$B \to \pi$ semileptonic form factors from Lattice QCD

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Fermilab Lattice and MILC Collaborations

Lattice 2013, Aug 2nd, Mainz, Germany

Motivation

- Discrepancy between inclusive/exclusive determination of $|V_{ub}|$. Will it stay?

- Continuous experimental effort on $B \to \pi\ell\nu$ decays .
  BaBar 1208.1253, Belle PRD83(071101)
  Expect $\sim 4\%$ exp. error at Belle II. **Browder, ANL snowmass**

- Possible NP contribution in rare decays $B \to K(\pi)\ell^+\ell^-$ (more by Andreas Kronfeld and Chris Bouchard).
  First observation of $B^+ \to \pi^+\mu^+\mu^-$ by LHCb. **1210.2645**
  But not seen in BaBar **1303.6010** and Belle **0804.3656v2 .** Desy?

- **No** published update from LQCD since 2009. **FNAL/MILC 2008**

Quantity	CKM element	Present expt. error	2007 forecast lattice error	Present lattice error	2018 lattice error
$B \to \pi\ell\nu$	$\lvert V_{ub}\rvert$	4.1%	–	8.7%	2%

2013 snowmass report (quark flavor)

Introduction and Notations

$$\frac{d\Gamma}{dq^2}(B \to \pi \ell \nu) = \text{Phase space} \times |V_{ub}|^2 |f_+|^2$$

$$\begin{aligned} \langle \pi | \mathcal{V}^\mu | B \rangle &= f_+(q^2)\left(p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu\right) + f_0(q^2)\frac{M_B^2 - M_\pi^2}{q^2} q^\mu \\ &= \sqrt{2M_B}\left[v^\mu f_\parallel(E_\pi) + p_\perp^\mu f_\perp(E_\pi)\right] \end{aligned}$$

Convenient for lattice

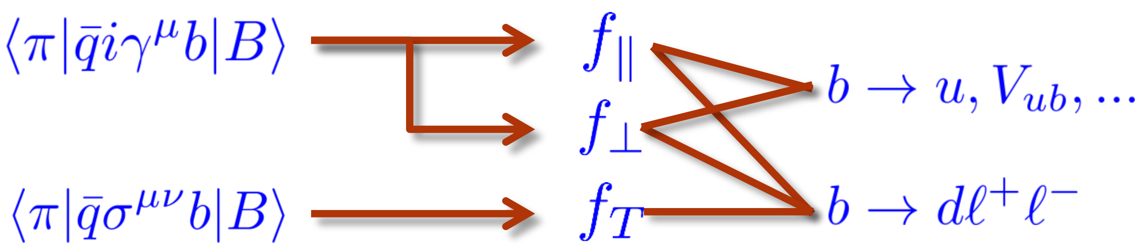
$$\frac{d\Gamma}{dq^2}(B \to \pi \ell^+ \ell^-) = \text{Phase space} \times |V_{tb}V_{td}^*|^2 \left\{ |C_9^{eff} f_+ - C_7 \frac{2m_b f_T}{M_B + M_\pi}|^2 + |C_{10} f_+|^2 \right\}$$

Aliev & Savci 1999

$$\langle \pi | \bar{d} \sigma^{\mu\nu} b | B \rangle q_\nu = -\frac{q^2 (p+k)^\mu - (M_B^2 - M_\pi^2) q^\mu}{M_B + M_\pi} f_T(q^2)$$

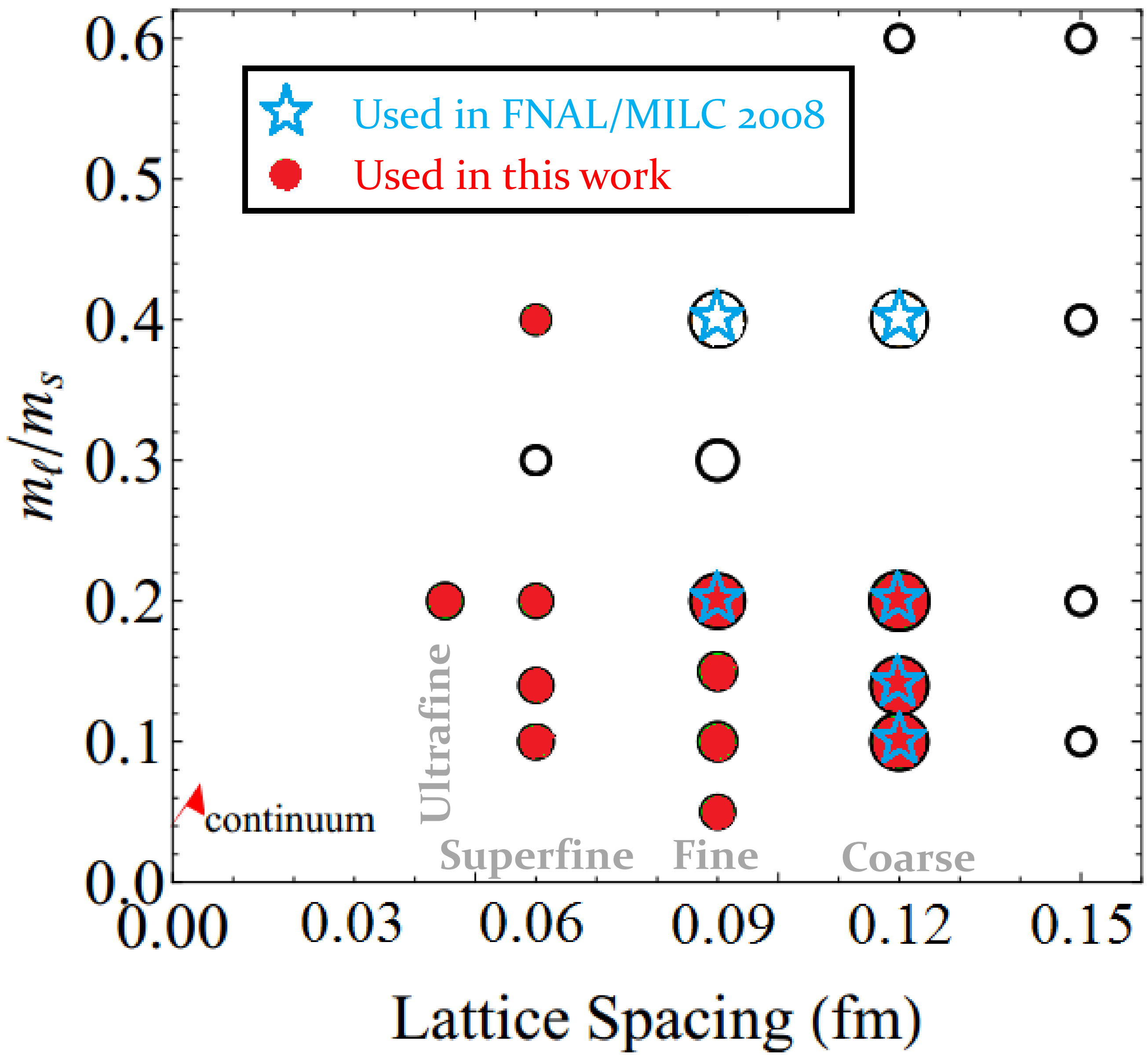
Overview

- What do we calculate?



- What is **new** (compared to FNAL/MILC 2008)?
  - ✓ Major set of the new asqtad data (no u_0 tuning error)
  - ✓ 12 ensembles vs 6
  - ✓ 4 lattice spacings vs 2
  - ✓ Mostly 3 times more configurations for the original ensembles still used
  - ✓ Better understanding of the simulation parameters and the systematic errors (heavy quark mass, Zv, etc)
  - ✓ New analysis methods (due to improved statistics)

MILC asqtad ensembles



Fitting correlation functions

- Method to extract the form factors $f_{\parallel}, f_{\perp}, f_T$
  - Use the 3pt/$\sqrt{2pts}$ ratio to avoid using wave function overlaps. Iteratively average over two sink-source separations to suppress the contributions from oscillating states (opposite parity to the physical particles). Fermilab-MILC 2008

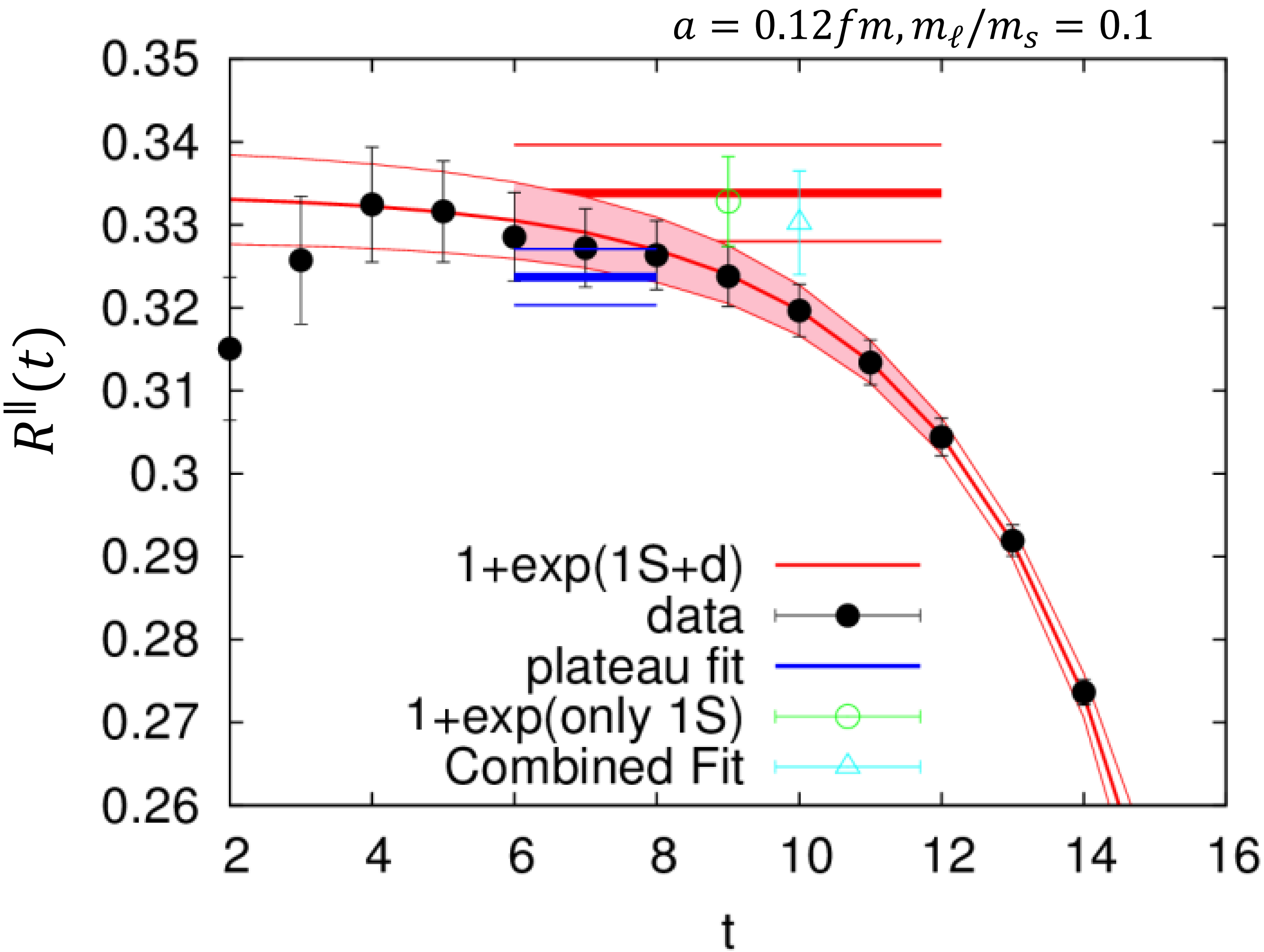
$$R(t) = \frac{\overline{C}_{3pt}(0,t,t_s;\mathbf{p}_\pi)}{\sqrt{\overline{C}^{\pi}_{2pt}(t;\mathbf{p}_\pi)\overline{C}^{B}_{2pt}(t_s-t)}}\sqrt{2E_\pi e^{E_{\pi,0}t+m_{B,0}(t_s-t)}}$$

  - Fit to **plateau + exponential** function reflecting the B meson excited state contribution.

$$R_{(\parallel,\perp,T)}(t) = f^{\mathrm{LAT}}_{(\parallel,\perp,T)} \times \left(1 + \mathcal{A}_{(\parallel,\perp,T)} e^{-\Delta M_B (t_s-t)}\right)$$

  - Combine the fit of ratio and B meson two-point function.

Fitting correlation functions



Chiral/continuum extrapolation

- The one-loop (NLO) HMsχPT expansion

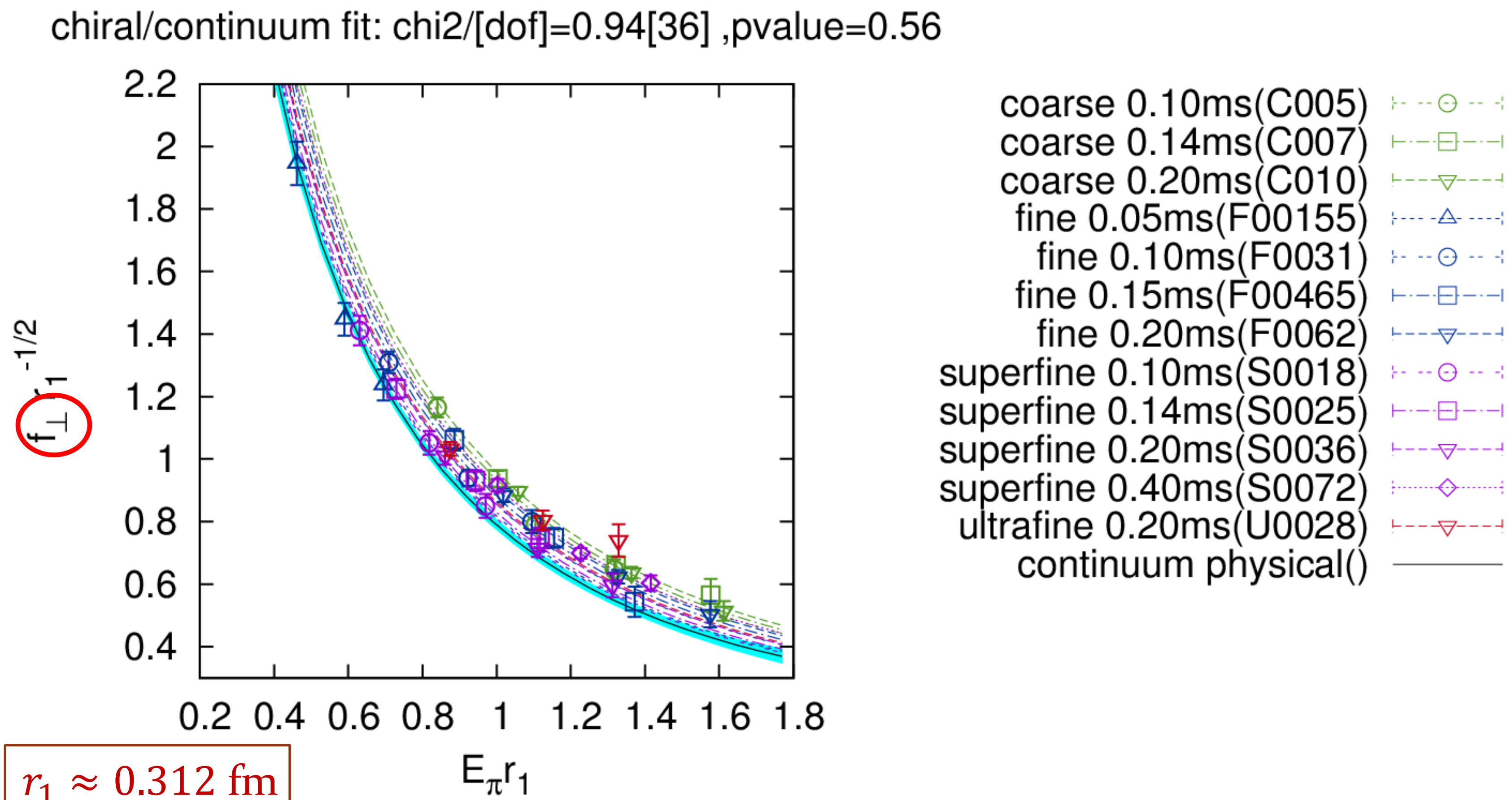
$$f = f^{(0)}\Big(c_0(1+\text{logs}_{SU3}) + c_1 m_{q,val} + c_2 m_{q,sea} + c_3 E_\pi + c_4 E_\pi^2 + c_5 a^2\Big)$$

hard pion SU(2) limit

Leading order: $f_{\parallel}^{(0)} = 1/f_\pi, f_{\perp,T}^{(0)} = (1/f_\pi)(g/(E_\pi + \Delta_B))$

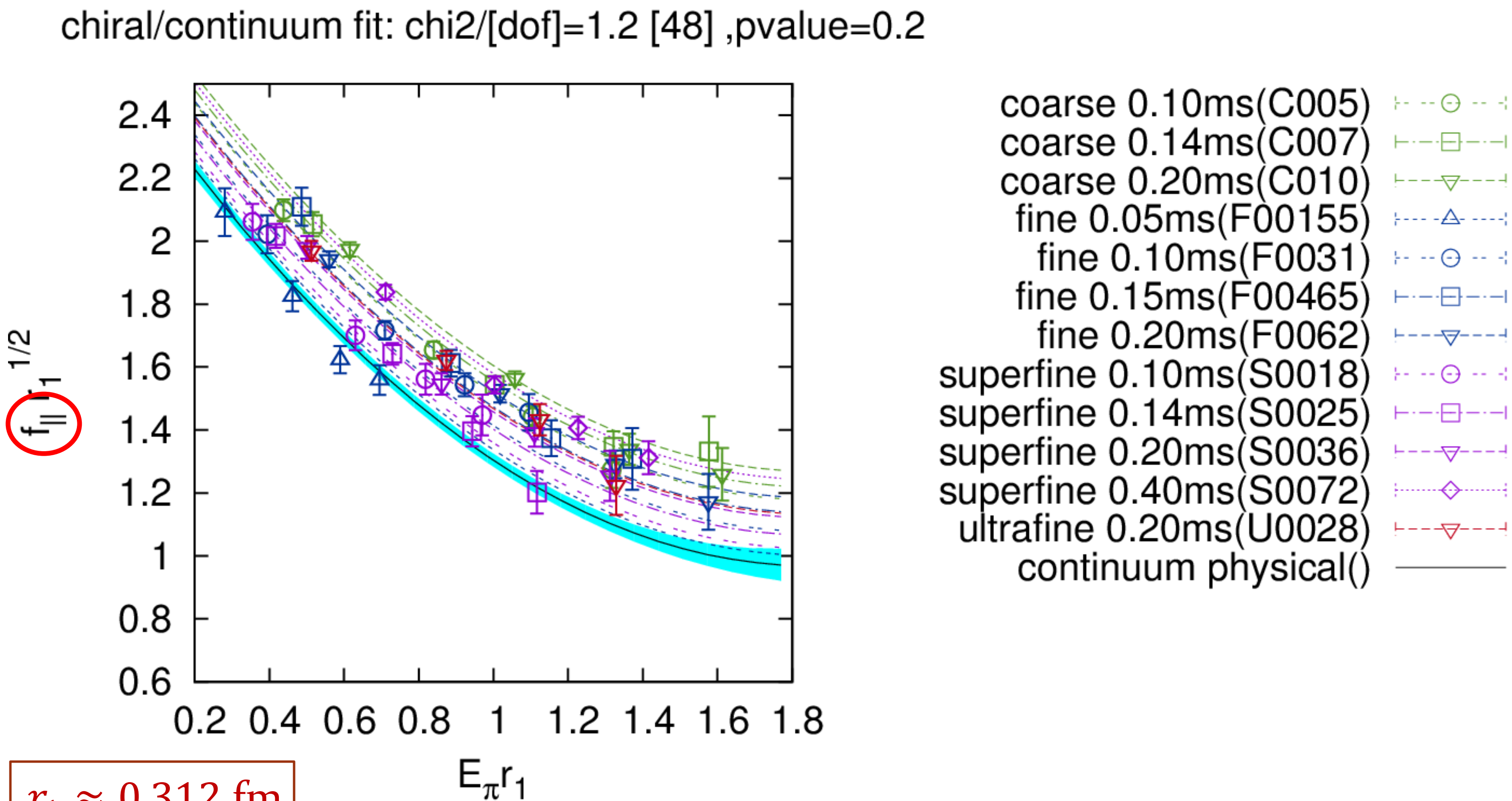
- The non-analytic term $logs$ are replaced by their hard pion 1011.6531 and SU(2) limit, because it fails to describe $f_{\parallel}$ data (even with NNLO analytic terms!)
- Pions in the simulation are to energetic. Hard pion χPT integrates them out. So the E_π dependence is essentially a phenomenological expansion.
- In hard pion limit, $f_{\parallel}, f_{\perp}, f_T$ share the same non-analytic term.
- The variables are normalized with the χPT breaking scale Λ_χ.
- The B-B^*-π coupling $g = 0.45(8)$. It has a small effect.

Chiral/continuum extrapolation



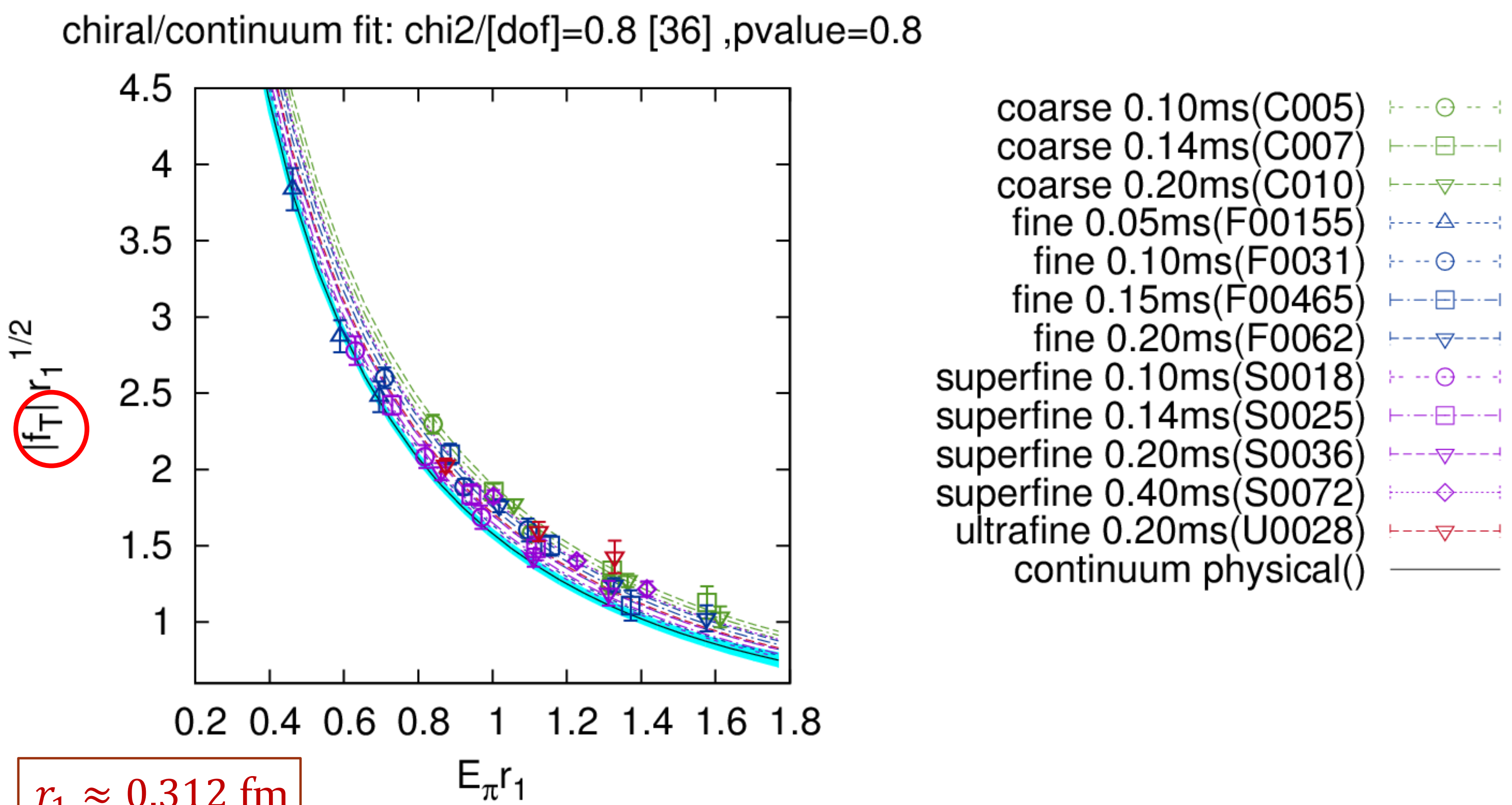
$r_1 \approx 0.312$ fm

Chiral/continuum extrapolation



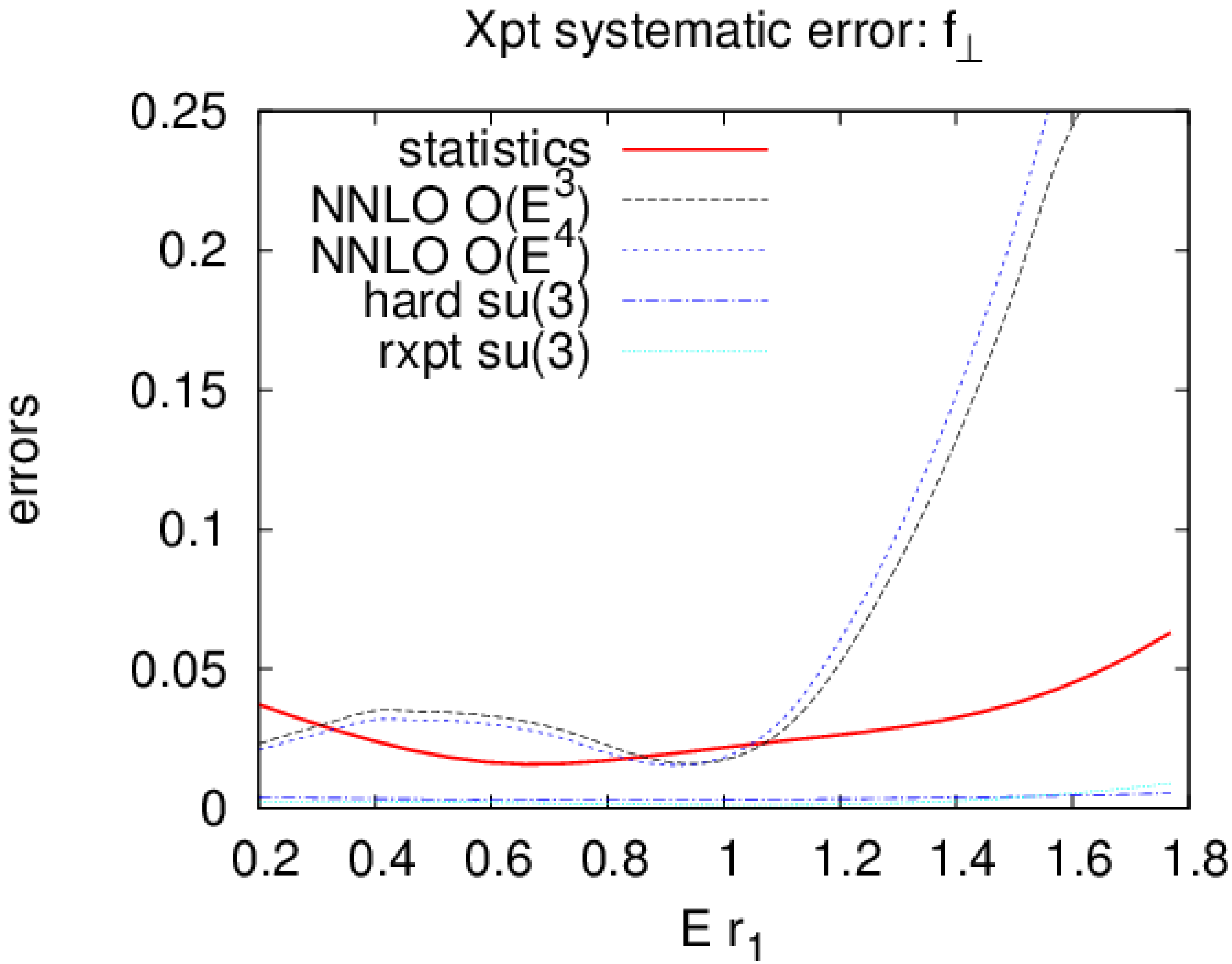
$r_1 \approx 0.312$ fm

Chiral/continuum extrapolation



$r_1 \approx 0.312$ fm

χPT systematic errors



z-expansion

- Measured partial rates are more reliable in low-q^2 region, while lattice has better confidence in large-q^2 region. To determine $|V_{ub}|$, need lattice results in full-q^2 range.
- The model-independent z-expansion is more natural than the χPT expansion ($|z| < 0.4$), plus important physical constraints (analyticity, unitarity, ...).
- A sequential fit (χPT/continuum → z expansion). Correcting an ill expansion with a well behaving expansion. **Fitting a function variable to another function form**!
- Synthetic data approach: sampling the χPT results at "certain" locations ⟶ a regular curve fitting problem.

Issues: how are points taken (number and locations)?

Is correlation correctly accounted? Systematic error?

z-expansion: a functional approach

- A generalization of data-point fitting to functional fitting.
- Covariance function (surface) $K_f(s,t)$ of a function $f(t)$ is a **Mercer kernel** (continuous, symmetry, positive semi-definite). By Mercer's theorem

$$K(s,t) = \sum_{i=1}^{L} \lambda_i \psi_i(s) \psi_i(t)$$

$$\mathcal{O}_K \psi_i = \lambda_i \psi_i$$

$$\mathcal{O}_K \phi(s) \equiv \int K(s,t) \phi(t) dt$$

If $f(t)$ is a finite asymptotic expansion → L finite.

(Pseudo) inverse of $K(s,t)$, $J(s,t)$

- Define an objective function ("χ^2")

"data" ↑ Fit function ↑

$$\begin{aligned} \mathcal{L} &= \int_{z_1}^{z_2} \int_{z_1}^{z_2} dsdt [f(s) - g_f(s)] J(s,t) [f(t) - g_f(t)] \\ &= \sum_{i=1}^{L} \frac{1}{\lambda_i} \Big[\int dt (f(t) - g_f(t)) \psi_i(t) \Big]^2 \end{aligned}$$

- Data point samples: $\{x_i, y_i + \delta y_i\}$
- Target function: $g(x; a_0, ...)$
- Covariance matrix Cov
- Objective function: χ^2

$$\chi^2 = \sum_{i,j} [y_i - g(x_i)] \mathrm{Cov}_{ij}^{-1} [y_j - g(x_j)]$$

- Summation over all data point $\{x_i\}$
- Principal decomposition:

  The eigen modes of Cov

$$\mathrm{cov}\ v_i = \lambda_i\ v_i$$

- Degrees of freedom: $N - n$

  N - # of non-singular modes of Cov

  n - # of fit parameters

- Function samples: $\{x, f(x) + \delta f(x)\}$
- Target function: $g_f(x; a_0, ...)$
- Covariance function $K(s,t)$
- Objective function: $\mathcal{L}$

$$\mathcal{L} = \iint dsdt [f(s) - g_f(s)] K^{-1}(s,t) [f(t) - g_f(t)]$$

- Integral over the data range $[a, b]$
- Principal decomposition:

  The eigen modes of $K(s,t)$

$$\int dt K(s,t) \psi_i(t) = \lambda_i\ \psi_i(s)$$

- Degrees of freedom: $L - n$

  L - # of non-zero eigenmodes of $K(s,t)$

  n - # of fit parameters

z-expansion: a functional approach

- Getting the covariance function $K(s,t)$.

$\langle \delta c_i \, \delta c_j \rangle$

$$f(t) = \sum_i c_i y_i(t) \qquad K_f(s,t) = \sum_{i,j} y_i(s) \mathrm{Cov}_{ij} \, y_j(t)$$

- z-expansion

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$t_+ = (M_B + M_\pi)^2$

t_0 choice to be optimized

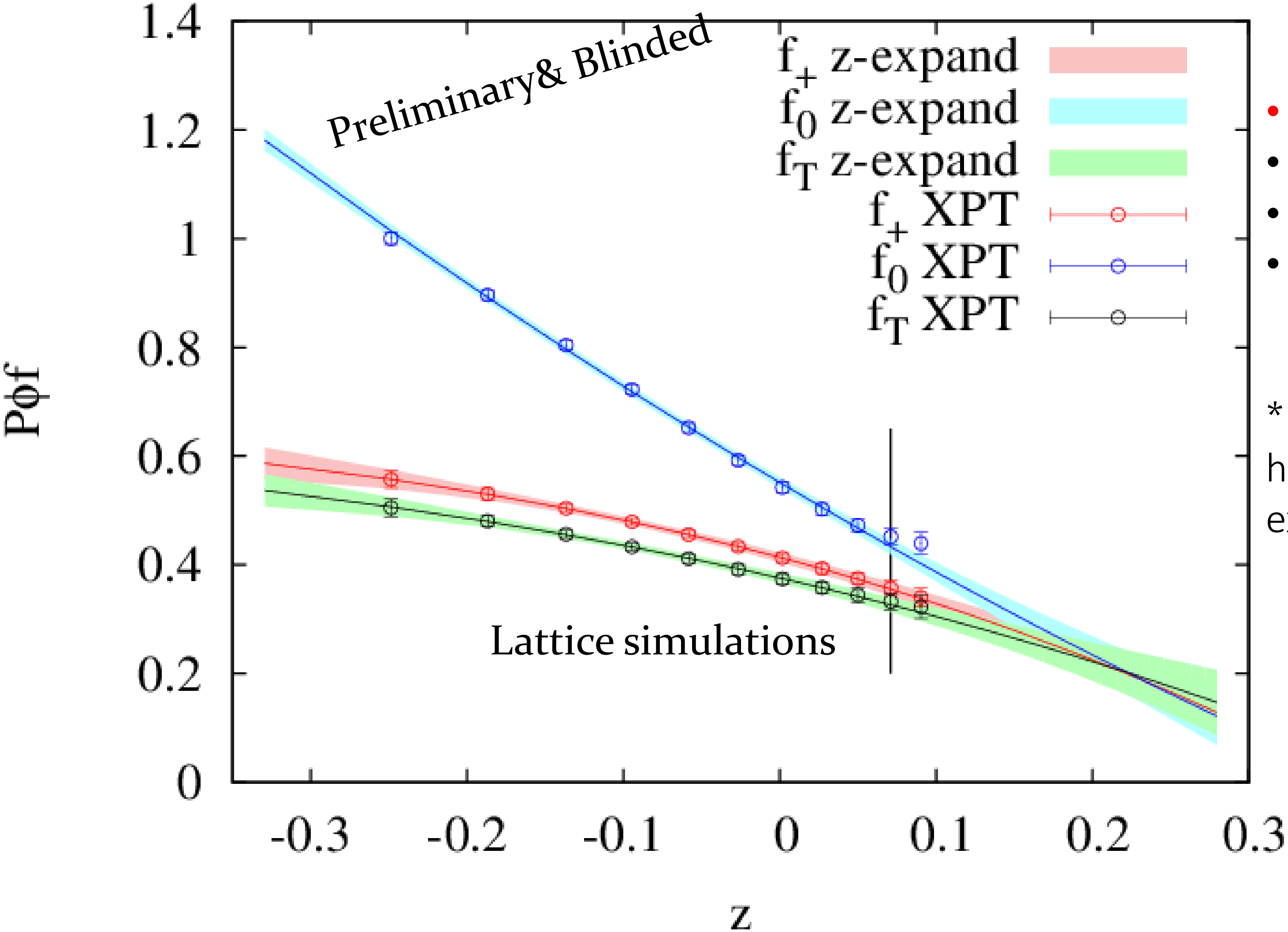
- **BGL** (Boyd-Grinstein-Lebed) expansion hep-ph/0504209

$$g_f(z) = \frac{1}{P(z)\phi(z)} \sum_{n=0} a_n z^n$$

- **BCL** (Bourrely-Caprini-Lellouch) expansion: correcting large q^2 behavior 0807.2722v3

$$g_f(z) = \frac{1}{1 - q^2/M_{B^*}^2} \sum_{n=0}^{K-1} a_n \left(z^n - (-1)^{n-K} \frac{n}{K} z^K \right)$$

z-expansion: a functional approach



- BCL expansion
- Individual fits
- Expansion up to z^3
- One singular mode is cut from $K(s,t)$.

* The expansion for f_0 here is a polynomial expansion with no pole.

Adding Constraints

- Use Lagrange multiplier to add bounds on expansion coefficients.

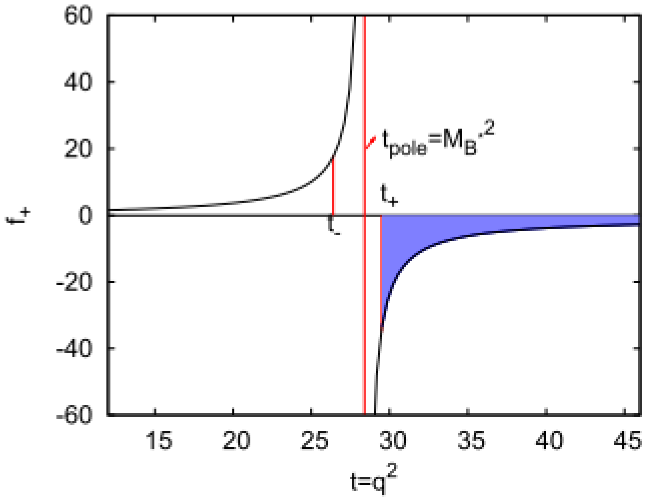
$$\sum_n |a_n|^2 \leq A$$ BGL

$$\sum_{m,n} B_{mn} b_m b_n \leq A$$ BCL

For **Unitarity** bound $A = 1$, which is usually not saturated (bound is loose).

- **Analyticity** of form factors constrains the coefficients more tightly!

Becher & Hill hep-ph/0509090



$$A_+ \sim \frac{m_b^2}{3} \int_{t_+}^{\infty} \frac{[(t-t_+)(t-t_-)]^{3/2}}{t^5} |f_+|^2$$

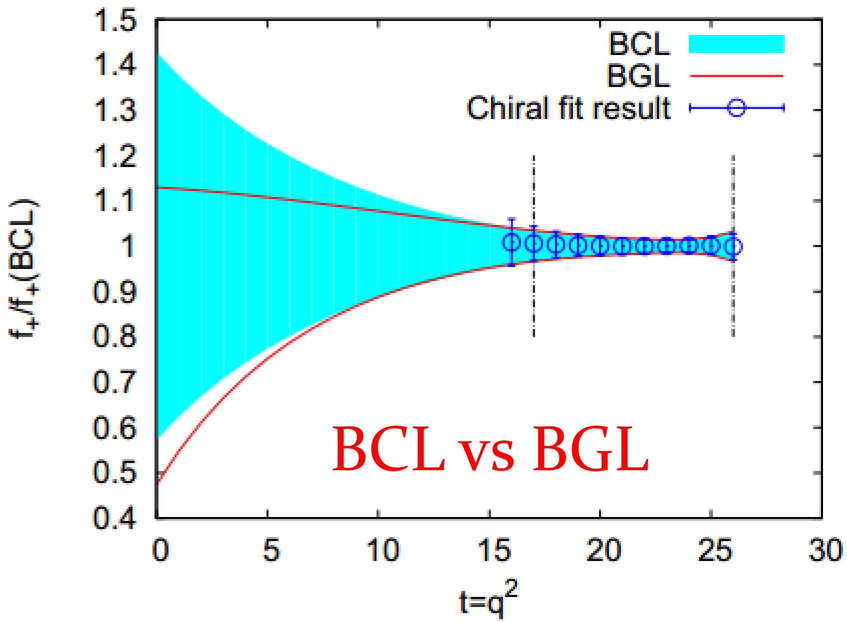
$$\sim 0.03$$

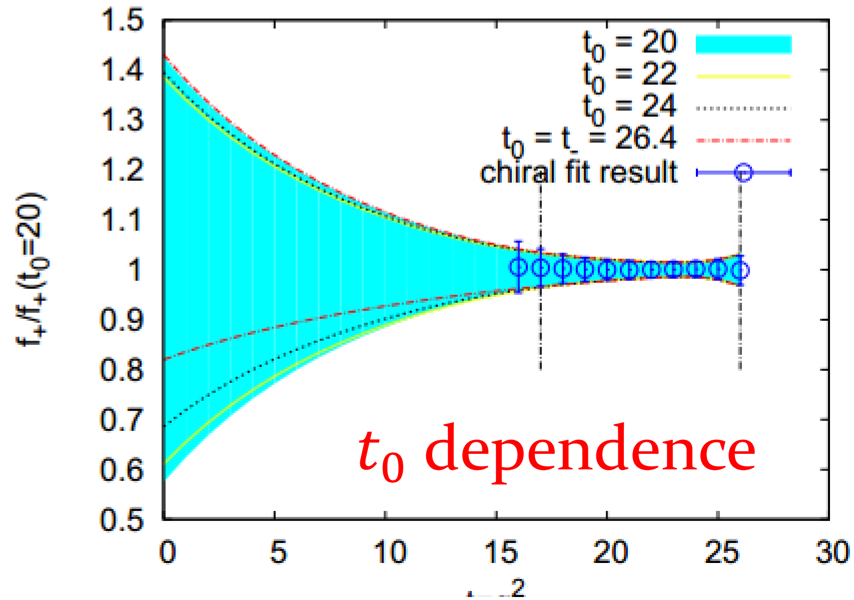
Replaced by leading order

A consistent check

Important for truncation error estimate.

z-expansion systematic errors

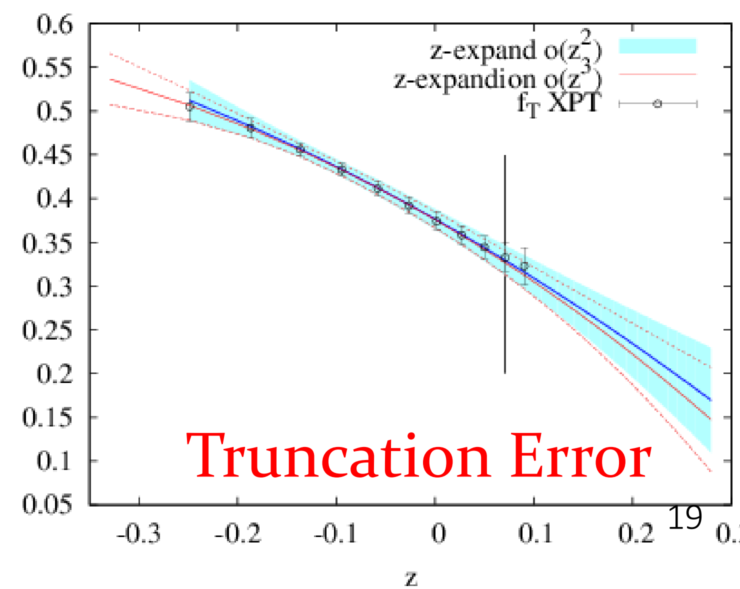




$[z_1, z_2]$	$[t_1, t_2]$	$\tilde{a}_0$	$\tilde{a}_1$	$\tilde{a}_2$	$f_+(q^2 = 0)$
[-0.25, 0.07]	[17,26]	0.414(8)	-0.78(7)	-0.8(5)	0.133(57)
[-0.19, 0.05]	[18,25]	0.412(9)	-0.74(7)	-0.6(6)	0.146(65)
[-0.14,0.03]	[19,24]	0.413(9)	-0.74(7)	-0.6(5)	0.155(70)
[-0.09,0.00]	[20,23]	0.411(9)	-0.78(7)	-0.5(6)	0.160(74)

Table I:

Integration Range



Error Estimate

In the region with lattice data:

- Statistics: 2~4%
- χPT systematic: 2~5% (the blowup is cured by z expansion)
- Kappa tuning: 0.5%
- z-expansion: < 1%
- Heavy quark discretization: ?%

Summary and outlook

- We are working on an update to the lattice form factor calculations for semileptonic $B \to \pi$ decays. We also include the tensor form factor in the calculation. The improvement on the error is promising.
- While χPT systematic is a dominant source of systematic errors in low-q^2 region, it can be improved by z-expansion.
- A new functional z-expansion method is used to reparameterize the χPT results.
- Full error budget is in progress.
- $|V_{cb}|$ and $B \to \pi \ell^+ \ell^-$ prediction.