Heavy Meson Semileptonic Form Factors:
For the Standard Model and Beyond

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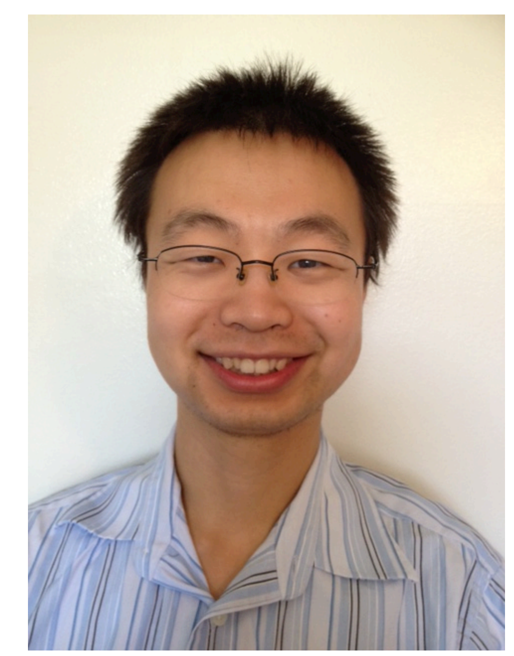
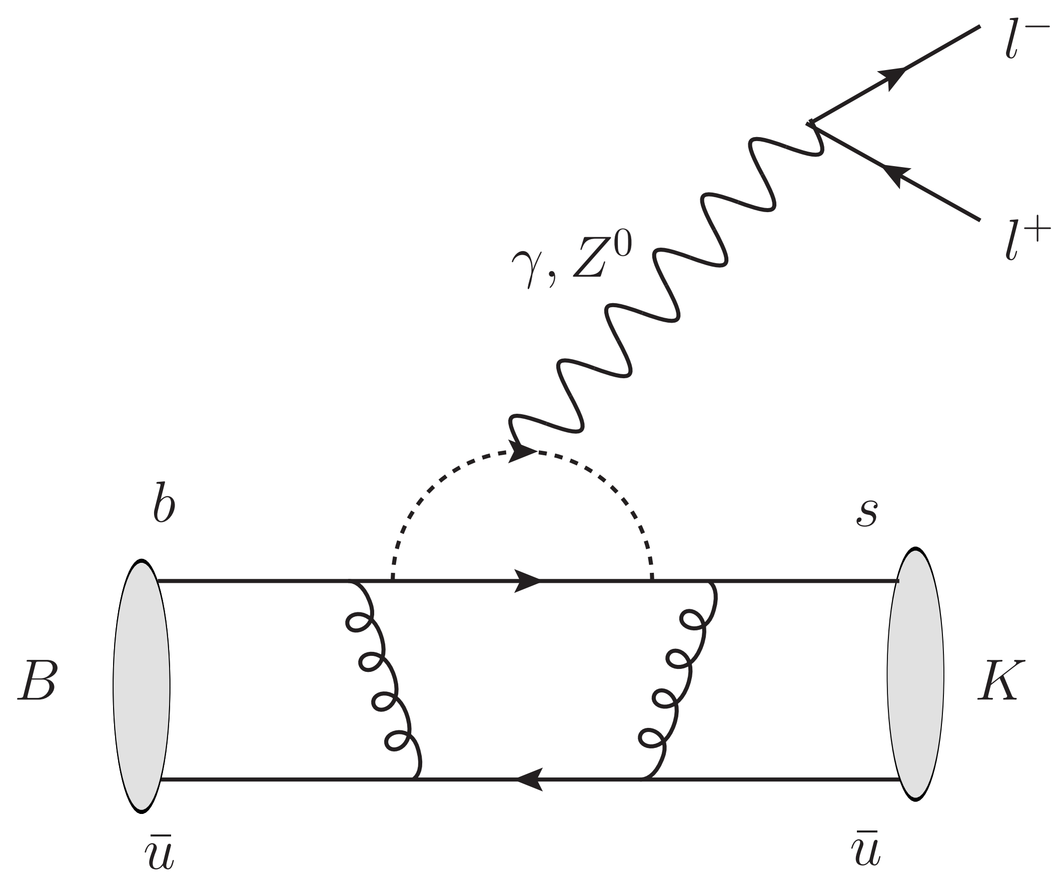
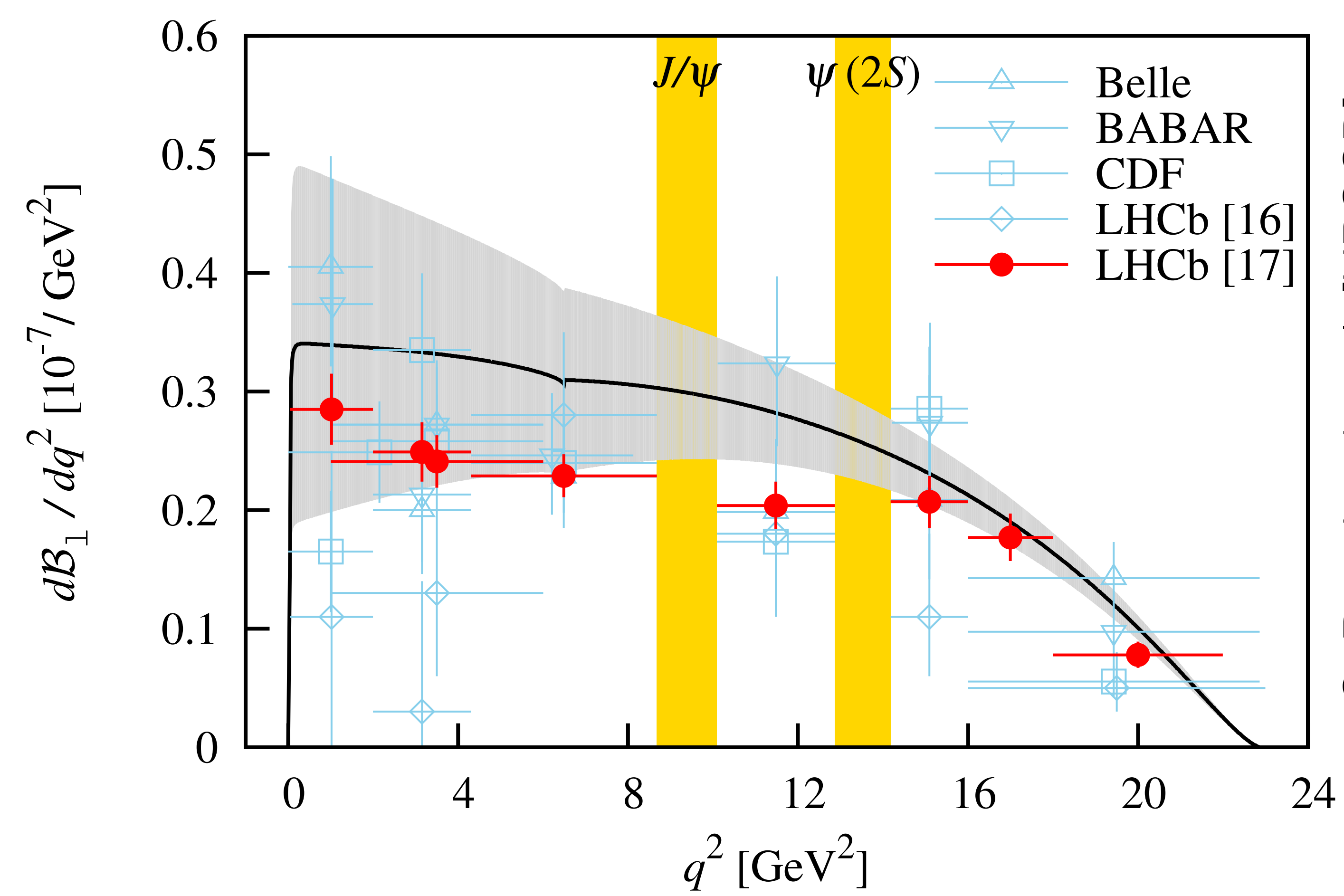


Physics Motivation

- The FCNC process $B \to Kl^+l^-$ proceeds via penguin diagrams in the Standard Model:



$B- > Kll$

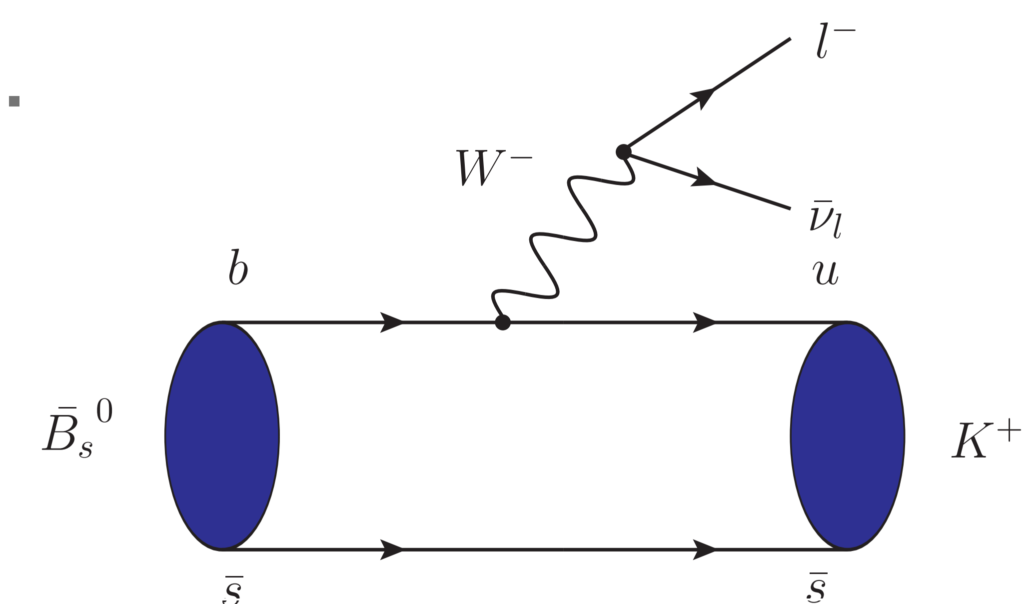


C. Bouchard et al. [HPQCD], arXiv:1306.0434

  - probe of physics beyond the SM (BSM), studied in experiments.

- The charged current $B_s \to K^-l^+\nu$ proceeds proportional to V^*_{ub}:

  - combining with future measurements at LHCb or Belle II (5S) would lead to a new determination of $|V_{ub}|$.



Matrix Elements and Form Factors

- Decompose amplitudes in form factors:

$$\langle K(k)|\bar{s}\gamma^\mu c|B_{(s)}(p)\rangle = \left(p^\mu + k^\mu - \frac{M_{B_{(s)}}^2 - M_K^2}{q^2} q^\mu\right) f_+(q^2) + \frac{M_{B_{(s)}}^2 - M_K^2}{q^2} q^\mu f_0(q^2),$$

$$\langle K(k)|\bar{s}\sigma^{\mu\nu} c|B_{(s)}(p)\rangle = -2i\frac{p^\mu k^\nu - p^\nu k^\mu}{M_{B_{(s)}} + M_K} f_T(q^2),$$

$$\langle K(k)|\bar{s}c|B_{(s)}(p)\rangle = \frac{M_{B_{(s)}}^2 - M_K^2}{m_c - m_s} f_0(q^2),$$

CVC: same

- For a heavy b quark:

$$f_T(q^2) = \frac{M_{B_{(s)}} + M_K}{\sqrt{2M_{B_{(s)}}}} f_\perp(E_K), \quad q^2 = M_{B_{(s)}}^2 + M_K^2 - 2M_{B_{(s)}} E_K$$

- In lattice QCD, and for heavy-meson chiral perturbation theory, it is convenient to write

$$\langle K(k)|\bar{s}\gamma^\mu c|B_{(s)}(p)\rangle = \sqrt{2M_{B_{(s)}}}\left(\frac{p^\mu}{M_{B_{(s)}}} f_\parallel(E_K) + k_\perp^\mu f_\perp(E_K)\right),$$

$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_{(s)}}}}\left[(M_{B_{(s)}} - E_K) f_\perp(E_K) + f_\parallel(E_K)\right],$$

$$f_0(q^2) = \frac{\sqrt{2M_{B_{(s)}}}}{M_{B_{(s)}}^2 - M_K^2}\left[(M_{B_{(s)}} - E_K) f_\parallel(E_K) + (E_K^2 - M_K^2) f_\perp(E_K)\right],$$

- Then construct three-point correlation functions to extract:

$$f_\parallel(E_K) = \frac{\langle K(k)|\bar{s}\gamma^4 c|B_{(s)}(0)\rangle}{\sqrt{2M_{B_{(s)}}}}, \quad f_\perp(E_K) = \frac{\langle K(k)|\bar{s}\gamma^i c|B_{(s)}(0)\rangle}{k^i\sqrt{2M_{B_{(s)}}}}, \quad f_T(E_K) = \frac{M_{B_{(s)}} + M_K}{\sqrt{2M_{B_{(s)}}}} \frac{\langle K(k)|\bar{s}i\sigma^{4i} c|B_{(s)}(0)\rangle}{k^i\sqrt{2M_{B_{(s)}}}},$$

Common Ingredients

- MILC asqtad ensembles (2+1 sea quarks simulated with rooted staggered determinant):
  - lattice spacing $a \approx 0.12, 0.09, 0.06$ fm; m_s near physical; $m_l/m_s = 0.05, 0.1, 0.14, 0.2, 0.4$.
- Valence quarks: asqtad for s, d, u; Fermilab method for b action and heavy-light current.
- Chiral-continuum extrapolation for $E_K r_1 < 1.8$ ($E_K < 1.2$ GeV) with rooted staggered HM χPT.
- Model-independent z expansion to extend results to whole kinematic range.
- Mostly nonperturbative matching, $\rho_{J_{bs}} = Z_{J_{bs}}(Z_{V^4_{bb}} Z_{V_{ss}})^{-1/2}$, computed at one loop. Blinded.
- *Similar work by HPQCD, using lattice NRQCD for the b quark, and giving phenomenology.*

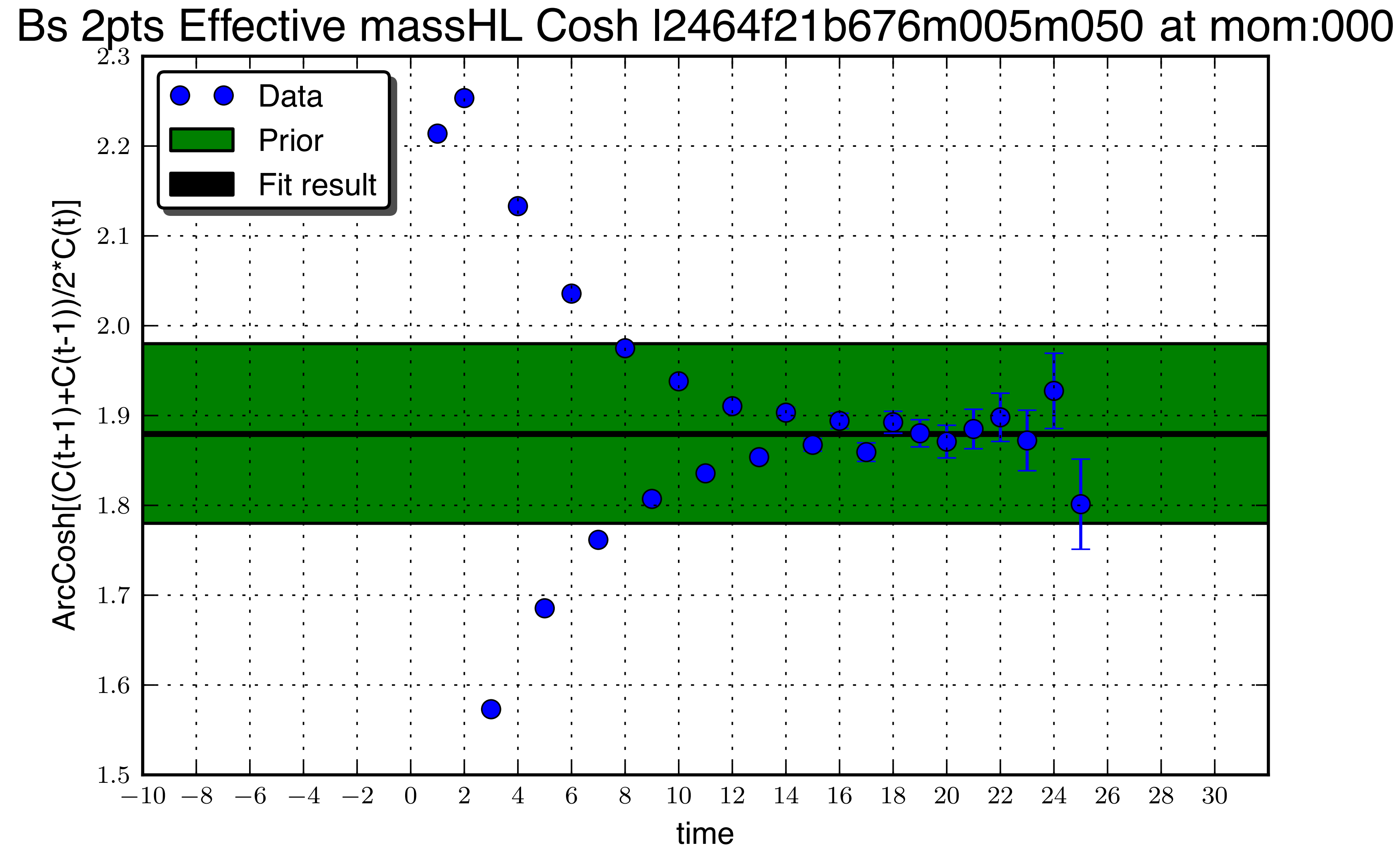
Ensembles

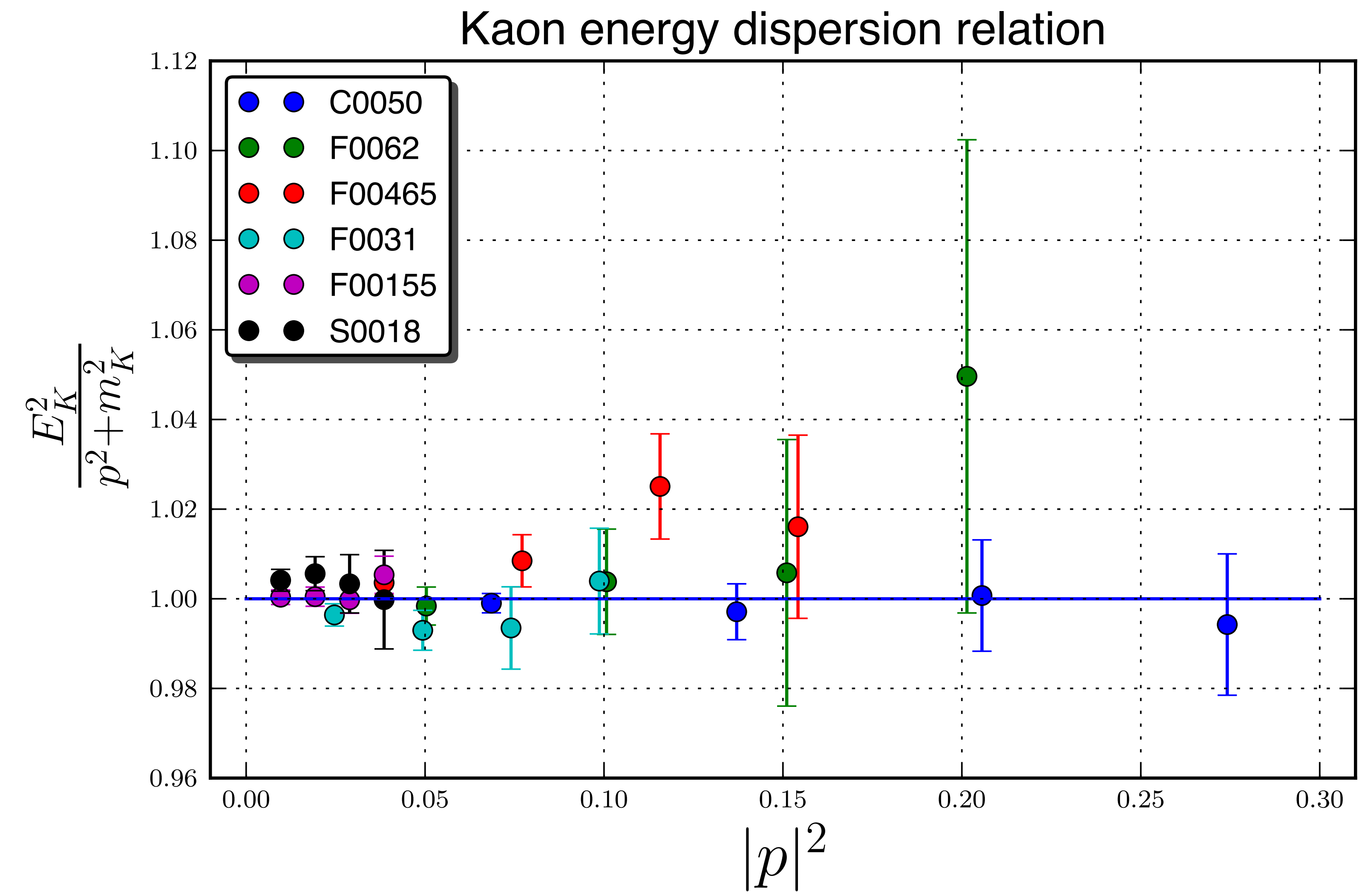
a (fm)	size	am_l/am_s	# confs	# sources	$am_s^{\mathrm{val}}\ (B_s)$
≈ 0.12	$20^3\times\ 64$	0.01/0.05	2259	4	
≈ 0.12	$20^3\times\ 64$	0.007/0.05	2110	4	
≈ 0.12	$20^3\times\ 64$	0.005/0.05	2099	4	0.0336
≈ 0.09	$28^3\times\ 96$	0.0124/0.031	1996	4	
≈ 0.09	$28^3\times\ 96$	0.0062/0.031	1931	4	0.0247
≈ 0.09	$32^3\times\ 96$	0.0465/0.031	984	4_{FCNC} 8_{CKM}	0.0247
≈ 0.09	$40^3\times\ 96$	0.0031/0.031	1015	4_{FCNC} 8_{CKM}	0.0247
≈ 0.09	$64^3\times\ 96$	0.00155/0.031	791	4	0.0247
≈ 0.06	$48^3\times144$	0.0036/0.018	673	4	
≈ 0.06	$64^3\times144$	0.0018/0.018	827	4	0.0177

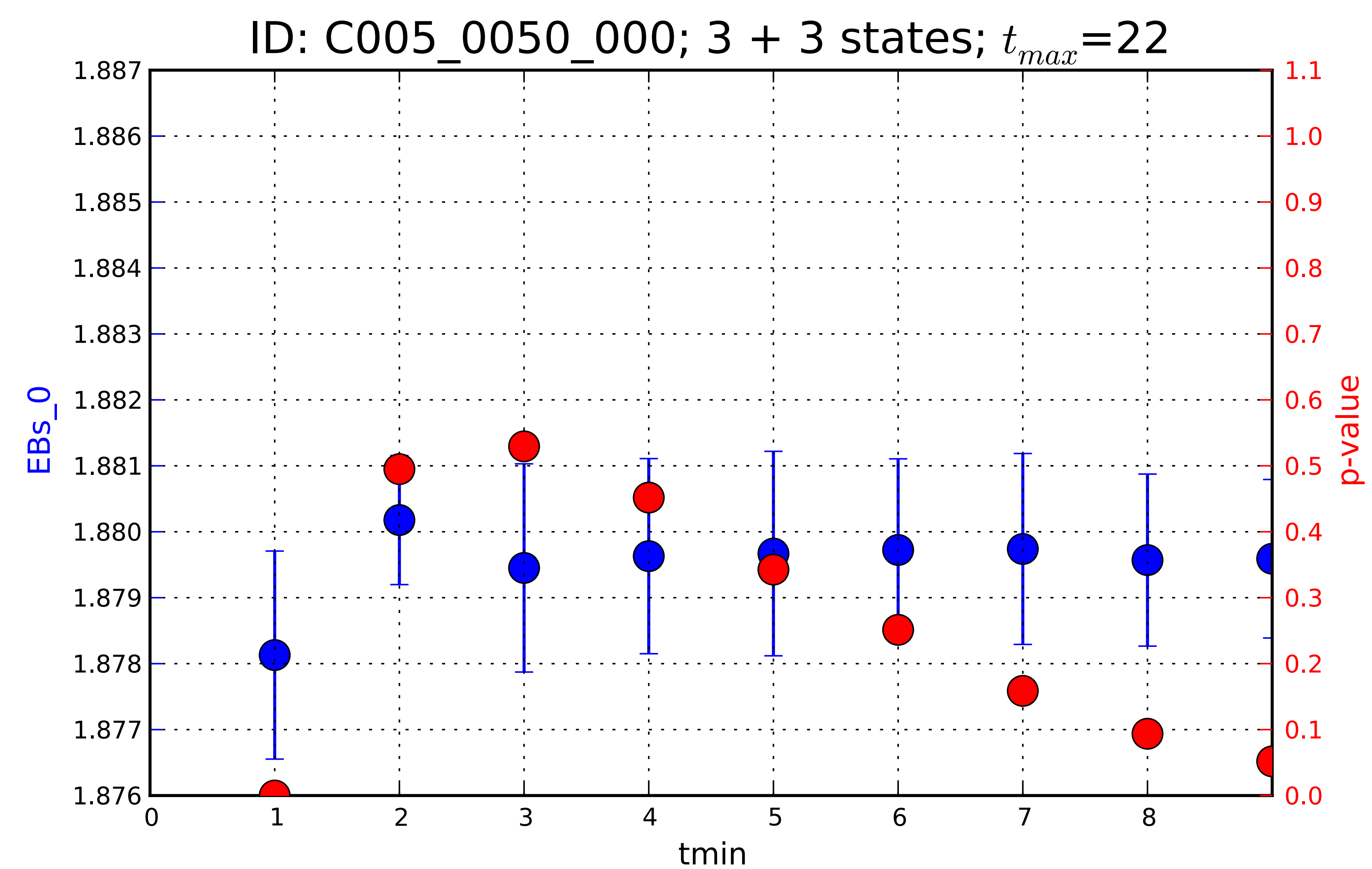
Outline

- CKM mode $B_s \to K^- l^+ \nu$:
    - ratio of two- and three-point correlators constructed to suppress excited states [arXiv: 0811.3640 [hep-lat]] *vs*. direct fit;
    - ~~read off plateau~~ fit to remove excited state contributions.
- FCNC mode $B \to K l^+ l^-$:
    - correlator fitting presented last year; [R. Zhou, PoS LATTICE2012 (2012) 120]
    - chiral-continuum extrapolation: SU(3) vs. SU(2); to pole or not to pole;
    - model-independent z expansion: BGL vs. BCL; to pole or not to pole.

CKM Mode $B_s \rightarrow K^- l^+ \nu$


Bs 2pts Effective massHL Cosh l2464f21b676m005m050 at mom:000
Data
Prior
Fit result
ArcCosh[(C(t+1)+C(t-1))/2*C(t)]
time


Kaon energy dispersion relation
C0050
F0062
F00465
F0031
F00155
S0018
$\frac{E_K^2}{p^2+m_K^2}$
$|p|^2$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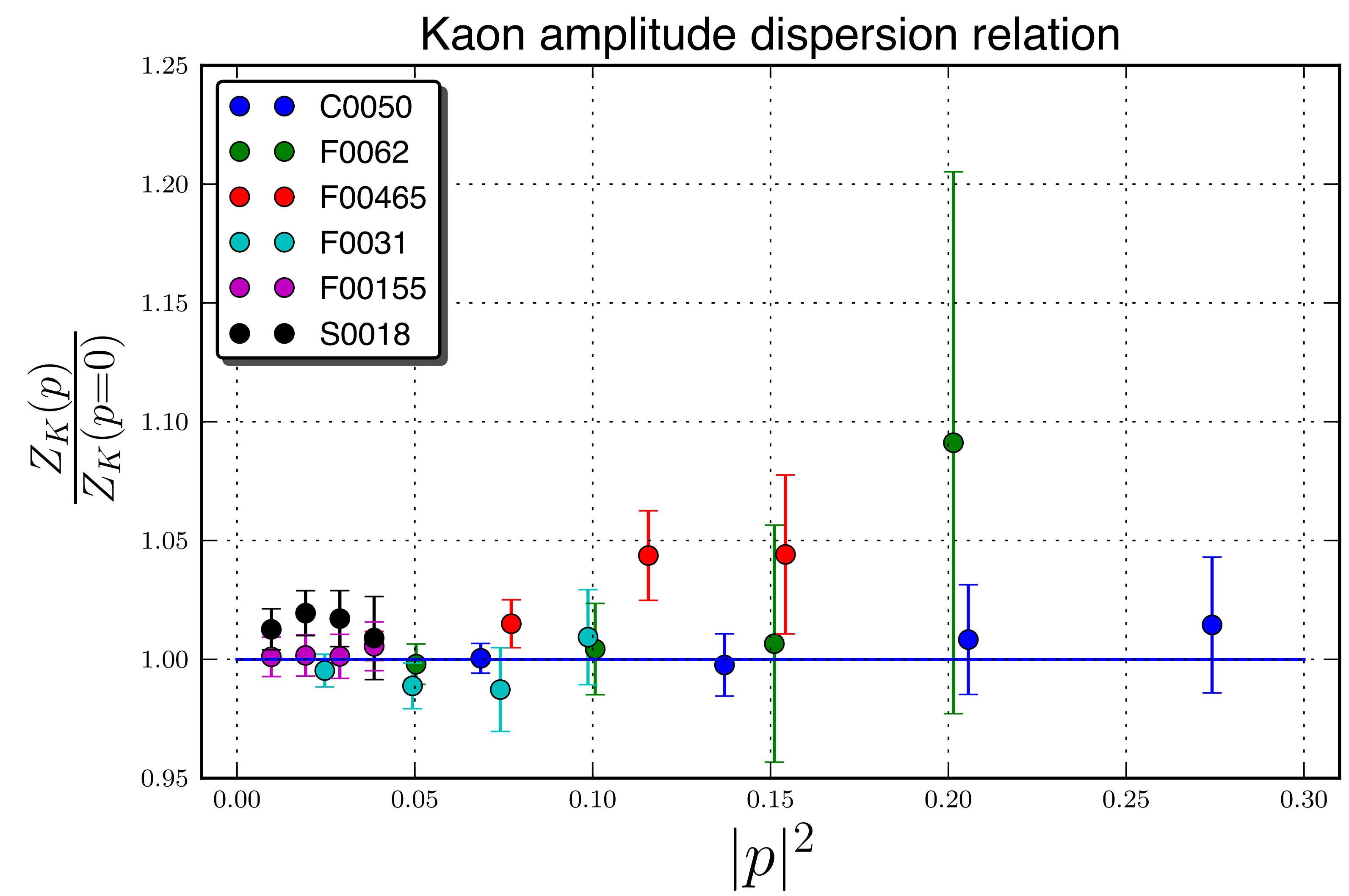
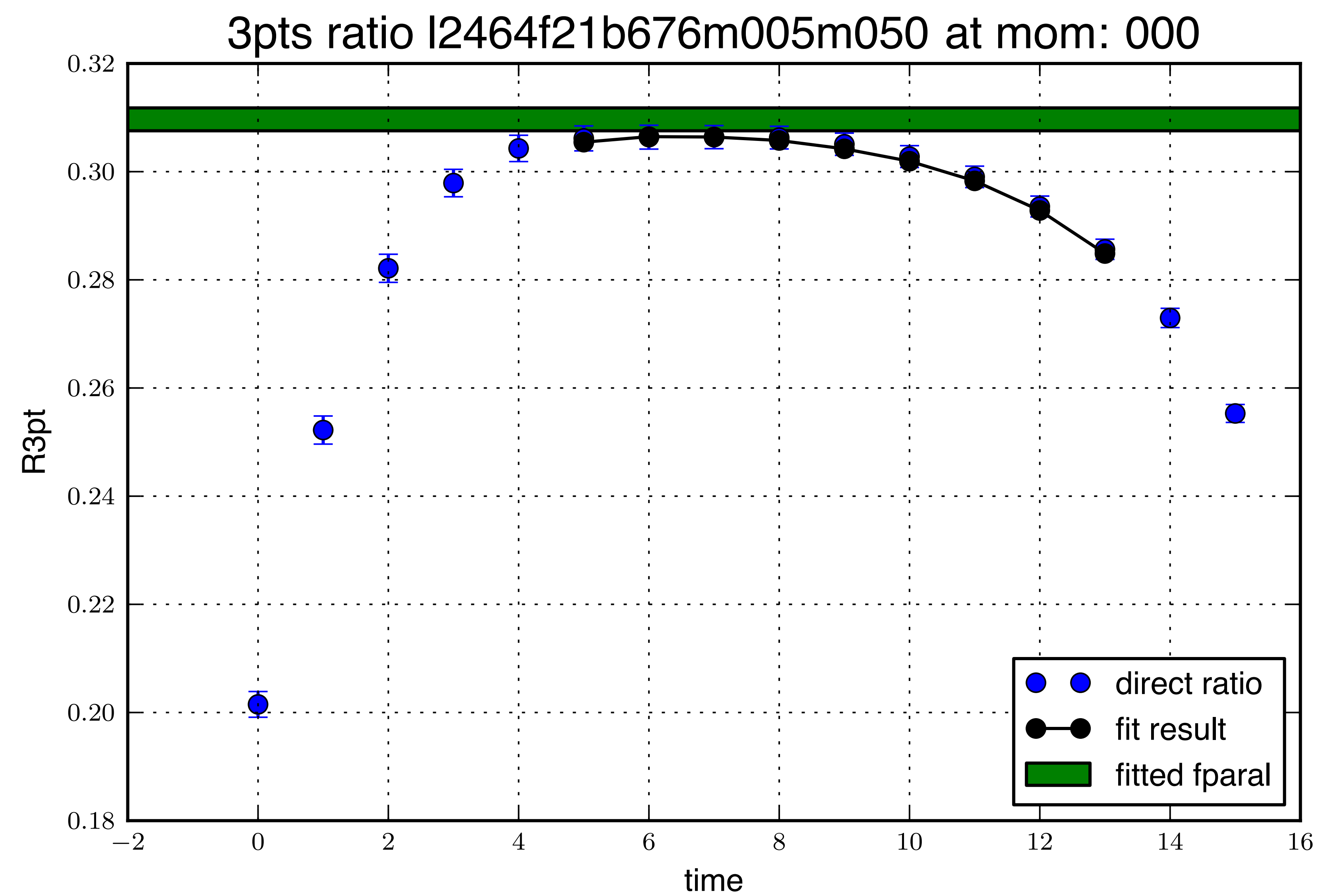
EBs_0
p-value
tmin

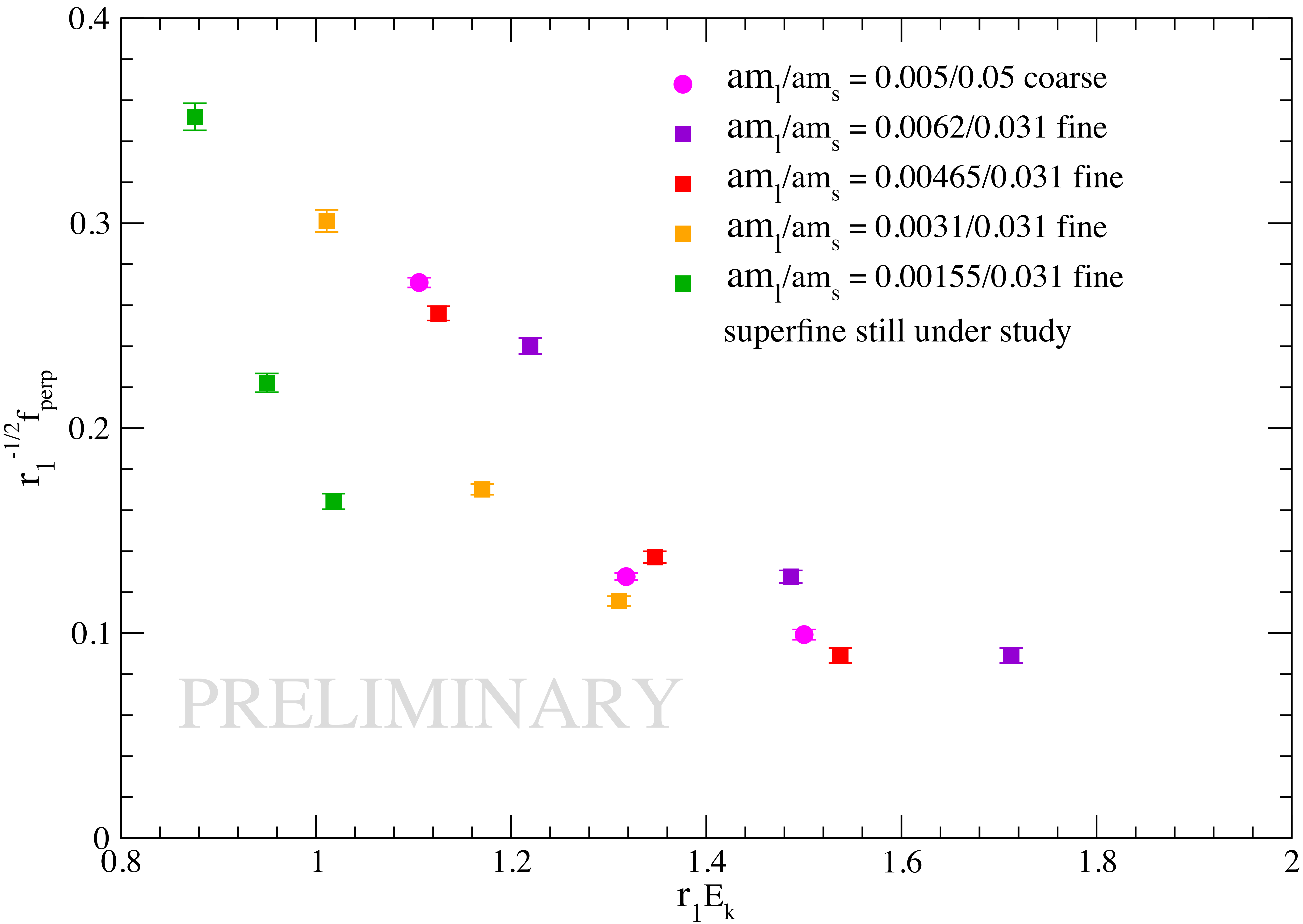

Kaon amplitude dispersion relation
C0050
F0062
F00465
F0031
F00155
S0018
$\frac{Z_K(p)}{Z_K(p=0)}$
$|p|^2$

Comparison: ratio plateau *vs*. fit

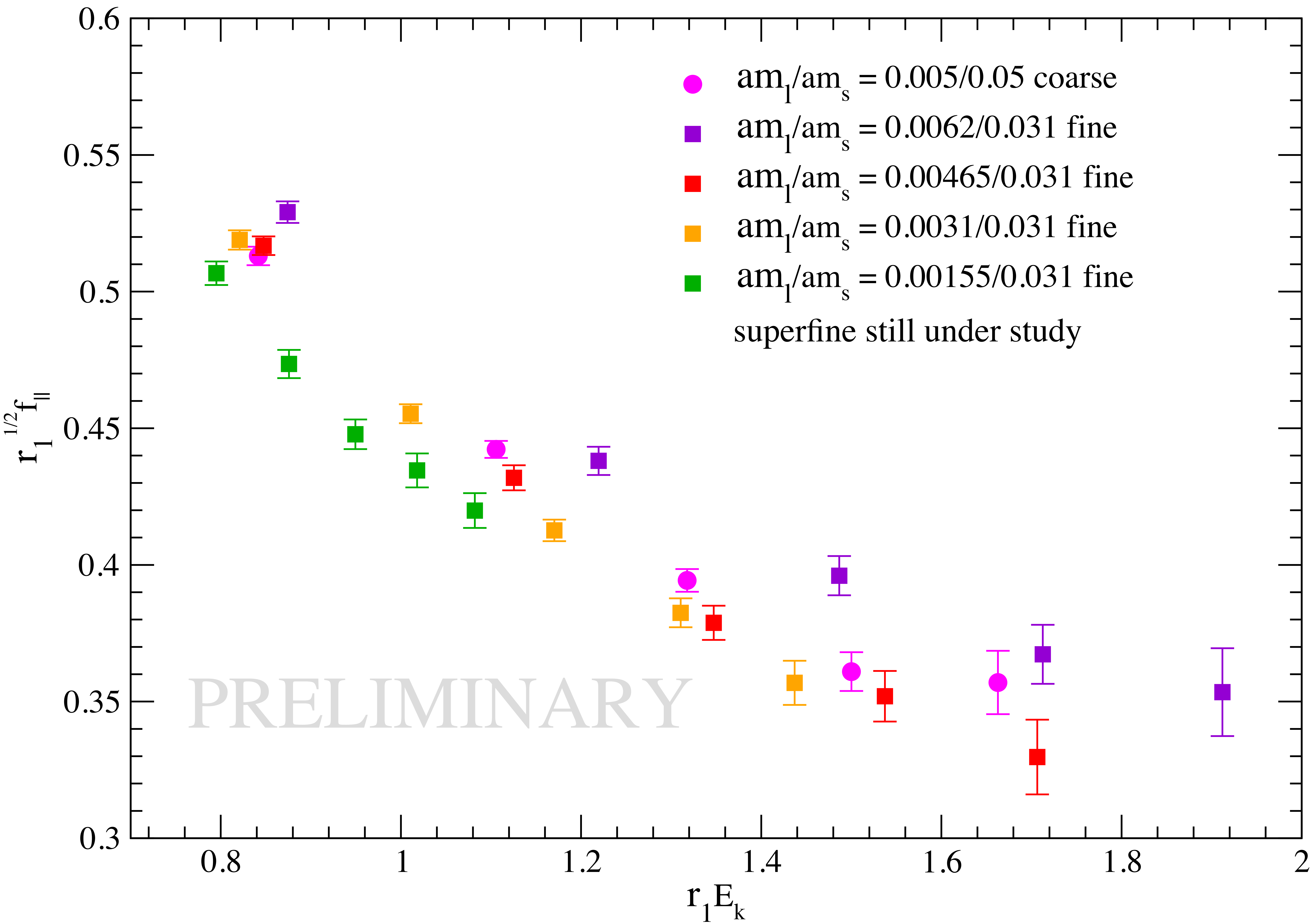
- Blue points indicate a ratio of two- and three-point correlators that suppress excited states.
- Statistics are good enough that small excited-state contributions are significant.
- Fit three-point correlators with two-point mass, $[M^2 + (2\pi \boldsymbol{n}/L)^2]^{1/2}$, and $Z(\mathbf{0})$ as priors, yielding black curve; note agreement with the ratio itself.
- Green band shows the $B_s \to K$ amplitude without excited states.



Preliminary results for B_s -> K: f_{perp}



Preliminary results for $B_s \to K$: $f_{\parallel}$



FCNC Mode $B \to Kl^+l^-$

R. Zhou, *PoS* **LATTICE2012** (2012) 120

Chiral continuum extrapolation

- SU(3) rSχPT leads to fits with poor χ^2/dof and—worse—unphysical shape for $f_\parallel$.

- SU(2) rSχPT:

$$f_\parallel(E) = \frac{C^{(0)}}{f_\pi}\left[1 + \mathrm{logs} + C^{(1)}\chi_l^{\mathrm{val}} + C^{(2)}\chi_E + C^{(3)}\chi_{a^2} + C^{(4)}\chi_E^2\right]$$

$$f_{\perp,T,\parallel}(E) = \frac{2M_B C^{(0)}}{f_\pi(M_{B_s^*}^2 - M_B^2 - M_K^2 - 2M_B E)}\left[1 + \mathrm{logs} + C^{(1)}\chi_l^{\mathrm{val}} + C^{(2)}\chi_E + C^{(3)}\chi_{a^2}\right]$$

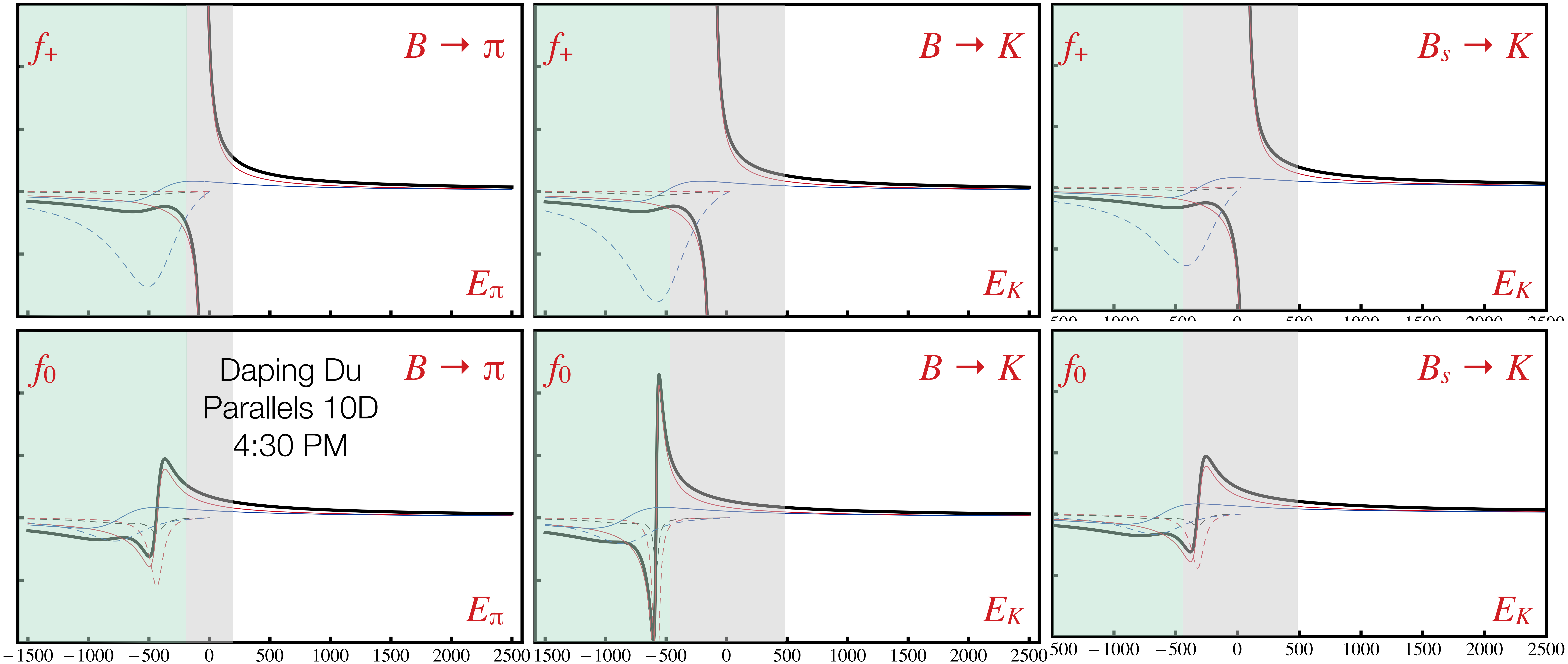
where the χs denote m_l, E, a^2 normalized by $4\pi f_\pi$ and other factors to that the natural χPT prior draws $C^{(j)}$, $j > 0$, from $\{0, \pm 1\}$. Pole position is given by the mass of the 1^- (0^+) B_s state.

- The data for f_0 show curvature in E. We model this either with a curvature term or a pole, even though the pole is not well known and (presumably) above threshold.

- We favor the pole fit, as explained on the next slide, with prior $\{5.72, \pm 0.50\}$ GeV for $M_{B_s(0^+)}$.

Polology

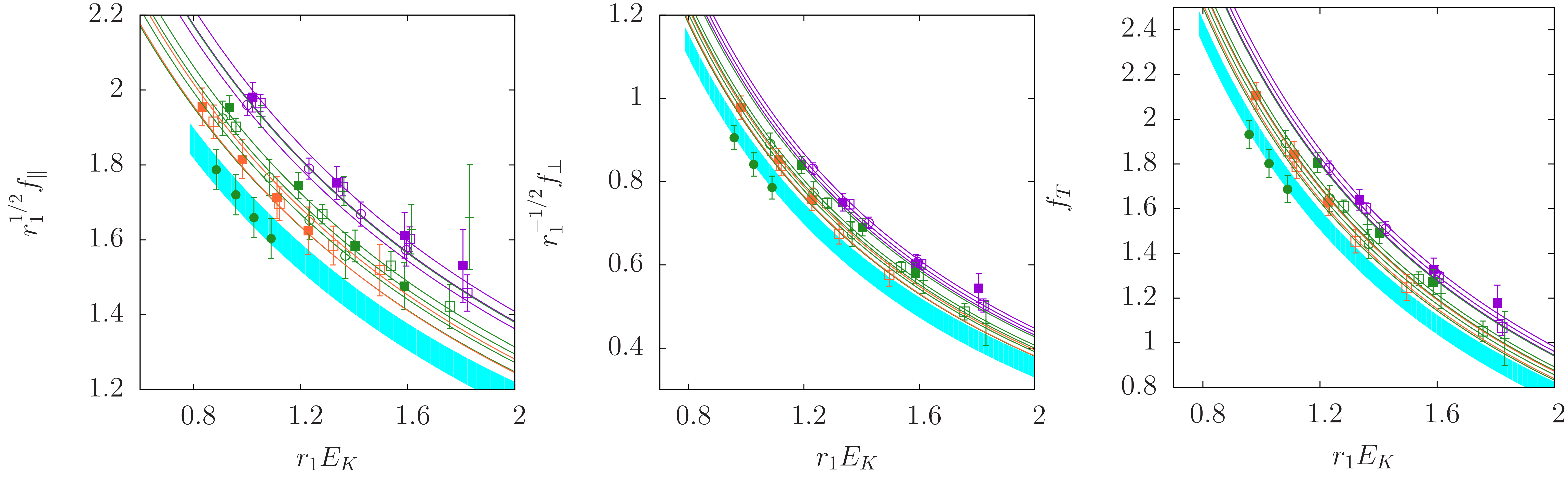
- Curvature in decay region ($E_{\pi|K} > m$) stems from poles in scattering region ($E_{\pi|K} < m$).



Chiral Continuum Extrapolation

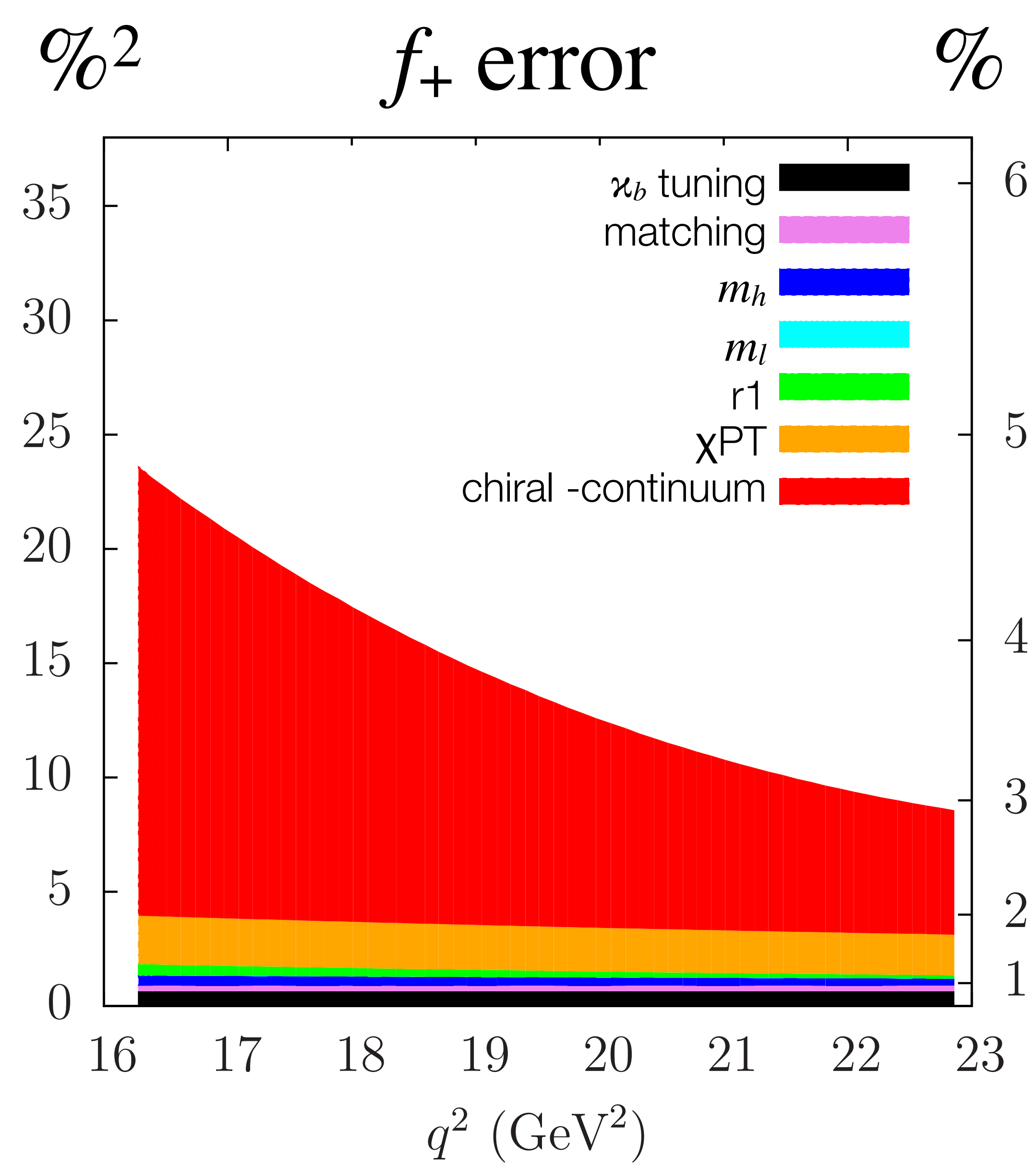
- Data with $m_q = m_l$ and $a \approx 0.12$ (coarse), $a \approx 0.09$ (fine), and $a \approx 0.06$ (superfine), with chiral-continuum extrapolation.

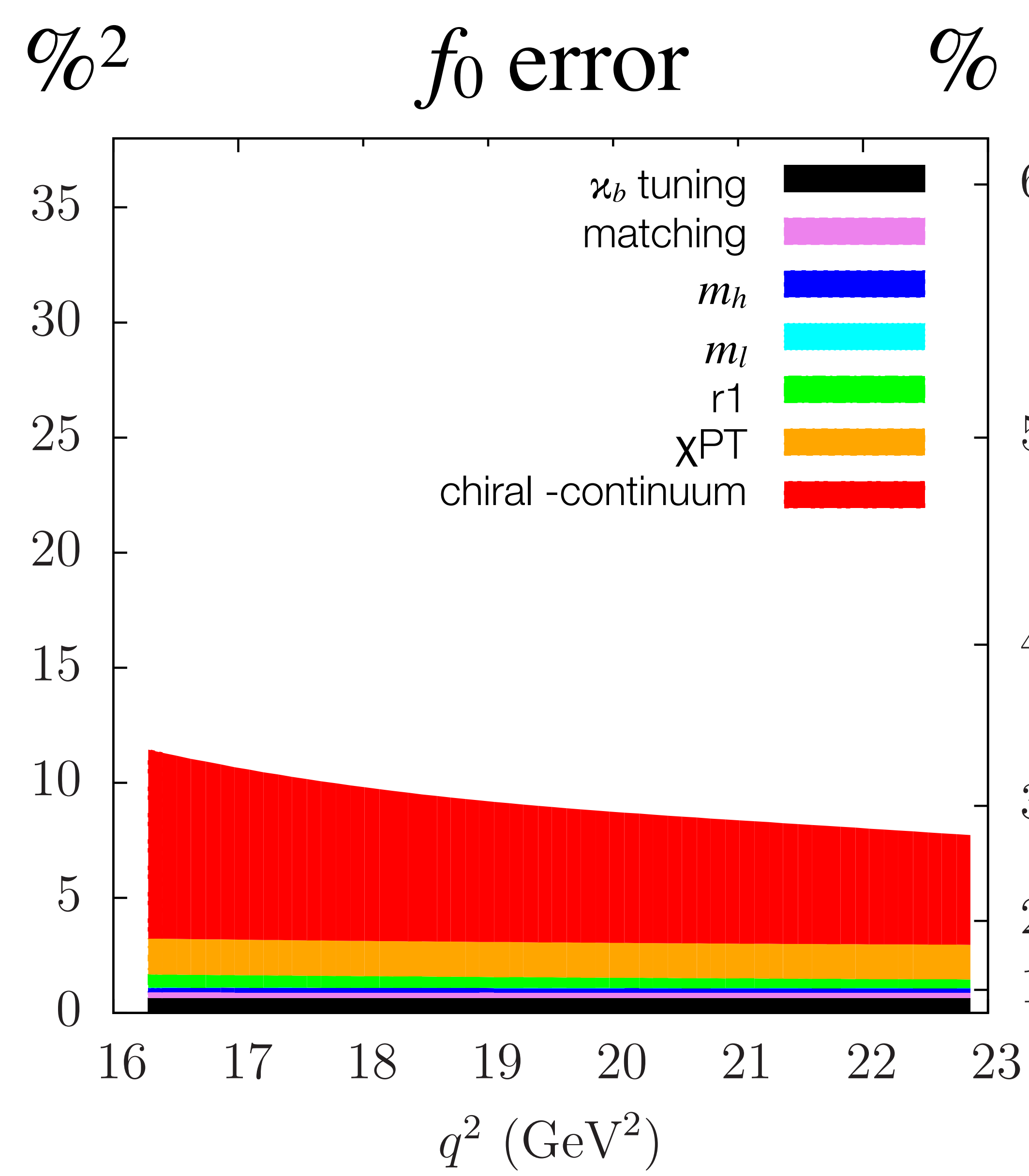
- Small but significant a and m_q dependence.

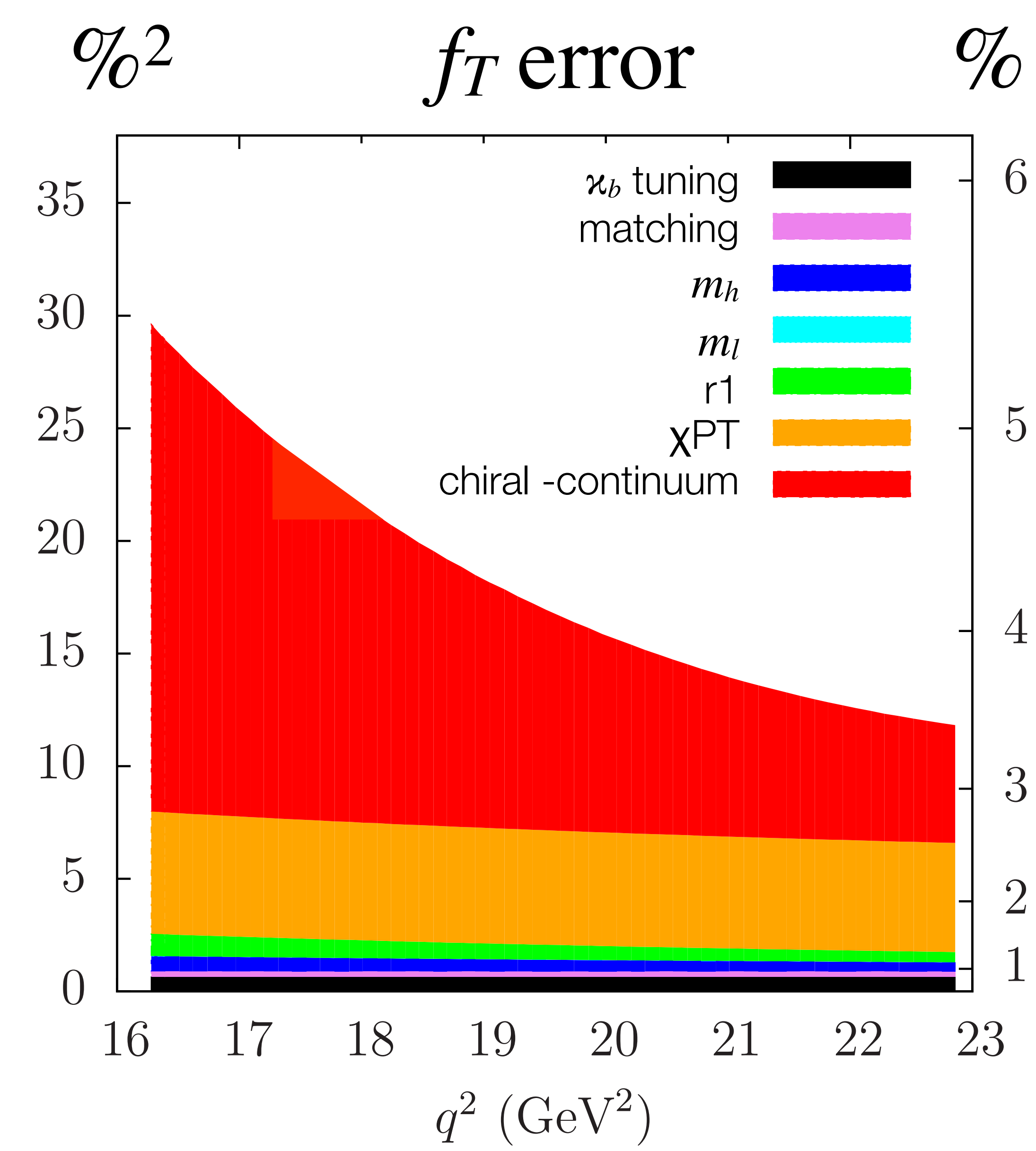


Preliminary Error Budgets—before z expansion

- Errors, esp. those from chiral-continuum extrapolation and discretization, depend on q^2.
- Over the range with explicit data, the (relative) error remains under 5, 3, 5% (for f_+, f_0, f_T) after the chiral-continuum extrapolation:

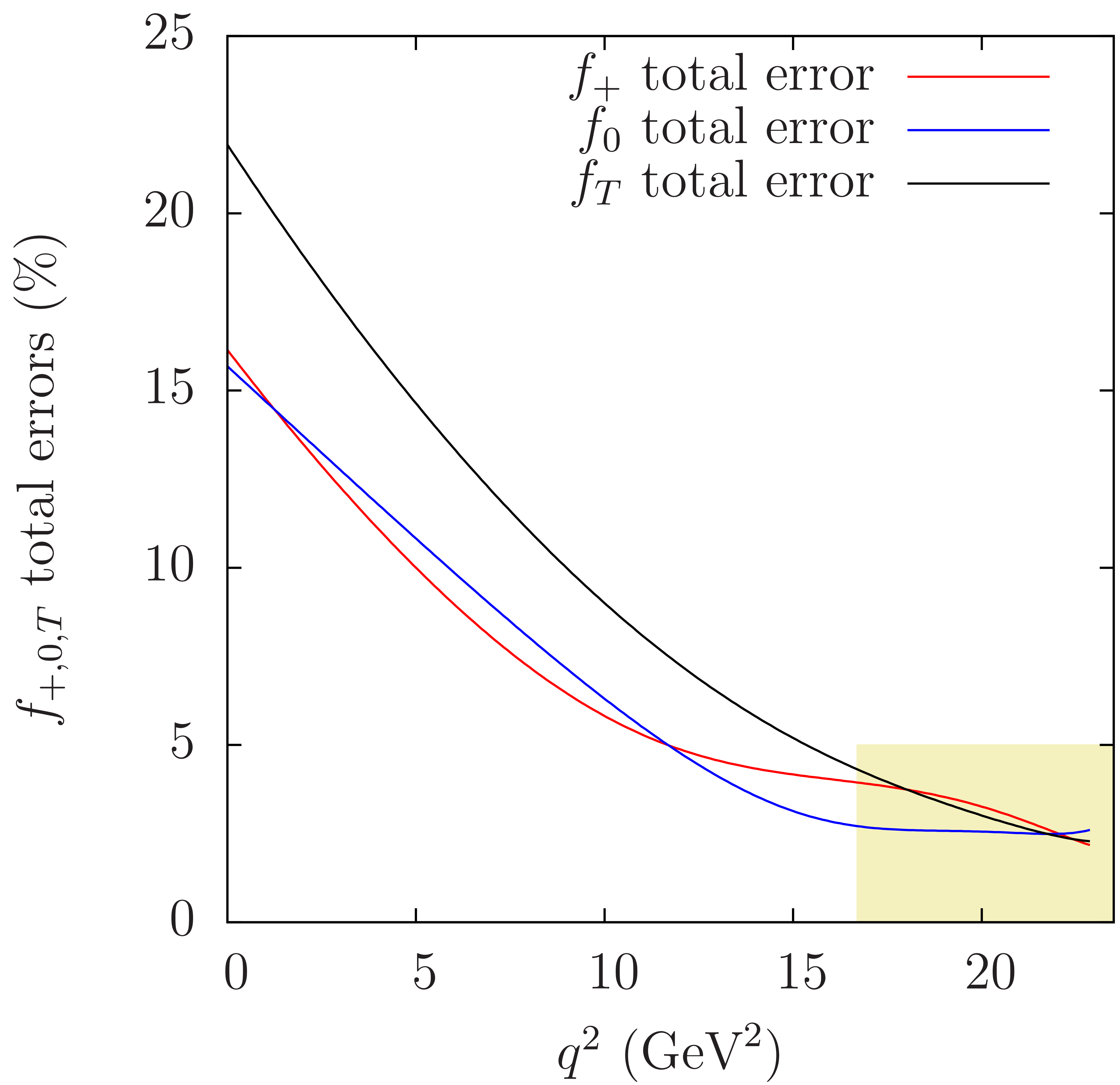






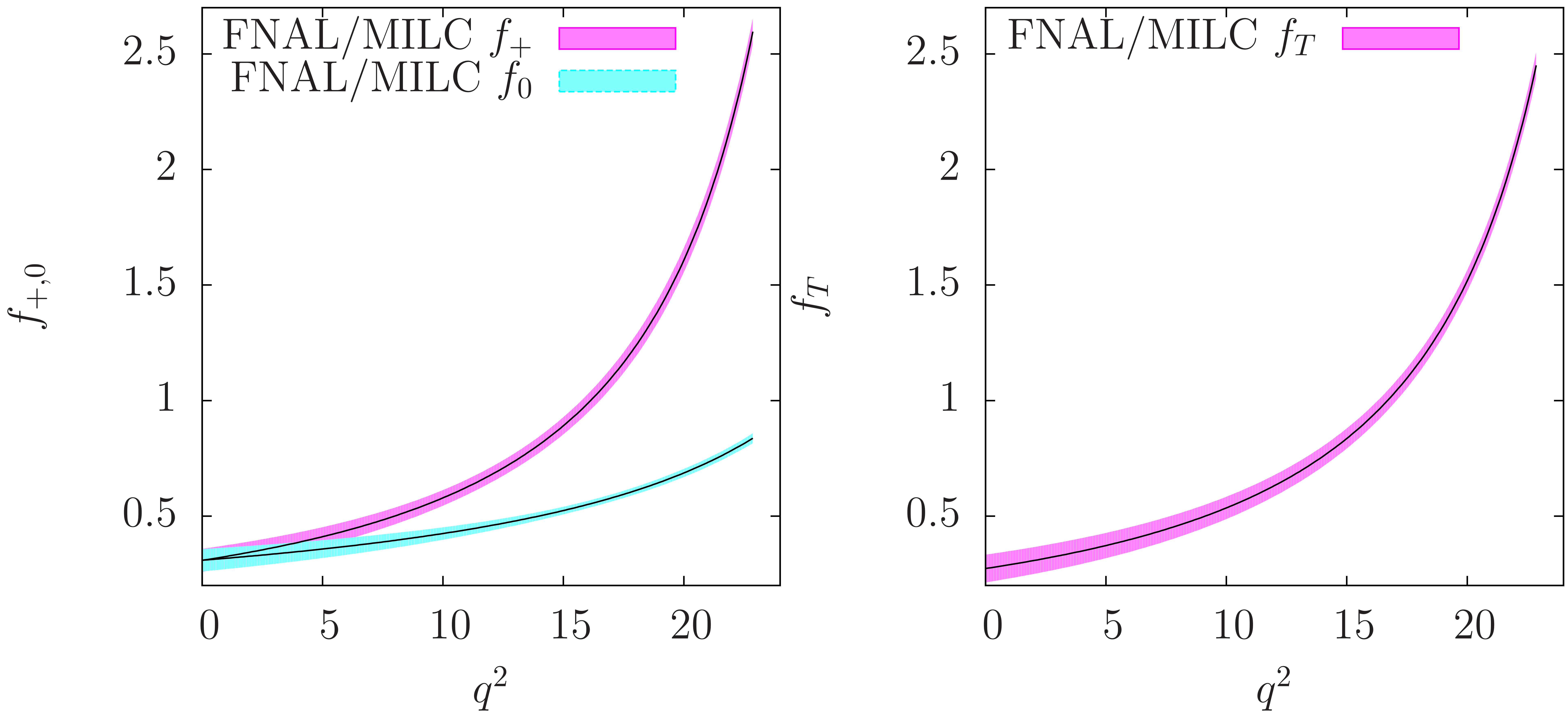
Preliminary Error Estimate—after z expansion

- $z(q^2, t_0) = \dfrac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$.
- $|z| \leq 0.16$.
- Fit using BCL outer function, $\phi = 1$.
- Verify the kinematic constraint from independent fits to f_+ & f_0.
- Final fit for f_+ & f_0 imposes kinematic constraint.
- The relative error grows to 16, 16, 22 % (for f_+, f_0, f_T) at $q^2 = 0$.



Preliminary Results for Form Factors for All q^2

- Normalization still blinded, but shape agrees with competitors' results.



Next Steps

- CKM mode $B_s \to K^- l^+ \nu$:
  - carry out chiral-continuum extrapolation and expansion;
  - package expansion z coefficients and correlation matrix for future use (LHCb, Belle2).
- FCNC mode $B \to K l^+ l^-$:
  - finalize error estimation (finalize favored fits and rank alternatives, *etc*.);
  - package expansion z coefficients and correlation matrix for phenomenology.
- Publish papers.