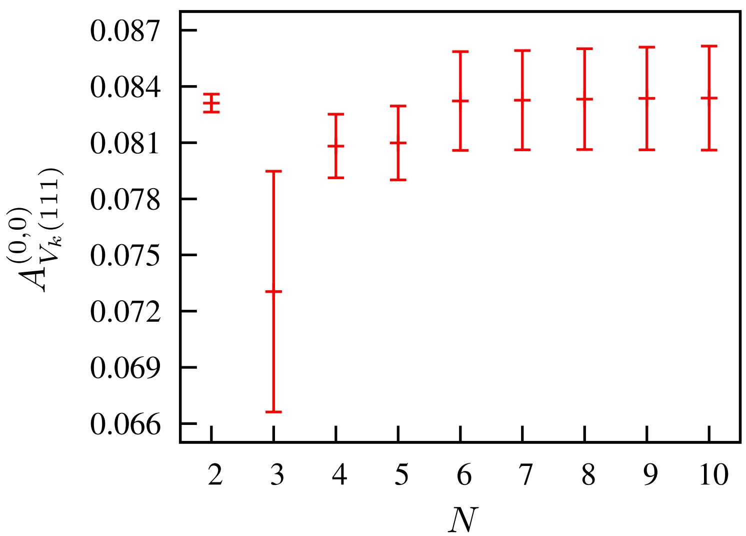
$B \to K\ell\bar{\ell}$ 3pt fits

- simultaneous, correlated, Bayesian fit to 2pt and 3pt data

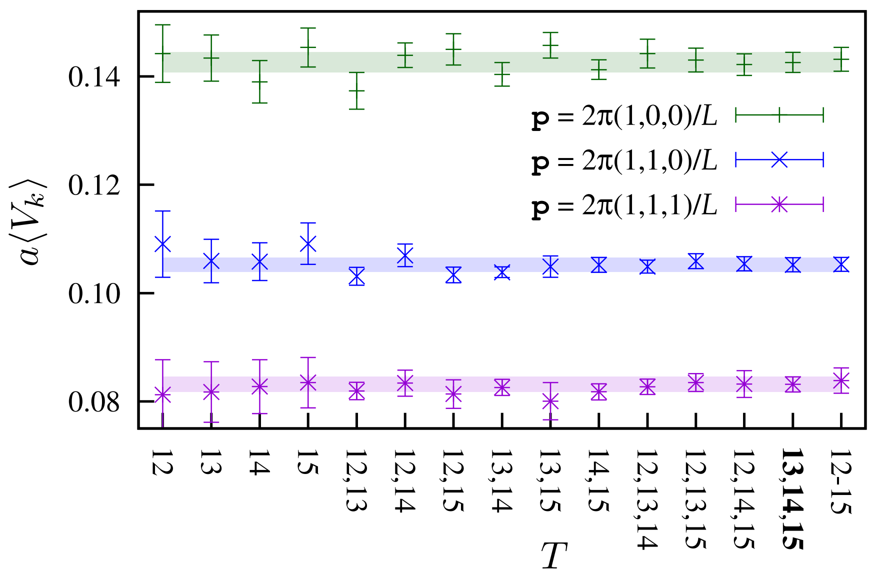
$$C^{\alpha}_{J(\mathbf{p})}(t,T) = \sum_{m,n=0}^{2N-1} d^{(n)}_{\mathbf{p}} A^{(n,m)}_{J(\mathbf{p})} b^{\alpha(m)\dagger} (-1)^{mt+n(T-t)} e^{-E_K^{(n)}(T-t)} e^{-E_B^{(m)} t}$$

where $$A^{(n,m)}_{J(\mathbf{p})} = \frac{a^3 \langle K^{(n)}_{\mathbf{p}} | J | B^{(m)} \rangle}{\sqrt{2a^3 E_K^{(m)}} \sqrt{2a^3 E_B^{(n)}}}$$

fits are stable vs. N, t_{min}



improvement from multiple T's



$B \to K\ell\bar{\ell}$ matching

- massless HISQ, one-loop PT [HPQCD, Monahan et al, PRD 87, 034017 (2013)]

- currents that contribute through $\mathcal{O}(\alpha_s, \alpha_s/(am_b), \Lambda_{\rm QCD}/m_b)$:

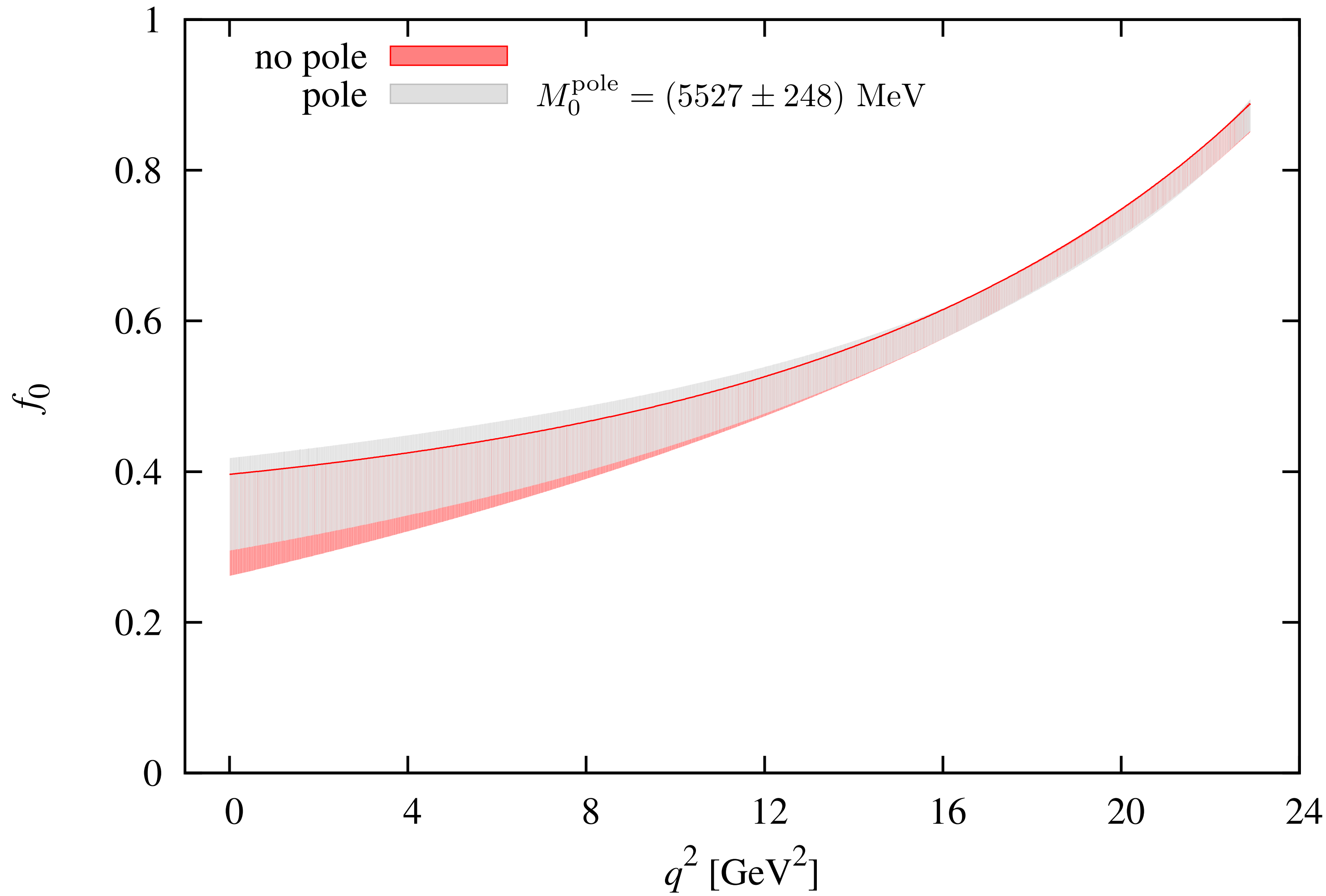
$$\begin{aligned} \mathcal{V}_\mu^{(0)} &= \overline{\Psi}_s\,\gamma_\mu\,\Psi_b & \mathcal{T}_{\mu\nu}^{(0)} &= \overline{\Psi}_s\,\sigma_{\mu\nu}\,\Psi_b \\ \mathcal{V}_\mu^{(1)} &= -\frac{1}{2am_b}\overline{\Psi}_s\,\gamma_\mu\,\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}\,\Psi_b & \mathcal{T}_{\mu\nu}^{(1)} &= -\frac{1}{2am_b}\overline{\Psi}_s\,\sigma_{\mu\nu}\,\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}\,\Psi_b \end{aligned}$$

- matching to continuum matrix elements

$$\langle V_\mu\rangle = \left[1+\alpha_s(\rho_0^V-\zeta_{10}^V)\right]\langle\mathcal{V}_\mu^{(0)}\rangle + \langle\mathcal{V}_\mu^{(1)}\rangle$$

$$\langle T_{k0}\rangle = \left[1+\alpha_s(\rho_0^T-\zeta_{10}^T)\right]\langle\mathcal{T}_{k0}^{(0)}\rangle + \langle\mathcal{T}_{k0}^{(1)}\rangle$$

Considering at scalar pole



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