Semileptonic decay $B \to D^{(*)} l\nu$ at nonzero recoil

Si-Wei Qiu, C. DeTar,
A.S. Kronfeld, J. Laiho, R. S. Van de Water
(Fermilab Lattice and MILC collaborations)

University of Utah

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Introduction

- A precise determination of $|V_{cb}|$ is needed to provide a stringent test of CKM unitarity and, perhaps, give clues about new physics. The Fermilab Lattice and MILC collaborations have been studying the exclusive semileptonic decay processes $B \to Dl\nu$ and $B \to D^* l\nu$. The study is also needed to clarify the present 2σ tension with inclusive determinations.
- We report results from a three-year project on gauge field ensembles with (2+1)-flavors of asqtad sea quarks.
- This talk is devoted to the $B \to Dl\nu$ project at nonzero recoil. Results for the companion $B \to D^* l\nu$ project at zero recoil are mentioned at the end.
- We have done a blind analysis to avoid biases. Our result is final and we have unblinded it for this conference.

$B \to Dl\nu$ decay rate and form factors

- The decay rate measured by experiment is related to $|V_{cb}|$ and the form factor G_D:

$$\frac{d\Gamma(B \to Dl\overline{\nu})}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{48\pi^2 m_B^2}(w^2 - 1)^{3/2}|G_D(q^2)|^2$$

- Lattice QCD provides a precise determination of the form factor G_D.
- The combination of experimental measurement and lattice calculation can then give a precise determination of $|V_{cb}|$.

Nonzero recoil

- The zero-recoil calculation was done using the double-ratio method (Hashimoto *et al.* 2002), where $h_+(1)$ is proportional to G_D:

$$|G_D|^2 = |h_+(1)|^2 = \frac{\langle D|\mathcal{V}_1|B\rangle \langle B|\mathcal{V}_1|D\rangle}{\langle D|\mathcal{V}_4|D\rangle \langle B|\mathcal{V}_4|B\rangle}$$

- Zero-recoil events for this process are suppresed by phase space. So we match lattice and experimental measurements at nonzero recoil.

Lattice form factor

- We use two conventions for the form factors:

$$\frac{\langle D(p')|\mathcal{V}^\mu|B(p)\rangle}{\sqrt{M_B M_D}} = h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu$$

$$\langle D(p')|\mathcal{V}^\mu|B(p)\rangle = f_+(q^2)\left[(p+p')^\mu - \frac{M_B^2 - M_D^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M_B^2 - M_D^2}{q^2}q^\mu$$

- They are linearly related

$$f_+(q^2) = \frac{1}{2\sqrt{r}}[(1+r)h_+(w) - (1-r)h_-(w)]$$

$$f_0(q^2) = \sqrt{r}\left[\frac{w+1}{1+r}h_+(w) - \frac{w-1}{1-r}h_-(w)\right]$$

- Form factors h_+ and h_- are more conveniently calculated in our simulation.
- Form factors f_+ and f_0 are used in our z-parameter fits.
- In the approximation that the lepton mass is negligible, the differential decay rate is proportional to only $|f_+|^2$.

Calculating the lattice form factors

- Our methods have been described elsewhere, so we just summarize here.
- For the b and c quarks we use clover fermions in the Fermilab interpretation. The spectator (degenerate) u and d quarks are asqtad fermions. Their masses match the masses of the sea quarks.
- We calculate matrix elements of the lattice vector current $\langle D(\mathbf{p})|V_4|B(0)\rangle$ and $\langle D(\mathbf{p})|V_1|B(0)\rangle$.
- To match the lattice current with the continuum current $\mathcal{V}^\mu$

$$\mathcal{V}^\mu_{cb} = Z_{cb,\mu} V^\mu_{cb}$$

  we use

$$Z_{cb,\mu} = \rho_{cb,\mu} \sqrt{Z_{cc} Z_{bb}}$$

  where Z_{cc} and Z_{bb} are computed nonperturbatively in the same simulation and $\rho_{cb,\mu}$ is computed in lattice perturbation theory.

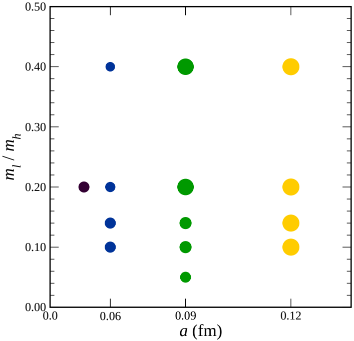
Calculating the lattice form factors

- We use ratios wherever possible to reduce statistical fluctuations:

$$
\begin{aligned}
R_+(\mathbf{p}) &= \langle D(\mathbf{p})|V_4|B(0)\rangle \\
R_-(\mathbf{p}) &= \frac{\langle D(\mathbf{p})|V_1|B(0)\rangle}{\langle D(\mathbf{p})|V_4|B(0)\rangle} \\
x_f(\mathbf{p}) &= \frac{\langle D(\mathbf{p})|V_1|D(0)\rangle}{\langle D(\mathbf{p})|V_4|D(0)\rangle} \\
h_+(w) &= R_+(\mathbf{p})(1 - x_f(\mathbf{p})R_-(\mathbf{p})) \\
h_-(w) &= R_+(\mathbf{p})(1 - R_-(\mathbf{p})/x_f(\mathbf{p}))
\end{aligned}
$$

- Here $w = v \cdot v'$ is the recoil parameter and $\mathbf{p}$ is the recoil momentum in the B-meson rest frame.

Lattice ensembles



Range of lattice spacings and light-quark masses used here. The area is proportional to the number of configurations in the ensemble, which ranges from 600 to 2200, approximately.

Adjustments and error propagation

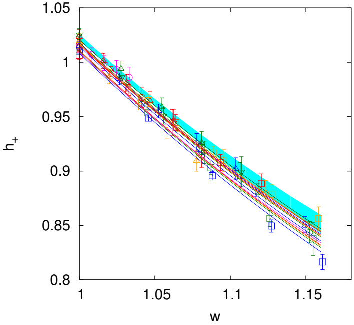
- Results were adjusted to our final values of κ_c and κ_b. This was done by simulating selected results at different κ's and calculating the numerical derivative.
- We used single-elimination jackknife to propagate errors and determine correlations in the h_+ and h_- form factors for each ensemble.

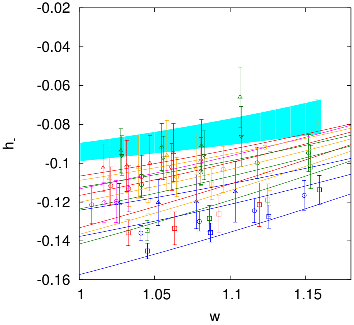
Chiral/continuum extrapolation

We fit h_+ and h_- as a function of the velocity transfer w, lattice spacing squared a^2, and light quark mass. Chiral logs are explained in [arXiv:1202.6346]

$$
\begin{aligned}
h_+(a, m_\ell, w) &= 1 - \rho_+^2(w-1) + k_+(w-1)^2 + \frac{X_+(\Lambda_\chi)}{m_c^2} \\
&+ c_{1,+} m_\ell + c_{a,+} a^2 + c_{a,w,+} a^2 (w-1) \\
&+ \frac{g_{D^* D\pi}^2}{16\pi^2 f^2} \mathrm{logs}_{1-\mathrm{loop}}(\Lambda_\chi, w, m_\ell, a) \\
h_-(a, m_\ell, w) &= \frac{X_-}{m_c} - \rho_-^2(w-1) + k_-(w-1)^2 \\
&+ c_{1,-} m_\ell + c_{a,-} a^2 + c_{a,w,-} a^2 (w-1)
\end{aligned}
$$

Chiral/continuum fit result





- Fit to form factors h_+ (left) and h_- (right) *vs* w, the recoil parameter w. The simultaneous fit gives $p = 0.27$.
- Blue band gives the physical continuum prediction.
- Rainbow color spectrum encodes m_l/m_s: blue to red for large to small.
- Symbol shapes encode lattice spacing: square, circle, triangle for coarser to finer.

z expansion formula

- The chiral/continuum extrapolation gives values of $f_+(w)$ and $f_0(w)$ at the physical point. To match with experiment we need a model for extrapolating/interpolating these results.
- We use the z expansion of Boyd, Grinstein, and Lebed [hep-ph/9508211] to parameterize the form factors (without explicit poles):

$$\begin{aligned} f_+(z)\phi_+(z) &= a_0 + a_1 z + a_2 z^2 + a_3 z^3 \\ f_0(z)\phi_0(z) &= a'_0 + a'_1 z + a'_2 z^2 + a'_3 z^3 \end{aligned}$$

  where the "outer functions" are

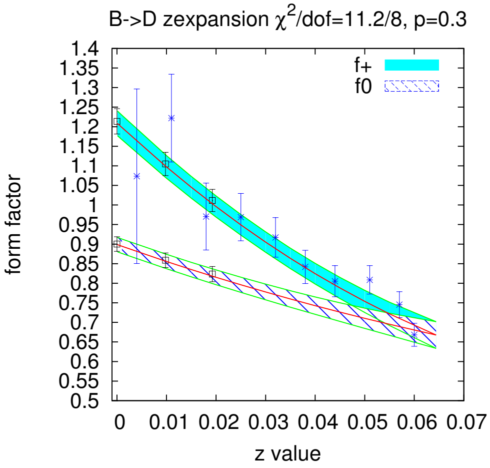
$$\begin{aligned} \phi_0(z) &= 0.5299(1+z)(1-z)^{3/2}[(1+r)(1-z) + 2\sqrt{r}(1+z)]^{-4} \\ \phi_+(z) &= 1.1213(1+z)^2(1-z)^{1/2}[(1+r)(1-z) + 2\sqrt{r}(1+z)]^{-5} \end{aligned}$$

- This is a model-independent parameterization based on analyticity and unitarity.
- We expand to order z^3.
- We impose the kinematic constraint $f_+ = f_0$ at $q^2 = 0$.
- We fit our lattice values and the BaBar experimental data simultaneously.

Experimental data

- We use data from the BaBar collaboration PRL 104, 011801 (2010) [arXiv:0904.4063].
- Systematic errors for large w were not published. BaBar quotes a 3% systematic error for low w. We assume 3% for the entire range. This needs more study.
- Thanks to Marcello Rotondo (BaBar) for clarification.

Fit using z-expansion



Fit theoretical values (squares) and experimental data (bursts) jointly. Errors shown combine statistical and systematic errors for all points.

Error Budget

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Table: Systematic error budget (preliminary)

source	$h_+(\%)$	$h_-(\%)$
κ-tuning adjustment	≤ 0.1	≤ 0.1
Lattice scale r_1	0.2	≤ 0.1
Heavy quark discretization	2.0	10.0
Light quark and gluon discretization	≤ 0.1	≤ 0.1
Finite volume	≤ 0.1	≤ 0.1
Electromagnetic effects	0.7	0.7
Isospin effects	≤ 0.1	≤ 0.1
Light quark mass tuning	≤ 0.1	≤ 0.1
ρ factor	0.4	20.

- The largest sources of error are the heavy quark discretization and the ρ factor for h_-.
- The truncation error in the z-expansion fit has been absorbed into the statistical error of the fit result.

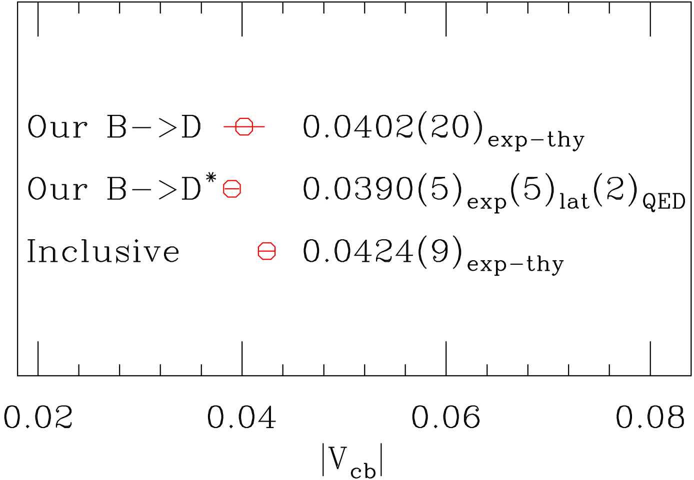
Result for $|V_{cb}|$

- Result from $B \to Dl\nu$ at nonzero recoil:

$$|V_{cb}| = 0.0402(20).$$

- The error depends in part on our assumptions about the experimental systematic error. With our present assumptions, this is the dominant source of error, followed by the error in our determination of the matching factor ρ_1.
- These are opportunities for further improvement of this result.

Comparison with other results

- 

- The exclusive determination of V_{cb} now differs from the Gambino-Schwanda inclusive (nonlattice) determination [arXiv:1307.4551] by 3σ.